Explosive growth of the tearing mode in a plasma slab with ruptured magnetic surfaces

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A dispersion relation is derived for the drift-tearing mode in a plane plasma slab for the case in which the magnetic fields of the excited modes cause a diffusion of magnetic field lines across ruptured magnetic surfaces. The growth rate of the instability increases with the magnetic-field diffusion coefficient. As a result, there is explosive growth of the mode.

1. INTRODUCTION

Effective plasma confinement in laboratory magnetic confinement devices and also in natural magnetoplasma formations observed in space is often achieved due to magnetic shear. This shear makes the plasma stable with respect to large-scale MHD instabilities. On the other hand, shearing of the magnetic field lines is associated with the flow of electric current along the magnetic field lines, so there is danger that the current in such configurations can develop a pinch instability. This instability has come to be known as the "disruptive" or "tearing-mode" instability.[1,2] The change which occurs in the topology of the magnetic field due to the pinch effect is a consequence of the breaking of magnetic field lines and their reconnection around the current involved in the pinch. The reconnection of magnetic field lines in the course of the tearing-mode instability results in heating of the plasma and acceleration of charged particles, at the cost of some dissipation of magnetic field energy. For this reason, the tearing-mode instability is invoked not only to explain the plasma dynamics in tokamaks[3] (the most promising confinement devices for fusion plasmas) but also to construct theoretical models for plasma heating in the loop-shaped magnetic structures of the solar corona[4,5] and for the transport of particles and energy of the solar wind in the magnetospheres of the earth and other planets.[6]

Of fundamental importance to all these problems is the quantitative description of the macroscopic consequences of the onset of the tearing-mode instability, e.g., the plasma heating, the transport of particles and heat, the acceleration of particles, and the restructuring of the magnetic field. Finding such a description requires in turn a description of the nonlinear stage of the instability, including the various features of the initial magnetic field configuration. In particular, in a magnetic field with sheared field lines the electromagnetic tearing mode is linearly coupled with the drift mode of an inhomogeneous plasma.[7,8] Two important effects arise as a result; they should be taken into consideration in the derivation of a nonlinear theory for this drift-tearing mode. First, if the shear of the magnetic field lines is sufficiently pronounced, the tearing mode stabilizes, as the result of an expenditure of energy on drift-mode excitations. Second, the longitudinal electric field (i.e., that along the magnetic field) of the drift-tearing mode is nonzero in a spatially bounded region where the interaction with plasma particles occurs. The reason is that outside this region the conductivity of the hot, collisionless plasma is very high, and the solenoidal electric field of the tearing mode is canceled by the electrostatic field of the drift mode, as the result of a free flow of charge.

In the derivation of a nonlinear theory for the drift-tearing mode, the greatest progress has been achieved for the magnetic field configuration which prevails in tokamak toroidal magnetic confinement devices. The reason is the requirement that the perturbations be periodic as functions of the toroidal and poloidal angles. When the finite size of the Larmor radius of the particles in comparison with the major and minor radii of the torus is taken into account, it turns out that the drift-tearing mode can be excited only for a finite set of discrete values of the toroidal and poloidal components of the wave vector. The resonant Čerenkov interaction of a mode with plasma particles occurs only near discretely spaced resonant surfaces, on which the phase velocity of the mode along the magnetic field is comparable to the particle thermal velocities. As long as the magnetic islands which form near the currents involved in pinching on these discretely spaced surfaces do not overlap, the development of each of the tearing modes which are excited can be treated as being independent of the other modes. In other words, the single-mode approximation can be used in this case. This approximation is of much assistance in simplifying the description of the nonlinear stage of the instability.

Calculations[9] on the interaction of the plasma particles with the finite-amplitude drift-tearing mode have shown that the mode amplitude reaches saturation in the stage in which the magnetic islands formed as a result of the tearing-mode instability fill the entire space where the longitudinal electric field of the mode differ significantly from zero. The reason is that once they go into magnetic islands the particles cease to interact with the wave. Interestingly, in the case in which the linearly drift-tearing instability is stabilized by a large magnetic shear there can be a hard excitation of the drift-tearing mode.[9]

In order to analyze the nonlinear stage of the drift-tearing instability in plasma formations in space with dimensions many orders of magnitude greater than the Larmor radius of the plasma particles, we need to develop a fundamentally new approach. The reason is that the number of modes excited in such formations is very large, and the distance between the discretely spaced resonant magnetic surfaces is very small. In this situation the magnetic islands which form near resonant surfaces overlap even while the amplitudes of the excited modes are still very small. According to the theory of Ref. 10, this overlap leads to a random walk of the magnetic field lines from one magnetic surface to another; i.e., it leads to breaking of these field lines.

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The behavior of the magnetic field lines may also become random as the result of the natural evolution of an initially ordered configuration. For example, the turbulent motion of the plasma at the bases of the loop-shaped magnetic structures in the solar corona (this motion is associated with thermal convection of plasma in the surface layer of the sun) entangles the magnetic field lines to the extent that their behavior can be thought of as random.

Analysis of the development of the tearing-mode instability in magnetic configurations with random field lines requires consideration of two fundamentally new physical effects: the decrease in the duration of the interaction of the particles with the excited mode, because the particles leave the vicinity of the resonant magnetic surface as they move along the field lines undergoing a random walk, and the diffusion of the longitudinal electric field of the drift-tearing mode across the magnetic surfaces.

Below we show that, as the amplitude of the drift-tearing modes increases, the efficiency of their interaction with the particles falls off, because of rapid spatial diffusion of the particles out of the region of the Čerenkov resonance with the excited mode. The mode growth rate increases. As a result, the development of the drift-tearing instability becomes explosive.

2. BASIC EQUATIONS

We analyze the drift-tearing instability of a plasma in a magnetic field with ruptured magnetic surfaces for the case in which the surfaces are planar:

$$\mathbf{B}=B_{1x}(y,\ z)\mathbf{e}_x+B_{0y}(x)\mathbf{e}_y+B_{0z}\mathbf{e}_z. \tag{1}$$

The small x component of the magnetic field ($B_{1x} \ll B_{0y} < B_{0z}$) fluctuates in space and can be represented by a Fourier series with components having random phases:

$$B_{1x}(y,z)=\sum_{\mathbf{k}} B_{x,\mathbf{k}} \exp(ik_y y+ik_z z). \tag{2}$$

Fluctuations of this sort may be caused by imperfections of the sources of the external magnetic field and also by the onset of the drift-tearing instability of the plasma configuration with the magnetic field (1) in which we have adopted. In the former case, the stability problem can be solved in the linear approximation, so it is this case which we consider in the present section of this paper. In the Conclusion to this paper, we will return to a self-consistent calculation of how the modes which grow as a result of the plasma instability affect the motion of the particles in the magnetic field (1).

As we have already mentioned, in the model of an infinite plasma slab the magnetic islands which form near magnetic surfaces overlap even while the amplitudes of the Fourier harmonics of the fluctuations are very small, as a result of the spatially fluctuating x component of the magnetic field. The displacement of magnetic field lines across the unperturbed magnetic surfaces is a diffusion with the properties

$$\langle x(s)-x(0)\rangle=0,$$
$$\langle [x(s)-x(0)]\cdot[x(s')-x(0)]\rangle=\begin{cases} Ds, & s<s' \\ Ds', & s>s'. \end{cases} \tag{3}$$

The angle brackets here mean an average over random displacements, and the diffusion coefficient of the magnetic field lines is found from the expression

$$D=\sum_{\mathbf{k}} (|B_{x,\mathbf{k}}|^2/B_0^2)\left\langle \int_0^\infty \exp\left\{-i\int_0^s k_\parallel[x(s')]ds'\right\}ds\right\rangle, \tag{4}$$

where s is the coordinate along the given field line, and $k_\parallel(x)$ is the longitudinal component of the wave vector. In our model of a magnetic field with sheared field lines, this longitudinal component depends on the displacement across the magnetic surfaces. Assuming that the displacement is small, we can expand the function $k_\parallel(x)$ in small random displacements from the given magnetic surface, with the coordinate $x=x(0)$, and we can carry out the averaging in expression (4) explicitly. As a result we find[11,12]

$$D=\sum_{\mathbf{k}} (|B_{x,\mathbf{k}}|^2/B_0^2)\int_0^\infty \exp\{-ik_\parallel[x(0)]s-k_\parallel'^2Ds^3/6\}ds. \tag{5}$$

To describe the drift-tearing instability in the magnetic field configuration (1), with planar magnetic surfaces, we use the drift approximation. In this case the complete system of equations includes the linearized drift kinetic equation for the particle distribution function,

$$\left[\frac{\partial}{\partial t}+v_\parallel\frac{\mathbf{B}_0}{B_0}\nabla+\frac{B_{1x}}{B_0}\frac{\partial}{\partial x}\right]f_{1j}=\left\{\frac{c[\nabla\varphi_1\mathbf{B}_0]_x}{B_0^2}-\frac{v_\parallel[\nabla\mathbf{A}_1]_x}{B_0}\right\}\frac{\partial f_{0j}}{\partial x}+\frac{e_j}{m_j}\left\{\frac{1}{c}\frac{\partial A_{1\parallel}}{\partial t}+\nabla_\parallel\varphi_1\right\}\frac{\partial f_{0j}}{\partial v_\parallel}, \tag{6}$$

Maxwell's equations for the longitudinal component of the vector potential,

$$\Delta A_{1\parallel}=-\sum_j (4\pi e_j/c)\int_{-\infty}^{\infty} v_\parallel f_{1j}\,dv_\parallel, \tag{7}$$

and the equation of plasma quasineutrality,

$$\sum_j e_j\int_{-\infty}^{\infty} f_{1j}\,dv_\parallel=0. \tag{8}$$

Here $f_{0j}(x,v_\parallel)$ is the equilibrium distribution of the particles of species j in the unperturbed plasma, $f_{1j}(r,v_\parallel,t)$ is the perturbation of the particle distribution function caused by the drift-tearing mode in the plasma, and φ_1 and A_1 are the perturbations of the scalar and vector potentials, to which we assign the coordinate and time dependence

$$\begin{aligned}\varphi_1(\mathbf{r},\ t)&=\varphi_1(x)\exp(-i\omega t+ik_y y+ik_z z),\\ \mathbf{A}_1(\mathbf{r},\ t)&=\mathbf{A}_1(x)\exp(-i\omega t+ik_y y+ik_z z).\end{aligned} \tag{9}$$

We assume that the unperturbed particle distribution is Maxwellian, shifted along the v_z axis by an amount equal to the current velocity of the given component, u_{jz}:

$$f_{0j}(x,v_\parallel)=n(x)\left(\frac{m_j}{2\pi T_j}\right)^{1/2}\exp\left[-\frac{m_j(v_\parallel-u_{j\parallel})^2}{2T_j}\right], \tag{10}$$

where $n(x)$ is the density of the inhomogeneous plasma, T_j is the uniform temperature of the particles of species j, m_j is the mass of the particles, and $u_{j\parallel}=u_{jz}B_{0z}/B_0$. The profile of the current density in the plasma is related to the profile of the y component of the magnetic field by Maxwell's equation

$$dB_{0y}(x)/dx=\sum_j (4\pi e_j/c)n(x)u_{jz}(x). \tag{11}$$

We find a solution of the kinetic equation (6) for the particles by integrating along the drift trajectories of the particles. These trajectories are straight lines in the plane of a magnetic surface $[y(t) = y + v_\parallel tB_{0y}/B_0;\quad z(t) = z + v_\parallel tB_{0z}/B_0)]$. In addition, we incorporate random walk of the particles across the magnetic surfaces in this integration. As a result we find

$$f_{1j}(\mathbf{r}, v_\parallel, t) = \frac{ie_j}{m_j}\exp(-i\omega t + ik_y y + ik_z z)\int_{-\infty}^{0}\left\{\left(\frac{k_\perp}{\omega_{cj}}\frac{\partial f_{0j}}{\partial x} + k_\parallel\frac{\partial f_{0j}}{\partial v_\parallel}\right)\left(\varphi_1[x(t')] - \frac{\omega}{k_\parallel c}A_{1\parallel}\right) + \frac{\omega - k_\parallel v_\parallel}{k_\parallel c}\frac{k_\perp}{\omega_{cj}}\frac{\partial f_{0j}}{\partial x}A_{1\parallel}\right\}$$
$$\times\exp\left(-i\omega t' + i\int_0^{t'} k_\parallel[x(t'')]v_\parallel dt''\right)dt', \qquad (12)$$

where $\omega_{cj} = e_j B_0/m_j c$ is the cyclotron frequency of the particles of species j, $k_\perp = [\mathbf{k}\mathbf{B}_0]_x/B_0$ is the component of the wave vector of the drift-tearing mode transverse with respect to the magnetic field, and $k_\parallel(x) = (k_y B_{0y}(x) + k_z B_{0z})/B_0$ is the longitudinal component of this vector. A point worth particular mention here is that the electrostatic potential $\varphi_1(x)$ of the drift-tearing mode has spatial fine structure near the resonant surface $[k_\parallel(X_s) = 0]$, so the integration of the terms which contain the electric potential is carried out with allowance for the random walk of the particles across the ruptured surfaces. In addition, in the limit in which these surfaces are highly ruptured, we need to consider the effect of the random walk of the particles on the integral of the terms containing the longitudinal component of the wave vector. For this reason, the quasineutral-plasma equation (8), which determines the structure of the electrostatic potential, takes different forms in the cases of slight and extensive ruptures of the magnetic surfaces (more on this below).

The longitudinal component of the vector potential $A_{1\parallel}(x)$ has a smoother structure, with far larger length scales. Consequently, its value near the resonant magnetic surface can be assumed constant. Far from the resonant surface, its profile is determined by Maxwell's equation (7). In evaluating the right side of this equation we should note that the first term in the integrand (12), which is proportional to the longitudinal electric field of the mode, vanishes far from the resonant surface, i.e., at $k_\parallel(x) \gg \omega/v_{Tj}$. The integral of the second term can be evaluated easily if we ignore the slight random walk of the particles across the magnetic surfaces. This integral describes an adiabatic perturbation of the particle distribution function:

$$f_{1j}^{ad} = -\frac{k_\perp}{k_\parallel(x)B_0}\frac{\partial f_{0j}}{\partial x}A_{1\parallel}(r, t). \qquad (13)$$

Substituting this expression into (7), and using (11), we find the well-known equation for the profile of the perturbation of the vector potential far from the resonant surface:[1,13]

$$\frac{d^2 A_{1\parallel}(x)}{dx^2} - \left(k^2 - \frac{1}{k_\parallel(x)}\frac{d^2 k_\parallel(x)}{dx^2}\right)A_{1\parallel}(x) = 0. \qquad (14)$$

Once we have found the profile of the scalar potential near the resonant surface (the inner region) from Eqs. (8) and (14), and once we have found the vector potential far from it (the outer region), we can derive a dispersion relation for the drift-tearing mode from Eq. (7). This relation takes the form of the condition for the matching of the solutions in the inner and outer regions:[2,12]

$$\Delta' = -\frac{1}{2A_{1\parallel}(X_s)}\sum_j\frac{4\pi e_j}{c}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_\parallel f_{1j}\,dv_\parallel\,dx. \qquad (15)$$

The left side of this equation is the jump in the derivatives of the solution of Eq. (14) at the resonant surface:

$$\Delta' = \int_{X_s-\varepsilon}^{X_s+\varepsilon}\frac{1}{2A_{1\parallel}(x)}\frac{d^2 A_{1\parallel}(x)}{dx^2}dx, \quad \varepsilon \ll X_s. \qquad (16)$$

The right side of (15) is the surface density of the perturbed current, which is localized near the resonant surface.

3. EVOLUTION OF THE INSTABILITY IN THE APPROXIMATION OF WEAK MAGNETIC DIFFUSION

The equations constructed in the preceding section of this paper to describe the development of the tearing-mode instability in a plane plasma slab with ruptured magnetic surfaces have an important feature: the current and charge densities in the plasma depend nonlocally on the electric field of the tearing mode which is excited [see, for example, expression (12) for the particle distribution function]. The meaning here is that the equations which we derived are generally integral equations. In the limit in which the diffusion of the magnetic field lines is weak, however, so that the displacement of the particles from the given magnetic surface over one oscillation period of the electric field of the mode is much smaller than the length scale of the spatial variation of the electric field, we can reduce these equations to differential equations by expanding in the small displacements of the particles from the magnetic surface. Specifically, we write the amplitude of the electric potential of the mode as a function of the particle displacement as an expansion in which we retain small terms of up to second order inclusively:

$$\varphi_1[x(t)] = \varphi_1(x) + \tilde{x}(t)\frac{d\varphi_1(x)}{dx} + \frac{\tilde{x}^2(t)}{2}\frac{d^2\varphi_1(x)}{dx^2}, \qquad (17)$$

where $x(t) = x + \tilde{x}(t)$, $\tilde{x}(0) = 0$, and $\tilde{x}(t)$ is a small random displacement from the magnetic surface with coordinate x. Under the assumption that the time scale of the random movements in the course of the diffusion of the particles is much smaller than the mode oscillation period, we can average (17) with the help of (3) and rewrite it in the form

$$\langle\varphi_1[x(t)]\rangle = \varphi_1(x) + {}^1/_2 D|v_\parallel t|\frac{d^2\varphi_1(x)}{dx^2}. \qquad (18)$$

Substituting this equation into (12) for the perturbation of the particle distribution function, and integrating along the drift trajectories of the particles, we find

$$f_{1j} = -\frac{e_j f_{0j}}{T_j}\left\{\frac{\omega_{*j} - k_\parallel v_\parallel}{\omega - k_\parallel v_\parallel + i\cdot 0}\left(\varphi_1 - \frac{\omega}{k_\parallel c}A_{1\parallel}\right) + \frac{i(\omega_{*j} - k_\parallel v_\parallel)}{2(\omega - k_\parallel v_\parallel + i\cdot 0)^2}\frac{d^2\varphi_1}{dx^2}D|v_\parallel| + \frac{\omega_{*j}}{k_\parallel c}A_{1\parallel}\right\}\exp(-i\omega t + i\mathbf{k}\mathbf{r}), \qquad (19)$$

where $\omega_{*j} = (k_\perp c T_j/e_j n B_0)dn/dx$ is the drift frequency of plasma component j. We are assuming here that the length scale δ_φ of the variations in the electric potential is much smaller than the distance to the resonant surface, $|x - X_s|$, so we have ignored the weak dependence of the longitudinal component of the mode wave vector on the particle displacement across the magnetic surface.

As in Refs. 7 and 8, we are interested primarily in the electric potential distribution between regions in which the mode undergoes Čerenkov interaction with electrons and ions:

$$\delta_e = \left| \frac{\omega_{*e}}{k_\parallel' v_{Te}} \right| < |x - X_s| < \delta_i = \left| \frac{\omega_{*e}}{k_\parallel' v_{Ti}} \right|, \tag{20}$$

where $k_\parallel' = dk_\parallel(x)/dx$, and $v_{Tj} = (T_j/m_j)^{1/2}$. The longitudinal phase velocity of the mode lies in the interval between the ion and electron thermal velocities:

$$v_{Ti} \ll \left| \frac{\omega}{k_\parallel(x)} \right| \ll v_{Te} \tag{21}$$

where the mode frequency is $\omega \approx \omega_{*e}$ (more on this below).

Integrating over velocity in quasineutrality equation (8) and expanding in the corresponding small parameters ($k_\parallel v_{Ti}/\omega \ll 1$, $\omega/k_\parallel v_{Te} \ll 1$), we can rewrite this equation as

$$\left[i\frac{D\omega_{*e} v_{Ti}}{(2\pi)^{1/2}\omega^2} + \frac{iD\omega}{(2\pi)^{1/2} k_\parallel'^2 v_{Te}} \ln\left(\frac{k_\parallel v_{Te}}{\omega} \right) + \left(1 + \frac{T_i}{T_e}\right)\frac{c_s^2}{\omega_{ci}^2} \right] \frac{d^2\varphi_1}{dx^2} = -\left\{ \frac{\omega - \omega_{*i}}{\omega}\frac{k_\parallel^2 c_s^2}{\omega^2} - \frac{\omega - \omega_{*e}}{\omega} \right\} \left(\varphi_1 - \frac{\omega}{k_\parallel c} A_{1\parallel} \right), \tag{22}$$

where $c_s = (T_e/m_i)^{1/2}$. We have added the last term on the left side of Eq. (22) for convenience in comparison with Refs. 7 and 8. This term incorporates the finite ion Larmor radius—an effect which we ignored in calculating the correction to the particle distribution function with the help of the drift equation (6).

To avoid unnecessary complications in the solution of this equation, we restrict the discussion here to the case in which the diffusion of the magnetic field lines is not too weak, so the length scale δ_φ of the potential variations is large:

$$\delta_i^2 > \delta_\varphi^2 > \delta_i \delta_e, \quad \delta_i \rho_i, \tag{23}$$

where $\rho_j = v_{Tj}/\omega_{cj}$ is the Larmor radius of the particles of species j. Since we have $\delta_j = \rho_j (L_s/L_n) \gg \rho_j$, where $L_s = |k_\perp / k_\parallel'| \sim (B_0/B_{0y}) L_n$ and $L_n = n/|dn/dx|$, the parameter interval (23) always exists. As a result, along with the corrections for the finite ion Larmor radius we can ignore the electron contribution on both sides of Eq. (22). Expanding the longitudinal component of the wave vector in the small distance to the resonant surface $k_\parallel(x) = k_\parallel'(x - X_s)$, we can put Eq. (22) in the well-studied form (reviewed, for example, in Ref. 13)

$$\frac{d^2\varphi_1(\xi)}{d\xi^2} - \xi^2 \varphi_1(\xi) = -\frac{\omega\xi}{k_\parallel' \delta c} A_{1\parallel}(X_s), \tag{24}$$

where

$$\xi = \frac{x - X_s}{\delta}, \quad \delta^4 = -\frac{i\omega_{*e} D}{(2\pi)^{1/2}(1 + T_e/T_i) k_\parallel'^2 v_{Ti}}. \tag{25}$$

The solution of this equation,[1]

$$\varphi_1(\xi) = \frac{\omega\xi}{2k_\parallel' \delta c} A_{1\parallel}(X_s) \int_0^1 (1 - \mu^2)^{-1/4} \exp\left(-\frac{\mu\xi^2}{2} \right) d\mu \tag{26}$$

means that there is an exponential decay of the longitudinal electric field far from the resonant surface when we choose the following root of Eq. (25):

$$\delta = \delta_\varphi \exp[-i(\pi/8)\,\mathrm{sign}\,\omega_{*e}], \quad \delta_\varphi = |\delta|. \tag{27}$$

We can now write explicit conditions under which the diffusion of magnetic field lines is weak—the conditions under which we derived expression (19) for the perturbation of the particle distribution function. In the latter derivation we assumed that the mean square displacements of the ions and electrons in the course of their random walk along the magnetic field lines were small in comparison with the length scale of the variations of the electric potential. Under the assumption that the integral over the particle trajectories is truncated at different times for the ions ($t_i \sim 1/\omega$) and for the electrons ($t_e \sim 1/k_\parallel v_\parallel$), we write the condition under which the displacements are small as follows:

$$\left(\frac{D v_{Ti}}{\omega} \right)^{1/2} \ll \delta_\varphi, \quad \left| \frac{D}{k_\parallel'} \right| \ll \delta_\varphi. \tag{28}$$

We have also assumed that the longitudinal phase velocity of the mode is higher than the ion thermal velocity:

$$\omega / |k_\parallel'| \delta_\varphi v_{Ti} \gg 1. \tag{29}$$

It turns out that all these inequalities reduce to the single inequality:

$$|D/k_\parallel'|^{1/3} \ll \delta_i. \tag{30}$$

Equation (7)—Maxwell's equation for the longitudinal component of the vector potential—must be written out explicitly in order to derive a dispersion relation for the drift-tearing mode. To do this, we use the same transformations as were used in writing the quasineutrality equation. As a result we find

$$\frac{d^2 A_{1\parallel}}{dx^2} - \left[k^2 - \frac{1}{k_\parallel}\frac{d^2 k_\parallel}{dx^2} \right] A_{1\parallel} = -\frac{\omega_{pe}^2 \omega}{k_\parallel c v_{Te}^2} \left\{ \left[\frac{\omega - \omega_{*i}}{\omega}\frac{k_\parallel^2 c_s^2}{\omega^2} - \frac{\omega - \omega_{*e}}{\omega}\left(1 + \frac{i\pi\omega}{n|k_\parallel|} f_{0e}\left(\frac{\omega}{k_\parallel} \right) \right) \right] \times \left(\varphi_1 - \frac{\omega}{k_\parallel c} A_{1\parallel} \right) + \frac{iD v_{Te}}{(2\pi)^{1/2}\omega}\frac{d^2\varphi_1}{dx^2} \right\}. \tag{31}$$

Here, in contrast with Eq. (22) for the electric potential, we have retained the contribution of the resonant electrons to the perturbation of the longitudinal current near the resonant magnetic surface ($|x - X_s| \lesssim \delta_e$). That surface is not included in the range of definition of the electric potential, (20). We can thus find the dispersion relation (15) for the drift-tearing mode by integrating both sides of Eq. (30) over the vicinity of the resonant magnetic surface, $|x - X_s| < \delta_i$. Using Eq. (22) to transform the right side of Eq. (31), we find

$$\Delta' = -\frac{i\omega_{pe}^2}{(2\pi)^{1/2} c^2} \left\{ \frac{\pi(\omega - \omega_{*e})}{|k_\parallel'| v_{Te}} - \frac{1}{2A_{1\parallel}(X_s)} \int_{-\infty}^{+\infty} \frac{d^2\varphi_1}{d\xi^2}\frac{d\xi}{\xi}\frac{D\omega_{*e} c}{k_\parallel' \delta^2 v_{Te}^2}\left(\frac{v_{Ti}}{\omega} - \frac{v_{Te}}{\omega_{*e}} \right) \right\}. \tag{32}$$

Carrying out the integration with the help of (26), we rewrite this expression as

$$\frac{c^2}{\omega_{pe}^2}\Delta' = -i\left(\frac{\pi}{2}\right)^{1/2}\frac{(\omega-\omega_{*e})}{|k_\parallel'|v_{Te}} + \frac{m_e}{2m_i}\left(1+\frac{T_i}{T_e}\right)\delta_\varphi I\left(1-\frac{\omega v_{Te}}{\omega_{*e}v_{Ti}}\right)\exp(-i(\pi/8)\,\mathrm{sign}\,\omega_{*e}), \quad (33)$$

where

$$I = \pi\Gamma(3/4)/\Gamma(1/4).$$

Under the condition $\delta_\varphi < \delta_i$, which we used in deriving the basic equations of this section, i.e., Eqs. (22) and (31), the solution of the dispersion relation is

$$\omega-\omega_{*e} = i\left(\frac{2}{\pi}\right)^{1/2}\frac{c^2\Delta'}{\omega_{pe}^2}|k_\parallel'|v_{Te} - \frac{im_e}{(2\pi)^{1/2}m_i}\left(1+\frac{T_i}{T_e}\right)\left(\Gamma-\frac{v_{Te}}{v_{Ti}}\right)|k_\parallel'|\delta_\varphi v_{Te} \times\exp\left(-\frac{i\pi}{8}\,\mathrm{sign}\,\omega_{*e}\right). \quad (34)$$

The first term in the expression for the instability growth rate describes the well-known tearing-mode instability or a spontaneous pinch of a distributed current. The first subterm of the second term is also known from Refs. 7 and 8. It describes saturation of the tearing-mode instability as a result of the effect of the longitudinal motion of the ions on the distribution of the longitudinal electric field of the tearing mode.

Since the dimensions of most of the magnetic configurations with current sheets observed in space are many orders of magnitude greater than the plasma length scales (the Larmor radius of the particles, ρ_j, and their inertial lengths c/ω_{pj}), the first term on the right side of Eq. (34) is negligible in comparison with the second. Consequently, if there is no rupture of the magnetic surfaces ($D=0$), the growth of the tearing mode is stopped. If, however, the magnetic surfaces are easily ruptured, nonuniform distribution of the electrons and ions across the resonant magnetic surface leads to diffusion of these particles. By virtue of current continuity, the faster diffusion of electrons sets up an additional longitudinal electric current, which is higher than the ion current and opposite in direction. The electron contribution to the dispersion relation is thus greater than the ion contribution, and it is furthermore destabilizing. As a result, the current pinch results not from a drawing of energy from the magnetic field but from a spatial redistribution of the current due to pressure forces.

4. INSTABILITY IN THE CASE OF A PRONOUNCED DIFFUSION OF MAGNETIC FIELD LINES

As we showed in the preceding section of this paper, if the inequalities (28) and (30) are violated, i.e., if

$$|D/k_\parallel'|^{1/3} > \delta_\varphi > \delta_i, \quad (35)$$

then random displacements of the particles across the ruptured magnetic surfaces are comparable to or greater than the length scale of the variations in the electric potential of the drift-tearing mode which is excited. In this case, the plasma quasineutrality equation (8) can be transformed into the following integral equation for the electric potential $\varphi(x)$ with the help of expression (12) for the perturbation of the particle distribution function:

$$\sum_j (e_j^2/T_j)\int_{-\infty}^{\infty} f_{0j}(X_s, v_\parallel)\int_{-\infty}^{0}\{i\omega_{*j}\varphi_1[x(t)] + iv_\parallel[(\omega/c)A_{1\parallel}(X_s) - k_\parallel'x(t)\varphi_1[x(t)] - (\omega_{*j}/c)A_{1\parallel}(X_s)]\} \times\exp\left[-i\omega t + ik_\parallel'v_\parallel\int_0^t x(t')dt'\right]dt\,dv_\parallel. \quad (36)$$

In contrast with the procedure of the preceding section of this paper, in the integration over the trajectories we took account of the change in the longitudinal component of the wave vector (along the trajectory) of the particles, using an expansion of the form

$$k_\parallel(x) = k_\parallel'(X_s)(x-X_s). \quad (37)$$

The coordinate x of the particles along the trajectory is given by

$$x(t) = \bar{x} + \tilde{x}(t), \quad (38)$$

where $\bar{x} = x - X_s$, and $\tilde{x}(t)$ is the random displacement of the particles across the magnetic surface $\tilde{x}(0) = 0$.

We see that there are three time scales in Eq. (37).

1) The time over which the particles diffuse out of the region in which the longitudinal electric field is nonzero and in which the particles interact with the mode, i.e., $\tau_1 \approx \delta_\varphi^2/Dv_{Tj}$.

2) The time taken by thermal particles to traverse the characteristic length on which the electric potential of the mode varies in the direction along the magnetic field, $\tau_2 \approx 1/k_\parallel'\delta_\varphi v_{Tj}$.

3) The oscillation period of the electric field of the mode, $\tau_3 \approx 1/\omega$.

Under the conditions (35), these times fall in the following order:

$$\tau_1 \ll \tau_2 \ll \tau_3. \quad (39)$$

Consequently, the integration along the trajectory of the terms of the integrand in (36) which contain the potential φ_1 or the longitudinal electric field is truncated at the diffusion time scale τ_1. It follows that the electron contribution to the quasineutrality equation is small in comparison with the ion contribution, and we will ignore it. In addition, the change in the wave phase over the time scale of the particle diffusion is small, and we can expand the exponential function in Eq. (36), retaining only the even terms. As a result, Eq. (36) becomes

$$\int_{-\infty}^{\infty} f_{0i}(X_s, v_\parallel)\int_{-\infty}^{0}\left\{i\omega_{*i}\varphi_1[x(t)] - k_\parallel'v_\parallel^2\int_0^t x(t')dt' \times[(\omega/c)A_{1\parallel} - k_\parallel'x(t)\varphi_1[x(t)]] - i\omega_{*i}v_\parallel A_{1\parallel}/c\right\}dt\,dv_\parallel = 0. \quad (40)$$

We seek a solution of this equation in the form

$$A_{1\parallel}(X_s) - \frac{k_\parallel'c}{\omega}x(t)\varphi_1[x(t)] = -\int_0^\infty \frac{\partial F(p,\bar{x})}{\partial p}\exp[-px^2(t)]dp = F(0,\bar{x}) - x^2(t)\int_0^\infty F(p,\bar{x})\exp[-px^2(t)]dp, \quad (41)$$

where the coefficients $F(p, \bar{x})$ depend on $\bar{x}$ as a parameter. To simplify the time integration, we take an average of the integrand, making use of relations which follow from the properties (3) of the random function $\bar{x}(t)$:

$$\langle x^2(t)\rangle = D|v_\parallel t|, \tag{42}$$

$$\langle \exp[-\tilde{x}^2(t)]\rangle = \exp[-p\bar{x}^2 - pD|v_\parallel t|(1-2p\bar{x}^2)], \tag{43}$$

$$\langle \tilde{x}(t)\exp[-2p\bar{x}\tilde{x}(t)]\rangle = -2p\bar{x}D|v_\parallel t|\exp(2p^2\bar{x}^2D|v_\parallel t|), \tag{44}$$

$$\left\langle \int_0^t \tilde{x}(t')dt' \exp[-2p\bar{x}\tilde{x}(t)]\right\rangle = -p\bar{x}D|v_\parallel t|\exp(2p^2D|v_\parallel t|). \tag{45}$$

As a result, the integration over the trajectories can be carried out explicitly, and Eq. (41) becomes

$$\int_0^\infty \frac{k_\parallel'\bar{x}(1+2p\bar{x}^2)}{p^2D^2(1-2p\bar{x}^2)^3}\left[\frac{\partial F}{\partial p} + \varkappa p(1-2p\bar{x}^2)F\right]dp = 0, \tag{46}$$

where

$$\varkappa = \int_{-\infty}^{\infty} \frac{i\omega_{*i}Df_{0i}(X_s, v_\parallel)dv_\parallel}{k_\parallel'' n[|v_\parallel| - i\omega/p(1-2p\bar{x}^2)D]}$$

$$\approx \frac{i\omega_{*i}D}{(2\pi)^{1/2}k_\parallel' v_{Ti}}\ln\left(\frac{D}{|k_\parallel'|\delta_i^3}\right). \tag{47}$$

We have eliminated the logarithmic divergence of the last integral in the limit $v_\parallel \to 0$ by incorporating the oscillations in the electric potential at the frequency ω.

The solution of Eq. (46) is obvious:

$$F(p, \bar{x}) = A_{1\parallel}(X_s)\exp[-\varkappa p^2/2 + 2\varkappa p^3\bar{x}^2/3]. \tag{48}$$

Substituting (48) into (41), we easily see that the longitudinal electric field falls off exponentially with distance from the resonant surface, over a length scale $\delta_\varphi \sim |\varkappa|^{1/4}$. The expressions for the length scale δ_φ of the region in which the longitudinal electric field of the mode is nonzero are parametrically the same in the cases of weak and strong diffusion of the magnetic field lines [compare (25) and (47)].

The dispersion relation (15) for the drift-tearing mode can be put in the following form with the help of the expression (12):

$$\Delta' = \sum_j (2\pi i e_j^2/c^2T_jA_{1\parallel}(X_s))$$

$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_\parallel f_{0j}(X_s, v_\parallel)\int_{-\infty}^{0}\{\omega_{*j}\varphi_1[x(t)]$$

$$+ v_\parallel[\omega A_{1\parallel}(X_s) - ck_\parallel' x(t)\varphi[x(t)]] - v_\parallel\omega_{*j}A_{1\parallel}(X_s)\}$$

$$\times\exp\left[-i\omega t + ik_\parallel' v_\parallel\int_0^t x(t')dt'\right]dt\, dv_\parallel\, dx. \tag{49}$$

The first term in the integrand is small in comparison with the others, and we will ignore it. The integral of the second term over the trajectories is evaluated with the help of (42)–(45). In integrating the third term over the trajectories we can ignore small displacements of the particles from the magnetic surface. As a result, the dispersion relation (49) becomes

$$\Delta' = -\frac{i\omega_{pe}^2\omega}{c^2}$$

$$\times\left\{\frac{\varkappa}{(2\pi)^{1/2}Dv_{Te}}\int_{-\infty}^{\infty}\int_0^{\infty}\exp\left[-p\bar{x}^2 - \frac{\varkappa p^2}{2} + \frac{2\varkappa p^3\bar{x}^2}{3}\right]dp\,d\bar{x}\right.$$

$$\left. -\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_{*e}}{\omega|k_\parallel'|v_{Te}}\right\}. \tag{50}$$

Changing the order of integration in the first term on the right, and then changing the integration path in the complex p plane, we rewrite this equation as the dependence of the frequency on the wave vector:

$$\omega = \frac{\pi^{3/2}D\exp[3\pi i\,\mathrm{sign}\,\omega_{*e}/8]}{6^{3/4}|\varkappa|^{3/4}\Gamma^2(1/4)\,{}_1F_1(1/4, 3/4, 3/4)|k_\parallel'|}$$

$$\times\left[\omega_{*e} + i\left(\frac{2}{\pi}\right)^{1/2}\frac{c^2\alpha'}{\omega_{pe}^2}|k_\parallel'|v_{Te}\right], \tag{51}$$

where ${}_1F_1(\alpha,\beta,z)$ is the confluent hypergeometric function. As in the preceding section of this paper, the second term in square brackets, which describes the growth of the perturbations at the expense of the decrease in magnetic field energy, can be ignored. As before, the current pinch stems from the work performed by plasma pressure forces.

5. EXPLOSIVE DEVELOPMENT OF THE INSTABILITY

Up to this point we have been assuming that this diffusion of the magnetic field lines which ruptures the magnetic surfaces itself results from fluctuations generated in the x component of the magnetic field by the external source. That assumption has made it possible to analyze the stability in linear perturbation theory. Actually, these fluctuations of the magnetic field result from the onset of the tearing-mode instability, and their intensity can exceed that of the fluctuations of the external magnetic field. In this case, the dispersion relations (34) and (51) can be put in the form of nonlinear equations for the intensity of the tearing modes. In particular, in the case of a pronounced diffusion of the magnetic field lines, we can reduce Eq. (51) to the following form, where we are using expression (47) for $\varkappa$, and where we are noting that the diffusion near the resonant surface is dominated by the tearing modes for which the longitudinal component of the wave vector lies within the broadening of the spatial resonance, i.e., $|k_\parallel(x)| \lesssim (k_\parallel'^2D)^{1/3}$ [see expression (5)]:

$$\frac{1}{2}\frac{\partial}{\partial t}\sum_{\mathbf{k}}\frac{|B_{x,\mathbf{k}}|^2}{B_0^2} = \alpha|\omega_{*i}|^{1/4}|k_\parallel'|^{3/4}v_{Ti}^{3/4}\left[\sum_{\mathbf{k}}\frac{|B_{x,\mathbf{k}}|^2}{B_0^2}\right]^{19/16}, \tag{52}$$

where

$$\alpha \approx \frac{\pi^{3/2}\sin(3\pi/8)}{6^{3/4}\Gamma^2(1/4)\,{}_1F_1(1/4, 3/4, 3/4)}\left[\frac{(2\pi)^{1/2}}{\ln(D/|k_\parallel'|\delta_i^3)}\right]^{3/4}\frac{T_e}{T_i}.$$

The summation in (52) is over those wave vectors which satisfy the condition $|k_\parallel(x)| < (k_\parallel'^2D)^{1/3}$.

Solving this equation, we find that the intensity of the excited modes increases faster than exponentially with the time, i.e., explosively:

$$\sum_{\mathbf{k}}|B_{x,\mathbf{k}}|^2/B_0^2 \approx \left[\sum_{\mathbf{k}}|B_{x,\mathbf{k}}(0)|^2/B_0^2\right](1 - 3/8\,\gamma t)^{-16/3}, \tag{53}$$

where

$$\gamma \approx \alpha |\omega_{*i}|^{1/4} |k_{\parallel}'|^{3/8} v_{Ti}^{3/4} \left[\sum_{\mathbf{k}} |B_{x,\mathbf{k}}(0)|^2 / B_0^2 \right]^{3/16}.$$

6. CONCLUSION

We have thus shown that even when the oscillations of the magnetic field of the tearing mode have a very low intensity, which satisfies the condition (23), the tearing-mode instability goes into a regime of explosive growth. As a result, despite the very small growth rate of the linear tearing-mode instability, this instability can explain the mechanism for the explosive releases of energy which are observed in current-carrying plasma configurations. A good example is the unexpectedly rapid brightening of isolated loop-shaped structures in the solar corona, which evidently results from the onset of an instability of a magnetic field configuration of this sort. The explosive tearing-mode instability can explain not only the rapid release of energy but also the observed[15] formation of a set of filamentary structures inside these loops, as a result of current pinching. A discussion of the various applications of the theory derived here goes beyond the scope of the present paper, however.

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