Transition radiation of relativistic particles in a magnetized plasma with random inhomogeneities

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The transition radiation which occurs in a cold, gyrotropic, magnetized plasma with random inhomogeneities in the electron density is analyzed. If the plasma is weakly gyrotropic, partially polarized radiation is emitted. Its degree of polarization is determined exclusively by the ratio of the gyrofrequency to the emission frequency. For an arbitrary degree of gyrotropy, there is a sharp difference in the emission of the two normal modes. As a result, highly polarized radiation is emitted. The intensity of the ordinary-wave radiation is basically the same as in the case of an isotropic plasma, except in a narrow interval of small pitch angles. The intensity of the extraordinary-wave radiation depends on the refractive indices for both normal modes. The range of applicability of these expressions and their possible applications under astrophysical and laboratory conditions are discussed.

1. INTRODUCTION

The radiation by particles in anisotropic and gyrotropic media has been under study for more than half a century now. A method for analyzing the radiation by particles in anisotropic media was proposed by Ginzburg[1] back in 1940. Ginzburg used this method to analyze Cherenkov radiation,[2] Kolomenskiĭ generalized this method to the case of gyrotropic media.[3] He also studied the formation of the Cherenkov radiation emitted by a particle moving along the magnetic field in a magnetized plasma.[4] Barsukov[5] studied the radiation emitted by oscillators moving along the symmetry axis of a gyrotropic crystal. The results of those early studies, as well as methods for calculating the radiation in anisotropic media, are described in Ref. 6. Here we mention only those papers which have dealt with the role played by an optical anisotropy in the process by which radiation is emitted in media. The possible presence of a crystal structure, i.e., an order in the arrangement of the particles of the medium, leads to a long list of interesting effects, which are themselves the subject of an extensive literature (see, for example, the review by Bazylev and Zhevago[7] and the bibliography there).

Questions concerning transition radiation in anisotropic media have also been studied previously. The transition radiation at an interface between an ordinary isotropic medium and an optically active isotropic medium was studied in Ref. 8. The radiation by particles as they pass through ferroelectric and crystalline plates was studied in Ref. 9. The transition radiation at the interface between a gyrotropic medium and an ideal conductor was calculated in Ref. 10 under the assumption of weak gyrotropy, $g/\varepsilon \ll 1$. However, the results of those studies look a bit strange, since in the limit $g/\varepsilon \to 0$ all the radiation turns out to be circularly polarized (rather than naturally polarized).

In this paper we discuss the transition radiation of relativistic particles which are moving through a gyrotropic magnetized plasma with a broad spectrum of random density inhomogeneities. This problem is of interest because magnetized plasmas are typical of both laboratory conditions and various astrophysical objects. Such plasmas are frequently in a turbulent state, having density fluctuations which are more or less intense.

An external magnetic field affects both the trajectory of a relativistic particle and the dispersion properties of the background plasma. The curvature of the trajectory of a relativistic particle in a magnetic field is important for sufficiently fast particles even in the case

$$\omega_p/\omega_{Be} > 1, \tag{1}$$

where ω_p is the plasma frequency, and $\omega_{\rm Be} = eB/mc$ is the electron gyrofrequency. This curvature can result in pronounced suppression of transition radiation.[11]

Under condition (1), a magnetic field has only a slight effect on the dispersion properties of a medium at frequencies $\omega \gg \omega_p$. Nevertheless, it is worthwhile to consider the magnetic field even in this case, since gyrotropy leads to a nonzero polarization of the radiation (while causing essentially no change in its intensity).

If inequality (1) does not hold, i.e., if

$$\omega_p/\omega_{Be} < 1, \tag{2}$$

the plasma is a highly gyrotropic medium, and the characteristics of the transition radiation change dramatically. Since the situation $\omega_{\rm Be}/\omega_p \gtrsim 1$ holds fairly frequently on the sun, the analysis here is also of applied interest, for studying radial emission during solar flares. The reason is that conditions in solar flares favor the generation of transition radiation.[12] The problem of the radiation by a particle moving along the magnetic field in a randomly inhomogeneous plasma was solved by Tamoĭkin[13] in terms of the reaction of the average field. That method, however, is valid only for inhomogeneities of very small scale, $L_0 \ll \lambda$. It furthermore suffers from several methodological drawbacks.[14]

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2. CALCULATION OF TRANSITION RADIATION IN A GYROTROPIC MEDIUM

Let us consider the transition radiation generated by one relativistic particle as it moves through a magnetized plasma at an arbitrary angle θ from the magnetic field. As in Ref. 11, we assume that there are random variations of the thermal-electron density in the plasma and that these fluctuations can be described by a power-law spectrum:

$$|\delta N|_{\mathbf{k}}^2 = \frac{\nu-1}{4\pi}\frac{k_0^{\nu-1}\langle\Delta N^2\rangle}{k^{\nu+2}}, \tag{3}$$

where ν is the spectral index of the inhomogeneities, $k_0 = 2\pi/L_0$, and L_0 and $\langle\Delta N^2\rangle$ are the basic length scale and the mean square size of the inhomogeneities. We know[14,15] that in order to calculate the transition radiation in a plasma with weak inhomogeneities, satisfying

$$\Delta N/N \ll 1 \tag{4}$$

we need to find the current $\mathbf{j}^{(2)}_{\omega,\mathbf{k}'}$ in the medium, which is bilinear in the field of the relativistic particle, $\mathbf{E}^q_{\omega,\mathbf{k}}$, and in the amplitude of the inhomogeneities, $\delta N_{\mathbf{k}}$. An expression for $\mathbf{j}^{(2)}_{\omega,\mathbf{k}}$ can be found by solving the kinetic equation by perturbation theory, as in the derivation of the dielectric tensor of a magnetized plasma:[16]

$$j^{(2),\alpha}_{\omega,\mathbf{k}} = \frac{ie^2}{m\omega}\chi_{\alpha\beta}\int d\mathbf{k}'\, E^{q,\beta}_{\omega,\mathbf{k}-\mathbf{k}'}\delta N_{\mathbf{k}'}. \tag{5}$$

The inhomogeneities are assumed to be quasistatic. The tensor $\chi_{\alpha\beta}$ is given in the cold-plasma approximation by

$$\chi_{\alpha\beta} = \begin{pmatrix} (1-u)^{-1} & -iu^{1/2}(1-u)^{-1} & 0 \\ iu^{1/2}(1-u)^{-1} & (1-u)^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

The diagonal components of the tensor $\chi_{\alpha\beta}$ describe the currents which are transverse and longitudinal with respect to the magnetic field ($\mathbf{B} = B\mathbf{e}_z$), while the off-diagonal components describe the Hall current of the plasma electrons. The dimensionless quantity u is determined by the ratio of the electron gyrofrequency to the radiation frequency:

$$u = \omega_{Be}^2/\omega^2. \tag{7}$$

The electric field of the relativistic particle, which appears in (5), is expressed in terms of the current of the particle by means of the Green's function $G_{\beta\sigma}(\omega,\mathbf{k})$:

$$E^{q,\beta}_{\omega,\mathbf{k}} = G_{\beta\sigma}(\omega,\mathbf{k})\, j^{q,\sigma}_{\omega,\mathbf{k}}. \tag{8}$$

The Green's function in a gyrotropic plasma, however, is not the same as that in an isotropic medium. It can be found by expanding the field in the anisotropic medium in normal modes (the Hamiltonian method[1,3,17]). In particular, the Green's function in a cold magnetized plasma can be written

$$G_{\alpha\beta}(\omega,\mathbf{k}) = 4\pi i\omega\left[\frac{a_{o\mathbf{k}}^{\alpha}a_{o\mathbf{k}}^{*\beta}}{\omega_{o\mathbf{k}}^2-\omega^2} + \frac{a_{e\mathbf{k}}^{\alpha}a_{e\mathbf{k}}^{*\beta}}{\omega_{e\mathbf{k}}^2-\omega^2}\right], \tag{9}$$

where $\mathbf{a}_{o,e}$ are the polarization vectors of the ordinary and extraordinary waves, and $\omega_{o,e}$ are the corresponding eigenfrequencies. These quantities were derived by Éĭdman[18] in a study of magnetobremsstrahlung in a gyrotropic plasma. They are reproduced in several monographs (e.g., Ref. 19), and we write them down here, for convenience in the discussion below:

$$\omega_{j,\mathbf{k}}^2 = k^2c^2/n_j^2 \quad (j=o,e), \tag{10}$$

$$n_{o,e}^2 = 1 - \frac{2v(1-v)}{2(1-v) - u\sin^2\theta \pm [u^2\sin^4\theta + 4u(1-v)^2\cos^2\theta]^{1/2}}, \tag{11}$$

where θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{B}$, and

$$v = \omega_p^2/\omega^2, \tag{12}$$

$$\mathbf{a}_{j,\mathbf{k}} = \frac{n_{j,\mathbf{k}}}{(1+\alpha_j^2+\beta_j^2)^{1/2}}(1, i\alpha_j, i\beta_j) \quad (j=o,e). \tag{13}$$

Here 1, $i\alpha_j$, and $i\beta_j$ are the x, y, and z components of the polarization vector. The coordinate system has been chosen in such a way that the magnetic field $\mathbf{B}$ is along the z axis, and the vector $\mathbf{k}$ lies in the (y,z) plane. Here

$$\alpha_j = K_j\cos\theta - \gamma_j\sin\theta, \quad \beta_j = K_j\sin\theta + \gamma_j\cos\theta \tag{14}$$

$$K_{o,e} = \frac{2u^{1/2}(1-v)\cos\theta}{u\sin^2\theta \mp [u^2\sin^4\theta + 4u(1-v)^2\cos^2\theta]^{1/2}}, \qquad K_oK_e = -1, \tag{15}$$

$$\gamma_{o,e} = -\frac{vu^{1/2}\sin\theta + uvK_{o,e}\cos\theta\sin\theta}{1-u-v(1-u\cos^2\theta)}. \tag{16}$$

These expressions give a correct description of the electric fields in a plasma except near the fundamental cyclotron resonance ($u = 1$) and possibly near multiples of it, where the thermal motion of the plasma particles (the spatial dispersion) must be taken into account.

The emission of electromagnetic waves can be described by means of various quantities. One could calculate the radiation intensity and the degree of polarization, find the polarization tensor or the Stokes parameters, etc. In our case, however, the most convenient approach is to calculate the intensity at which the normal modes—ordinary and extraordinary—are radiated.

The energy radiated by a particle into one of the normal modes during the entire time which the particle spends moving through the plasma can be written[20]

$$E_{j,\mathbf{n},\omega} = (2\pi)^6\frac{\omega^2}{c^3}\langle|(\mathbf{a}^*_{j,\mathbf{k}}\mathbf{j}^{(2)}_{j,\omega,\mathbf{k}})|^2\rangle. \tag{17}$$

For definiteness we consider the energy radiated into the ordinary wave; the final expressions will also hold for the extraordinary wave when the subscripts are changed: $o \rightleftarrows e$. In the calculations we use the approximation that the relativistic particles move rectilinearly. In reality, this approximation may not be valid. Under the condition $\omega_{Be}/\omega_p > 1$, for example, the curvature of the trajectory has a very strong effect on the transition radiation by relativistic particles of any energy. Nevertheless, it is still worthwhile to examine the rectilinear motion of the particles in this case, since we would like to see the effect of the plasma gyrotropy on the radiation in its pure form. Furthermore, for protons and heavier nuclei, this approximation may be completely valid even in fairly strong magnetic fields. With these general comments, we end our discussion of the role of trajectory curvature for the time being. We will return to this topic in Sec. 6, where we look at some specific calculations and find the applicability conditions.

The Fourier component of the current of a particle in rectilinear motion is

$$\mathbf{j}^{q}_{\omega,\mathbf{k}-\mathbf{k}'}=\frac{q\mathbf{v}}{(2\pi)^3}\delta[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}], \tag{18}$$

where q is the charge of the relativistic particle. Substituting (18) into (8), and then substituting (8) into (5), we find the plasma current, which is the source of the transition radiation. Substituting it into (17), we find

$$\begin{aligned}E_{o,\mathbf{n},\omega}=&\frac{8\pi e^4q^2\omega^2}{c^3}T\Big\{|(\overset{*\alpha}{a}_{o,\mathbf{k}}\chi_{\alpha\beta}\overset{\beta}{a}_{o,\mathbf{k}})|^2|(\overset{*}{\mathbf{a}}_{o,\mathbf{k}}\mathbf{v})|^2\\&\times\int d\mathbf{k}'|\delta N|_{\mathbf{k}'}^2\frac{\delta[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]}{[(\mathbf{k}-\mathbf{k}')^2c^2/n_o^2-\omega^2]^2}\\&+(|\overset{*\alpha}{a}_{o,\mathbf{k}}\chi_{\alpha\beta}\overset{\beta}{a}_{e,\mathbf{k}})|^2|(\overset{*}{\mathbf{a}}_{e,\mathbf{k}}\mathbf{v})|^2\int d\mathbf{k}'|\delta N|_{\mathbf{k}'}^2\frac{\delta[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]}{[(\mathbf{k}-\mathbf{k}')^2c^2/n_e^2-\omega^2]^2}\\&+2\,\mathrm{Re}(\overset{*\alpha}{a}_{o,\mathbf{k}}\chi_{\alpha\beta}\overset{\beta}{a}_{o,\mathbf{k}})(\overset{\gamma}{a}_{o,\mathbf{k}}\chi_{\gamma\delta}{}^{*}\overset{*\delta}{a}_{e,\mathbf{k}})|(\overset{*}{\mathbf{a}}_{o,\mathbf{k}}\mathbf{v})(\mathbf{a}_{e,\mathbf{k}}\mathbf{v})|\cdot\\&\times\int d\mathbf{k}'|\delta N|_{\mathbf{k}'}^2\frac{\delta[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]}{[(\mathbf{k}-\mathbf{k}')^2c^2/n_o^2-\omega^2][(\mathbf{k}-\mathbf{k}')^2c^2/n_e^2-\omega^2]}\Big\},\end{aligned} \tag{19}$$

where T is the total duration of the radiation. This is the time which we should use as the divisor in making the conversion to a radiation intensity (the energy radiated per unit time). The other quantities in (19) were defined above. In general, the intensity of the radiation of the ordinary wave is determined by the refractive indices for both normal modes. The formal reason for this is the difference between the tensor $\chi_{\alpha\beta}$ in (6) and $\delta_{\alpha\beta}$. The Green's function in (9) is the electric field of the particle, expressed as the sum of virtual ordinary and extraordinary waves. The scattering of the virtual ordinary wave (and of the extraordinary wave) by inhomogeneities of the medium leads to the emission of both ordinary and extraordinary waves. Correspondingly, we will call the two contributions to the radiation of each normal mode the "radiation through a virtual ordinary wave" (or through a "virtual extraordinary wave").

Since the radiation by relativistic particles is highly directional, we will make use of the circumstance that the angle ϑ between $\mathbf{k}$ and $\mathbf{v}$ is small, as before.[11,21] In this case we write the velocity of the particle as[22]

$$\mathbf{v}=\mathbf{n}v(1-\vartheta^2/2)+v\boldsymbol{\vartheta}, \tag{20}$$

where $\mathbf{n}=\mathbf{k}/k$, and the vector ϑ is equal in magnitude to the angle ϑ. Combining (20) with (13) and (14), we find

$$\begin{aligned}|(\mathbf{a}^*_{o},\mathbf{kV})|^2&\approx c^2\frac{K_o^2\vartheta^2\sin^2\varphi+\vartheta^2\cos^2\varphi}{1+K_o^2}\\&=c^2\vartheta^2\frac{\sin^2\varphi+K_e^2\cos^2\varphi}{1+K_e^2},\\|(\mathbf{a}^*_{e,\mathbf{k}}\mathbf{v})|^2&\approx c^2\vartheta^2\frac{\cos^2\varphi+K_e^2\sin^2\varphi}{1+K_e^2},\\2(\mathbf{a}^*_{o,\mathbf{k}}\mathbf{v})(\mathbf{a}_{e,\mathbf{k}}\mathbf{v})&\approx-2K_ec^2\vartheta^2\frac{\cos2\varphi}{1+K_e^2}.\end{aligned} \tag{21}$$

Here we have used the condition $K_oK_e\equiv-1$. We have also discarded some terms which contain γ_j and which describe the longitudinal component of the electric field, since under the conditions of interest here the relations $|\gamma_j|\ll1$, $|K_j|$ hold. Here φ is the azimuthal angle of the vector $\boldsymbol{\vartheta}$, we have $\cos\varphi=0$ when $\boldsymbol{\vartheta}$ lies in the $(\mathbf{B},\mathbf{v})$ plane, and we have $\sin\varphi=0$ when $\boldsymbol{\vartheta}$ is perpendicular to this plane.

Substituting (21) into (19), and using $\mathbf{kk}'/kk'\simeq\mathbf{vk}/vk'$, we can integrate (19) over the angles of the vector $\mathbf{k}'$. Here we make use of the δ-function. We convert to a radiation intensity by dividing (19) by T:

$$\begin{aligned}I_{o,\mathbf{n},\omega}=&\frac{16\pi^2e^4q^2}{m^2\omega^2c^2}\int_{k'^{o}_{min}}^{\infty}k'\,dk'|\delta N|^2_{\mathbf{k}'}\frac{\vartheta^2}{(1+K_e^2)^3}\\&\times\Big\{\Big[\frac{\cos^2\theta-2K_eu^{1/2}\cos\theta+K_e^2}{1-u}\\&+\sin^2\theta\Big]^2\frac{\sin^2\varphi+K_e^2\cos^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]^2}\\&+\Big[\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{1-u}\Big]^2\\&\times\frac{\cos^2\varphi+K_e^2\sin^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_e)]^2}\\&-2K_e\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{(1-u)[\vartheta^2+\gamma^{-2}+2(1-n_e)]}\\&\times\frac{1-2K_eu^{1/2}\cos\theta+K_e^2-u\sin^2\theta}{(1-u)[\vartheta^2+\gamma^{-2}+2(1-n_o)]}\cos2\varphi\Big\},\end{aligned} \tag{22}$$

where

$$k'^{j}_{min}=\frac{\omega}{2c}[\vartheta^2+\gamma^{-2}+2(1-n_j)]. \tag{23}$$

In deriving (22) we ignored the difference between the directions of $\mathbf{k}$ and $\mathbf{v}$ everywhere, except in expressions of the form $|(\mathbf{a}^*_{j,\mathbf{k}}\mathbf{v})|^2\sim[\mathbf{nv}]^2$. In particular, the angle θ in (22) is assumed to be equal to the angle between the velocity of the particle and the external magnetic field. Integrating (22) over the inhomogeneity spectrum (3), we find

$$\begin{aligned}I_{o,\mathbf{n},\omega}=&\frac{8\pi(\nu-1)}{\nu}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\frac{(2k_0c/\omega)^{\nu-1}}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]^\nu}\frac{\vartheta^2}{(1+K_e^2)^3}\\&\times\Big\{\Big[\frac{\cos^2\theta-2K_eu^{1/2}\cos\theta+K_e^2}{1-u}\\&+\sin^2\theta\Big]^2\frac{\sin^2\varphi+K_e^2\cos^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]^2}\\&+\Big[\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{1-u}\Big]^2\frac{\cos^2\varphi+K_e^2\sin^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_e)]^2}\\&-2K_e\cos2\varphi\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{(1-u)[\vartheta^2+\gamma^{-2}+2(1-n_e)]}\\&\times\frac{(\cos^2\theta-2K_eu^{1/2}\cos\theta+K_e^2)(1-u)^{-1}+\sin^2\theta}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]}\Big\}.\end{aligned} \tag{24}$$

The expression for the spectrum and angular distribution of the radiation of the extraordinary wave is calculated similarly. The result is

$$\begin{aligned}I_{e,\mathbf{n},\omega}=&\frac{8\pi(\nu-1)}{\nu}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\frac{(2k_0c/\omega)^{\nu-1}}{[\vartheta^2+\gamma^{-2}+2(1-n_e)]^\nu}\frac{\vartheta^2}{(1+K_e^2)^3}\\&\times\Big\{\Big[\frac{1+2K_eu^{1/2}\cos\theta+K_e^2\cos^2\theta}{1-u}\\&+K_e^2\sin^2\theta\Big]^2\frac{\cos^2\varphi+K_e^2\sin^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_e)]^2}\\&+\Big[\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{1-u}\Big]^2\frac{\sin^2\varphi+K_e^2\cos^2\varphi}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]^2}\\&-2K_e\cos2\varphi\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{(1-u)[\vartheta^2+\gamma^{-2}+2(1-n_e)]}\\&\times\frac{[(1+2K_eu^{1/2}\cos\theta+K_e^2\cos^2\theta)(1-u)^{-1}+K_e^2\sin^2\theta]}{[\vartheta^2+\gamma^{-2}+2(1-n_o)]}\Big\}.\end{aligned} \tag{25}$$

Expressions (24) and (25) solve the problem of the transition radiation by a relativistic particle moving through a magnetized plasma with random density variations. These expressions will be examined in detail in the following sections of this paper.

3. CASE OF A SLIGHTLY GYROTROPIC PLASMA: $\omega_{Be}/\omega \ll 1$

We consider fairly high frequencies,

$$\omega_{Be}/\omega \ll 1, \tag{26}$$

at which the plasma gyrotropy has only a weak effect on the intensity of the transition radiation. In the case of an isotropic spectrum of fluctuations of the medium, however, the weak magnetic field is the primary source of polarization of the radiation. The polarization of the radiation is a pertinent problem because of possible applications of the transition-radiation mechanism in astrophysical problems, in particular, in solar flares.[12] Observing and analyzing the polarization characteristics of radiation frequently prove to be of decisive importance in identifying the mechanisms for electromagnetic radiation in astrophysical problems.[23]

To find the degree of polarization of the transition radiation under condition (26), we proceed in the following way. We calculate the intensities of the radiation in the normal modes, (24) and (25), and then expand them in power series in u. Since the quantity u appears in combinations with $\sin\theta$ and $\cos\theta$ in the expressions for the refractive indices, (11), and for the polarization vectors, (15), the corresponding expansions differ, depending on the relation between $\cos^2\theta$ and u. If

$$\cos^2\theta \gg u, \tag{27}$$

the polarization of the normal modes in the plasma is nearly circular,[23] and the intensities of the ordinary and extraordinary waves are

$$I_{o,\omega} = \frac{I_\omega^m}{2}\left[1-2u^{1/2}\cos\theta\left(1-\frac{\nu}{2(1+\omega^2/\omega_p^2\gamma^2)}\right)\right], \tag{28}$$

$$I_{e,\omega} = \frac{I_\omega^m}{2}\left[1+2u^{1/2}\cos\theta\left(1-\frac{\nu}{2(1+\omega^2/\omega_p^2\gamma^2)}\right)\right], \tag{29}$$

where I_ω^m is the intensity of the transition radiation in an isotropic plasma.[11,15,21] Obviously, the extraordinary wave is radiated slightly less efficiently than the ordinary wave. The difference between the radiation intensities in (28) and (29) (on the one hand) and those in the case of an isotropic plasma (on the other) stems primarily from the existence of the Hall current of the electrons of the medium (the terms $\pm 2\sqrt{u}\cos\theta$). Since the direction in which the electrons revolve is the same as the direction in which the electric field rotates in the extraordinary wave, the corresponding Hall current increases the radiation of these waves and reduces the radiation of ordinary waves. The other terms stem from the small difference between the refractive indices for the normal modes. In the case $\nu < 2$, they are smaller than the Hall contributions. The degree of polarization of the resultant radiation is

$$P = \frac{I_{o,\omega}-I_{e,\omega}}{I_{o,\omega}+I_{e,\omega}} = -2\frac{\omega_{Be}}{\omega}\cos\theta\left[1-\frac{\nu}{2(1+\omega^2/\omega_p^2\gamma^2)}\right]. \tag{30}$$

According to the definition of the degree of polarization, (30), a positive (negative) value of P corresponds to the predominant emission of ordinary (extraordinary) waves.

A point of importance for astrophysical applications is that the degree of polarization in (30) at a given frequency is determined almost exclusively by the strength of the magnetic field (more precisely, its longitudinal component). In principle, we thus have another method for determining the magnetic field in radiation sources, e.g., in solar flares.

We now consider a quasitransverse motion of the particle with respect to the magnetic field:

$$\cos^2\theta \ll u. \tag{31}$$

Using expansions as in the preceding case, we find

$$I_{o,\omega} = \frac{I_\omega^m}{2}\left(1+4\,\mathrm{ctg}^2\,\theta+\frac{\nu\cos^2\theta}{1+\omega^2/\omega_p^2\gamma^2}\right), \tag{32}$$

$$I_{e,\omega} = \frac{I_\omega^m}{2}\left(1+2u-4\,\mathrm{ctg}^2\,\theta-\frac{\nu u\sin^2\theta}{1+\omega^2/\omega_p^2\gamma^2}\right). \tag{33}$$

In this case the polarization of the ordinary wave is nearly linear along the magnetic field, while the extraordinary wave is elliptically polarized, in the plane perpendicular to the magnetic field.[24] For $\theta = \pi/2$ the quantity $I_{o,\omega}$ is exactly half the radiation intensity in an isotropic plasma, since the electric field of the ordinary wave is directed strictly along the external magnetic field. This field has no effect on the processes by which ordinary waves are radiated and propagate. The degree of polarization is

$$P = -u+4\,\mathrm{ctg}^2\,\theta+\frac{\nu(\cos^2\theta+u\sin^2\theta)}{2(1+\omega^2/\omega_p^2\gamma^2)}. \tag{34}$$

In the case of strictly transverse motion ($\cos\theta = 0$), this expression becomes

$$P = -\left(\frac{\omega_{Be}}{\omega}\right)^2\left[1-\frac{\nu}{2(1+\omega^2/\omega_p^2\gamma^2)}\right]. \tag{35}$$

In contrast with (27), the polarization is now quadratic in the magnetic field, and at the typical values $1 < \nu < 2$ we have $P < 0$. In other words, the extraordinary waves are radiated more intensely, as before. The reason for the quadratic dependence of the degree of polarization on the magnetic field is that the polarization is now dominated by the corresponding transverse current, rather than by the Hall current of the plasma electrons.

4. HIGHLY GYROTROPIC PLASMA, WITH $\omega_{Be}/\omega \gg 1$: LIMITING CASES

4.1. Motion along the magnetic field ($\sin\theta = 0$)

For wave propagation along and across the magnetic field, the expressions for the refractive indices and the polarization vectors of the normal modes simplify considerably.[19] For longitudinal propagation, $\sin\theta = 0$, we find from (11) and (15)

$$n_{o,e}^2 = 1\mp v/(u^{1/2}\pm 1), \qquad K_{o,e} = \mp 1, \qquad \gamma_{o,e} = 0, \tag{36}$$

i.e., the normal modes are circularly polarized. Under the condition $\sqrt{u} \gg 1$, the refractive indices are

$$n_{o,e}^2 = 1\mp\omega_p^2/\omega_{Be}\omega. \tag{37}$$

When we substitute (36) and (37) into (24) and (25), we find that the contributions from the emission of the ordinary wave through a virtual extraordinary wave and vice versa vanish exactly. The spectrum and angular distribution of the radiation of the normal modes are

$$I_{\mathbf{n},\omega}^{(o,e)}=\frac{4\pi(\nu-1)}{\nu}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\left(\frac{\omega}{\omega_{Be}}\right)^2$$
$$\times\frac{\vartheta^2(2k_0c/\omega)^{\nu-1}\Theta(\vartheta^2+\gamma^{-2}\pm\omega_p{}^2/\omega_{Be}\omega)}{(\vartheta^2+\gamma^{-2}\pm\omega_p{}^2/\omega_{Be}\omega)^{\nu+2}}. \quad (38)$$

Here Θ is the unit step function. It has been introduced in the numerator because for

$$\gamma^{-2}-\omega_p{}^2/\omega_{Be}\omega<0 \quad (39)$$

the Cherenkov condition for the radiation of extraordinary waves is satisfied. For

$$\vartheta^2+\gamma^{-2}-\omega_p{}^2/\omega_{Be}\omega=0 \quad (40)$$

expression (38) for the extraordinary wave diverges. There is a simple physical explanation for this divergence. Under condition (39), the electric field of a particle moving through a gyrotropic medium has two components: a quasisteady self-field and a radiation field (Cherenkov radiation). The scattering of the quasisteady field (in other words, the scattering of the virtual photons emitted by the particle) occurs over a finite distance, equal to the size of the zone in which the transition radiation is formed.[14] There is also a scattering of real Cherenkov photons by irregularities of the medium. In an infinite, nonabsorbing medium, however, the distance over which the photons interact with irregularities is infinite. This is the reason for these divergences.

Under actual conditions, the intensity of the scattered Cherenkov radiation is determined by the geometric dimensions of the system and by the photon absorption length. In specific problems, the corresponding contribution to the overall radiation intensity can be dealt with in a straightforward manner. We will not go into that matter here, however. We restrict the discussion to the transition radiation proper, which we understand as the result of a conversion of the quasisteady (virtual) field of the particle into electromagnetic radiation at irregularities of the medium.[14] In an analysis of the transition radiation in a plate in which the Cherenkov condition holds,[15] the corresponding contribution is simply the result of repeated refraction and reflection of the Cherenkov radiation at the boundaries.

An important feature of expression (38), which distinguishes it from the expression for the case of an isotropic medium, is the presence of the small factor $(\omega/\omega_{Be})^2$. This factor appears because the radiation is dominated in this case by the Hall component of the plasma current, which is described by the off-diagonal terms of the tensor $\chi_{\alpha\beta}$ in (6). The reason is that the transverse field of the relativistic particle does not contain a component along the magnetic field in this geometry. Accordingly, in strong fields, with $(\omega/\omega_{Be})^2\to0$, the transition radiation associated with the longitudinal electric field of the relativistic particle may become important (this is the radiation through a virtual longitudinal wave). The longitudinal field of the particle was incorporated in Ref. 25 in a study of polarized bremsstrahlung. This type of radiation was studied in a weakly gyrotropic plasma in Ref. 26. The basic distinguishing feature of the transition-radiation problem is that we can ignore the spatial dispersion of the plasma, while in the case of polarized bremsstrahlung the thermal motion of the particles is a matter of fundamental importance.

Let us find the component of the transition radiation associated with the longitudinal field of the relativistic particle. This field is

$$\mathbf{E}_{\omega,\mathbf{k}}^{q,l}=-\frac{4\pi iqc\delta(\omega-\mathbf{kv})}{(2\pi)^3\omega\varepsilon_l(\omega)}\frac{\mathbf{B}}{B}. \quad (41)$$

Calculating the plasma current excited by the field (41) along the magnetic field, we find the radiation intensity, working by the method used in deriving (24) and (25):

$$I_{\mathbf{n},\omega,l}^{(o,e)}=\frac{4\pi(\nu-1)}{\nu}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3\varepsilon_l{}^2}\frac{\vartheta^2(2k_0c/\omega)^{\nu-1}}{(\vartheta^2+\gamma^{-2}\pm\omega_p{}^2/\omega_{Be}\omega)^{\nu}}. \quad (42)$$

Comparison of expressions (38) and (42) shows that under the condition

$$\gamma<(\omega_{Be}/\omega_p)^{1/2} \quad (43)$$

the contribution (42) from radiation through the longitudinal field, dominates at frequencies

$$\omega_p<\omega<\omega_{Be}\gamma^{-2}, \quad (44)$$

while at higher frequencies the radiation through the transverse field, (38), dominates. Under condition (43), no Cherenkov radiation appears at any frequency. As $\omega\to\omega_p$, however, the intensity in (42) diverges, since the condition for Cherenkov emission of longitudinal waves (plasma waves) begins to hold.

Integrating (38) and (42) over angle, we find the corresponding emission spectra. It is found that the integration over $d\vartheta^2$ in (42) cannot be carried out between infinite limits, since the result turns out to be infinite. The meaning here is that the radiation through a virtual longitudinal wave does not have a tight directional pattern along the velocity of the particle. However, expression (42) is valid only in a small interval of angles with respect to the magnetic field, since in deriving this expression we used the specific expressions (36) and (37) for the refractive indices and the polarization vectors of the normal modes. To get a rough idea of the situation, we accordingly estimate the radiation intensity (42) within an angle $\vartheta_*=\gamma^{-1}$ under condition (43):

$$I_{\omega,l}^{(o,e)}\sim\frac{4\pi^2(\nu-1)}{\nu}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\frac{(2k_0c/\omega)^{\nu-1}}{(\gamma^{-2}\pm\omega_p{}^2/\omega_{Be}\omega)^{\nu-2}}. \quad (45)$$

The integration in (38) can be carried out between infinite limits, because of the rapid convergence:

$$I_\omega^{(o,e)}=\frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\left(\frac{\omega}{\omega_{Be}}\right)^2\frac{(2k_0c/\omega)^{\nu-1}}{(\gamma^{-2}\pm\omega_p{}^2/\omega_{Be}\omega)^{\nu}}. \quad (46)$$

Under condition (43), we thus have $I_\omega^{(o,e)}\propto\omega^{-\nu-2}$ at frequencies $\omega_p\ll\omega\ll\omega_{Be}\gamma^{-2}$ and $I_\omega^{(o,e)}\propto\omega^{-\nu}$ at frequencies $\omega_{Be}\gamma^{-2}\ll\omega\ll\omega_{Be}$.

For higher-energy particles, i.e., under the condition

$$(\omega_{Be}/\omega_p)^{1/2}<\gamma<\omega_{Be}/\omega_p, \quad (47)$$

the radiation through the longitudinal wave is unimportant. In this case we find a frequency interval in which the condition for Cherenkov radiation holds. We consider only that region of parameter values in which this condition does not hold. For the ordinary wave, this condition always holds, while for the extraordinary wave it holds at frequencies

$$\omega > (\omega_p^2/\omega_{Be})\gamma^2. \tag{48}$$

In these cases, expression (46) remains valid. For the ordinary wave, the radiation spectrum can be decomposed into two power-law regions: $I_\omega^{(o)} \propto \omega^{-\nu}$ under condition (48) and $I_\omega^{(o)} \propto \mathrm{const}(\omega)$ in the opposite case.

If $\gamma > \omega_{Be}/\omega_p$, the intensity of the ordinary-wave radiation is constant over the entire frequency interval $\omega_p \ll \omega \ll \omega_{Be}$, and the extraordinary waves are generated by the Cherenkov mechanism in this interval. It is meaningful to speak in terms of transition radiation of extraordinary waves only outside the Cherenkov angle, i.e., only at $\vartheta > \vartheta_* = \omega_p^2/\omega_{Be}\omega$. As ϑ approaches ϑ_*, the intensity of the transition radiation diverges. A divergence of this sort, occurring as the corresponding parameters of the system approach the Cherenkov threshold, was noted by Kapitsa[27] back in 1960. As the Cherenkov threshold is approached, a virtual photon becomes progressively more "similar" to a real photon. Its range in a nonabsorbing medium ultimately becomes infinite, and the virtual photon itself converts into a real Cherenkov photon. Under real conditions, this divergence may be limited when the following factors are taken into consideration: the finite size of the basic length scale of the inhomogeneities, L_0; the curvature of the particle's trajectory; and the energy losses in the medium. In other words, any physical factor which acts to limit the coherence length to some finite value prevents the divergence of the transition radiation as the Cherenkov threshold is approached.

4.2. Motion across the magnetic field ($\cos\theta = 0$)

We now consider the motion of a particle across the magnetic field. In this case, the curvature of the trajectory of a relativistic particle may in general have an important effect on the spectrum of the transition radiation.[11] For protons and heavier nuclei, however, there is a certain range in which the approximation of rectilinear motion is applicable; this range will be determined below.

The refractive indices for normal modes in the case $\cos\theta = 0$ are

$$n_o^2 = 1 - \omega_p^2/\omega^2, \quad n_e^2 = 1 + \omega_p^2/\omega_{Be}^2. \tag{49}$$

The spectrum and angular distribution of the ordinary waves are

$$I_{n,\omega}^{(o)} = \frac{8\pi(\nu-1)}{\nu} \frac{e^4 q^2 \langle \Delta N^2 \rangle}{cm^2\omega^3} \frac{(2k_0 c/\omega)^{\nu-1}\vartheta^2 \sin^2\varphi}{(\vartheta^2+\gamma^{-2}+\omega_p^2/\omega^2)^{\nu+2}}. \tag{50}$$

The factor $\sin^2\varphi$ shows that the directional pattern for the ordinary wave is pointed along the direction of the magnetic field. The frequency spectrum of the radiation in this case is

$$I_\omega^{(o)} = \frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)} \frac{e^4 q^2 \langle \Delta N^2 \rangle}{cm^2\omega^3} \frac{(2k_0 c/\omega)^{\nu-1}}{(\gamma^{-2}+\omega_p^2/\omega^2)^{\nu}}, \tag{51}$$

i.e., half the total intensity of the transition radiation in an isotropic medium. The magnetic field has absolutely no effect on expressions (50) and (51), since in this case the direction of the electric field in the wave is the same as the direction of the external magnetic field.

The angular distribution found for the extraordinary-wave radiation from (25) is

$$I_{n,\omega}^{(e)} = \frac{8\pi(\nu-1)}{\nu} \frac{e^4 q^2 \langle \Delta N^2 \rangle}{cm^2\omega^3} \left(\frac{\omega}{\omega_{Be}}\right)^4 \frac{(2k_0 c/\omega)^{\nu-1}\vartheta^2 \cos^2\varphi}{(\vartheta^2+\gamma^{-2}-\omega_p^2/\omega_{Be}^2)^{\nu+2}}. \tag{52}$$

In this case the pattern of the radiation points in a direction perpendicular to the magnetic field. It is clear from the discussion in Subsection (4.1) that in this case it is meaningful to consider the radiation by particles whose energy is not too high,

$$\gamma < \omega_{Be}/\omega_p, \tag{53}$$

For such particles, the condition for Cherenkov radiation does not hold. Under condition (53), we can ignore the term $(\omega_p/\omega_{Be})^2$ in comparison with γ^{-2} in the denominator in (52). For the radiation spectrum we then find

$$I_\omega^{(e)} = \frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)} \frac{e^4 q^2 \langle \Delta N^2 \rangle}{cm^2\omega_{Be}^4} \omega\gamma^{2\nu}\left(\frac{2k_0 c}{\omega}\right)^{\nu-1}. \tag{54}$$

This expression shows that the radiation by particles of any energies which satisfy (53) increases in the frequency interval $\omega_p \ll \omega \ll \omega_{Be}$ in accordance with

$$I_\omega^{(e)} \propto \omega^{2-\nu} \tag{55}$$

with $\nu < 2$. The ordinary-wave radiation is of a different nature: The spectrum in (51) falls off slowly, $I_\omega^{(o)} \propto \omega^{\nu-2}$, at frequencies $\omega_p \ll \omega \ll \omega_p\gamma$ and then falls off sharply, $I_\omega^{(o)} \propto \omega^{-\nu-2}$, at higher frequencies. Comparing (51) and (54), we easily find that the ordinary-wave radiation dominates except in a narrow frequency interval near ω_{Be}. The conclusion that ordinary waves are radiated more effectively in a highly gyrotropic plasma was reached previously[12] on the basis of a qualitative analysis of the refractive indices for the normal modes.

5. MOTION AT AN ARBITRARY ANGLE FROM THE MAGNETIC FIELD

We turn now to the properties of the transition radiation of the normal modes for the case in which the particle is moving at an arbitrary angle from the magnetic field. We first consider the properties of the angular distribution of the transition radiation, working from the general expressions (24) and (25).

Figure 1a shows a typical directional pattern for the ordinary wave for the case in which the relativistic particle is moving at an angle from the external magnetic field which is not too small. We see that this distribution is generally pointed along the projection of the magnetic field onto the plane of the figure. The reason for this simple shape of the directional pattern is that the component of the ordinary-wave radiation due to the scattering of the virtual extraordinary wave is always small in comparison with the main component, coming from the scattering of the virtual ordinary wave.

For extraordinary waves, however, the radiation through a virtual ordinary wave may be very important. For example, Fig. 1, b–d, shows intensity distributions of the radiation of extraordinary waves by a particle with $\gamma = 5$ at the frequency $\omega = 5\omega_p$ for various values of the magnetic field: $\omega_{Be}/\omega_p = 300$, 500, and 700. In the first case, the radiation is due primarily to the scattering of the virtual extraor-

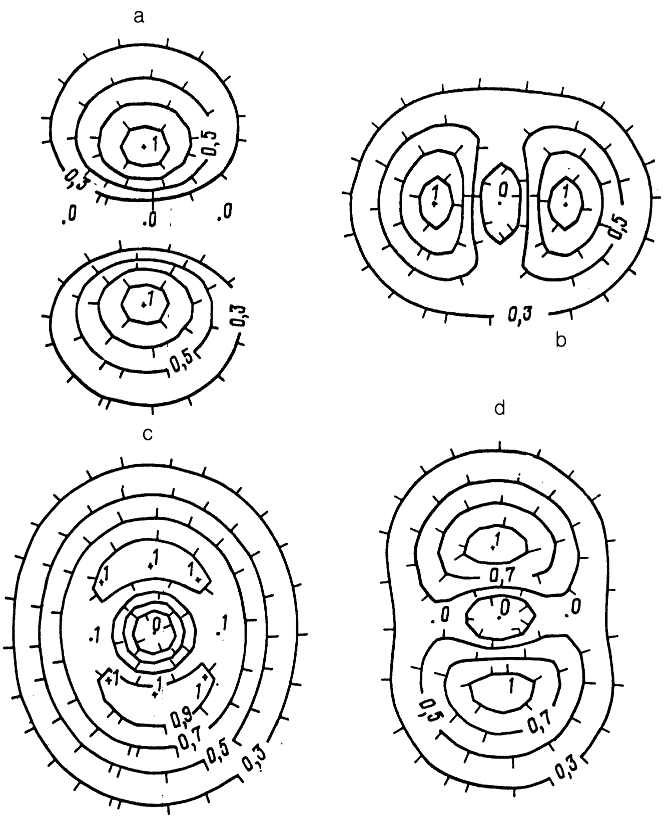

FIG. 1. Directional patterns of the radiation of normal modes for $\gamma = 5$, $\omega/\omega_p = 5$, and $\theta = \pi/4$. a—Ordinary wave; b—extraordinary wave, $\omega_{Be}/\omega_p = 300$; c—extraordinary wave, $\omega_{Be}/\omega_p = 500$; d—extraordinary wave, $\omega_{Be}/\omega_p = 700$.

dinary wave (the pattern is pointed in a direction transverse with respect to the magnetic field). In the second case the two components are comparable, and in the third the component from the virtual ordinary wave dominates. The meaning of this result is that the ordinary-wave radiation, integrated over angles, is determined exclusively by the refractive index corresponding to this wave, while the intensity of the extraordinary waves generally depends on the refractive indices for both normal modes. In this regard the transition radiation is quite different from radiation of other types (e.g., Cherenkov radiation[2,4] and magnetobremsstrahlung[18]), for which the intensity of each of the normal modes is determined exclusively by the refractive index corresponding to the given mode. This property of the transition radiation has the same physical origin as the mutual conversion of normal modes in a gyrotropic plasma.[19,24]

When (24) and (25) are integrated over angles, the terms containing $\cos 2\varphi$ vanish (these terms describe interference between two components of the radiation). Ignoring the small component of the intensity of the ordinary-wave radiation associated with n_e, we find

$$I_\omega^{(o)} = \frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\,\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\left(\frac{2k_0c}{\omega}\right)^{\nu-1} \times \frac{[(\cos^2\theta-2K_eu^{1/2}\cos\theta+K_e^2)(1-u)^{-1}+\sin^2\theta]^2}{[\gamma^{-2}+2(1-n_o)]^\nu(1+K_e^2)^2} \tag{56}$$

for the ordinary wave and

$$I_\omega^{(e)} = \frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\,\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\,\frac{(2k_0c/\omega)^{\nu-1}}{[\gamma^{-2}+2(1-n_e)]^\nu(1+K_e^2)^2} \times\left\{\left[\frac{1+2K_eu^{1/2}\cos\theta+K_e^2\cos^2\theta}{1-u}+K_e^2\sin^2\theta\right]^2 + \left[\frac{(1-K_e^2)u^{1/2}\cos\theta-uK_e\sin^2\theta}{1-u}\right]^2 \times F\left(2,\nu,\nu+2;\,1-\frac{\gamma^{-2}+2(1-n_o)}{\gamma^{-2}+2(1-n_e)}\right)\right\} \tag{57}$$

for the extraordinary wave. Here $F(2,\nu,\nu+2;1-z)$ is the hypergeometric function.

Let us first find the asymptotic representations of (56) and (57) at $u \gg 1$. More precisely, we assume

$$4\cos^2\theta/u\sin^4\theta \ll 1. \tag{58}$$

For the refractive indices and the polarization vectors we then find

$$2(1-n_o)=\frac{\omega_p^2}{\omega^2}\sin^2\theta,\qquad 2(1-n_e)=-\frac{\omega_p^2}{\omega_{Be}^2\sin^2\theta},$$
$$K_o\approx -u^{1/2}\frac{\sin^2\theta}{\cos\theta},\qquad K_e\approx\frac{\cos\theta}{u^{1/2}\sin^2\theta}. \tag{59}$$

Since the condition for Cherenkov radiation holds for extraordinary waves for particles with $\gamma > \omega_{Be}\sin\theta/\omega_p$, we consider only particles with lower energies. In this case the arguments of the hypergeometric function in (57) is limited to values $|z| \gg 1$. We can thus approximate the function F by the analytic expression[28]

$$F(2,\nu,\nu+2;1-z)\approx\left[1+\frac{(z^\nu-1)}{\Gamma(2+\nu)\Gamma(2-\nu)}\right]^{-1}. \tag{60}$$

Substituting (59) into (56), adopting condition (58), and retaining only the largest terms, we find

$$I_\omega^{(o)} = \frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\,\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\,\frac{(2k_0c/\omega)^{\nu-1}\sin^4\theta}{[\gamma^{-2}+\omega_p^2\sin^2\theta/\omega^2]^\nu}. \tag{61}$$

This expression differs from that for an isotropic plasma (or for strictly transverse motion of the particle) in having a

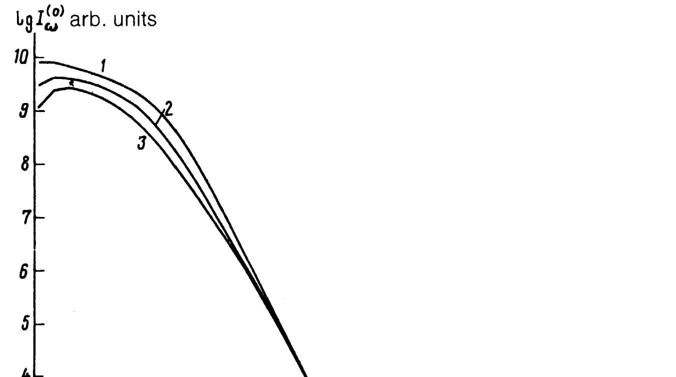

FIG. 2. Family of spectra of the transition radiation of ordinary waves for various pitch angles of the relativistic particle, for the parameter values $\gamma = 25$, $\omega_{Be}/\omega_p = 100$, and $\nu = 1.5$. *1*—$\theta = \pi/2$; *2*—$\theta = \pi/4$; *3*—$\theta = \pi/6$.

factor of $\sin^2\theta$ in the dispersion law for the ordinary waves, and a factor of $\sin^4\theta$ in the numerator in (61). The latter factor describes the projection of the electric field of the ordinary wave onto the external magnetic field.

Figure 2 shows a family of ordinary-wave radiation spectra plotted from (56) for various angles θ. We see that the magnetic field causes no qualitative change in the radiation of the ordinary waves.

Correspondingly, the transition radiation of extraordinary waves is

$$I_\omega^{(e)}=\frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\gamma^{2\nu}\left(\frac{2k_0c}{\omega}\right)^{\nu-1}\times\left[\frac{1}{u^2\sin^4\theta}+\frac{v^2\cos^2\theta}{u}F\left(2,\nu,\nu+2;\,-\frac{\omega_p^2\gamma^2}{\omega^2}\sin^2\theta\right)\right]. \quad (62)$$

Taking (60) into account, we can easily analyze this expression in various frequency regions. At $\omega\ll\omega_p\gamma\sin\theta$, for example, we have

$$I_\omega^{(e)}=\frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\left(\frac{2k_0c}{\omega}\right)^{\nu-1}\times\left[\frac{\omega^4\gamma^{2\nu}}{\omega_{Be}^4\sin^4\theta}+\frac{\Gamma(2+\nu)\Gamma(2-\nu)\omega_p^{4-2\nu}\omega^{2(\nu-1)}\cos^2\theta}{\omega_{Be}^2\sin^{2\nu}\theta}\right]. \quad (63)$$

The second term, which is associated with the radiation through the virtual ordinary wave, may become dominant at low frequencies, close to ω_p. In the opposite limit,

$$\omega_p\gamma\sin\theta\ll\omega\ll\omega_{Be} \quad (64)$$

the spectrum is

$$I_\omega^{(e)}=\frac{4\pi^2(\nu-1)}{\nu^2(\nu+1)}\frac{e^4q^2\langle\Delta N^2\rangle}{cm^2\omega^3}\gamma^{2\nu}\left(\frac{2k_0c}{\omega}\right)^{\nu-1}\times\frac{\omega^4}{\omega_{Be}^4\sin^4\theta}\left(1+\frac{\omega_p^4\omega_{Be}^2}{\omega^6}\cos^2\theta\sin^4\theta\right). \quad (65)$$

The second term may appear for particles whose energy is not too high, at frequencies which are low [but which satisfy (64)]. Figure 3 shows a family of extraordinary-wave spectra for $\omega_{Be}/\omega_p=300$, $\theta=\pi/4$, and various values of γ. With increasing radiation frequency and/or with increasing energy of the particle, the component of the radiation through the virtual ordinary wave decreases. This conclusion agrees with the circumstance that the mutual conversion of normal modes in a gyrotropic medium occurs most efficiently for parameter values corresponding to $\omega_p/\omega\approx 1$.

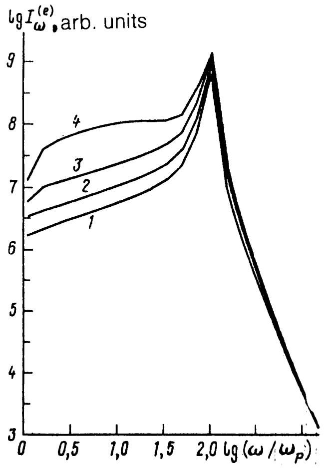

FIG. 4. Family of extraordinary-wave emission spectra for various pitch angles of the particle, for the parameter values $\gamma=25$, $\omega_{Bc}/\omega_p=100$, and $\nu=1.5$. 1—$\theta=\pi/2$; 2—$\theta=\pi/3$; 3—$\theta=\pi/4$; 4—$\theta=\pi/6$.

Figure 4 shows a family of extraordinary-wave radiation spectra for the same parameter values as in Fig. 2. For $\omega<\omega_{Be}$ the nature of the transition radiation differs sharply from that in the case of an isotropic plasma, in that the radiation intensity decreases with decreasing frequency, $I_\omega^{(e)}\propto\omega^{2-\nu}$, instead of increasing, as it does for an ordinary wave, $I_\omega^{(o)}\propto\omega^{\nu-2}$.

Figure 5 shows the polarization of the radiation for the same parameter values. In the frequency interval $\omega_p<\omega<\omega_{Be}$, the ordinary wave is predominantly radiated, while at $\omega>\omega_{Be}$ it is predominantly the extraordinary wave. These conclusions agree with the results in Sec. 3: The frequency dependence of the polarization at $\theta=\pi/2$ is quadratic, while at other angles it is linear.

Although the radiation of ordinary waves is usually

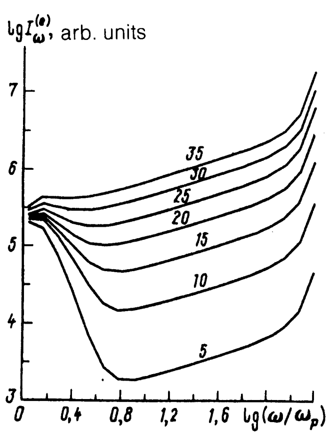

FIG. 3. Family of emission spectra of extraordinary waves for various values of the energy (the curves are labeled with the values of γ) of the relativistic particle, for the parameter values $\omega_{Bc}/\omega_p=300$, $\nu=1.5$, and $\theta=\pi/4$.

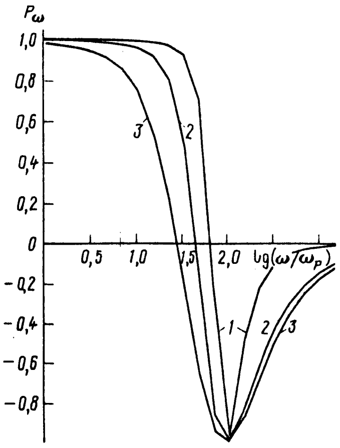

FIG. 5. Polarization of the transition radiation versus the frequency for various pitch angles of the particle. The parameter values are $\gamma=25$, $\omega_{Be}/\omega_p=100$, and $\nu=1.5$. 1—$\theta=\pi/2$; 2—$\theta=\pi/4$; 3—$\theta=\pi/6$. Values $P>0$ correspond to ordinary waves, and values $P<0$ to extraordinary waves.

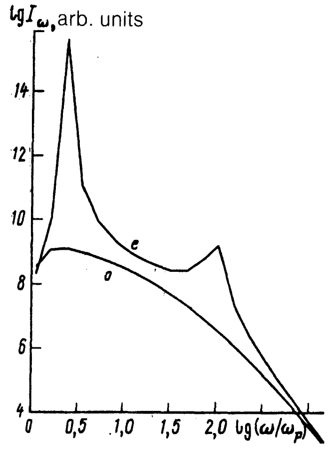

FIG. 6. Emission spectra of ordinary and extraordinary waves as the threshold for Cherenkov generation of extraordinary waves is approached. The parameter values here are $\gamma = 25$, $\omega_{Be}/\omega_p = 100$, $\nu = 1.5$, and $\theta = \pi/15$.

more efficient at $\omega < \omega_{Be}$, this may not always be the case. For example, Fig. 6 shows the radiation spectra for ordinary and extraordinary waves as the parameter values approach the Cherenkov threshold. In this case, the radiation of extraordinary waves may be several orders of magnitude more intense than the radiation of ordinary waves at certain frequencies. These curves were plotted from (56) and (57), which ignore all the factors that limit the growth of the radiation, as we mentioned in Subsection (4.1).

6. CONCLUSION

Let us briefly discuss some features which arise in an analysis of transition radiation in a magnetized plasma. These features are seen, in particular, in the divergence of the expressions under certain conditions, e.g., if the condition for Cherenkov radiation holds for the ordinary wave. The reason here is that the probability for the scattering of a Cherenkov photon over an infinite distance (in an unbounded, nonabsorbing medium) is infinite. When specific bounded media are discussed, this divergence disappears.

As the Cherenkov threshold is approached (but before the Cherenkov condition becomes satisfied), there may be an anomalous enhancement of the radiation (of extraordinary waves). This has been recognized for a long time now.[27] It stems from an increase in the coherence length as the Cherenkov threshold is approached (virtual photons become "nearly" real). Nevertheless, under actual conditions the increase in the radiation may be limited by many factors: the finite size of the basic length scale of the inhomogeneities in the plasma density, the curvature of the particle's trajectory, the slowing of the particle in the plasma, etc.

Furthermore, our expressions for the extraordinary waves diverge as $\omega_{Be}/\omega \to 1$, i.e., near the cyclotron resonance. We know[16] that the cold-plasma approximation, which we have used here, is not valid in this region. The correct description would require consideration of the thermal motion of the plasma particles (spatial dispersion). We have not considered that factor, since the description presented here becomes incorrect before the spatial-dispersion effects become important. The reason is that, as we approach the cyclotron resonance, the condition

$$|1-n_e^2| \ll 1 \qquad (66)$$

becomes violated at a certain frequency. The meaning here is that the phase velocity of the normal mode starts to become very different from the velocity of light in vacuum, c. Consequently, the distinctive features characteristic of the radiation by relativistic particles fade away; in particular, we no longer have the sharp directionality of the radiation along the direction of the particle's velocity [expression (20)]. For this reason, the distinctive features in the region $\omega \approx \omega_{Be}$ in Figs. 4 and 6 do not give a quantitative description of the radiation at these frequencies; they merely indicate that there are certain peculiarities in the spectra and that the entire analysis of the transition radiation in this region should take a different approach [in the narrow frequency $\omega_{Be}(1 + \omega_p^2 \sin^2\theta/2\omega_{Be}^2) < \omega < \omega_{Be}(1 + \omega_p^2/\omega_{Be}^2)$, extraordinary waves cannot propagate at all in a magnetized plasma]. From the practical standpoint, however, such an analysis would not seem to be particularly interesting, since at $\omega \approx \omega_{Be}$ the transition radiation would be masked by the thermal cyclotron radiation of the plasma.

Finally, we consider the range of applicability of the approximation of a rectilinear motion of the relativistic particle. As was shown in Ref. 11, transition radiation by electrons moving along a helix is strongly suppressed under the condition

$$\gamma > \omega_p/\omega_{Be} \sin\theta. \qquad (67)$$

The transition radiation by electrons is thus described by the equations of the present paper only in weakly gyrotropic plasma (Sec. 3) and in the case of quasilongitudinal motion (Subsection 4.1). For the radiation of protons and other heavy particles, the situation is more favorable. According to Refs. 11 and 14, the curvature of the particle's trajectory strongly influences the transition radiation (specifically, it suppresses this radiation) if the particle gyrates through an angle greater than the characteristic radiation-emission angle over a distance equal to the coherence length of the radiation. The condition for the applicability of the approximation of a rectilinear motion of the heavy particles can thus be written as

$$(2c/\omega)\,[\gamma^{-2}+2(1-n_j)]^{-1} < 2c(\gamma^2/\omega\omega_{Bp}^2 \sin\theta)^{1/3}, \qquad (68)$$

or, more compactly,

$$(\omega_{Bp} \sin\theta/\omega\gamma)^{2/3} < \gamma^{-2}+2(1-n_j). \qquad (69)$$

For estimates we consider the least favorable case, in which the particle is moving strictly across the magnetic field. For ordinary waves we then find the same applicability condition as before:

$$\gamma < \omega_p/\omega_{Bp}. \qquad (70)$$

For extraordinary waves we find [using (53)]

$$\omega > \omega_* = \omega_{Bp}\gamma^2. \qquad (71)$$

The curvature of the trajectory thus does not affect the radiation of extraordinary waves at any frequency if

$$\gamma < (\omega_p/\omega_{Bp})^{1/2}. \qquad (72)$$

At

$$\gamma < (\omega_{Be}/\omega_{Bp})^{1/2} = (m_p/m_e)^{1/2} \approx 43 \qquad (73)$$

the curvature of the trajectory is unimportant at frequencies $\omega_* < \omega < \omega_{Be}$, but it is seen at lower frequencies. However, the suppression of the extraordinary-wave radiation for $\omega < \omega_*$ has only a slight effect on the total energy of the transition radiation in a magnetized plasma. Assuming the parameter values typical of solar flares, we find that conditions (70)–(73) usually hold, since the energies of the protons which appear rarely exceed 20 GeV (Ref. 29; $\gamma \sim 20$), even in the most powerful flares. In the laboratory, on the other hand, one could apparently arrange various regimes for transition radiation, in particular, regimes in which the curvature of the trajectory of the heavy particles is important.

7. SUMMARY

1. The production of transition radiation by relativistic particles in a magnetized plasma with random density inhomogeneities has been analyzed.

2. It has been shown that the ordinary and extraordinary waves are generally excited in essentially different ways. As a result, highly polarized radiation is emitted.

3. As a particle moves along the magnetic field, the transition radiation is significantly damped at frequencies $\omega/\omega_{Be} < 1$, because the transverse motion of the plasma electrons is suppressed by the magnetic field.

4. In the case of transverse motion of a relativistic particle (a proton), ordinary waves are emitted considerably less efficiently than extraordinary waves (under the condition $\omega_{Be}/\omega_p \gg 1$). The transition radiation by electrons under these conditions is considerably reduced by the curvature of the trajectories of the electrons caused by the magnetic field.

5. At frequencies $\omega/\omega_{Be} > 1$, the polarization of the radiation can again be extremely high. The degree of polarization in this case depends essentially exclusively on the magnitude of the magnetic field. We thus might be looking at a method for finding an independent estimate of magnetic fields, e.g., by observing the radio emission from solar flares.

6. For motion of a particle at an arbitrary angle from the magnetic field, we find a qualitatively new effect. Scattering "through a virtual ordinary wave" can contribute significantly to the intensity of the emission of extraordinary waves. As a result, the intensity of the emission of extraordinary waves depends on the refractive indices for both normal modes. At $\theta = 0$, $\pi/2$, and π, the contribution from such processes vanishes.

The results of this analysis may find use in interpreting the radio emission generated in solar flares. However, special laboratory studies of these transition-radiation processes in a plasma in a magnetic field would also be of considerable interest in their own right.

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[1] V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **10**, 601 (1940).
[2] V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **10**, 608 (1940).
[3] A. A. Kolomenskiĭ, Zh. Eksp. Teor. Fiz. **24**, 167 (1953).
[4] A. A. Kolomenskiĭ, Dokl. Akad. Nauk SSSR **106**, 982 (1956) [Sov. Phys. Dokl. **1**, 133 (1956)].
[5] K. A. Barsukov, Zh. Eksp. Teor. Fiz. **36**, 1485 (1959) [Sov. Phys. JETP **9**, 1052 (1959)].
[6] V. L. Ginzburg, Usp. Fiz. Nauk **69**, 537 (1960) [Sov. Phys. Usp. **2**, 874 (1960)].
[7] V. A. Bazylev and N. K. Zhevago, Usp. Fiz. Nauk **160**, 47 (1990) [Sov. Phys. Usp. **33**, 1021 (1990)].
[8] B. M. Bolotovskiĭ and O. S. Mergelyan, Opt. Spectrosc. **18**, 3 (1965) [Opt. Spectrosc. (USSR) **18**, 1 (1965)].
[9] V. E. Pafomov, Zh. Eksp. Teor. Fiz. **39**, 134 (1961) [Sov. Phys. JETP **12**, 97 (1961)].
[10] É. D. Gazayan and O. S. Mergelyan, Izv. Akad. Nauk Arm. SSR **17**, 87 (1964); Dokl. Akad. Nauk Arm. SSR **38**, 143 (1964).
[11] G. D. Fleĭshman, Zh. Eksp. Teor. Fiz. **99**, 488 (1991) [Sov. Phys. JETP **72**, 272 (1991)].
[12] G. D. Fleishman and S. W. Kahler, Astrophys. J. (1991) (in press).
[13] V. V. Tamoĭkin, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **8**, 42 (1965).
[14] G. D. Fleĭshman, Usp. Fiz. Nauk **161**, 165 (1991) [Sov. Phys. Usp. **34**, 86 (1991)].
[15] V. L. Ginzburg and V. N. Tsytovich, *Transition Radiation and Transition Scattering*, Amer. Inst. Physics, New York, 1990.
[16] E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Nauka, Moscow, 1979 (Pergamon, New York, 1981).
[17] V. L. Ginzburg, *Theoretical Physics and Astrophysics*, Pergamon, Oxford, 1979 (Russ. Transl. Nauka, Moscow, 1981).
[18] V. Ya. Éĭdman, Zh. Eksp. Teor. Fiz. **34**, 131 (1958) [Sov. Phys. JETP **7**, 91 (1958)]; Zh. Eksp. Teor. Fiz. **36**, 1335 (1959) [Sov. Phys. JETP **9**, 947 (1959)].
[19] V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas*, Nauka, Moscow, 1967 (Pergamon, Oxford, 1970).
[20] V. A. Basylev and N. K. Zhevago, *Radiation by Fast Particles in Matter and in External Fields*, Nauka, Moscow, 1987.
[21] I. N. Toptygin and G. D. Fleishman, Astrophys. Space Sci. **132**, 213 (1987).
[22] A. B. Migdal, Dokl. Akad. Nauk SSSR **96**, 49 (1954).
[23] A. Pacholczyk, *Radio Astrophysics: Nonthermal Processes in Galactic and Extragalactic Sources*, W. H. Freeman, San Francisco, 1970 (Russ. transl. Mir, Moscow, 1970).
[24] V. V. Zheleznyakov, *Electromagnetic Waves in Astrophysical Plasmas*, Nauka, Moscow, 1977.
[25] A. V. Akopyan and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **72**, 1824 (1977) [Sov. Phys. JETP **45**, 957 (1977)].
[26] K. Yu. Platonov and I. N. Toptygin, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **32**, 735 (1989).
[27] S. P. Kapitsa, Zh. Eksp. Teor. Fiz. **39**, 1367 (1961) [Sov. Phys. JETP **12**, 954 (1961)].
[28] M. Abramowitz and I. A. Stegun (editors), *Handbook of Mathematical Functions*, Dover, New York, 1964 (Russ. Transl. Nauka, Moscow, 1979).
[29] T. Mathews and D. Venkatesan, Nature **345**, 600 (1990).

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