Stark effect for a two-level atom in a strong polyharmonic field

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An analytical description of the quasienergy spectrum for a two-level atom in a strong deeply modulated polyharmonic field is obtained. It is shown that with modulation close to 100% the presence of "forbidden bands" of values for the quasienergy is possible, and a criterion for the existence of such bands is proposed. The relationship of this effect to the presence of sharp singularities in the absorption spectrum of the physical system being studied is discussed.

1. Recently, in connection with applied problems in laser spectroscopy, quantum optics, and chemical physics, intensive studies have been made of the spectroscopic characteristics of a two-level atom situated in a polyharmonic field:[1–8]

$$G(t)=\operatorname{Re}\{R(t)\},\qquad R(t)=\sum_n A_n \exp\{i[(n\Delta+\Omega)t+\psi_n]\},\tag{1}$$

where Ω is the "central" frequency of the external field, ψ_n are the phases of the corresponding harmonics, and Δ is a parameter characterizing the spacing between the harmonics. The interest in this form of external field arises from the following circumstances. First, the expression (1) contains, as a particular case, a biharmonic field. Second, the field in a laser resonator has such a structure. The quasienergy spectrum of this physical system, as follows from general considerations,[9] is given by the expression

$$(E_1+E_2\pm\Omega)/2+\Delta(\nu_{1,2}+m),\quad m=0,\ \pm1,\ \pm2,\dots,\tag{2}$$

where E_1 and E_2 are the energy levels of the atom and $\nu_{1,2}$ are the Floquet indices for the dimensionless system of equations, which corresponds in this case to the Schrödinger equation.[5,10] Thus, both initial levels of the atom are split into two infinite series of quasilevels. The Stark effect for this physical system reduces to the dependence of these quasilevels on the intensity of the incident field for fixed values of the other parameters, including the form of the external field and the detuning $\varkappa=(E_2-E_1-\Omega)/2\Delta$ of the "central" frequency from the frequency of the atomic transition. In Ref. 10 the quasienergy spectrum for a strong external field was calculated. It was shown that the Stark effect can vary qualitatively, depending on the dpeth of modulation of the external field. For a large (in the appropriate asymptotic sense) detuning of the "central" frequency from the transition frequency, or for weak modulation of the external field, the Stark effect is simple: As the intensity of the external field increases the series of quasilevels for the two initial levels are displaced uniformly in opposite directions. However, for small detuning and 100% modulation the quasilevels oscillate as the intensity of the external field increases, with each series remaining in a bounded interval. "Forbidden bands" arise, into which the quasilevels cannot fall. From the results of experimental[2,4] and calculational work[1,3] it follows that it is the case of deep modulation, close to 100%, that possesses a number of interesting physical features. The results of Ref. 10 do not permit us to study the "asymptotic region" (in the space of the parameters of the problem) of an external field with 100% modulation. In the present paper, we present an analytical description of the quasienergy spectrum of our physical system in this case.

2. Let us proceed to more-precise formulations. We assume that the external field is strong, i.e., that the asymptotic representation

$$\mu R(\tau)/\hbar\Delta=\rho q(\tau)+\rho^{1/2}r(\tau)+p(\tau),\quad \rho\gg1$$

is valid, where $\tau=\Delta t$ is the dimensionless time and μ is the dipole moment of the atom. The functions $q(\tau)$, $r(\tau)$, and $p(\tau)$ are periodic, with period 2π; they, and their first and second derivatives, are quantities of order 1; this condition fixes, in an appropriate sense, the "central" frequency Ω. In these terms, deep modulation of the external field implies that $q(\tau)$ has zeros $\tau_k\,(l\leqslant k\leqslant n)$ in the period: $0<\tau_1<\tau_2<\dots<\tau_n<2\pi$; $\tau_{n+1}=\tau_1+2\pi$. The 100% modulation of the external field implies that, in addition, $r(\tau)=p(\tau)\equiv0$; this is the case that was considered in Ref. 10. It is not difficult to see that, if, for all k, $r(\tau_k)=p(\tau_k)=0$, this case reduces to the situation with $r(\tau)=p(\tau)\equiv0$. Here, we assume that, at least for one k, $|r(\tau_k)|^2+|p(\tau_k)|^2\neq0$.

Remark 1. If q, r, and p are real functions, then, by changing, only the notation, we can arrive at the case $r=p\equiv0$. We consider the general case, when the functions q, r, and p are complex-valued.

Remark 2. For a biharmonic field, when only A_0 and A_1 are nonzero, deep modulation in our terms implies that $1-A_0/A_1=o(1)$, while 100% modulation implies that $A_0=A_1$. Thus, in the given case, our terminology coincides with the usual terminology.

We assume henceforth that the detuning is small, i.e., the parameter $\varkappa$ satisfies the relation

$$\varkappa^2=a\rho,\quad a=O(1).\tag{3}$$

The situation $\varkappa=B\rho$ with $B\neq0$ was analyzed in Ref. 10. As follows from the above account, the determination of the quasienergy spectrum reduces to the calculation of the Floquet indices for the system of equations that corresponds to the Schrödinger equation.[5,10] To determine the values of the quantities $\nu_{1,2}$ we used the technique of asymptotic calculations based on the reference-equation method.[10,11] We give the asymptotic forms of the quantities $\nu_{1,2}$ in the large parameter ρ, including the terms of order unity.

With each point τ_k we associate a matrix

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$$M(k)=\begin{vmatrix} \exp[i(\theta_k+\varphi_k)]\cos\delta_k & \exp(-i\theta_k)\sin\delta_k \\ -\exp(i\theta_k)\sin\delta_k & \exp[-i(\theta_k+\varphi_k)]\cos\delta_k \end{vmatrix}.$$

Here,

$$\delta_k=\arcsin\{\exp[-\pi(a_k+\beta_k^2)/2]\},$$

$$\varphi_k=\arg\{\Gamma[1/2-i(a_k+\beta_k^2)/4]\Gamma[-i(a_k+\beta_k^2)/4](a_k^{1/2}-i\beta_k)\}-\pi/4,$$

$$\theta_k=\int_{\tau_k}^{\tau_{k+1}} d\tau\{\rho|q|+\rho^{1/2}\,\mathrm{Re}(q^*r)/|q|$$
$$+[2\mathrm{Re}(q^*p)+|r|^2-(\mathrm{Re}(q^*r)/|q|)^2]\}$$
$$+{}^1/_2(a_k+\beta_k^2)\ln[\rho^{1/2}|y(\tau,k)|]$$
$$+{}^1/_2(a_{k+1}+\beta_{k+1}^2)\ln[\rho^{1/2}|y(\tau,k+1)|]$$
$$+\int_{\tau_k}^{\tau} ds[a(2|q(s)|)^{-1}-(a_k+\beta_k^2)y'(s,k)/2y(s,k)]$$
$$+\int_{\tau}^{\tau_{k+1}} ds[a(2|q(s)|]^{-1}$$
$$+(a_{k+1}+\beta_{k+1}^2)y'(s,k+1)/2y(s,k+1)]+(\alpha_k^2+\alpha_{k+1}^2-\pi)/2,$$
$$\alpha_k+i\beta_k=r(\tau_k)\exp\{-i\arg[q'(\tau_k)]\},$$
$$y^2(\tau,k)=2\,\mathrm{sgn}(\tau-\tau_k)\int_{\tau_k}^{\tau}|q(s)|ds,\qquad a_k=a[|q'(\tau_k)|]^{-1}. \tag{4}$$

Then

$$\nu_{1,2}=\pm(2\pi)^{-1}\arccos\{{}^1/_2\mathrm{Tr}[M(1)M(2)\dots M(n)]\}.$$

3. We discuss the consequences of the formulas presented here. Above all, they make it possible to describe the Stark effect for a deeply modulated external field. We shall consider, e.g., the case $n=1$. Then

$$\nu_{1,2}=\pm(2\pi)^{-1}\arccos[\cos\delta_1\cos(\theta_1+\varphi_1)].$$

Only the quantity θ_1 depends on the intensity of the external field, i.e., on ρ, the quantities δ_1 and φ_1 are determined by the form of the external field. For $a_1+\beta_1^2>0$ the quantities $\nu_{1,2}$ oscillate as φ increases, each remaining within a bounded interval; $\nu_{1,2}$ does not take values in the intervals $(k/2-\delta_1(2\pi)^{-1}, k/2+\delta_1(2\pi)^{-1}$ $(k=0,\pm 1)$. Thus, when the modulation of the external field is close to 100% there exist "forbidden bands" for the quasilevels. The width of these bands is determined, according to (4), by the quantity $a_1+\beta_1^2$. We note that our formulas are valid to within terms $O(\rho^{-1/2})$ $[O(\rho^{-1})$, if $\beta_1=0]$. This makes it possible to propose the following criterion for the vanishing of the "forbidden bands": They drop out when a_1 and β_1 are so large that

$$\exp[-\pi(a_1+\beta_1^2)/2]=O(\rho^{-1/2})\quad(O(\rho^{-1})\ \text{при}\ \beta_1=0). \tag{5}$$

It follows from this that the presence of "forbidden bands" for the quasilevels is associated with the fulfillment of rather stringent restrictions on the parameters of the external field. Analogous conclusions also hold for $n=2,3,\dots$.

Next, the formulas obtained for the quasienergy spectrum can also be applied for a qualitative analysis of the absorption spectrum of a two-level atom in an external field. We assume, for simplicity, that the external field is biharmonic and intense, and we are interested in the dependence on A_0 (the amplitude of one of the harmonics) of the coefficient of absorption of a probe wave at a fixed frequency $\Omega+(m+\xi)\Delta$; to make the situation simple, we assume that

$$(2\pi)^{-1}\exp[-\pi a_1/2]<\xi<{}^1/_2 \tag{6}$$

and that A_1 (the amplitude of the second harmonic) is fixed. We assume that the detuning is small, i.e., condition (3) is fulfilled. Suppose that, initially $A_0\approx A_1/2$ (the modulation is weak); we let A_0 increase gradually, approaching deep modulation. The absorption coefficient has a peak when the difference of the energies of the two quasilevels corresponds to the frequency of the probe field; it follows from the condition (6) that a transition can be realized only for quasilevels moving in opposite directions. For a weakly modulated external field the corresponding quasienergy spectrum is described in Ref. 10, and, according to the results obtained there, the coefficient of absorption of the probe wave as a function of A_0 will have an approximately periodic character. The absorption maxima correspond to values of A_0 such that

$$2\nu_1=\xi+l\Delta,\quad l-\text{unity}. \tag{7}$$

Gradually increasing A_0, we reach the region of parameter values corresponding to deep modulation of the external field. As follows from our formulas, for modulaton close to 100% the relation (7) cannot be fulfilled, and there is no absorption at the frequency of the probe wave. Therefore, in the absorption spectrum we observe a dip that is asymptotically narrow, since its width is determined by the width (in the parameter A_0) of the "forbidden band." The presence of narrow dips in such situations was noted in Ref. 3. With a different choice of probe-wave frequency analogous phenomena occur, and for $\xi=1/2$ and $\xi=1$ sharp peaks can appear in the absorption spectrum. Of course, a rigorous quantitative description of these phenomena must be given within the framework of the density-matrix technique.

In practice, generally speaking, it is possible to measure how the absorption spectrum of the physical system under consideration depends on an arbitrary parameter, such as, e.g., the frequency-detuning parameter $\varkappa$. It follows from our formulas that if, upon variation of this parameter, the "forbidden bands" [determined by the relations (5)] in the space of the parameters cross, then sharp singularities will appear in the absorption spectrum.

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