Peculiarities of the propagation of nonequilibrium phonons in doped compensated semiconductors: phonon magnetothermoconductivity and the possibilities of phonon spectroscopy

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The propagation of phonons of arbitrary frequency in a doped compensated semiconductor has been considered at low temperatures when the resonant interaction of phonons with electronic two-level systems (pairs comprising one occupied and one free impurity center drawn together) becomes important. It is shown that the phonon absorption (scattering) has a pronounced frequency-selective character and is the greatest for phonons of wavelength of the order of the Bohr radius, whereas at higher frequencies the absorption drastically decreases. The effect of external magnetic field on the absorption caused by variation of the electronic wave functions at the centers has been analyzed. Weak magnetic fields lead to corrections quadratic in the field without changing the frequency dependence of the absorption, whereas strong magnetic fields shift the cutoff frequency into the higher frequency range. The mutual influence of phonon and microwave pulses, which gives rise to new possibilities in phonon spectroscopy, is discussed.

1. INTRODUCTION

It is known that one of the important features of doped compensated semiconductors, which, to a considerable extent, determines their low-temperature properties, is the presence of electronic two-level systems (TLS) formed by one unoccupied and one occupied impurity centers that are close to each other (see Ref. 1). In particular, as it has been shown both theoretically and experimentally (see Refs. 2 and 3), such systems make an important contribution to sound absorption. An interesting feature of this contribution is a special dependence on magnetic field, caused by its corresponding influence on the wave functions of the electrons localized at these centers.[3] Due to this dependence, it is possible to separate this contribution from the contribution of both band electrons and nonelectronic mechanisms of sound absorption.

Evidently, the existence of the TLS should also influence the propagation of nonequilibrium phonons. Note that the spatial dispersion of phonons plays an essential role, since, on one hand, for low-frequency phonons of wavelength much larger than the characteristic pair spacing the absorption increases with frequency, similar to the resonant sound absorption, whereas for phonons of wavelength much smaller than the size of the electron wave functions at the centers the interaction matrix element decreases drastically with decreasing wavelength. Due to this fact, the resonant phonon absorption proves to be frequency-selective. On the other hand, as in the case of sound absorption, the given mechanism of the phonon absorption should depend on the applied magnetic field. In particular, such a dependence should give rise to a dependence of the phonon thermal conductivity of corresponding samples on the magnetic field (the phonon magnetothermoconductivity).

It can also lead to an influence of the magnetic field on the phonon propagation in the sample. One can expect that, due to this, it would be possible to separate the mentioned contribution from the contributions of the usual mechanisms of phonon scattering (Rayleigh scattering by defects), even in the case of relatively small TLS concentrations. Note that, due to spectral sensitivity of the discussed absorption mechanism (or, more precisely, of scattering, since re-emission follows an act of resonant absorption), relevant experiments could allow to single out the contribution of phonons having certain frequencies.

Another important fact is that the resonant absorption of both phonons and photons is sensitive to the TLS occupation numbers (see, e.g., Ref. 4). On the other hand, the presence of nonequilibrium phonons of a given frequency should result in variation of the TLS occupation numbers resonantly interacting with them and determined by the phonon distribution function. Such a variation can be recorded by the change of the absorption of electromagnetic radiation of a relevant frequency. The last factor makes it possible to "detect" the spectrum of nonequilibrium phonons with the help of electromagnetic methods. In its turn, a reverse experiment is possible, when an external electromagnetic field causes a change in occupation numbers of the electronic TLS, thus leading to a change in the sample transmittance for the phonons of the same frequency as the field. This also gives a possibility of creating a spectrally sensitive phonon detector, using conventional bolometers.

2. INTERACTION OF ACOUSTICAL PHONONS OF ARBITRARY FREQUENCY WITH ELECTRONIC TWO-LEVEL SYSTEMS

Consider the resonant absorption of phonons by electronic TLS in doped semiconductors, with phonon spatial dispersion taken into account. The Hamiltonian of a TLS interacting with phonons can evidently be written in the form (cf. Ref. 4)

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{int}, \tag{1}$$

where

$$\mathcal{H}_0=\begin{pmatrix}\varphi_1 & I\\ I & \varphi_2\end{pmatrix}, \tag{2}$$

$$\mathcal{H}_{int}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} D_{nm}\int \psi^* u_{nm}\psi d^3r(1-e^{i\mathbf{qR}}). \tag{3}$$

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Here φ_1 and φ_2 are single-site energies, $I(\mathbf{R})$ is the energy overlap integral (in the absence of the magnetic field $I(\mathbf{R}) = I_0\exp(-R/a)$, a is the Bohr radius), D_{nm} and u_{nm} are the components of the tensors of the deformation potential and the deformation respectively (we omit henceforth the tensor indices), ψ is the electron wave function at a center (the centers are considered to be alike), $\mathbf{q}$ is the phonon wave vector, and $\mathbf{R}$ is the size of a pair. Here we consider the deformation mechanism of the phonon interaction with the TLS; generalization to the case of piezointeraction offers no difficulty.

After a standard diagonalization of the Hamiltonian $\mathscr{H}_0$ we find the following expression for the nondiagonal contribution to $\mathscr{H}_{\text{int}}$ responsible for the resonant transitions:

$$\mathscr{H}'_{int}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\mathscr{D}\mathscr{A}(1-e^{i\mathbf{qR}})\frac{\Lambda}{E}, \tag{4}$$

$$E=[(\varphi_2-\varphi_1)^2+\Lambda^2]^{1/2}, \quad \Lambda=2I(\mathbf{R}). \quad \mathscr{D}=D\left(\frac{\hbar}{2\rho V}\right)^{1/2}\cdot\frac{q}{\omega_q^{1/2}},$$

$$\mathscr{A}=\int\psi^*e^{i\mathbf{qr}}\psi\,d^3r.$$

Following Ref. 6, we use the Fermi "golden rule" to get the following expression for the phonon absorption coefficient:

$$\Gamma=\sum_i\frac{2\pi}{\hbar}|\mathscr{H}'^{(i)}_{int}|^2V\delta(\hbar\omega-E_i)Z_i^{-1}\times\exp\left(\frac{-E_{1i}^--\mu}{T}\right)\text{th}\left(\frac{\hbar\omega}{T}\right),$$

where

$$Z_i=1+\exp\left(-\frac{E_{1i}^--\mu}{T}\right)+\exp\left(-\frac{E_{1i}^+-\mu}{T}\right)+\exp\left(-\frac{E_{2i}-2\mu}{T}\right). \tag{5}$$

Here the sum is over all pairs and V is the normalization volume.

Note that the factor $\tanh(\chi\omega/T)$ gives the level-occupation difference in resonant pairs in the approximation of equilibrium distribution of the occupation numbers. When $\hbar\omega\gg T$, Eq. (5) yields

$$\Gamma=\frac{32\pi^2\mathscr{D}^2V}{\hbar}\mathscr{A}^2J, \tag{6}$$

where

$$J=\int dr\int d\varphi\int\frac{d\mathbf{n}}{4\pi}r(1-e^{i\mathbf{qr}})^2\frac{I^2(\mathbf{r})}{(\hbar\omega)^2}F(\varphi,r)\delta(\hbar\omega-E),$$

and $F(\varphi,r)$ is the TLS pair distribution function.

It is seen that expanding $\mathscr{H}'_{\text{int}}$ in powers of q, as $q\to0$, and retaining the lowest terms, we can obtain the well-known expression for the sound absorption coefficient, which obeys the law $q^3\tanh(\hbar\omega/T)$ with growing frequency. The cause of such a relatively high power of q is that we consider a crystalline semiconductor, for which the deformation potentials of pair-forming centers are assumed to be equal; the interaction is due only to the phase difference of the sonic wave at the centers.

For arbitrary q, passing on in (5) from the summation over pairs to the integration over single-site energies φ_1, φ_2 and the pair separation $\mathbf{R}$, and integrating further over the directions of $\mathbf{R}$, one can find the following expression for the absorption coefficient:

$$\Gamma=\frac{\pi^2D^2\varepsilon a}{v\rho e^2}\mathscr{A}^2qr_\omega\left|1-2\frac{\sin(qr_\omega)}{qr_\omega}+\frac{\sin(2qr_\omega)}{2qr_\omega}\right|N_D^{1/3}Kf(K), \tag{7}$$

where ε is the dielectric constant, N_D is the donor concentration, K is the degree of compensation, $f(K)$ is the function giving the density of electronic states (see Ref. 6) and varies as $0.26K^{1/4}$ for $K\ll1$ and as $(1-K)^{1/3}$ for $(1-K)\ll1$, r_ω is defined by the condition $2I(r_\omega)=\hbar\omega$, and

$$\mathscr{A}=\int\psi^*e^{i\mathbf{qr}}\psi d^3r=\frac{1}{[1+(qa/2)^2]^2}.$$

It is seen that in the range of high frequencies Γ rapidly decreases as $(1/q^7)\tanh(\hbar\omega/T)$. The absorption coefficient versus frequency is plotted in Fig. 1.

Thus, the phonon absorption has a frequency-selective character in the sense that the phonons with q in the range from $1/r_\omega$ to $1/a$ are absorbed most effectively, whereas in the low and high frequency ranges the absorption decreases rapidly.

3. PHONON ABSORPTION BY TWO-LEVEL SYSTEMS IN THE PRESENCE OF A MAGNETIC FIELD

Let us discuss now how the expression for the absorption coefficient is modified in the presence of the magnetic field. In this situation the electron wave functions are known to have the form:

weak fields, $H\ll H_0$ for $a\ll\rho\ll\lambda^2/a$;

$$\psi_H(r,\rho)=K_1\exp\left(-\frac{r}{a}-\frac{\rho^2ra}{24\lambda^4}\right), \tag{8}$$

strong fields, $H\gg H_0$:

$$\psi_H(r,\rho)=K_2\exp\left(-\frac{\rho^2}{4\lambda^2}-\frac{|z|}{a_H}\right). \tag{9}$$

Here $\lambda\equiv(c\hbar/eH)^{1/2}$ is the magnetic length, H_0 is defined by the relation $\lambda(H_0)=a$, K_1 and K_2 are the normalization factors, $a_H\equiv\hbar/\sqrt{2mE_H}$ and E_H is the binding energy of an electron with a donor in the magnetic field ($E_H\propto H^{1/3}$).

The effect of the field on the absorption can be connected with two factors. The first one is the change in overlap

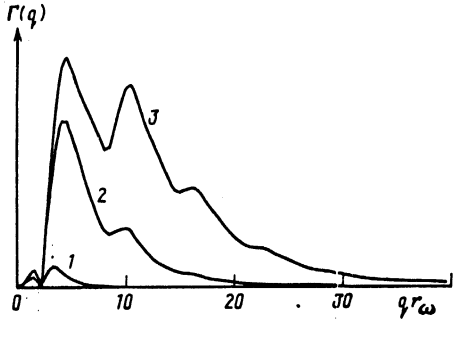

FIG. 1. The phonon absorption coefficient versus the phonon wave vector for the parameter r_ω/a equal to 2(1), 6(2) and 10(3).

integral. The corresponding effect has been considered by Galperin *et al.*[3] in the problem of sound absorption. The other factor, important for large q, is the change in deformational interaction of a phonon with a center (given by the parameter $\mathscr{A}$) due to the change in spatial scale of the electron wave functions at the centers in the presence of the magnetic field. Let us discuss the latter factor separately for weak and strong fields.

a) Weak fields ($H \ll H_0$)

Taking into account the corrections both to the parameter of one-center interaction $\mathscr{A}$ and to the quantity J, estimated in Appendix I, we find

$$\frac{\Gamma(H)-\Gamma(0)}{\Gamma(0)} \sim C\left(\frac{a}{\lambda}\right)^4 .$$

Here the constant C is of order unity and depends both on the relation between q and $1/r_\omega$ and the orientation of the vector $\mathbf{q}$ with respect to the vector $\mathbf{H}$. Note that for $qr_\omega \gg 1$ the main contribution comes from the change in overlapping intergral.

b) Strong fields ($H \gg H_0$)

Let us write the expression for the coefficient of phonon absorption in a strong magnetic field, using Appendix 2 and Ref. 3:

1) If $qr \ll 1$, then, for $\mathbf{q} \| \mathbf{H}$, we have

$$\frac{\Gamma(H)}{\Gamma(0)} = 3\left(\frac{a_H}{a}\right)^2\left(\frac{\lambda}{a}\right)^2 \frac{1}{\mathscr{L}}, \tag{10}$$

where $\mathscr{L} \sim r_\omega/a$. For $\mathbf{q} \perp \mathbf{H}$ we have

$$\frac{\Gamma(H)}{\Gamma(0)} = 6\left(\frac{\lambda}{a}\right)^4 \frac{1}{\mathscr{L}^2} \ln\left(\frac{a_H}{\lambda}\mathscr{L}\right). \tag{11}$$

2) Let $qr_\omega \gg 1$. Then, for $\mathbf{q} \perp \mathbf{H}$, we have two regions: $q\lambda \ll 1$ and $q\lambda \gg 1$. For $q\lambda \ll 1$ we can write

$$\frac{\Gamma(H)}{\Gamma(0)} = \left[1+\left(\frac{qa}{2}\right)^2\right]^4 C\left(\frac{a_H}{r_\omega}\right)\left(\frac{a_H}{a}\right), \tag{12}$$

whereas for $q\lambda \gg 1$ we have

$$\frac{\Gamma(H)}{\Gamma(0)} = \frac{[1+(qa/2)^2]^4}{\exp[-(q\lambda)^2]} C\left(\frac{a_H}{r_\omega}\right)\left(\frac{a_H}{a}\right). \tag{13}$$

For $\mathbf{q} \| \mathbf{H}$ we have

$$\frac{\Gamma(H)}{\Gamma(0)} = \frac{[1+(qa/2)^2]^4}{[1+(qa_H/2)^2]^2} C\left(\frac{a_H}{r_\omega}\right)\left(\frac{a_H}{a}\right). \tag{14}$$

4. MUTUAL INFLUENCE OF NONEQUILIBRIUM OCCUPATION NUMBERS OF PHONONS AND TLS

Up to now we have supposed that the TLS occupation numbers were at equilibrium. Now let us discuss the situation when these occupation numbers are not in equilibrium owing to the interaction with the considered pulses of nonequilibrium phonons or with some other external field (e.g., with microwave pulses).

Assuming that the TLS interact mainly with phonons, we write the following kinetic equation for the distribution function f^s of the TLS occupation numbers:

$$\frac{\partial f^s}{\partial t} = \frac{2\pi}{\hbar}\frac{V}{(2\pi)^3}\int d^3q\,|\mathscr{H}'_{int}|^2[(1-f^s)n_q - f^s(n_q+1)] \times \delta(E^s-\hbar\omega),$$

where n_q is the phonon distribution function, or

$$\frac{\partial f^s}{\partial t} = \frac{1}{\tau}[n_\omega - f^s(1+2n_\omega)], \tag{15}$$

where n_ω is the distribution function of nonequilibrium phonons averaged over the angles and $1/\tau \sim (1/\hbar^2 v)(E^s/\hbar w)^2|\mathscr{H}'_{\text{int}}|^2 V$ is the relaxation rate.

To the lowest approximation in interaction with the TLS (i.e., in fact, neglecting the phonon reabsorption by the TLS) the distribution function of nonequilibrium phonons can be regarded as specified. In this case, as seen from Eq. (15), for a sufficiently large phonon pulse duration $t_0 \gg \tau$ the TLS occupation numbers are governed by the phonon distribution function

$$f^s = \frac{n_\omega}{1+2n_\omega}. \tag{16}$$

If we introduce the partial temperature T_ω of the phonons of frequency ω, so that $n_\omega = N_0(\hbar\omega, T_\omega)$, where N_0 is the Planck distribution function, then it is easy to see that Eq. (16) means a "fine adjustment" of the effective temperature of the TLS with $E^s = \hbar\omega$ to the corresponding partial temperature of phonons.

Thus, it is possible to consider the temperature T in the initial expression (5) as equal to the thermostat temperature only if $t_0 \ll \tau$. In the opposite case the temperature T_ω, which is generally speaking frequency-dependent, should enter in Eq. (5). On the other hand, to be able to neglect the factor $\tanh(\hbar\omega/T_\omega)$ in subsequent expressions, we should have $\hbar\omega/T_\omega \gg 1$, i.e. the occupation numbers of nonequilibrium phonons should be relatively small. In the opposite case, it is necessary to include this factor into the obtained expression of the phonon absorption coefficient. As a result, the TLS contribution into the absorption coefficient proves to depend on the occupation numbers of the nonequilibrium phonons and decreases as n_ω^{-1} (nonlinear bleaching of a sample).

Let us discuss now a possible effect of the microwave field on the phonon absorption. As is known, electromagnetic pumping of a sufficiently high intensity leads to equalization of the level population in the TLS. Such an equalization is important[4] if

$$\frac{(er_\omega \mathscr{E})^2 \tau_1 \tau_2}{\hbar^2} \geqslant 1, \tag{17}$$

where $\mathscr{E}$ is the electric field and τ_1 and τ_2 are the relaxation times of the diagonal and off-diagonal components of the TLS density matrix respectively (in our case for $t_0 \gg \tau$ and $n_\omega > 1$ we have $\tau_1 \approx \tau n_\omega^{-1}$). Using the analogy with electromagnetic and acoustic absorption, it is easy to understand that for $P \gg P_0$, where P is the intensity of the electromagnetic radiation and P_0 is the threshold value corresponding to the moment when the condition (17) begins to hold, the phonon absorption coefficient for frequencies close to the frequency of electromagnetic excitation is proportional to P_0/P. The phonon frequency band for which this holds is

defined by the estimate[7]

$$\delta E \sim \frac{\hbar}{\tau_2}\left(\frac{P}{P_0}\right)^{1/2}. \qquad (18)$$

5. CONCLUSION

Let us discuss the experimental consequences of our results. First of all, note that the contribution of the electronic TLS to the absorption (scattering) of nonequilibrium phonons in a semiconductor can be sufficiently large. Thus, for a semiconductor with the GaAs parameters ($\varepsilon_0 = 12$, $m^* = 0.06m_0$, $E_B = 5.7$ meV, $a = 100$ Å, $N_D = 10^{16}$ cm^{-3}) the characteristic value of Γ for relevant phonon frequencies is of order 10^8 s^{-1}, which corresponds to mean free paths of order $3\cdot 10^{-3}$ cm. It is not difficult to see that this mechanism prevails over the Rayleigh scattering by the impurities and can even compete with the isotopic Rayleigh scattering.

As we have shown, Γ has a pronounced frequency dependence with an absolute maximum at $qr_\omega \sim 4$. At the same time, in the high-frequency range Γ decreases rapidly like ω^{-7}.

The possibility of observing the considered absorption mechanism experimentally is made considerably easier by the fact that the absorption depends on the magnetic field. In weak magnetic fields ($H \ll H_0$) the field does not alter the frequency dependence of the absorption, giving, however, rise to corrections quadratic in field. Therefore, switching on the field can alter the signal of the phonon detector, which can be recorded even if the phonon propagation is governed mainly by other scattering mechanisms.

Note that a magnetic field can thus influence the ordinary phonon thermal conductivity of semiconductors in quasiequilibrium conditions (magnetothermoconductivity).

It is interesting that strong magnetic fields ($H \gg H_0$) can result in a change in frequency dependence of the absorption [according to Eqs. (12)–(14)]. In particular, they can shift the cutoff frequency to a higher frequency.

As to the H_0 estimate, $H_0 \sim 10$ T for the material with typical parameters listed above, therefore it is difficult to observe the regime $H > H_0$. However, for semiconductors with narrower bands the value of H_0 is considerably smaller (e.g., $H_0 \approx 5$ kOe for InSb).

Exposing samples to electromagnetic field opens, as it seems to us, new possibilities for phonon spectroscopy. As we have shown, the spectrum of the TLS occupation numbers reproduces, under certain conditions, the spectrum of the occupation numbers of nonequilibrium phonons. On the other hand, the TLS contribution to the absorption γ of a weak ($P \ll P_0$) microwave radiation is governed by relevant TLS occupation numbers ($\gamma \sim \tanh(\hbar\omega/T_\omega)$, where T_ω is the partial temperature introduced above). Therefore study of the change in absorption spectrum of the microwave radiation under the action of a phonon pulse permits analysis of the spectral composition of this pulse.

In its turn strong microwave pumping ($P \gg P_0$), equalizing the TLS occupation in the corresponding spectral band (18), leads to a decreasing contribution of the TLS to the phonon absorption coefficient at corresponding frequencies, which also permits a spectral analysis of the phonon pulse using ordinary bolometers.

We emphasize that the analysis above assumes a uniform distribution of the TLS over energies in the relevant spectral range (which practically means that the phonon energy should be much smaller than the width of the impurity band).

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APPENDIX 1

CALCULATIONS OF CORRECTIONS TO THE PHONON ABSORPTION COEFFICIENT IN A WEAK MAGNETIC FIELD

The parameter of one-center interaction can be written in the form

$$\mathscr{A}_H = \mathscr{A}_0 + \Delta\mathscr{A}.$$

Substituting the expression (8) for ψ_H into the expression for $\mathscr{A}$ we find the following corrections:

1) For $\mathbf{q}\|\mathbf{H}$ we have

$$\Delta\mathscr{A}^{\|} = -\frac{\pi}{12}K_1^2 a^3\left(\frac{a}{\lambda}\right)^4 \frac{5-b^2}{(1+b^2)^4},$$

where $b \equiv qa/2$. In the limiting cases we have

$$\Delta\mathscr{A}^{\|} = -\frac{5\pi}{12}K_1^2 a^3\left(\frac{a}{\lambda}\right)^4 \approx -\frac{5\pi}{12}\left(\frac{a}{\lambda}\right)^4 \quad \text{for } b \ll 1,$$

$$\Delta\mathscr{A}^{\|} = \frac{\pi}{12}K_1^2 a^3 \frac{1}{b^6} \propto \frac{1}{b^6} \quad \text{for } b \gg 1.$$

2) For $\mathbf{q}\perp\mathbf{H}$ we find in the limiting cases

$$\Delta\mathscr{A}^{\perp} = -\frac{5\pi}{12}K_1^2 a^3\left(\frac{a}{\lambda}\right)^4 \approx -\frac{5\pi}{12}\left(\frac{a}{\lambda}\right)^4 \quad \text{for } b \ll 1,$$

$$\Delta\mathscr{A}^{\perp} = -\frac{\pi}{6}K_1^2 a^3\left(\frac{a}{\lambda}\right)^4 \frac{1}{b^6} \quad \text{for } b \gg 1.$$

Consider now the corrections to J. For weak magnetic fields we find, in accordance with (8)

$$I = I_0 \exp\left[-\left(\frac{r}{a} + \frac{r^3 a \sin^2\theta}{24\lambda^4}\right)\right],$$

where θ is the angle between $\mathbf{H}$ and $\mathbf{r}$.

As is easy to see, the corrections to J arise when we expand (6) in a power series of $\lambda^{-2} \propto H^2$. In the limiting cases $qr_\omega \gg 1$ and $qr_\omega \ll 1$ they do not result in an extra dependence on $|q|$:

$$\frac{J(H)-J(0)}{J(0)} \approx -C\left(\frac{a}{\lambda}\right)^4,$$

where the constant $C \sim 1$ and, generally speaking, depends on the angle between $\mathbf{q}$ and $\mathbf{H}$.

APPENDIX 2

THE PHONON ABSORPTION COEFFICIENT IN A STRONG MAGNETIC FIELD

Consider the case of strong magentic fields. For the parameter of deformational one-center interaction $\mathscr{A}$ we find the following expressions:

1) For $\mathbf{q}\|\mathbf{H}$ we have

$$\mathscr{A}_{cH}^{\|} = K_2^2 \pi\lambda^2 2a_H \frac{1}{1+(qa_H/2)^2}.$$

The expressions in the limiting cases $qa_H \ll 1$ and $qa_H \gg 1$ are obvious.

2) For $\mathbf{q} \perp \mathbf{H}$ we have in the limiting cases:

$$\mathscr{A}_{cH}{}^{\perp} = K_2{}^2 \pi \lambda^2 2a_H \approx 1 \quad \text{for } q\lambda \ll 1,$$

$$\mathscr{A}_{cH}{}^{\perp} = K_2{}^2 \pi \lambda^2 2a_H \exp\left[-\frac{(q\lambda)^2}{2} \right] \quad \text{for } q\lambda \gg 1.$$

To estimate the integral J, we take it into account that in the case of strong fields

$$I = I_0 \exp\left[-\frac{r|\mu|}{a_H} - \frac{r^2(1-\mu^2)}{4\lambda^2} \right],$$

where $\mu = \cos\theta$ and $\theta = (\mathbf{r},\mathbf{H})$.

It is easy to show that the integration over $\mathbf{n}$ in (6) is governed, first of all, by an exponential factor entering the expression for $I(\mathbf{H},\mathbf{r})$. The main contribution comes from the vicinity of $\theta \approx \pi/2$, since the value of $I(r,\mu)$ decreases rapidly with growing μ. The latter fact allows us to estimate the result of integration over $\mathbf{n}$. Expanding all the functions of μ in the vicinity of $\mu = 0$ and restricting overselves to the lowest-order terms, we find the following estimates:

1) For $qr_\omega \ll 1$ (see Ref. 3) we have two expressions:

$$\frac{J^{\parallel}(H)}{J(0)} = 3\left(\frac{a_H}{a}\right)^2 \left(\frac{\lambda}{a}\right)^2 \frac{1}{\mathscr{L}},$$

where $\lambda \cong r_\omega / a$, if $\mathbf{q} \| \mathbf{H}$, and

$$\frac{J^{\perp}(H)}{J(0)} = 6\left(\frac{\lambda}{a}\right)^4 \frac{1}{\mathscr{L}^2} \ln\left[\left(\frac{a_H}{\lambda}\right)^2 \mathscr{L} \right].$$

if $\mathbf{q} \perp \mathbf{H}$.

2) For $qr_\omega \gg 1$ we have

$$\frac{J(H)}{J(0)} \sim \left(\frac{a_H}{a}\right)^2 \frac{1}{\mathscr{L}} \sim \left(\frac{a_H}{a}\right)\left(\frac{a_H}{r_\omega}\right),$$

i.e., the ratio $J(H)/J(0)$ does not depend on the angle between $\mathbf{q}$ and $\mathbf{H}$.

[1] M. Pollak and T. H. Geballe, Phys. Rev. **12**, 1742 (1961).
[2] H. Tokumoto and R. Mansfield, Japan. J. Appl. Phys. **22** (3), 196 (1983).
[3] Yu. M. Gal'perin and É. Ya. Priev, Fiz. Tverd. Tela **28**, 692 (1986) [Sov. Phys. Solid State **28**, 385 (1986)].
[4] Yu. M. Galperin, V. L. Gurevich, and D. L. Parshin, Zh. Eksp. Teor. Fiz. **85**, 1757 (1983) [Sov. Phys. JETP **58**, 1023 (1986)].
[5] Nguen Van Lien, B. I. Shklovskiĭ, and A. L. Éfros, Fiz. Tekh. Poluprovodn. **13**, 2192 (1979) [Sov. Phys. Semicond. **13**, 1281 (1979)].
[6] A. L. Efros and B. I. Shklovskii, in *Electron–Electron Interactions in Disordered Systems*, ed. by A. L. Efros and M. Pollak, North Holland, 1985, pp. 201–281.
[7] D. A. Parshin and V. I. Rzaev, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 344 (1987) [JETP Lett. **45**, 437 (1987)].

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