Anisotropy of electron-phonon umklapp processes and galvanomagnetic properties of single-crystal tungsten in strong magnetic fields

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We have measured the temperature dependences of the galvanometric coefficients of single-crystal tungsten with $\rho_{293.2\,K}/\rho_{4.2\,K}$ up to 95 000 in the temperature interval 4.2 to 50 K and in magnetic fields up to 140 kOe, for two directions of the magnetic field along the ⟨110⟩ and ⟨111⟩ axes. The magnetoresistance measurements were made on samples in the form of Corbino disks, while samples of rectangular form were used for measuring the Hall emf. We show that the temperature dependence of the electron magnetoconductivity contains an exponential contribution due to intersheet electron-phonon umklapp processes. When this mechanism for scattering of conduction electrons is dominant, the Hall resistance exhibits strong temperature variations that follow a power law.

1. INTRODUCTION

The contemporary picture of electron-phonon umklapp processes and their role in the low-temperature electrical conductivity of metals is primarily based on the ideas of Peierls.[1] One of the key conclusions of his theory is the prediction of an exponential decay of the electrical resistivity with decreasing temperature for $T<\Theta_D$, i.e., $\rho \propto \exp(-\hbar \cdot s \cdot \Delta k / k_B T)$ (where Θ_D is the Debye temperature, s is the velocity of sound, Δk is the gap in k-space between adjacent sheets of the Fermi surface (FS), and k_B is Boltzmann's constant). In recent years, exponential temperature dependences of the electrical resistivity have been observed in alkali metals at temperatures below 4 K.[2,3] These experimental results are interesting for two reasons: first of all, they provide excellent confirmation of the results of recent theoretical investigations[4] in which the ideas of Peierls were elaborated to a qualitatively new level of modelistic and mathematical description; secondly, they provide the stimulus for further investigations into the role of umklapp processes in the kinetic properties of metals with complex Fermi surfaces, including metals subjected to high magnetic fields. In particular, strong experimental evidence for the identification of electron-phonon umklapp processes as one of the fundamental mechanisms for the scattering of electrons by phonons comes from studies of the magnetoresistance in uncompensated metals[5] and the Hall effect in compensated metals.[6]

Observation of an exponential temperature-dependent contribution to the electrical conductivity of metals is plagued by a number of experimental difficulties. Therefore, alkali metals are considered to be the most promising objects to look for the Peierls exponentials, due to the simplicity of their FS. However, in these metals the velocity of sound in the directions of probable electron-phonon umklapp processes is low, and therefore the exponential contributions to the electrical conductivity can be observed only at temperatures so low (between 4 K) that the dominant contribution to ρ is most often due to impurity scattering, as was observed, e.g., in Refs. 2 and 3.

In the majority of multivalent metals the FS geometry is very complicated, consisting of various sheets separated by gaps of different magnitudes. This greatly complicates the task of separating out exponential temperature-dependent contributions from the experimental temperature dependence of the electrical resistivity $\rho(T)$. Furthermore, according to Ref. 7, electrons in compensated metals, i.e., metals with equal concentrations of electron and hole carriers, $n_e = n_h$, are not subject to the phonon drag effect in zero magnetic field; this makes intrasheet scattering of conduction electrons more efficient than intersheet scattering, causing the power-law temperature-dependent contribution of the former to dominate.

The study of the role of umklapp processes in the behavior of the electrical resistance of metals is also accompanied by difficulties of a purely methodological nature. Among these is the presence of size-quantization effects: because the mean free path of conduction electrons is long in metals with a high degree of purity at low temperatures, scattering by the surfaces of crystals becomes the primary scattering mechanism. The precise measurement of the small values of resistivity of such metals is hindered by apparatus-related difficulties.

According to Refs. 4 and 8 the search for an exponential contribution due to umklapp processes can be made easier in high magnetic fields by measuring the temperature dependence of the magnetoresistance, because in this case it is possible to avoid competition between the process of diffusion of electrons along the Fermi surface and the intersheet umklapp process. This is done by orienting the magnetic field in such a way that the Larmor orbits of the electrons on the FS sheets are separated in k-space by small gaps lying in the plane perpendicular to **H**. In this case the transfer of an electron from one hot point in k-space to another takes place along a Larmor orbit, and the diffusion contribution is small. The systems that are most likely to exhibit observable Peierls exponents are compensated metals with closed Fermi surfaces, because in high magnetic fields they have extremely high values of magnetoresistance. However, in high-purity compensated metals subjected to high magnetic fields (i.e.,

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with $\omega_c\tau \gg 1$) scattering of electrons by the sample surface can lead to strong nonuniformity of the distribution of electrical current along the crystal owing to the presence of the static skin effect (SSE).[9,10] It is obvious that the redistribution of current throughout the volume of the crystal as the temperature changes will distort the shape of the temperature dependence of the magnetoresistance. In Refs. 10 and 11 it was shown that we can avoid this complication if the measurements of the magnetoresistance are made on samples in the shape of Corbino disks, because in this geometry there is no static skin effect.

Thus, a survey of previous papers reveals that exponential temperature-dependent contributions to the electrical resistance of potassium[2,3] have been observed in the dirty limit, although to date such contributions have never been seen in the magnetoresistance. Therefore, obtaining more reliable data on the existence of the Peierls exponents remains a timely problem.

In our view, one of the most promising systems to study with regard to identifying the role played by electron-phonon umklapp processes in the temperature dependence of the magnetoresistance, and the one in which the search for Peierls exponents is most likely to succeed, is single-crystal high-purity tungsten. Our choice of tungsten rests both on the fact that tungsten satisfies the considerations set forth above and the fact that the authors of Refs. 6 and 12 have already established the existence of electron-phonon umklapp processes in tungsten, and have analyzed the reasons for its strong influence on the Hall effect by studying the temperature dependence of the latter for intermediate magnetic fields ($\omega_c\tau \gtrsim 1$).

The goal of this paper is to investigate the contribution of intersheet electron-phonon umklapp processes to the temperature dependence of the magnetoresistance and of the Hall emf of single-crystal tungsten in high magnetic fields, and to search for Peierls exponential temperature-dependent contributions to the resistivity.

In this paper we investigate the temperature dependence of the galvanomagnetic coefficients of single-crystal tungsten samples with values of the resistivity ratio $\rho_{293\,K}/\rho_{4.2\,K}$ up to 95 000 in the temperature interval 4.2 to 50 K and in magnetic fields up to 140 kOe. In our experiments, these values of T and H correspond to values of the quantity $\omega_c\tau$ between 5 and $8\cdot 10^3$. We measured the temperature dependences of the galvanomagnetic properties for two directions of the magnetic field with respect to the crystallographic axes, in such a way that we were able to separate out the contributions of isotropic and anisotropic electron-phonon scattering to the temperature dependences.

In Sec. 2 we describe the basic characteristics of the samples and the distinctive features of the measurement techniques used. The local peculiarities of the Fermi surface of tungsten and its phonon spectrum are described in Sec. 3. In Sec. 4 we present the results of our measurements of the temperature dependences of the magnetoresistance, and discuss possible mechanisms for the scattering of electrons by phonons that could explain these results. Taking into account the data from these studies of the magnetoresistance, we analyze in Sec. 5 the results of our measurements the temperature dependences of the Hall resistance.

TABLE I.

No.	Samples and their dimensions (mm)	Direction of sample axis	Sample Facet	RRR_{bulk}	RRR_{real}
1	Corbino disk $\varnothing = 5.8$ mm $d = 0.36$ mm	-	(110)	80000	-
2	Corbino disk $\varnothing = 9.0$ mm $d = 0.99$ mm	-	(111)	95000	-
3	Slab $0.34 \times 1.38 \times 12$ mm^3	⟨100⟩	large (110) small (110)	80000	39790
4	Slab $0.61 \times 2.76 \times 12$ mm^3	⟨110⟩	large (111) small (112)	95000	59970

2. SAMPLES AND EXPERIMENTAL METHOD

To exclude the influence of the static skin effect on the temperature dependence of the magnetoresistance of tungsten, our measurements were carried out on samples having the shape of a Corbino disk. In this geometry the magnetic field **H** is directed perpendicular to the disk plane, and the electric current **J** flows radially outward from the center of the disk to its edges. Thus, the sample has no boundaries that are simultaneously parallel to both **H** and **J**, and consequently the conditions under which the SSE can occur are satisfied nowhere. Scattering of conduction electrons can occur in the Corbino disk at surfaces perpendicular to **H**, and this scattering can be considerable in a disk with small thickness $d \ll 1$. However, this scattering does not lead to a redistribution of electrical current as T varies, and its contribution to the conductivity is temperature-independent.[13] In this case we can separate out the temperature-dependent contribution to the magnetoconductivity from that caused by the scattering of conduction electrons by phonons in the bulk of the sample.

Our measurements of the Hall emf U_H were carried out for samples in the form of rectangular slabs in which we were unable to avoid the static skin effect. These measurements of U_H were carried out using the 6-contact method described in Refs. 10 and 14, which allowed us to suppress the contribution to U_H from the transverse even field.

The magnetoresistance was measured at constant current using the 4-contact method. The potential contacts were placed on the sample in keeping with the specifics of Corbino-disk measurement, i.e., along with the radial direction.

The samples were prepared from single-crystal tungsten with a resistivity ratio $\rho_{293\,K}/\rho_{4.2\,K} = RRR = 80\,000$ and 95 000. The orientation of the sample planes coincided with crystallographic planes to an accuracy no worse than $\pm 1°$. The basic characteristics of the samples are listed in Table I.

The temperature was stabilized using a capacitive thermometer, accurate to ± 0.02 K; the temperature measurements were carried out using a carbon thermometer, which was calibrated in the magnetic field. The temperature dependences of the magnetoresistance and Hall emf were recorded in fixed magnetic fields, which were chosen in such a way that the condition $\omega_c \tau > 1$ was fulfilled over the entire temperature interval from 4.2 to 50 K. The current density in the samples did not exceed 50 A/cm^2.

To eliminate completely contributions that were even in the magnetic field we measured the Hall emf for two opposite directions of the magnetic field. The thermoelectric and thermomagnetic contributions were eliminated by commuting the directions of the electric current. The measurement errors did not exceed 0.1% for the magnetoresistance and 4% for the Hall emf. The Hall resistance was determined with an error of no more than 8 to 10%.

The sources of magnetic field were IGC-150 superconducting solenoids with magnetic field intensities up to 150 kOe. The measurements were carried out at the International Laboratory for High Magnetic Fields and Low Temperatures in Wroclaw, Poland.

3. PECULIARITIES OF THE FERMI SURFACE AND PHONON SPECTRUM OF TUNGSTEN

Tungsten is a compensated metal with equal electron n_e and hole n_h current densities ($n_e = n_h = 0.25$ electrons per atom). Detailed data on the electron spectrum and topology of the FS of tungsten were presented in Refs. 15 and 16. As asserted in Ref. 16, the FS of tungsten consists of closed sheets: (1) an electron sheet $\Gamma 4e$ (a "jack"); (2) a hole sheet $H3h$ (an "octahedron"); and (3) rather small hole "ellipsoids" $N3h$.

In Fig. 1 we show two central cross sections of the FS of tungsten in the planes (110) and (111). These are in fact cross sections for the experiments described in this paper; the measurements of the magnetoresistance and Hall emf were carried out for **H** $\parallel \langle 110 \rangle$ and **H** $\parallel \langle 111 \rangle$. It is seen that in the (110) plane (Fig. 1a) the electron "jack" $\Gamma 4e$ and the hole "octahedron" $H3h$ are separated by a gap Δk_1 in momentum space. This gap corresponds to an energy gap 0.4 eV such that the effect of magnetic breakdown is impossible for the magnetic fields used in this paper.Studies of the radio-frequency size effect[15] imply that $\Delta k_1 = 0.15 \pm 0.02$ Å^{-1}. According to Ref. 15, the distances between the electron "jack" and the hole ellipsoid $N3h$ in momentum space equals $\Delta k_2 = 0.80 \pm 0.02$ Å^{-1}, while the distance between the hole "octahedron" and the hole ellipsoids is $\Delta k_2' = 0.684 \pm 0.014$ Å^{-1}, i.e., close to Δk_2 in magnitude. The (111) plane (Fig. 1b) has no FS cross sections in which a gap Δk_1 might exist, but retains the gaps Δk_3 and $\Delta k_3'$ which agree with Δk_2 and $\Delta k_2'$ in magnitude. In Fig. 1b we show the FS cross section passing through the point Γ and to which the gap Δk_3 belongs. The gap $\Delta k_3'$ is located in a parallel plane passing through the point H of the Brillouin zone.

Thus, the FS of tungsten exhibits a pronounced anisotropy of its intersheet momentum-space gaps. Because of this, we can expect the electron-phonon scattering with participation of intersheet umklapp processes to be anisotropic in the region of low temperatures. This may be reflected in differing behaviors of the galvanomagnetic properties for the magnetic field orientations **H** $\parallel \langle 110 \rangle$ and **H** $\parallel \langle 111 \rangle$.

The phonon spectrum of tungsten is reconstructed to a

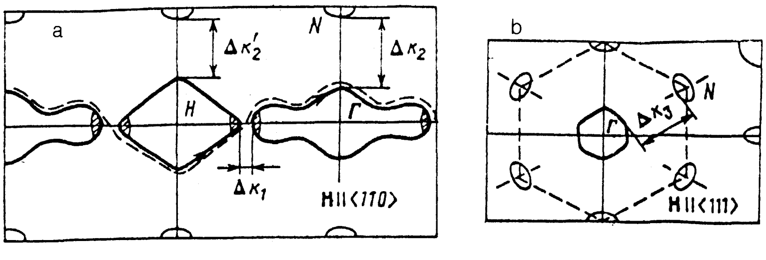

FIG. 1. Extremal cross sections of the Fermi surface of tungsten: (a) the plane (110); (b) the plane (111). Δk_1, Δk_2, $\Delta k_2'$, and Δk_3 are the gaps between sheets of the Fermi surface.

high degree of accuracy in Refs. 17 and 18. From these references it follows that at energies corresponding to thermal excitation of phonons with frequencies $\nu \leqslant 3 \cdot 10^{12}$ Hz the dispersion law $\nu(q)$ in tungsten is linear, both for the transverse and longitudinal phonons. Therefore, when $T < 130$ K, rather than using the full phonon spectrum to estimate the phonon wave vectors we need only the velocities of sound, since in the range of q where the function $\nu(q)$ is linear we can write $q = k_B T/\hbar s$. The values of s are given in Ref. 19. From this latter paper it follows that the branches T_1 and T_2 of the phonon spectrum of tungsten in the direction $\langle 0\xi\xi \rangle$ agree for phonons with transverse polarization to the limits of experimental error. Thus, tungsten is an elastically isotropic metal.

The Debye temperature of tungsten $\Theta_D = 379$ K.[20] Therefore, the condition $q \approx \Delta k$ is realized for $T \ll \Theta_D$ ($q_1 \approx \Delta k_1$ for $T \approx 33$ K, while $q_2 \approx \Delta k_2$ for $T \approx 136$ K). The features of the FS and phonon spectrum of tungsten mentioned above show that tungsten is a convenient system for investigating electron-phonon umklapp processes and for looking for exponential temperature dependences of the magnetoresistance of the sort predicted in Ref. 8; however, up to now these have not been observed experimentally.

4. MAGNETORESISTANCE

Figure 2 shows the temperature dependences of the magnetoresistance $\rho_{xx}(T)$ in a magnetic field $H = 140$ kOe for the two orientations $\mathbf{H} \| \langle 110 \rangle$ and $\mathbf{H} \| \langle 111 \rangle$. Here $\rho_{xx}(T) = \rho(H,T) - \rho(H=0,T)$, where $\rho(H,T)$ is the resistivity of the sample in a magnetic field, while $\rho(H=0,T)$ is the same for $H = 0$. Especially noteworthy is the extremely high value of the resistivity (for metals) $\rho_{xx} \approx 2000$ $\mu\Omega \cdot$cm in a magnetic field $H = 140$ kOe at $T = 4.2$ K, and the decrease of $\rho_{xx}(T)$ by three orders of magnitude as the temperature increases. It is remarkable that ρ_{xx} for a Corbino disk is almost an order of magnitude larger than for a pure crystal in the form of a rectangular bar, since in the Corbino disk there is no static skin effect.[10]

In analyzing the mechanisms that mediate the scattering of conduction electrons by phonons, the magnetoconductivity σ_{xx} is more convenient to use than the magnetoresistivity ρ_{xx}, since the contributions of the various scattering mechanisms to σ_{xx} are additive. For samples in the shape of a Corbino disk the magnetoresistivity ρ_{xx} and the electron magnetoconductivity σ_{xx} are related by the equation $\sigma_{xx} = \rho_{xx}^{-1}$, because in this geometry the Hall current lines are closed and the Hall effect cannot influence the values of the diagonal components of the magnetoconductivity tensor. In Fig. 3 we plot the dependence of $[\sigma_{xx}(T) - \sigma_{xx}^0]$ on T, using a base-10 logarithmic scale. Here σ_{xx}^0 is the temperature-independent contribution to $\sigma_{xx}(T)$ due to scattering of electrons by impurities, defects, and the surface of the disc; clearly σ_{xx}^0 can be interpreted as the electronic magnetoresistivity at $T = 0$. In our experiments, we determine this quantity by extrapolating $\sigma_{xx}(T)$ to zero temperature. Although this is a crude way to determine σ_{xx}^0, it turns out to be entirely satisfactory when further processing is done on the rapidly varying quantity $\sigma_{xx}(T)$.

From Fig. 3 it is clear that $\sigma_{xx}(T) - \sigma_{xx}^0$ cannot be described by a simple power-law function of temperature over the entire temperature interval 4.2 to 50 K, and that we can only, and very artificially, identify a segment of the curve within a narrow interval 20 to 30 K in the neighborhood of its point of inflection with the dependence $\sigma_{xx}(T) \propto T^{-4.6}$. Taking into account that, according to Ref. 21, a dependence $\rho(T) \propto T^5$ is observed in the electrical resistivity for $H = 0$ in this region of temperatures, interpreting the observed departure of $\sigma_{xx}(T)$ from the function $\sigma_{xx}(T) \propto T^{-5}$ in terms of an isotropic mechanism for electron-phonon scattering is far from simple.

From Fig. 3 it is also clear that in the low-temperature region $T < 20$ K the electron magnetoconductivity for $\mathbf{H} \| \langle 110 \rangle$ falls off considerably more rapidly with tempera-

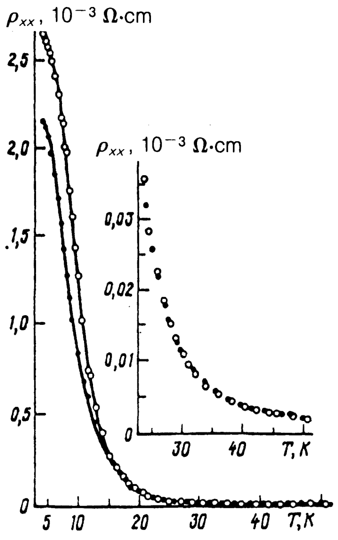

FIG. 2. Dependence of the magnetoresistance $\rho_{xx}(T)$ on temperature when $H = 140$ kOe. (○)—$\mathbf{H} \| \langle 111 \rangle$; (●)—$\mathbf{H} \| \langle 110 \rangle$. In the inset we show high-temperature segments of these curves, in expanded scale.

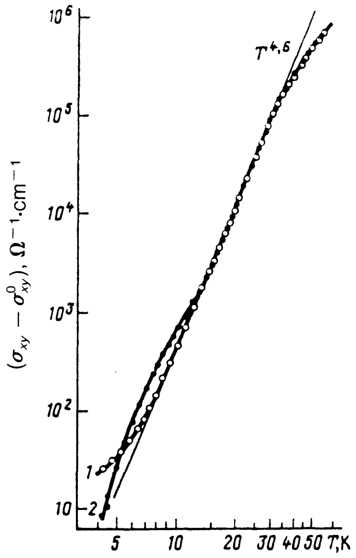

FIG. 3. Temperature dependence of the electron magnetoconductivity of tungsten samples Nos. 1 and 2 ($\sigma_{xx}(T) - \sigma_{xx}{}^0$) in base 10 logarithmic coordinates when $H = 140$ kOe: (○)—$\mathbf{H} \| \langle 111 \rangle$; (●)—$\mathbf{H} \| \langle 110 \rangle$.

ture (curve 2) than for $\mathbf{H}\|\langle 111\rangle$. This fact also requires an explanation.

a) **MAGNETIC FIELD H$\|\langle 110\rangle$.** In Fig. 4 the quantity $\ln(\sigma_{xx} - \sigma^0_{xx})$ is plotted as a function of T^{-1} for three values of the magnetic field: 60, 75, and 140 kOe. It is clear from this plot that the high-temperature and low-temperature intervals for which $\ln(\sigma_{xx} - \sigma^0_{xx})$ depends linearly on T^{-1} are clearly separated. Consequently, $\sigma_{xx} - \sigma^0_{xx}$ can be described by an exponential function for each of these intervals. In view of this, the dependence of σ_{xx} on temperature can be described by the empirical formula

$$\sigma_{xx}(T) = \sigma_{xx}^0 + \sigma_1 \exp(-\Delta_1/T) + \sigma_2 \exp(-\Delta_2/T). \quad (1)$$

For a magnetic field $H = 140$ kOe the parameters σ^0_{xx}, σ_1, σ_2 have the values $\sigma^0_{xx} = 457.87\ (\Omega\cdot\text{cm})^{-1}$, $\sigma_1 = 19.926\cdot 10^3\ (\Omega\cdot\text{cm})^{-1}$, $\sigma_2 = 7.49\cdot 10^6\ (\Omega\cdot\text{cm})^{-1}$, $\Delta_1 = 32.9 \pm 0.5$ K, and $\Delta_2 = 136.3 \pm 1.5$ K. The numerical values of the coefficients σ_1 and σ_2 are determined from the experimental data by extrapolating the linear segments of the upper curve in Fig. 4 ($H = 140$ kOe) to infinite temperature. σ_1 corresponds to the low-temperature linear portion, and σ_2 to the high-temperature portion. The values of Δ_1 and Δ_2 are determined from the slopes, of these linear segments; it is clear from Fig. 4 that they do not depend on H. No special fitting of the parameters σ^0_{xx}, σ_1, σ_2, Δ_1, and Δ_2 was carried out.

The exponential character of the temperature dependence of the electrical magnetoconductivity in the magnetic fields we used is probably not caused by transitions of conduction electrons between Landau bands on the FS mediated by phonon scattering. An estimate of the temperature at which the phonon wave vector q is comparable to the distance p_H between Landau bands on the FS indicates that for $T > 4$ K we have $q > P_H$. Consequently, despite the discreteness of the electron states on the FS, scattering of electrons by phonons may be considered quasiclassical over the entire temperature we are looking at.

According to Refs. 4 and 8, exponential contributions to the temperature dependence of the electron magnetoconductivity of compensated metals can arise in high magnetic fields as a consequence of "freezeout" of the electron-phonon umklapp processes between closely-spaced sheets of the FS. Umklapp processes that transfer current carriers between FS sheets can occur only when phonons are present for which the wave vector q is larger or comparable to the

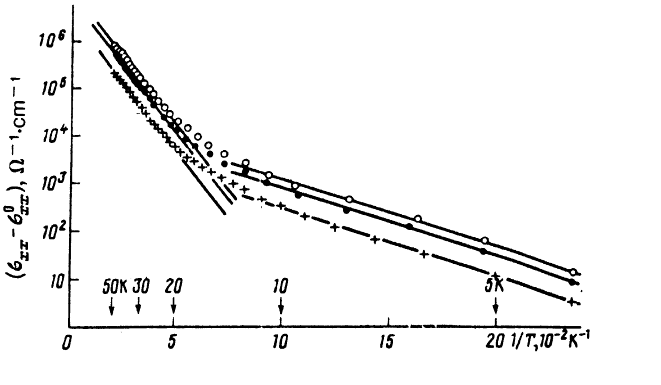

FIG. 4. Temperature dependence of the electron magnetoconductivity of sample No. 1 for $H = 60$ (○), 75 (●), and 140 kOe (+) for $\mathbf{H}\|\langle 110\rangle$.

minimum separation between sheets in k-space Δk (i.e., $q \geqslant \Delta k$). As the temperature decreases, the number of these phonons falls off exponentially.

In tungsten (see Fig. 1) there is a small gap between the FS sheets Δk_1 and Δk_2 across which umklapp processes can occur. The form of the distribution function of phonons with respect to energy over the linear segment of their energy spectrum implies that the number of phonons with momentum $q_0 = \Delta k$ is proportional to $\exp(-\hbar s q_0/k_B)$, which according to Refs. 4,8 can lead to an exponential temperature dependence of the magnetoresistance of compensated metals for $T < T_0 = \hbar s q_0/k_B$. Since the FS of tungsten has sheets separated in momentum space by gaps, it may be assumed that the quantities Δ_1 and Δ_2 in Eq. (1) are due to the two different gaps Δk_1 and Δk_2 (see Fig. 1), which implies that $\Delta_1 = \hbar s \Delta k_1/k_B$ and $\Delta_2 = \hbar s \Delta k_2/k_B$. From the phonon spectrum and sound velocities given in Refs. 18 and 19, based on the measured values of Δ_1 and Δ_2, we determine the sizes of the gaps to be $\Delta k_1 = 0.15 \pm 0.03$ Å^{-1}, and $\Delta k_2 = 0.72 \pm 0.06$ Å^{-1}. These values of Δk_1 and Δk_2 are in good agreement with the measurements of the gap determined by the method of radio-frequency size effects in Ref. 15: $\Delta k_1 = 0.15 \pm 0.02$ Å^{-1}, and $\Delta k_2 = 0.8 \pm 0.02$ Å^{-1}. A rather small disagreement, of the same order as the measurement error, is observed in the quantity Δk_2 between that obtained in our paper and in Ref. 15. Possibly this is due to the fact that the gap between the hole ellipsoid and the electron "jack" Δk_2 is close in magnitude to the gap $\Delta k'_2$ between the hole ellipsoid and the hole "octahedron" (see Fig. 1) $\Delta k'_2 = 0.684 \pm 0.014$ Å^{-1} (Ref. 15), across which electron-phonon umklapp processes may also occur. Because of this closeness of the quantities Δk_2 and $\Delta k'_2$, the exponents connected with them are not resolved in the experimental data, and give an average value for Δ_2.

On the basis of general considerations, it appears improbable that a high-temperature exponential contribution would be present in the range of magnetic fields used in our experiment. The reason that this is doubtful is connected with the fact that electrons capable of radiating and absorbing phonons with energies $\hbar qs/k_B \approx \Delta_2 \approx 130$ K should have a relaxation time corresponding to this 130 K temperature. At this temperature, the effective magnetic fields for such electrons are weak, because $\omega_c\tau \approx 0.3$ for $T = 130$ K. However, our experimental data reliably reveal a distinct high-temperature exponential contribution. In order to explain why this contribution does not vanish, it is necessary to know not the average value of the relaxation time over the FS, but rather the relaxation time for electrons on that local portion of the FS from which the umklapp processes originate, and also to know the umklapp times for the gaps Δk_2. Unfortunately, at the present time there is no data on these times for tungsten, and the question as posed remains open.

The data presented above on the temperature dependence of the electron magnetoconductivity of tungsten for $\mathbf{H}\|\langle 110\rangle$ indicates that it can be explained by the presence of intersheet electron-phonon umklapp processes over the entire temperature interval, 4.2 to 50 K, for which $\omega_c\tau > 1$. We note that there is no contribution, which is usually proportional to T^5, to the temperature dependence of $\sigma_{xx}(T)$ from intrasheet electron-phonon scattering. This is obviously related to the fact that when the points of closest approach of the FS sheets are located on a single open "quasiorbit" intrasheet scattering is not efficient, because there is no diffusion of electrons between "hot" points belonging to a single sheet.

Of course, the assumptions above require confirmation. In particular, it appears that by choosing another orientation of the magnetic field **H** such that the plane of the Larmor orbits and the crystallographic directions along which the gaps Δk_1 are located do not coincide, we will be able to track whether or not there is a change in the ratio of the contributions from intrasheet and intersheet scattering. This possibility is realized when the magnetic field **H** is directed along the $\langle 111 \rangle$ crystallographic axis.

b) **MAGNETIC FIELD H**$\|\langle 111 \rangle$. In Fig. 1b we show the cross section of the FS of tungsten in the $\langle 111 \rangle$ direction. It is apparent that, in contrast to the cross section in the plane (110), there are no small gaps Δk_1 in the plane (111). In fact, these gaps are absent not only from the plane (111) passing through the point Γ of the Brillouin zone, but also from all the other equivalent planes, including the plane passing through the H-point. When the direction of **H** is $\|\langle 111 \rangle$, the electron "jack" and the hole ellipsoid are separated by a gap $\Delta k_3 = \Delta k_2$ (see Fig. 1b). According to Ref. 15, the gap Δk_3 is equal to 0.80 ± 0.02 Å^{-1}. In the plane (111) passing through the H-point there is a gap $\Delta k_3'$ between the hole "octahedron" and the hole ellipsoid with a value 0.684 ± 0.014 Å^{-1}, i.e., the same as $\Delta k_2'$. It is clear that in this experimental geometry, which differs in principle from the **H**$\|\langle 110 \rangle$ geometry, we can expect that there will be no contribution due to umklapp processes across the gap Δk_1 to the temperature dependence of the electron magnetoconductivity.

It is also clear from Fig. 2 that whereas the quantities σ_{XX} practically coincide for **H**$\|\langle 110 \rangle$ and **H**$\|\langle 111 \rangle$ within the limits of measurement error when $T > 20\,K$, for $T < 20$ K the magnetoresistance increases more rapidly for **H**$\|\langle 111 \rangle$ than for **H**$\|\langle 110 \rangle$ as the temperature decreases. This indicates that for $T > 20$ K the temperature dependences of the magnetoresistances for **H**$\|\langle 110 \rangle$ and **H**$\|\langle 111 \rangle$ can be described by like functions.

For $T < 20$ K, the functional descriptions of these dependences $\sigma_{xx}(T)$ differ substantially. In fact, it is not possible to describe the temperature dependence $\sigma_{xx}(T)$ for **H**$\|\langle 111 \rangle$ using a function of the type (1), i.e., a sum of two exponential contributions. Figure 5 shows the dependence of the quantity $\ln(\sigma_{xx}(T) - \sigma_{xx}^0)$ on T^{-1} over the temperature interval 15 to 50 K when **H**$\|\langle 111 \rangle$. It is clear that in the interval 20 to 50 K, $\ln(\sigma_{xx} - \sigma_{xx}^0)$ depends linearly on the inverse temperature. Consequently, the high-temperature segment of the temperature dependence $\sigma_{xx}(T)$ has an exponential character, and can be written as follows:

$$(\sigma_{xx} - \sigma_{xx}^0)|_{T>20\text{ K}} \sim \sigma_3 \exp(-\Delta_3/T). \qquad (2)$$

where $\sigma_{xx}^0 = 378.1$ $(\Omega\cdot\text{cm})^{-1}$, $\sigma_3 = 8.04 \cdot 10^6$ $(\Omega\cdot\text{cm})^{-1}$, and $\Delta_3 = 136.9 \pm 1.5$ K, when $H = 140$ kOe. Within the limits of measurement error, the quantities Δ_3 (for **H**$\|\langle 110 \rangle$) and Δ_3 (for **H**$\|\langle 111 \rangle$) coincide, implying that these exponential contributions to σ_{xx} have the same origin. Actually, when **H** is parallel to $\langle 111 \rangle$ the gap Δk_3, in agreement with Ref. 15, coincides with the gap Δk_2 and comes to 0.80 ± 0.05 Å^{-1}.

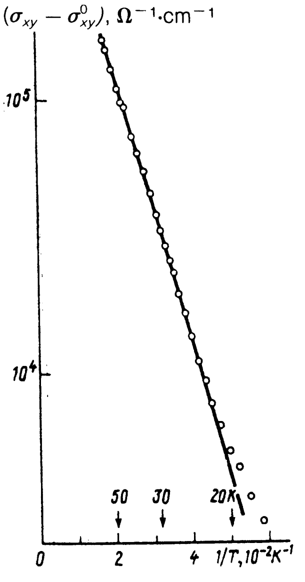

FIG. 5. Temperature dependence of the electron magnetoconductivity of sample No. 2 for $T > 20$ K, $H = 140$ kOe, **H**$\|\langle 111 \rangle$.

Thus, for **H**$\|\langle 111 \rangle$, just as for **H**$\|\langle 110 \rangle$, the high-temperature portion of the temperature dependence of the electron magnetoconductivity of tungsten is most likely to be determined by electron-phonon umklapp processes across the gaps Δk_3 and $\Delta k_3'$ between the hole ellipsoids of the FS located at the boundary of the Brillouin zone at the N points and the FS sheets centered at the points Γ and H, respectively.

A different picture is observed for the low-temperature segment of the curve $\sigma_{xx}(T)$ when **H**$\|\langle 111 \rangle$. In Fig. 6 we plot, in the coordinates $(\sigma_{xx} - \sigma_{xx}^0)/T^2 = f(T^3)$, the segment of the temperature dependence of the electron magnetoconductivity in the interval 4.2 to 25 K. It is clear that in these coordinates the experimental points all lie very close to a straight line when $T < 20$ K. This indicates that the low-

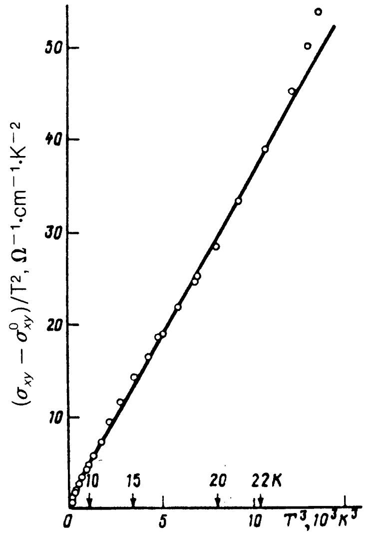

FIG. 6. Temperature dependence of the electron magnetoconductivity of sample No. 2 for $T < 20$ K, $H = 140$ kOe, **H**$\|\langle 111 \rangle$.

temperature segment of the function $\sigma_{xx}(T)$ can be written in the form

$$(\sigma_{xx}-\sigma_{xx}^{0}) \approx AT^2+BT^5. \quad (3)$$

For this curve $A = 0.83$ $(\Omega\cdot\text{cm}\cdot\text{K}^2)^{-1}$ and $B = 3.53\cdot10^{-3}$ $(\Omega\cdot\text{cm}\cdot\text{K}^5)^{-1}$. The contributions AT^2 and BT^5 are comparable when $T\approx6$ K.

It is obvious that the observed contribution $\sim BT^5$ is caused by intrasheet scattering of conduction electrons by phonons. Its appearance can be related to diffusion of electrons along the sheets of the Fermi surface up to regions next to the gaps Δk_1. The presence of a power-law diffusion contribution in this experimental geometry, and its dominance over the umklapp exponential contribution across the gap Δk_1, is probably associated with the fact that in the $\mathbf{H}\langle110\rangle$ geometry the gaps Δk_1 are located on a single "quasiorbit" lying in a plane perpendicular to **H**, and there is no diffusion of carriers along the FS sheet between "hot" points.

The contribution $\sim AT^2$ could be due to several causes: electron-electron scattering, scattering of electrons by the disk surface, and scattering of electrons by thermal oscillations of impurities. In Ref. 22, the presence of a contribution proportional to T^2 to the temperature dependence of the electron magnetoconductivity of beryllium was related to electron-electron scattering. As for electron-hole scattering, the authors of Ref. 23 showed that in high magnetic fields this process does not contribute to the electron magnetoconductivity. In the present study, we have not investigated the origin of this T^2 contribution; however, it is noteworthy that in the absence of a magnetic field the contribution of electron-electron scattering to the temperature dependence of the electrical conductivity of tungsten is extremely small.[21]

Thus, in the low-temperature range the electron magnetoconductivity of single-crystal tungsten in high magnetic fields $\mathbf{H}\|\langle110\rangle$ is determined to a considerable degree by electron-phonon umklapp processes across the gap Δk_1 between the electron "jack" $\Gamma4e$ and the hole "octahedron" $H3h$, and also across the gaps Δk_2 and $\Delta k_2'$ between the hole ellipsoids $N3h$ and the sheets $\Gamma4e$ and $H3h$, respectively. For $\mathbf{H}\|\langle111\rangle$ the electron-phonon umklapp processes across Δk_3 and $\Delta k_3'$ between the sheets $N3h$ and $\Gamma4e$, or $N3h$ and $H3h$ at temperatures $T<20$ K are almost completely frozen out; therefore at temperatures below 20 K the character of the temperature dependence of the electron magnetoconductivity is primarily due to intrasheet scattering of electrons by phonons.

5. THE HALL EFFECT

The results of our investigations of the magnetoresistance indicate that at low temperatures $T\ll\theta_D$ intersheet electron-phonon umklapp processes are to a considerable degree responsible for the character of its temperature dependence. In view of this, we can expect to see evidence of this scattering mechanism in the temperature dependence of the Hall coefficient under analogous experimental conditions. A study of the role played by umklapp processes in the Hall effect at high magnetic fields is also interesting because investigations carried out previously in Refs. 6 and 12 at intermediate magnetic fields showed that there is a connection between anomalies in the temperature dependences of the Hall coefficient in the form of sharp maxima on the curve $R_H(T)$ and anisotropy of the electron-phonon scattering.

Figure 7 shows the temperature dependences of the Hall resistance $\rho_{xy}(T)$ in a magnetic field 140 kOe for the two directions $\mathbf{H}\|\langle111\rangle$ and $\mathbf{H}\|\langle110\rangle$. In contrast to the magnetoresistance, the measurements of the Hall emf were carried out not on a Corbino disk, but rather on samples of rectangular shape (see Table I). The method of measurement was described in Sec. 2. It is clear from Fig. 7 that as the temperature decreases from 50 to 4.2 K, the quantity ρ_{xy} increases rapidly and monotonically, by a factor of 120 when $\mathbf{H}\|\langle111\rangle$ and a factor of 50 when $\mathbf{H}\|\langle110\rangle$. We note first of all that this strong increase in the Hall resistance is itself unusual for compensated metals when $\omega_c\tau\gg1$. Secondly, unlike the measured temperature dependence of the Hall coefficient at intermediate magnetic fields (i.e., when $\omega_c\tau\approx1$; see Ref. 6), when $\mathbf{H}\|\langle110\rangle$, there is no maximum in $\rho_{xx}(T)$ in the vicinity of 30 K, which the authors of Ref. 6 associated with intersheet umklapp processes. Also noteworthy is the difference by almost a factor of 5 in ρ_{xy} for the directions $\mathbf{H}\|\langle110\rangle$ and $\mathbf{H}\|\langle111\rangle$ at $T=4.2$ K, while for $T>100$ K there is no anisotropy in ρ_{xy}.[6,12]

Since we have found that the character of the temperature dependence of the magnetoresistance is strongly influenced by intersheet electron-phonon scattering in the temperature range we investigated, we are fully justified in postulating that it influences also the Hall effect. For $\omega_c\tau\gg1$, the Hall effect in compensated metals is determined both by the difference in mobilities of the electrons and holes and by the degree of anisotropy in the character of their scattering.[24] We might expect that exponential contributions from the umklapp processes are also manifest in the function $\rho_{xy}(T)$. However, semilog plots of the Hall resistivity $\rho_{xy}-\rho_{xy}^{\infty}$ versus T^{-1} using lead to no curves with linear segments that would indicate the presence of contributions proportional to $\exp(-\Delta/T)$ (here ρ_{xy}^{∞} is the Hall resistivity of tungsten measured at $T>150$ K, at which temperature the electron-phonon scattering is isotropic:

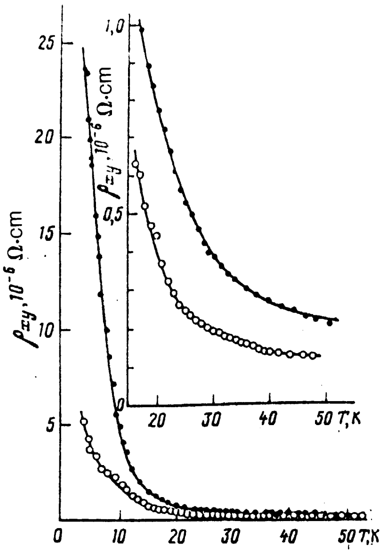

FIG. 7. Temperature dependence of the Hall resistivity of sample Nos. 3 and 4 (●)—$\mathbf{H}\|\langle111\rangle$; (○)—$\mathbf{H}\|\langle110\rangle$ for $H=140$ kOe. The inset shows in an expanded scale the high-temperature segments of these curves.

$\rho_{xy}^{\infty}(\mathbf{H}\|\langle 110\rangle) = \rho_{xy}^{\infty}(\mathbf{H}\|\langle 111\rangle) = 0.15\cdot 10^{-6}$ ($\Omega\cdot$cm).

In Fig. 8 we have plotted the temperature-dependent portion of $\rho_{xy}(T)$ for both directions of the magnetic field in decimal logarithmic scale. It is clear that in the temperature interval 15 to 50 K these temperature dependences can be satisfactorily described by the power law function $\rho_{xy} \propto T^{-n}$. For $\mathbf{H}\|\langle 110\rangle$, just as for $\mathbf{H}\|\langle 111\rangle$, we find that $n = 2.8 \pm 0.2$. The decrease in the exponent for $T < 15$ K is most likely a manifestation of the static skin effect in this region of temperatures for samples of rectangular form.[10] Because of the temperature variation of the transport mean-free path for electrons, the relation between l, r, and d changes and leads, under the conditions favorable to the SSE, to a redistribution of electric current between the subsurface layer and the bulk of the sample.

In the temperature interval 20 to 50 K there is no SSE, and the temperature dependence of $\rho_{xy}(T)$ should be due predominantly to electron-phonon scattering in this range of T. Our investigations of the magnetoresistance show clearly that in this temperature interval the form of the temperature dependence is determined by the process of freezing-out of the intersheet scattering of electrons by phonons. This raises the question of why the $\rho_{xy}(T)$ dependence in this temperature interval is described by a relation of the form

$$\rho_{xy}(T) = \rho_{xy}^{\infty} + C_{xy}T^{-n} \quad (n \approx 3). \tag{4}$$

The calculations of the conductivity tensor in a magnetic field given in Refs. 4 and 8, in which intersheet electron-phonon scattering was taken into account, are applicable to uncompensated metals only, i.e., $n_e \neq n_h$. For these metals, the nondiagonal components of this tensor, which describe the Hall effect, are temperature-independent. In Ref. 25 the Hall effect was investigated theoretically for compensated metals in high magnetic fields. These authors found that when the nonequilibrium part of the electron distribution function is anisotropic, which could arise from the anisotropic character of electron-phonon scattering, it turns out

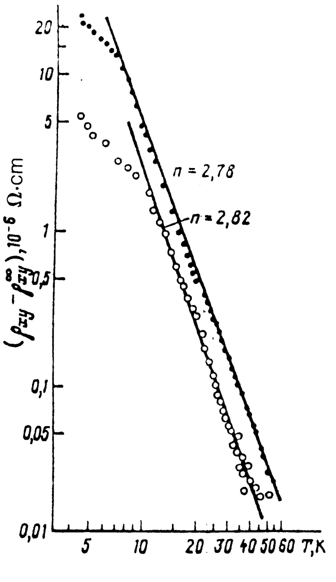

FIG. 8. Temperature dependence of the Hall resistivity of sample Nos. 3 and 4 for $H = 140$ kOe (●)—$\mathbf{H}\|\langle 111\rangle$; (○)—$\mathbf{H}\|\langle 110\rangle$. For the meaning of ρ_{xx}^{∞} consult the text.

that for $\omega_c\tau \gg 1$ the symmetrization of the distribution function of electrons leads to a change in the value of the Hall coefficient of a compensated metal compared to its value as $H \to 0$, and to a strong temperature dependence. If we include in our considerations the results of Ref. 26, where expressions were obtained for the field dependences of the Hall resistivity $\rho_{xy}(H)$ at various temperatures under conditions of intersheet electron-phonon scattering assuming that the transport relaxation time of electrons depended on temperature as $\tau = A + BT^5$, then we can conclude that ρ_{xy} can vary strongly with temperature in a compensated metal when $T \ll \theta_D$. Clearly, the dependence $\rho_{xy}(T) \propto T^{-3}$ for tungsten cannot possibly be a universal function that characterizes the temperature behavior of the Hall effect for electron-phonon umklapp processes in compensated metals, since according to Ref. 26 the form of the function $\rho_{xy}(T)$ should to a considerable degree be determined by the geometry of the FS, by the placement of the FS sheets relative to one another, and by the characteristics of the electron and hole carriers. The reason why scattering of electrons by phonons leads to a temperature dependence of $\rho_{xy}(T)$ close to T^{-3} in the temperature interval 20 to 50 K for tungsten in particular will most likely be understood only after calculating $\rho_{xy}(T)$ from first principles, taking into account anisotropy of the electron-phonon scattering in a magnetic field.

We have already noted that in intermediate magnetic fields ($\omega_c\tau \sim 1$) the authors of Ref. 6 observed a maximum in the temperature dependence of the Hall coefficient $R_H(T)$ in single-crystal tungsten for $\mathbf{H}\|\langle 110\rangle$ near 30 K. In the present study we have measured $\rho_{xy}(T)$ for the same direction of $\mathbf{H}$ but for $\omega_c\tau \gg 1$; we have found that the temperature dependence of $\rho_{xy}(T)$ is smooth, and that there are no extrema. The authors of Ref. 6 noted that increasing the magnetic field leads to a decrease in the magnitude of the anomalies in $R_H(T)$; the absence of extrema in our case confirms this tendency. However, the theoretical arguments in Ref. 25 predict that increasing H cannot lead to the disappearance of extrema in $\rho_{xy}(T)$ over the temperature range where the umklapp processes are most effective in making the electron distribution function anisotropic. Nevertheless, the results of our paper indicate that this prediction is violated for the case of tungsten, for reasons as yet not understood.

CONCLUSION

Our low-temperature investigations of the temperature dependence of the galvanomagnetic properties of high-purity single-crystal tungsten in magnetic fields up to 140 kOe, under the conditions $\omega_c\tau \gg 1$, $T \ll \theta_D$, allows us to draw the following conclusions.

1. For the first time, we have shown experimentally, with tungsten as an example, that at low temperatures and in high magnetic fields the temperature dependence of the transverse magnetoresistance of compensated metals with closed Fermi surfaces contains an exponential temperature-dependent contribution, and that it is apparently determined by intersheet electron-phonon umklapp processes.

2. The anisotropic character of the intersheet electron-phonon scattering and its competition with intrasheet scattering of electrons by phonons cause anisotropy in the temperature dependent contributions to the temperature dependences of the galvanomagnetic coefficients.

3. Under conditions where the intersheet umklapp processes dominate over other scattering mechanisms for conduction electrons, the Hall resistance does not contain an exponential temperature-dependent contribution. However, observation of its strongly monotonic temperature dependence, which is proportional to T^{-n} (where $n \gtrsim 3$) may also be connected with the anisotropy of the intersheet electron-phonon scattering.

4. Our measurements of the magnetoresistance of single-crystal tungsten samples in the form of Corbino disks persuade us that this experimental geometry can be successfully used to study the low-temperature bulk properties of high-purity compensated metals in high magnetic fields, i.e., in those cases where it is necessary to avoid the surface scattering of conduction electrons that leads to the static skin effect.

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