Optical polarization of nuclei during stimulated recombination of atoms

D. F. Zaretskiĭ and É. A. Nersesov

I. V. Kurchatov Institute of Atomic Energy, USSR Academy of Sciences, Moscow

(Submitted 2 July 1991)
Zh. Eksp. Teor. Fiz. **101**, 35–43 (January 1992)

The phenomenon of polarization of nuclei in the process of stimulated recombination of atoms in the field of circularly polarized laser radiation is considered. This effect is considered for the case of the proton-electron beams used in the method of electron cooling. An estimate is obtained for the maximum degree of polarization of the protons on components of the hyperfine structure of the $2s$ state of the hydrogen atom.

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

Laser spectroscopy of hyperfine-structure (HFS) states is presently being applied in investigations of the properties of atoms and nuclei.[1,2] The polarization of nuclei that arises in multiphoton resonance ionization of atoms was considered in Refs. 3 and 4. In a similar formulation, the problem of the polarization of the photoelectrons that appear as a result of resonance two-photon ionization of unpolarized alkali-metal atoms was solved in Ref. 5.

In connection with experiments on electron cooling of ions in storage rings[6] it is of interest to consider the inverse process—namely, stimulated recombination of atoms (ions) with subsequent population of HFS components. Effects involving stimulated recombination of atoms in proton-electron beams were considered in Refs. 7 and 8. It was shown that under certain conditions, ensured by the method of electron cooling, the rate of stimulated recombination considerably exceeds the rate of spontaneous recombination. In view of this, it is natural to expect that in the case of stimulated recombination in the field of a circularly polarized wave it will be possible to observe effects involving the optical polarization of nuclei.

The process of the polarization of the nuclei occurs in two stages. In the initial state there are unpolarized proton and electron beams, propagating in the cooling part with equal average velocities. We shall consider the situation that arises at the end of the cooling, when, in the co-moving reference frame, the temperature of the protons in all their degrees of freedom is comparable to the longitudinal temperature of the electrons.[6]

Parallel to these beams (along the z axis) is transmitted a laser wave, circularly polarized in the transverse (xy) plane and tuned to resonance with certain free-to-bound transitions. As a result of stimulated recombination in the field of this wave, selective population of certain states of an intermediate level of the bound system occurs. In particular, the role of this level could be played by the fine-structure components $3p_{3/2}$ and $3p_{1/2}$ of the hydrogen atom.

In the case when one wave is used, as a result of spontaneous transitions from the states $3p_{3/2,1/2}$ the HFS components of the $2s$ and $1s$ states of the atom become populated. The rate of these transitions is determined by the natural width γ of the $3p$ states, but the degree of polarization of the nuclei in the final states is found to be negligibly small. It is proportional to the factor $\Delta\varepsilon_{\rm HFS}/\Delta E_{nm}$, where $\Delta\varepsilon_{\rm HFS}$ is the energy of the hyperfine splitting of the appropriate states and ΔE_{nm} is the energy of the transition from the intermediate to the final state. As will be shown below, in the case of stimulated transitions, from the $3p_{3/2,1/2}$ states to components of the HFS of the metastable level $2s_{1/2}$ in the field of a second resonance wave, the degree of polarization turns out to be much greater and reaches values ~ 1.

The degree of polarization of the nuclei can be calculated for three variants, differing in the type of polarization of the second wave and in the geometry of the experiment: a) a laser wave with linear (along the z axis) polarization and with propagation direction transverse to the beams; b) a wave with circular polarization in the xy plane, propagating along the beams (along the z axis); c) a wave with linear polarization, transmitted along the beams.

In principle, case (c) does not differ in any way from case (b), since linear polarization in the plane perpendicular to the z axis can be described by a superposition of two waves with circular polarizations in which the directions of rotation of the vector **E** in the waves are opposite. The difference then reduces entirely to the appropriate recalculation of the intensities of the waves.

We note that variant (a) is the least effective from the point of view of the rate of recombination of the atoms into the final states, since for this variant the volume of the region of interaction of the beams with the waves turns out to be substantially smaller. In addition, in view of the relatively high transverse temperature of the electrons in the beam, the Doppler shift $\Gamma_{D\perp}$ of the radiation in the directions perpendicular to the z axis is considerably greater than the natural width γ of the $3p$ level. For these reasons, we shall confine ourselves henceforth to analyzing the case of two waves propagating along the beams [variant (b)].

Figure 1 shows the scheme of the transitions in the field of two resonance waves, leading to stimulated recombination to intermediate levels of the bound system with subsequent stimulated selective population of HFS components. The numerical values of the parameters of the hydrogen-atom states are taken from Ref. 9.

2. THE BASIC EQUATIONS

In the center-of-mass frame of the recombining particles we shall describe their interaction with the first wave by the operator (the wave propagates along the z axis and is circularly polarized in the xy plane; $\hbar = c = 1$):

$$V^{(1)}(t)=\frac{eE_{01}}{2}\left(\frac{8\pi}{3}\right)^{1/2} rY_{1\pm1}(\theta,\varphi)\exp(i\omega_1 t)+\text{c.c.}\,, \quad (1)$$

where E_{01} is the amplitude of the electric-field intensity and

 0038-5646/92/010018-05$05.00

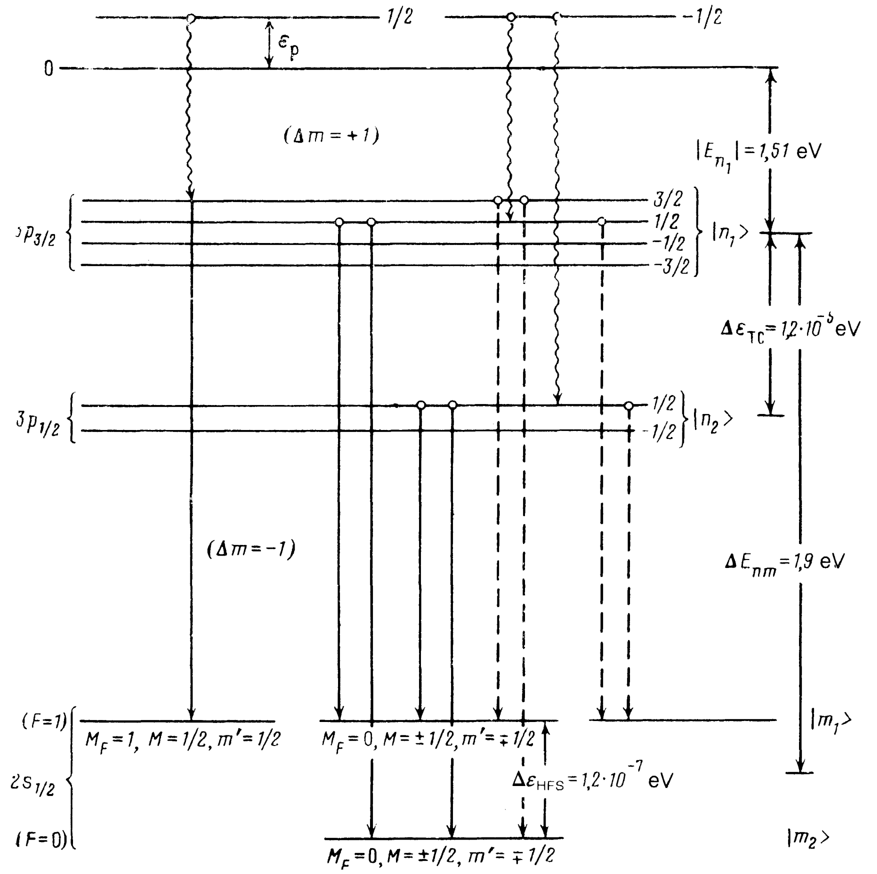

FIG. 1. The wavy lines correspond to transitions that lead to recombination of the hydrogen atom to the states $3p_{3/2,1/2}$ ($\Delta m = +1$); the solid lines show transitions that populate components of the HFS with nuclear-spin projection $M = 1/2$; the dashed lines show transitions to components of the HFS with $M = -1/2$ ($\Delta m = -1$); the quantities F and M_F specify the values of the total angular momentum of the atom and its projection along the quantization axis (the z axis); the quantities $\Delta\varepsilon_{\rm FS}$ and $\Delta\varepsilon_{\rm HFS}$ represent the energies of the fine and hyperfine splittings, respectively, of the levels.

ω_1 is the frequency of the wave (for definiteness, we later confine ourselves to the case of a wave with right polarization; see Fig. 1).

With regard to the second wave, which leads to induced $3p \to 2s$ transitions, we shall assume that it is quasimonochromatic, with frequency width $\Delta\omega_2$ satisfying the condition $\Delta\omega_2 \gtrsim \Delta\varepsilon_{\rm HFS}$. This inequality originates from the fact that, for typical beam parameters in the method of electron cooling, the Doppler width $\Gamma_{D\parallel}$ of the $3p$ level for emissions in the direction along the beams attains the magnitude of the energy interval $\Delta\varepsilon_{\rm HFS}$ between the components $F = 1$ and $F = 0$ of the HFS of the $2s$ level. Thus, the condition $\Delta\omega_2 \gtrsim \Delta\varepsilon_{\rm HFS} \sim \Gamma_{D\parallel}$ makes it possible to use the recombined atoms more effectively during the subsequent polarization of the nuclei.

At first sight, it might appear admissible to have a degree of nonmonochromaticity of the wave such that $\Delta\omega_2 > \Delta\varepsilon_{\rm FS}$, where $\Delta\varepsilon_{\rm FS}$ is the energy of the fine splitting of the components $3p_{3/2}$ and $3p_{1/2}$. In this case, both components of the fine structure of the $3p$ level would participate, on an equal basis, in the stimulated population of the HFS components of the $2s$ level. However, as will be shown below, the resonance condition for recombination of atoms leads to the result that this process effectively proceeds only through one component of the fine structure of the intermediate level (under the condition that for the first wave the spectral width $\Delta\omega_1 < \Delta\varepsilon_{\rm FS}$).

Certain restrictions, associated with the condition for optimization of the effects to be observed, are also imposed on the intensities of the waves used. It is necessary that the ionization width Γ_i and field width Γ_f of the $3p$ level in the resonance waves be of the order of the total width $\Gamma_n = \gamma_n + \Gamma_{D\parallel}$ of the level.

We shall describe the interaction of an atom with the second wave, which we shall assume to be circularly polarized, by the operator

$$V^{(2)}(t) = 2\pi^2 e\left(\frac{2}{V\omega_2^2} I_{\mathbf{n},\omega_2}\right)^{1/2}\left(\frac{8\pi}{3}\right)^{1/2} r Y_{1\mp1}(\theta,\varphi)\exp(i\omega_2 t) + \text{c.c.}, \tag{2}$$

where $I_{\mathbf{n},\omega_2}$ is the angular spectral density of the radiation intensity in the wave, normalized by the condition

$$\int\int I_{\mathbf{n},\omega_2}\, d\omega_2\, d\Omega_{\mathbf{n}} = I_0$$

(I_0 is the total intensity of the second wave); V is the normalization volume. We draw attention to the fact that, since we henceforth confine ourselves to an s state of the incident electrons in the continuous spectrum, and the final state of the recombined atom is $2s_{1/2}$, the field-intensity vectors in the first and second waves rotate in opposite directions. We note also that the second wave can be linearly polarized along any direction lying in the plane perpendicular to the z axis.

To solve the problem of the behavior of the system in the fields of two resonance waves it is appropriate to use the method of Heitler,[10] which leads to the following system of equations for the Fourier transforms of the amplitudes of the individual states:

$$(E-E_0)C_p(E)=1+\sum_{n_1,\mathbf{k}} V^{(1)}_{p/n_1}\tilde{C}_{n_1}(E)C_p(E)\zeta(E-E_1)$$

$$+\sum_{n_2,\mathbf{k}'} V^{(1)}_{p/n_2}\tilde{C}_{n_2}(E)C_p(E)\zeta(E-E_2),$$

$$(E-E_1+{}^1/_2 i\Gamma_{n_1}+{}^1/_2 i\Gamma_{in_1})\tilde{C}_{n_1}(E)\zeta(E-E_1)=V^{(1)*}_{n_1/p}$$

$$+\sum_{m_1,\mathbf{k}_1} V^{(2)}_{n_1/m_1\mathbf{k}_1}\tilde{C}_{m_1\mathbf{k}_1}(E)\zeta(E-E_3)$$

$$+\sum_{m_2,\mathbf{k}_1'} V^{(2)}_{n_1/m_2\mathbf{k}_1'}\tilde{C}_{m_2\mathbf{k}_1'}(E)\zeta(E-E_4),$$

$$(E-E_2+{}^1/_2 i\Gamma_{n_2}+{}^1/_2 i\Gamma_{in_2})\tilde{C}_{n_2}(E)\zeta(E-E_2)$$

$$=V^{(1)*}_{n_2/p}+\sum_{m_1,\mathbf{k}_2} V^{(2)}_{n_2/m_1\mathbf{k}_2}\tilde{C}_{m_1\mathbf{k}_2}(E)\zeta(E-E_5)$$

$$+\sum_{m_2,\mathbf{k}_2'} V^{(2)}_{n_2/m_2\mathbf{k}_2'}\tilde{C}_{m_2\mathbf{k}_2'}(E)\zeta(E-E_6), \qquad (3)$$

$$\tilde{C}_{m_1\mathbf{k}_1}(E)=V^{(2)*}_{m_1\mathbf{k}_1/n_1}\tilde{C}_{n_1}(E)\zeta(E-E_1),$$

$$\tilde{C}_{m_2\mathbf{k}_1'}(E)=V^{(2)*}_{m_2\mathbf{k}_1'/n_1}\tilde{C}_{n_1}(E)\zeta(E-E_1),$$

$$\tilde{C}_{m_1\mathbf{k}_2}(E)=V^{(2)*}_{m_1\mathbf{k}_2/n_2}\tilde{C}_{n_2}(E)\zeta(E-E_2),$$

$$\tilde{C}_{m_2\mathbf{k}_2'}(E)=V^{(2)*}_{m_2\mathbf{k}_2'/n_2}\tilde{C}_{n_2}(E)\zeta(E-E_2).$$

In Eqs. (3) we have used the following notation: $C_p(t)$ is the amplitude of the system in the continuous spectrum with energy $E_0=\varepsilon_p$ [the initial state, with incident-electron energy ε_p and initial condition $C_p(0)=1$]; $\tilde{C}_{n_1}$ and $\tilde{C}_{n_2}$ are the amplitudes of the intermediate bound states of the system (with energies $E_1=E_{n_1}+\omega_{\mathbf{k}}$ and $E_2=E_{n_2}+\omega_{\mathbf{k}'}$, respectively) that are formed from the initial state under the action of the perturbation $V^{(1)}$ (see Fig. 1); $\tilde{C}_{m_1\mathbf{k}_1}$ and $\tilde{C}_{m_2\mathbf{k}_1'}$ are the amplitudes of the final states (with energies $E_3=E_{m_1}+\omega_{\mathbf{k}}+\omega_{\mathbf{k}_1}$ and $E_4=E_{m_2}+\omega_{\mathbf{k}}+\omega_{\mathbf{k}_1'}$, respectively) corresponding to the HFS components of the $2s_{1/2}$ level (the amplitudes $\tilde{C}_{m_1\mathbf{k}_2}$ and $\tilde{C}_{m_2\mathbf{k}_2'}$ and energies E_5 and E_6 have an analogous meaning); $V^{(1)}_{p/n_1}$ and $V^{(1)}_{p/n_2}$ are the matrix elements for the stimulated recombination from the s state of the continuous spectrum to the states $3p_{3/2}$ and $3p_{1/2}$, respectively; $V^{(2)}_{n_1/m_1\mathbf{k}_1}$ and $V^{(2)}_{n_1/m_2\mathbf{k}_1'}$ are the matrix elements for the stimulated transitions of the atom in the field of the second wave to the HFS components of the $2s_{1/2}$ level; Γ_{n_1} and Γ_{n_2} are the total widths of the $3p_{3/2}$ and $3p_{1/2}$ levels, including the natural and Doppler widths; Γ_{in_1} and Γ_{in_2} are the photoionization widths of the corresponding levels in the field of the first wave; $\zeta(x)=\mathscr{P}/x-i\pi\delta(x)$. The system of equations (3) was obtained in the resonance approximation, which assumes fulfillment of the standard conditions: The total widths of the states taking part in the transition, and also the frequency detunings of the waves from resonance, are smaller than the energy distances to the nearest levels and wave frequencies.

The probabilities of transitions of the system to the different final states are determined by the values of the amplitudes of the corresponding individual states.[10] Solution of the system of equations (3) leads to the following expressions for the probabilities of a transition in unit time to states in which the nuclear spin has a specified projection M along the quantization axis (the z axis):[11,12]

$$w\left(M=\frac{1}{2}\right)$$

$$=(2\pi)^2\frac{1}{3}\alpha\,|\langle 2s|r|3p\rangle|^2\cdot\left\{\frac{|V^{(1)}_{p/n_1}|^2}{(\varepsilon-\varepsilon_{01})^2+\tilde{\Gamma}^2_{n_1}/4}\cdot\right.$$

$$\times\left[\left(-2^{1/2}+\frac{1}{3}\right)^2 I_{\omega_{20}}+\frac{1}{9}I_{\omega_{20}'}\right]$$

$$\left.+\frac{4}{9}\frac{|V^{(1)}_{p/n_2}|^2}{(\varepsilon-\varepsilon_{02})^2+\tilde{\Gamma}^2_{n_2}/4}(I_{\tilde{\omega}_{20}}+I_{\tilde{\omega}_{20}'})\right\}, \qquad (4)$$

$$w\left(M=-\frac{1}{2}\right)$$

$$=(2\pi)^2\frac{1}{3}\alpha\,|\langle 2s|r|3p\rangle|^2\left\{\frac{|V^{(1)}_{p/n_1}|^2}{(\varepsilon-\varepsilon_{01})^2+\tilde{\Gamma}^2_{n_1}/4}\right.$$

$$\times\left[\left(-1+\frac{2^{1/2}}{3}\right)^2 I_{\omega_{20}}+I_{\omega_{20}'}\right]$$

$$\left.+\frac{8}{9}\frac{|V^{(1)}_{p/n_2}|^2}{(\varepsilon-\varepsilon_{02})^2+\tilde{\Gamma}^2_{n_2}/4}I_{\tilde{\omega}_{20}}\right\}, \qquad (5)$$

where $\varepsilon\equiv\varepsilon_p$; $\varepsilon_{01}=\omega_1-|E_{n_1}|$ and $\varepsilon_{02}=\omega_1-|E_{n_2}|$ are parameters specifying the amount by which the energy ω_1 of a quantum of the first wave exceeds the threshold for ionization from the corresponding level; $\langle 2s|r|3p\rangle$ is the radial matrix element of the $3p\to 2s$ transition; $I_{\omega_{20}}$ and $I_{\tilde{\omega}_{20}}$ are the spectral densities of the second wave, taken at the frequencies $\omega_{20}=\varepsilon-E_{m_1}-\omega_1\approx\Delta E_{n_1m_1}$ [the transition $3p_{3/2}\to 2s_{1/2}$ $(F=1)$] and $\tilde{\omega}_{20}=\varepsilon-E_{m_1}-\omega_1\approx\Delta E_{n_2m_2}$ [the transition $3p_{1/2}\to 2s_{1/2}$ $(F=1)$]; $\omega_{20}'\approx\omega_{20}+\Delta\varepsilon_{\text{HFS}}$ and $\tilde{\omega}_{20}'\approx\tilde{\omega}_{20}+\Delta\varepsilon_{\text{HFS}}$; $\tilde{\Gamma}_{n_1}$ and $\tilde{\Gamma}_{n_2}$ are the total widths of the $3p_{3/2}$ and $3p_{1/2}$ levels, including the natural width γ_n, the Doppler width $\Gamma_{D\parallel}$, the photoionization width Γ_i in the field of the first wave, and the field width Γ_f in the field of the second wave: $\tilde{\Gamma}_n=\gamma_n+\Gamma_{D\parallel}+\Gamma_i+\Gamma_f$; $\alpha=e^2/\hbar c$ is the fine-structure constant. The expressions (4) and (5) are written under the assumption that $\Delta\varepsilon_{\text{HFS}}\lesssim\Delta\omega_2$, and the interference of the amplitudes of the corresponding transitions has been taken into account in them.

The formulas (4) and (5) determine the difference of the probabilities of population of $2s_{1/2}$ states with different projections M of the nuclear spin:

$$\Delta w\equiv w(M={}^1/_2)-w(M=-{}^1/_2)$$

$$=-{}^8/_{27}(2\pi)^2\alpha\,|\langle 2s|r|3p\rangle|^2\,\Delta\varepsilon_{\text{HFS}}\cdot$$

$$\times\left[\frac{|V^{(1)}_{p/n_1}|^2}{(\varepsilon-\varepsilon_{01})^2+\tilde{\Gamma}^2_{n_1}/4}\left.\frac{dI}{d\omega_2}\right|_{\omega_{20}}\right.$$

$$\left.-\frac{1}{2}\frac{|V^{(1)}_{p/n_2}|^2}{(\varepsilon-\varepsilon_{02})^2+\tilde{\Gamma}^2_{n_2}/4}\left.\frac{dI}{d\omega_2}\right|_{\tilde{\omega}_{20}}\right]. \qquad (6)$$

As we should expect, when the condition $\Delta\omega_2\gtrsim\Delta\varepsilon_{\text{HFS}}$

is fulfilled, the probability difference (6) has turned out to be proportional to the energy $\Delta\omega_{\rm HFS}$ of the hyperfine splitting of the $F=1$ and $F=0$ components of the total angular momentum of the $2s_{1/2}$ state of the atom and is determined by the values of the derivatives of the spectral density of the radiation in the second wave at the frequencies of the transitions $3p_{3/2}\to 2s_{1/2}$ $(F=1)$ and $3p_{1/2}\to 2s_{1/2}$ $(F=1)$.

We define the degree of polarization of the nuclei of the recombined atoms by the expression

$$P=\frac{\Delta w}{w(M={}^1/_2)+w(M=-{}^1/_2)}, \qquad (7)$$

where the denominator is the total probability of formation of hydrogen atoms in all the HFS components of the $2s_{1/2}$ state.

If in all the expressions (4), (5), and (6) for the transition probabilities we retain only the terms corresponding to transitions via the intermediate level $3p_{3/2}$, it follows from the definition (7) that

$$P\approx 0{,}34\,\frac{\Delta\varepsilon_{\rm HFS}(dI/d\omega_2)_{\omega_{20}}}{I_{\omega_{20}}}. \qquad (8)$$

We shall describe the frequency spectrum of the second wave by the model dependence

$$I_{\omega_2}=I_0\frac{\Delta\omega_2}{2\pi}\frac{1}{(\omega_2-\omega_0)^2+\Delta\omega_2{}^2/4}, \qquad (9)$$

where $\Delta\omega_2$ is the half-width of the distribution function and ω_0 is the frequency at the maximum. For an upper bound on the degree of polarization of the nuclei we obtain

$$P_{max}\approx\frac{\Delta\varepsilon_{\rm HFS}}{\Delta\omega_2}, \qquad (10)$$

and, thus, in the framework of the approximations used, this quantity is not small.

The effectiveness of the method proposed in this paper for optical polarization of nuclei in beams is determined by the absolute rate of recombination of the atoms. For a correct estimate of this rate it is necessary to take into account the energy spread of the colliding particles, and also the pulsed character of the operation of the laser and storage ring. An expression for the matrix element for stimulated recombination from an s state of the continuous spectrum of the incident electron to a $3p$ state of the hydrogen atom was obtained in Ref. 11. We shall confine ourselves here to the limiting case of this expression when ε_{01}, $\varepsilon_{02}\ll{\rm Ry}$ $({\rm Ry}=m_e e^4/2\hbar^2=13.6\ {\rm eV})$:

$$|V^{(1)}_{p/n}|^2\approx(2\pi)^2\left(\frac{32\cdot 9}{e^6}\right)^2(eE_{01}a_0)^2\frac{|C|^2a_0{}^3}{4}\left(\frac{\rm Ry}{\varepsilon}\right)^{1/2}, \qquad (11)$$

where C is the normalization coefficient of the ψ function of the electron in the continuous spectrum: $a_0=\hbar^2/m_e e^2$ is the first Bohr radius of the hydrogen atom; in the denominator of the numerical factor, $e=2.718...$.

At the end of the cooling process the spectrum of the incident electrons in the center-of-mass frame of the colliding particles is given by the "flat" Maxwell distribution

$$F(\varepsilon_\parallel,\varepsilon_\perp)=\frac{1}{(\pi kT_\parallel)^{1/2}}\frac{1}{2kT_\perp}\exp\left(-\frac{\varepsilon_\parallel}{kT_\parallel}\right)\exp\left(-\frac{\varepsilon_\perp}{kT_\perp}\right)\varepsilon_\parallel^{-1/2}, \qquad (12)$$

where the spread of the transverse velocities of the electrons is determined by the transverse temperature $T_\perp\sim T_c$ (T_c is the temperature of the electron-beam cathode); the spread of the longitudinal velocities as a result of the potential acceleration of the electrons is substantially smaller and is characterized by a temperature $T_\parallel\approx T_c/(kT_c/4E_e)$, where E_e is the kinetic energy of the electrons in the laboratory frame.

The rate of recombination of atoms, averaged over the distribution (12), is determined by the integral

$$\left\langle\frac{dN}{dt}\right\rangle\propto\iint\frac{|V^{(1)}_{p/n_1}|^2}{(\varepsilon-\varepsilon_{01})^2+\Gamma_{n_1}{}^2/4}I_{\omega_{20}}(\varepsilon)F(\varepsilon_\parallel,\varepsilon_\perp)\,d\varepsilon_\perp\,d\varepsilon_\parallel, \qquad (13)$$

in which the integrand can be represented by a product of three delta-like functions of the incident-electron energy $\varepsilon=\varepsilon_\parallel+\varepsilon_\perp$. The value of the integral depends on which of these functions is the sharpest. In a real situation, for typical parameters of the electron-cooling method, the conditions $kT_\perp\gg\Delta\omega_2>kT_\parallel>\Gamma_n$ are realized. For the averaged rate of recombination of atoms into $2s_{1/2}$ states with a particular projection M of the nuclear spin we obtain (in the usual units)

$$\left\langle\frac{dN}{dt}\right\rangle=\frac{(2\pi)^3}{\hbar}\frac{1}{3}\left(\frac{3{,}1\cdot 24}{e^6}\right)^2\frac{(eE_{01}a_0)^2(eE_{02}a_0)^3}{kT_\perp\Gamma_n\hbar\Delta\omega_2}$$
$$\times n_e n_p V\left(\frac{l}{L}\right)\left(\frac{\rm Ry}{\pi kT_\parallel}\right)^{1/2}\left[5\left(\frac{\pi kT_\parallel}{\varepsilon_{01}}\right)^{1/2}\right.$$
$$\times\exp\left(-\frac{\varepsilon_{01}}{kT_\parallel}\right)\Phi\left(\left[\frac{\varepsilon_{01}}{kT_\parallel}\right]^{1/2}\right)$$
$$\left.+4\left(\frac{\pi kT_\parallel}{\varepsilon_{02}}\right)^{1/2}\exp\left(-\frac{\varepsilon_{02}}{kT_\parallel}\right)\Phi\left(\left[\frac{\varepsilon_{02}}{kT_\parallel}\right]^{1/2}\right)\right]\frac{1}{Q}. \qquad (14)$$

In (14) we have used the following notation: n_e and n_p are the concentrations of electrons and protons in the beams; V is the volume of the region of intersection of the waves with the beams; l is the length of that part of the storage ring in which cooling of the protons occurs; L is the perimeter of the storage ring; $\Phi(x)$ is the probability integral; $Q=T/\tau$ is the off-duty factor of the pulsed laser creating the first wave; E_{02} is the amplitude of the field intensity in the second wave.

We draw attention to a number of features of the expression (14):

1) The quantity $\langle dN/dt\rangle$ is proportional to the volume V of the region of interaction of the beams with the waves, and for this reason a scheme with parallel transmission of both waves along the beams is preferable.

2) The optimal conditions for observation of the effects under consideration arise when the ionization width and field width of the intermediate level $3p$ are of the order of the width Γ_n: Γ_i, $\Gamma_f\sim\Gamma_n=\gamma+\Gamma_{D\parallel}$. This imposes upper limits on the admissible values of the electric-field intensities in the waves (for estimates, see below).

3) The condition for resonance of the stimulated recombination requires that the amount by which the energy $\hbar\omega_1$ of a quantum exceeds the threshold for ionization from the intermediate level be very small: ε_{01}, $\varepsilon_{02}\lesssim kT$. Otherwise, the effect is found to be exponentially small. But since, in a real situation, $\Delta\varepsilon_{\rm FS}\gg kT_\parallel$, only one term operates effectively in the square brackets in Eq. (14). In other words, the recombination of atoms proceeds in practice via one particu-

lar component of the fine structure of the $3p$ level (under the condition $\Delta\omega_1 \ll \Delta\varepsilon_{FS}$).

We shall formulate the conditions and obtain estimates for the admissible values of the field intensities in the waves. An expression for the photoionization width of the $3p$ level was found in Ref. 11. In ionization to the edge of the absorption band ($\varepsilon_0/kT_\parallel \to 0$) the formula for Γ_i takes the form

$$\Gamma_i = \frac{\pi (eE_{01})^2 c a_0^5 m_e c^2 (2m_e c^2 \text{ Ry})^{1/2}}{(\hbar c)^4}.$$

The condition $\Gamma_n \gtrsim \Gamma_i$ can be formulated in the form of the inequality

$$eE_{01} \lesssim (\hbar c)^2 \left[\frac{\Gamma_n}{\pi c a_0^5 m_e c^2 (2m_e c^2 \text{ Ry})^{1/2}}\right]^{1/2}. \tag{15}$$

Hence, $E_{01} \lesssim 2\times 10^5$ V/cm (the tabulated value of γ is $\gamma = 30$ MHz, and the Doppler width corresponding to the longitudinal temperature $kT_\parallel \sim 10^{-6}$ eV is estimated as $\Gamma_{D\parallel} \approx 1.4\times 10^{-7}$ eV). Such field intensities can be achieved by means of pulsed lasers with tunable frequency.

The field width of the intermediate level $3p$ in the field of the second wave is given, to within a numerical factor of order unity, by the formula

$$\Gamma_f = \frac{(2\pi)^2}{\hbar} \alpha a_0^2 I_{\omega_{20}} \sim \frac{(2\pi)^2}{\hbar} \alpha a_0^2 \frac{I_0}{\Delta\omega_2}. \tag{16}$$

Here the condition $\Gamma_n \gtrsim \Gamma_f$ acquires the form

$$eE_{02} \lesssim \left(\frac{\hbar\Gamma_n \hbar\Delta\omega_2}{\pi a_0^2}\right)^{1/2}. \tag{17}$$

If for the characteristic spectral width $\Delta\omega_2$ in the second wave we take $\Delta\omega_2 \sim \Delta\varepsilon_{HFS}$, then from (17) we obtain the bound $E_{02} \lesssim 30$ V/cm.

3. ESTIMATES. CONCLUSION

In this section we obtain numerical estimates for the effects considered in the paper, using data on electron cooling of protons from Ref. 6. When the electron energy is $E_e = 50$ keV, the energy of protons with the same average velocity as the electrons is $E_p = E_e(M/m_e) = 100$ MeV. In the co-moving reference frame, the spread of the transverse velocities of the electrons is determined by the temperature $kT_\perp \approx kT_c \approx 0.2$ eV, and the spread of the longitudinal velocities is determined by the temperature $kT_\parallel \approx 10^{-6}$ eV.

The length l of the region in which the cooling of the protons occurs is $l \approx 2$ m, and the length of the storage ring is $L \approx 200$ m; the volume of the region of interaction of the beams with the waves is $V = (\pi d^2/4) l \approx 160$ cm^3 ($d = 1$ cm is the diameter of the cross section of the first wave); $n_e = n_p \approx 10^8$ cm^{-3}.

For the estimate, in (14) we set $\hbar\Delta\omega_2 \approx \Delta\varepsilon_{HFS} \approx 1.2\times 10^{-7}$ eV, $\tilde{\Gamma}_n \approx \Gamma_n \approx 1.4\times 10^{-7}$ eV, $E_{02} = 30$ V/cm, and $E_{01} = 2\times 10^5$ V/cm (when the diameter of the light wave emerging from the laser is $d \approx 1$ cm and the pulse duration is $\tau \approx 10^{-8}$ sec, this gives a pulse energy $W \approx 0.3$ J). When the laser has pulse frequency $\nu = 10$ kHz we obtain an off-duty factor $Q = 10^4$.

Retaining for the estimate just the first term in (14), in which we set $\varepsilon_{01} \approx 0$, we obtain $\langle dN/dt \rangle \approx 10^2$ sec^{-1}. This estimate has been found under the assumption that the proton beam continuously enters the region of electron cooling. If the size of a proton bunch is of the order of the length of the cooling region, this estimate can be increased by two orders of magnitude.

The proposed scheme of optical polarization gives us the possibility of populating the metastable $2s_{1/2}$ level of the hydrogen atom. Subsequent ionization of this state makes it possible to obtain a polarized proton beam. The estimates found in the paper for the degree of polarization of the nuclei are also valid, of course, in the case when the first wave is linearly polarized in the plane perpendicular to the z axis and the second wave is circularly polarized in the xy plane.

[1] E. W. Otten, Laser Spectroscopy of Radioactive Beams, in *Proceedings of the International Seminar on Heavy-Ion Physics* [in Russian], Alushta, 14–21 April 1983 (JINR, Dubna, 1983).
[2] E. W. Otten, Nucl. Phys. **A354**, 471 (1981).
[3] N. B. Delone, B. A. Zon, and M. V. Fedorov, Pis'ma Zh. Tekh. Fiz. **4**, 229 (1978) [Sov. Tech. Phys. Lett. **4**, 94 (1978)].
[4] N. B. Delone, B. A. Zon, and M. B. Fedorov, Zh. Eksp. Teor. Fiz. **76**, 505 (1979) [Sov. Phys. JETP **49**, 255 (1979)].
[5] A. I. Andryushin and A. E. Kazakov, Kvantovaya Elektron. (Moscow) **7**, 2371 (1980) [Sov. J. Quantum Electron. **10**, 1381 (1980)].
[6] A. N. Skrinskiĭ and V. V. Parkhomchuk, Fiz. Elem. Chastits At. Yadra **12**, 557 (1981) [Sov. J. Part. Nucl. **12**, 223 (1981)].
[7] D. F. Zaretskiĭ, A. V. Kozlinskiĭ, and V. V. Lomonosov, Kvantovaya Elektron. (Moscow) **9**, 478 (1982) [Sov. J. Quantum Electron. **12**, 283 (1982)].
[8] D. F. Zaretsky and E. A. Nersesov, Phys. Lett. **147A**, 304 (1990).
[9] A. A. Radtsig and B. M. Smirnov, *Handbook of Parameters of Atoms and Atomic Ions* [in Russian] (Énergoatomizdat, Moscow, 1986).
[10] W. Heitler, *The Quantum Theory of Radiation*, 3rd ed. (Clarendon, Oxford, 1954) [Russ. transl., IL, Moscow, 1956].
[11] D. F. Zaretskiĭ and E. A. Nersesov, Preprint 093-88, Moscow Engineering-Physics Institute (1988).
[12] D. F. Zaretskiĭ, V. V. Lomonosov, and V. A. Lyul'ka, Zh. Eksp. Teor. Fiz. **77**, 867 (1979) [Sov. Phys. JETP **50**, 437 (1979)].

Translated by P. J. Shepherd