Nonlinear dynamics of packets of intense electromagnetic fields in plasmas with linear and parabolic density barriers

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The dynamics of packets of intense electromagnetic fields in inhomogeneous plasmas with linear and parabolic density barriers and an ion-acoustic nonlinearity is analyzed. Dynamic phase-conjugate states of the field which are frequency-tunable may exist. In a plasma with a linear density barrier, states of this sort correspond to reflection, while in a plasma with a parabolic barrier they correspond to reflection and transmission, separated by a regime in which the packets are "pulled up" to the crest of the barrier. The behavior of the depth to which the field penetrates into the dense, supercritical plasma in this pulling regime is analyzed; the thermal losses are taken into account. Most of the energy may be absorbed near the crest of the barrier.

1. INTRODUCTION

In the theoretical work on the propagation of nonlinear waves, a substantial effort is being made to find various mechanisms for the penetration of rf waves into the supercritical regions of inhomogeneous plasma media and for the penetration of these waves through such media.[1-9] In particular, for rf fields describable by the nonlinear Schrödinger equation one such mechanism may involve the existence of frequency-tunable dynamic states.[7-9]

In the present paper we find a new class of dynamic phase-conjugate states of intense electromagnetic fields in inhomogeneous plasmas with linear and parabolic density profiles and with an ion-acoustic nonlinearity. For a plasma with a linear density profile, conjugate states are found which correspond to a reflection from dense layers with the properties of the packets before and after the reflection differing in the sign of the phase. We will clarify the existence of this class of states in a model problem of the evolution of a $1D$ wave field $\varphi(z,t)$, which is describable in dimensionless variables by the Schrödinger equation in a medium with a linear or parabolic density profile and with an ion-acoustic nonlinearity:

$$-2i\frac{\partial\varphi}{\partial t}+\frac{\partial^2\varphi}{\partial x^2}-n\varphi+(-x)^p\varphi=0, \qquad (1)$$
$$\frac{\partial^2 n}{\partial t^2}-\frac{\partial^2 n}{\partial x^2}=\frac{\partial^2}{\partial x^2}(|\varphi|^2).$$

The values $p=1$ and 2 correspond to inhomogeneous media with, respectively, linear and parabolic density profiles. Transforming the independent variables,

$$\xi=x-\int a(t)\,dt, \quad t'=t$$

and the unknown function,

$$\varphi=\Phi(\xi,t)\exp\left\{-ia\xi-i\frac{d(a^2)}{dt}\right\}$$

and assuming that the velocity of the motion, $a(t)$, is given by

$$\frac{da}{dt}+1=0, \quad p=1, \qquad (2)$$

$$\frac{d^2a}{dt^2}-a=0, \quad p=2, \qquad (3)$$

we can reduce Eqs. (1) to the system of equations

$$-2i\frac{\partial\Phi}{\partial t}+\frac{\partial^2\Phi}{\partial\xi^2}+\{-n+\delta_{p,2}\xi^2\}\Phi=0, \qquad (4a)$$

$$(a^2-1)\frac{\partial^2 n}{\partial\xi^2}+\frac{\partial^2 n}{\partial t^2}-a\frac{\partial^2 n}{\partial t\,\partial\xi}-\frac{\partial^2(an)}{\partial t\,\partial\xi}=\frac{\partial^2(|\Phi|^2)}{\partial\xi^2}. \qquad (4b)$$

where $\delta_{p,2}$ is the Kronecker delta.

In media with a linear density profile ($p=1$), the solution $a=-t$ of Eq. (2) corresponds to a reflection of the packets from the dense layers. The time $t=0$ in this case corresponds to the crossing of the turning point by the center of the packet. In media with a parabolic density barrier ($p=2$), the velocity of the packet satisfies Eq. (3). One of the independent solutions of this equation, $a_1=\cosh t$, corresponds to transmission of the packets through the barrier, while another, $a_2=\sinh t$, corresponds to reflection of the packets from the barrier. In the former case, the time $t=0$ corresponds to the passage of the crest of the barrier by the center of the packet, while in the latter case it corresponds to the crossing of the turning point.

In reflection regimes, with $a(t)=-a(-t)$, Eqs. (4) are invariant under the simultaneous replacements

$$t\to -t, \quad \Phi\to\Phi^*, \qquad (5)$$

so for the wave packets (4), which can be described by the real functions $\Phi(\xi,0)=\Phi^*(\xi,0)$ at the time at which the center of the packet passes the reflection point, there exist conjugate solutions:

$$\Phi(\xi,t)=\Phi^*(\xi,-t), \quad n(\xi,t)=n(\xi,-t). \qquad (6)$$

In this case the properties of the packets before and after the reflection differ in the sign of the phase. We call those solutions of (4) which satisfy (6) "conjugate" solutions.

In the transmission regime, with $a(t)=a(-t)$, Eqs. (4) are invariant under the simultaneous replacements

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$$t\to-t,\ \xi\to-\xi\ (\text{в } (4.6)),\ \Phi\to\Phi^*, \tag{7}$$

where the field Φ^* is the complex conjugate of Φ. It follows that for the wave packets in (4) whose field Φ and density n satisfy the relations

$$\Phi(\xi,\ 0)=\Phi^*(-\xi,\ 0),\ n(\xi,\ 0)=n(-\xi,\ 0), \tag{8}$$

when the center of the packet passes the crest of the barrier [relations (8) are satisfied, in particular, by even real functions], there may exist solutions

$$\Phi(\xi,\ t)=\Phi^*(-\xi,\ -t),\ n(\xi,\ t)=n(-\xi,\ -t), \tag{9}$$

which relate the amplitude A of the field $\Phi = Ae^{i\varkappa}$ and the perturbations of the packet density n before and after the barrier by the specular relation $A(-\xi,-t)=A(\xi,t)$, $n(-\xi,-t)=n(\xi,t)$: The leading edges of the envelope of the packet and the perturbations of the density of the packet in front of the barrier correspond to the trailing edges beyond the barrier. The phases $\varkappa$ of the packets, before and after the barrier are related in this case by the mirror-conjugate relation $\varkappa(\xi,t)=-\varkappa(-\xi,-t)$. We call solutions of (4) which satisfy (9) "mirror-conjugate" solutions.

In this paper we consider dynamic states of packets of intense electromagnetic fields in plasmas with an ion-acoustic nonlinearity and with a density irregularity in the form of a linear or parabolic barrier. We begin with a study of the equations of motion of the packets. In a plasma with a linear density barrier, we find reflection regimes which correspond to conjugate dynamic states. In a plasma with a parabolic density barrier, we find (depending on the initial conditions) reflection and transmission regimes, which are separated by a regime in which the packets of intense electromagnetic field are "pulled up" to the crest of the barrier and "slide down" from it. We show that dynamic states may be excited by an electromagnetic pulse incident on the plasma from vacuum. We then take up the evolution of the packets as they move along the barrier. We study both mirror-conjugate and conjugate dynamic states, which are realized in the regimes of, respectively, transmission of the packets through the density barrier and reflection of the packets from this barrier. In analyzing the evolution of the packets when they are pulled up to the crest of the barrier, which separates the regimes of reflection and transmission, we allow for thermal losses. We show that packets of intense electromagnetic field may be absorbed near the crest of a parabolic barrier in dense plasma layers.

2. BASIC EQUATIONS

We consider a self-consistent $1D$ field of intense electromagnetic waves in a plasma slab with an ion-acoustic nonlinearity. We assume that the wave vectors of the electromagnetic waves are collinear with the plasma gradient. We represent the electric field $E(z,t)$ of the electromagnetic waves as a wave packet with a frequency $\omega(t)$ which varies slowly in time:

$$E(z,\ t)=E_0(z,\ t)\ \exp\{i\int\omega(t)dt\}.$$

We consider plasma density perturbations N_s which are caused by the electromagnetic field. We assume that the length scale L_s of the variations in these perturbations is small in comparison with the length scale L of the variations in the unperturbed plasma. We then find the following system of equations for the field envelope $E_0(z,t)$ and for the density perturbation N_s (Ref. 10):

$$c^2\frac{\partial^2E_0}{\partial z^2}-2i\omega(t)\frac{\partial E_0}{\partial t}+\left\{-i\frac{d\omega}{dt}+\omega^2(t)-\omega_p^2(z,t)\right\}E_0=0, \tag{10}$$

$$\frac{1}{c_s^2}\frac{\partial^2N_s}{\partial t^2}-\frac{\partial^2N_s}{\partial z^2}=\frac{1}{16\pi T_e}\frac{\partial^2|E_0|^2}{\partial z^2}, \tag{11}$$

where

$$\omega_p^2(z,t)=\frac{4\pi e^2N(z,t)}{m_e},\quad N(z,t)=N(z)+N_s(z,t),$$

$N(z)$ is the unperturbed value of the plasma density, c is the velocity of light, c_s is the velocity of ion acoustic waves, and T_e is the electron temperature. We assume that the profile of the unperturbed plasma $N(z)$ density is

$$N(z)=N_0\left[1-\left(\frac{-z}{\Delta}\right)^p\right],$$

where $p=1,2$. We introduce the dimensionless variables

$$t_H=\frac{c^2}{\omega_p(0)\Delta^2}t,\quad z_H=\frac{z}{\Delta},\quad E_H=\frac{E_0}{(16\pi N_0T_e)^{1/2}},$$

$$\omega_H(t)=\frac{\omega(t)}{\omega_p(0)},\quad n=\frac{N_s}{N_0},$$

where

$$\omega_p^2(0)=\frac{4\pi e^2N_0}{m_e},\quad k_0=\frac{\omega_p(0)}{c},\quad \alpha=k_0\Delta,$$

We transform the unknown function (below, we omit the subscript H):

$$E_0(z,\ t)=\tilde{E}_0(z,\ t)\,(\omega(t))^{1/2}$$

Equations (7) and (8) then become

$$-2i\omega(t)\frac{\partial\tilde{E}_0}{\partial t}+\frac{\partial^2\tilde{E}_0}{\partial z^2}+\alpha^2\{[\omega^2(t)-1]+(-z)^p-n\omega(t)\}\tilde{E}_0=0, \tag{12}$$

$$G^2\frac{\partial^2n}{\partial t^2}-\frac{\partial^2n}{\partial z^2}=\omega(t)\frac{\partial^2|\tilde{E}_0|^2}{\partial z^2}, \tag{13}$$

where $G=c/(c_s\alpha)$. In Eqs. (12) and (13), we switch to a coordinate system which is moving at a varying velocity $V(t)$:

$$\xi=(\omega(t))^{1/2}\left\{z-\int_{t_0}^{t}V(t)dt\right\},\quad t'=t,$$

We write the solution of these new equations in the form

$$\tilde{E}_0(z,\ t)=\varphi(\xi,\ t)\exp\{-iV(t)(\omega(t))^{1/2}\xi\}.$$

Ignoring terms of order V^2 in the relation found from (12) in this manner, ignoring the term with the second derivative with respect to the time from (13), and integrating over ξ from $-\infty$ to ξ in the relation found from (13), under the conditions $n(\xi\to-\infty, t)\to 0$ and $\varphi(\xi\to-\infty, t)\to 0$, (these conditions correspond to nonlinear wave packets which are localized along the ξ scale), we find

$$-2i\frac{\partial\varphi}{\partial t}+\frac{\partial^2\varphi}{\partial\xi^2}+\alpha^2\left\{\frac{1}{\omega(t)}[\omega^2(t)-\omega_p^2(t)]-\frac{n}{\omega(t)}+\frac{\xi^2}{\omega^2(t)}\delta_{p,2}-\frac{2\xi}{\omega^{3/2}(t)\alpha^2}\times\left[2\omega\frac{d}{dt}\left(\omega(t)\frac{dz(t)}{dt}\right)-2\alpha^2 z(t)\delta_{p,2}+\alpha^2\delta_{p,1}\right]\right\}\varphi=0, \quad (14)$$

$$\left(V^2-\frac{1}{G^2}\right)\frac{\partial n}{\partial\xi}-\frac{1}{n(\omega(t))^{1/2}}\frac{\partial}{\partial t}(n^2V)=\frac{\omega(t)}{G^2}\frac{\partial}{\partial\xi}(|\varphi|^2), \quad (15)$$

Here

$$z(t)=\int_{t_0}^{t} V(\tilde t)\,d\tilde t$$

is the coordinate of the center of the wave packet ($\xi=0$) in the laboratory coordinate system, and $\omega_p(t)=[1-(-z)^p]^{1/2}$ is the value of the unperturbed plasma frequency at the center of the wave packet. We seek a solution of (14), (15) under the condition that the packet frequency $\omega(t)$ and its velocity $V(t)=dz(t)/dt$ satisfy

$$\omega^2(t)-\omega_p^2(t)=\frac{\mu}{\alpha^2}, \quad (16a)$$

and either

$$2\omega\frac{d}{dt}\left(\omega(t)\frac{dz(t)}{dt}\right)+\alpha^2=0, \quad (16b)$$

in the case $p=1$ or

$$2\omega\frac{d}{dt}\left(\omega(t)\frac{dz(t)}{dt}\right)-2\alpha^2 z(t)=0, \quad (16c)$$

in the case $p=2$. Here μ is an arbitrary constant. In this case the change

$$\varphi(\xi,t)=\Phi(\xi,t)\exp\left\{-i\frac{\mu}{2}\int\frac{dt}{\omega(t)}\right\}$$

in the unknown function puts the equation for the envelope of the packet in the form

$$-2i\frac{\partial\Phi}{\partial t}+\frac{\partial^2\Phi}{\partial\xi^2}+\alpha^2\left\{\frac{\xi^2}{\omega^2(t)}\delta_{p,2}-\frac{n}{\omega(t)}\right\}\Phi=0. \quad (17)$$

Equations (17) and (15) describe the envelope Φ and the perturbation of the density n for electromagnetic wave packets whose frequency $\omega(t)$ is determined by (16).

With $p=1$, the equations of motion (16) are invariant under the replacement of t by $-t$. It follows that for $z(t)$ there exist solutions which are even in the time t and which correspond to a reflection of the electromagnetic field packets from a plasma with a linear density profile.

With $p=2$, equations of motion (16) are invariant under the simultaneous replacement of t by $-t$ and of z by $\pm z$. It follows that for $z(t)$ there exist solutions which are even and odd in the time $t=0$. These solutions correspond to reflection of the electromagnetic field packets from the parabolic density barrier and transmission of the packets through the barrier, respectively. In each case the packet frequency $\omega(t)$ is even in the time t:

$$\omega(t)=\omega(-t). \quad (18)$$

In the regime in which the packets are transmitted through the density barrier, for which the packet velocity $V(t)$ is an even function, $V(-t)=V(t)$, Eqs. (14) and (16) are invariant under the following simultaneous replacements in (7): the replacement of t by $-t$, of ξ by $-\xi$, and of Φ by Φ^*. It follows that for the wave packets in (15) and (17) whose field Φ and density n satisfy relations (8) at the time the center of the packet passes the crest of the barrier there may exist mirror-conjugate dynamic states as in (9).

When the packets are reflected from the barrier, and the packet velocity $V(t)$ is an odd function $[V(t)=-V(-t)]$ with respect to the time ($t=0$) at which the center of the packet crosses the turning point, Eqs. (15) and (17) are invariant under the following simultaneous replacements in (5): the replacement of t by $-t$ and the replacement of Φ by Φ^*. In this case, conjugate solutions (6) exist for the wave fields (15) and (17), which are described by real functions at the time $t=0$. We will first analyze the equations of motion of the packets in (16); then we will take up the evolution of the properties of the packets as they move through the density barrier.

3. EQUATIONS OF MOTION OF THE PACKETS

1. Plasma with a linear density barrier ($p=1$). In this case, the equations of motion of the packets in (16) have the first integral

$$\left(\frac{dz(t)}{dt}\right)^2=\alpha^2\frac{z_r-z(t)}{\omega_m^2+z(t)}, \quad (19)$$

where $\omega_m^2=1+\theta$, $\theta=\mu/\alpha^2$, and z_r is the coordinate of the turning point. Figure 1 shows the phase plane for (19). Part *a* corresponds to values $\mu>0$, and part *b* to $\mu<0$. The hatching along the z axis represents the plasma region. In the former case ($\mu>0$) the wave packet moves in the region $-1\leqslant z\leqslant z_r$, reaching the plasma boundary with velocity

$$V_p=\pm\alpha\left\{\frac{1+z_r}{\theta}\right\}^{1/2}$$

and frequency $\omega_0=\theta^{1/2}$. In the latter case (Fig. 1a), the wave packet moves inside the plasma and does not reach the boundaries of the plasma: $1<-\omega_m^2\leqslant z\leqslant z_r$.

2. Plasma with a parabolic density barrier. In this case ($p=2$), the equations of motion (16) of the packets have the integral

$$\left(\frac{dz}{dt}\right)^2=\frac{C+\alpha^2z^2}{\omega_m^2-z^2}. \quad (20)$$

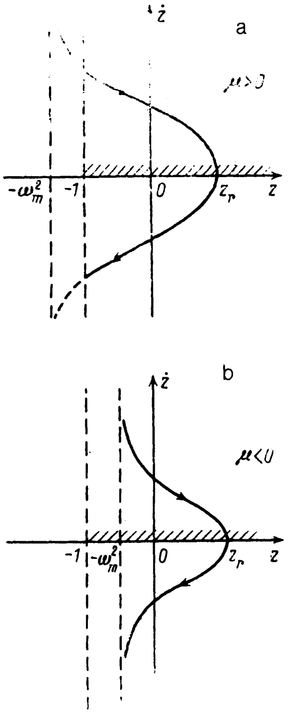

FIG. 1. Phase plane of Eq. (19), which describes the motion of packets of intense electromagnetic field in a plasma with a linear density barrier. a—$\mu>0$; b—$\mu<0$.

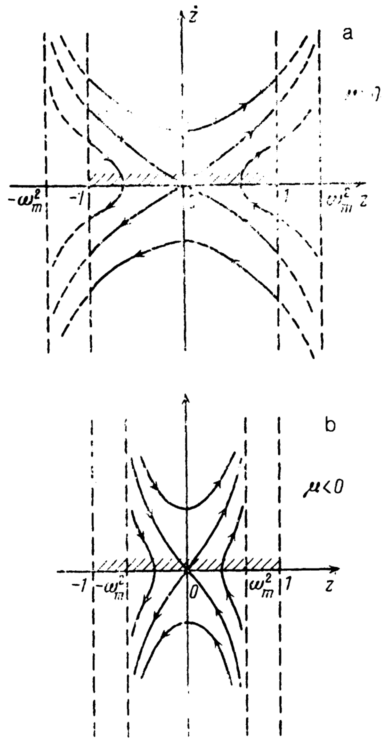

FIG. 2. Phase plane of Eq. (20), which describes the motion of packets of intense electromagnetic field in a plasma with a parabolic density profile. a—$\mu>0$; b—$\mu<0$.

Figure 2 shows the phase plane for (20). Part *a* corresponds to values $\mu>0$, and part *b* to values $\mu<0$. The curves 1 correspond to transmission of the packets through the density barrier. The curves 3 correspond to reflection. The separatrices 2, which separate these two regimes of motion, describe pulling of the electromagnetic field packets up to the crest of the barrier and "sliding down" from the crest. Let us examine these regimes of motion in more detail.

The transmission of the packets through the density barrier corresponds to (20) with $C=\omega_m^2 V_m^2$, where V_m is the packet velocity when the center of the packet, $\xi=0$, crosses the crest of the barrier. When the packet reaches the boundaries of the plasma layer, $z=\pm1$ (Fig. 2a), its velocity at these points satisfies

$$V_p^2(\pm1)=\frac{\alpha^2+V_m^2(1+\theta)}{\theta}. \qquad (21)$$

The frequency ω_0 of the packet at the boundary of the plasma layer is $\theta^{1/2}$. When an electromagnetic pulse of frequency ω_0 is incident from vacuum on a plasma slab with a parabolic density barrier, nonlinear wave packets will evidently be excited in the plasma. The velocity of these packets at the boundary of the plasma will be equal to the velocity of light. If the packet velocity near the crest of the barrier is sufficiently high, $V_m \gg \alpha/\omega_m$, Eq. (20) can be integrated in terms of elementary functions. Under the condition $z(0)=0$, which corresponds to crossing of the crest of the barrier by the center of the packet at the time $t=0$, and under the condition $V_m \gg \alpha/\omega_m$, for example, the solution of (20) is

$$\frac{z}{2}(\omega_m^2-z^2)^{1/2}+\frac{\omega_m^2}{2}\arcsin\frac{z}{\omega_m}=\omega_m V_m t. \qquad (22)$$

The time t_s at which the packets slide down from the crest of the barrier is defined as the time at which the packet reaches the boundary of the barrier: $z(t_s)=\omega_m$. Using (22), we thus find $t_s=\pi\omega_m/4V_m$. The time t_s decreases with increasing velocity of the packet at the crest, $t_s\sim V_m^{-1}$. The time t_n at which the packets of electromagnetic field cross the barrier is twice the sliding time, $2t_s$.

Vanishing of the packet velocity at the crest of the barrier ($V_m=0$) corresponds to a value of zero for the parameter C in (20). In this case the trajectory of the packet in the phase plane of (20) is the separatrix 2 (Fig. 2), on which the velocities V_c and the coordinates z of the center of the packet are related by

$$V_c^2=\frac{\alpha^2 z^2}{\omega_m^2-z^2}. \qquad (23)$$

At the plasma boundaries ($z=\pm1$) in the case $\mu>0$, the packet velocity is given by (21) with $V_m=0$. The value of $V_c^2(\pm1)$ decreases with increasing θ. The part of the separatrix with the negative slope, $\dot{z}/z<0$, corresponds to pulling of the packets to the barrier and stopping of the packets at the crest of the barrier. The region with $\dot{z}/z>0$ corresponds to sliding of the packet down from the crest. From (20) we see that pulling of the packets up to the crest corresponds to the equation

$$\frac{d\eta}{dt} = -\frac{\alpha}{\omega_m}\frac{\eta}{(1-\eta^2)^{1/2}}, \tag{24}$$

where $\eta = z/\omega_m$.

In the case $V < V_c$ the wave packet is reflected from the density barrier. Corresponding to this regime of motion in the phase plane of (20) are curves 3, for which we have $C = -\alpha^2 z_r^2$, where z_r is the coordinate of the turning point found from the condition that the packet velocity vanish: $V(z_r) = 0$. For z_r we thus find

$$z_r^2 = z_0^2 - \frac{V_0^2}{\alpha^2}(\omega_m^2 - z_0^2),$$

where V_0 is the packet velocity at the point z_0. The depth to which the packets penetrate into the dense plasma layers thus increases with increasing velocity V_0.

4. CHANGES IN THE PROPERTIES OF THE PACKETS

We turn now to an analysis of the evolution of the properties of the electromagnetic-field packets described by Eqs. (15) and (17). In view of the finite time spent by the packets in moving through the density barrier in these regimes, and under the assumption that the damping is weak, we can use the dissipationless approximation to analyze the mirror-conjugate and conjugate states of the packets, which are realized when the packets are transmitted through and reflected from the density barrier, respectively. In the pulling regime, the packet reaches the crest of the barrier after an infinitely long time, so dissipation of the electromagnetic field must be taken into account.

We first analyze the evolution of the properties of the packets in the transmission and reflection regimes in a plasma without thermal losses. We consider both mirror-conjugate and conjugate states of the electromagnetic field. We will then analyze the changes in the packet properties in the pulling regime in the case with thermal losses, which result from collisions of plasma particles.

4.1. Conjugate and mirror-conjugate states of the electromagnetic field. We are interested in mirror-conjugate and conjugate solutions of (15), (17), which are realized in the transmission and reflection regimes, respectively. The mirror-conjugate solutions are described at the time $t = 0$ (at which the center of the packet passes the crest of the barrier) by functions which satisfy relations (8). The conjugate solutions which are realized in the case of reflection from the barrier are described at the time $t = 0$ (at which the center of the packet crosses the reflection point) by real functions.

Equations (15) and (17) have been analyzed by numerical methods on the time interval $0 \leqslant t \leqslant t_s$, where t_s is the time at which the packet slides down from the crest of the barrier in the transmission regime or the time at which the packet slides down from the turning point in the reflection regime. The distributions of the density and the field for $t < 0$ for these states can be found from the distributions of the density and the field for $t > 0$ with the help of (9) for the mirror-conjugate states or with the help of (6) for the conjugate states.

The field distribution $\Phi(\xi, 0)$ is specified as an isolated Gaussian pulse

$$\Phi(\xi, 0) = \Phi_0 \exp\left\{-\frac{\xi^2}{L_0^2}\right\}, \tag{25}$$

whose velocity $V(0)$ and frequency $\omega(0)$ are determined by the relations

$$dV(0)/dt = 0,\ V(0) \neq 0,\ \omega(0) = \omega_m, \tag{26}$$

in the transmission case and by the relations

$$V(0) = 0,\ dV(0)/dt \neq 0,\ \omega(0) < \omega_m. \tag{27}$$

in the reflection case. The density distribution $n(\xi,0)$ is found from (15) along with (26) and (27). In particular, if the packet velocity near the crest of the barrier is sufficiently low, $V \ll G^{-1}$, we can ignore the term with the time derivative in (15). For wave packets which are localized in ξ we find

$$n = -\omega(t)\,|\Phi|^2. \tag{28}$$

Equations (15) and (17) were solved numerically for the values $L_0 = 1$ and $G = 1$ and for various values of the packet amplitude Φ_0 and the packet velocity $V(0)$. The nature of the evolution of the packets depends strongly on the relation between V_m and G^{-1}. Figure 3 shows distributions of the envelope A and of the density perturbation n of wave packets $\Phi = Ae^{i\varphi}$ versus ξ at various times for $\Phi_0 = 3$ and $G = 1$ in the transmission regime. The velocity of the packet at the crest of the barrier is $V_m = 0.6 < G^{-1}$ here. The curves 1 correspond to the envelope A, and the curves 2 to the density n. The time t_c at which the packet velocity is equal to the ion acoustic velocity, $V(t_c) = G^{-1}$, is 1.0 for the value of V_m which we chose.

As the packet moves near the crest of the barrier for $V < G^{-1}$ (this value corresponds to the time interval $t < t_c$), for a given value of L_0 there exists a critical value Φ_c: Above this value, waveguide propagation of the wave packet occurs. In other words, the properties of the packet oscillate around slowly varying average values (Fig. 3a, b). If $\Phi_0 > \Phi_c$ holds, the size of the wave packets increases in this

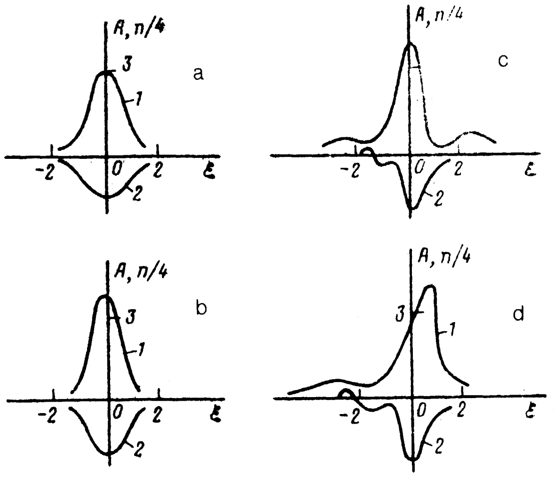

FIG. 3. The envelope A (curve 1) and the perturbation of the density n (curve 2) of wave packets $\Phi = Ae^{i\varphi}$ versus ξ at various times t in the regime in which the packets are transmitted through a parabolic density barrier. The wave packet $\Phi(\xi,0)$ is specified as a Gaussian pulse as in (25) with $\Phi_0 = 3$, $L_0 = 1$, and a velocity $V_m = 0.6 < G^{-1}$ at the time at which the center of the packet passes the crest of the barrier ($t = 0$). a—The time $t = 0$; b—$t = 0.23$; c—$t = 1.16$; d—$t = 1.86$.

time interval. Near the crest of the barrier, the evolution preserves the shape of the pulse. With increasing distance from the packet to the crest of the barrier, distortions of the pulse shape become important (Fig. 3c). At $t \approx t_c$, the packet velocity is close to the velocity of ion acoustic waves, so the latter waves are excited. It follows from the calculated results that the ion acoustic waves are excited on the time interval defined by the inequality

$$\left| V^2(t) - \frac{1}{G^2} \right| \leqslant 0{,}3.$$

After an ion acoustic wave has been excited, it lags behind the core of the packet. As a result of recoil, the core of the packet acquires an additional acceleration (Fig. 3d). In addition, for $t > t_c$ the core of the packet spreads out.

Without going into a detailed analysis of the evolution of packets in the regime of conjugate states, we would like to point out that the dynamics of the packet in this case is qualitatively similar to that observed in the regime of mirror-conjugate states for $V(0) \ll 1/G$.

It should also be noted that for conjugate states in a plasma with a linear density barrier, and at velocities $V(t) \ll 1/G$, which correspond to velocities of nonlinear wave packets which are low in comparison with the velocity of ion acoustic waves, an exact description of these states is possible as well as an approximate nonaberrational description. In this case, Eq. (14) for the envelope Φ, with a local nonlinearity of the ponderomotive type [see (28)], has the form of the nonlinear Schrödinger equation in a homogeneous medium:

$$-2i\frac{\partial \Phi}{\partial t} + \frac{\partial^2 \Phi}{\partial \xi^2} + \alpha^2 |\Phi|^2 \Phi = 0.$$

One solution of this equation is the well-known soliton

$$\Phi(\xi, t) = \frac{\Phi_0}{\alpha \operatorname{ch}(2^{1/2}\Phi_0 \xi)} \exp\{i\Phi_0^2 t\}.$$

The solution for the electric field amplitude in this case is

$$E(\xi, t) = \frac{\Phi_0(\omega(t))^{1/2}}{\alpha \operatorname{ch}(2^{1/2}\Phi_0 \xi)} \exp\{-i\chi(\xi, t)\},$$

$$\chi(\xi, t) = -\Phi_0^2 t - V(t)(\omega(t))^{1/2}\xi + \int \left(\omega(t) - \frac{\mu}{2\omega(t)}\right) dt$$

and is a generalization of the solution derived by Chen[7] to the case in which there is a significant variation in the soliton carrier frequency $\omega(t)$.

4.2. Evolution of packets in the pulling regime. Equation (24) corresponds to pulling of the electromagnetic-field packets to the crest of the density barrier. As the center of the packet approaches the crest, its velocity tends toward zero. This tendency means that thermal losses must be taken into account for field packets near the crest of the barrier. In a plasma with a parabolic density profile and thermal losses due to particle collisions, the equation for the envelope Φ of the field packets becomes

$$-2i\frac{\partial \Phi}{\partial t} + \frac{\partial^2 \Phi}{\partial \xi^2} + \alpha^2\left\{\frac{\xi^2}{\omega^2(t)} - \frac{n}{\omega(t)} - iq\omega^2(t)\right\}\Phi = 0. \qquad (29)$$

Here

$$q = N_0^{1/2}\,(\text{cm}^{-3}) \cdot T_e^{-3/2}(\text{K}).$$

Multiplying (29) by Φ^*, subtracting the complex conjugate of the result from the result, and integrating over ξ from $-\infty$ to $+\infty$, we find the following relation for wave packets which are localized in ξ [$\Phi(\xi \to \pm\infty, t) \to 0$]:

$$\frac{dW}{dt} = -\alpha^2 q \omega^2(t) W(t). \qquad (30)$$

Here

$$W(t) = \int_{-\infty}^{\infty} |\Phi|^2 d\xi$$

is the energy of the wave packet. In (30), we switch from the variable t to the variable coordinate of the center of the wave packet, η. As a result we find from (30), with (24),

$$W(\eta) = W_0 \exp\left\{-\alpha q \omega_m^4 \int_{\eta_0}^{\eta} \frac{(1-\eta^2)^{3/2}}{\eta} d\eta\right\}. \qquad (31)$$

It follows from (31) that as the packet moves far from the crest (in the case $|\eta - 1| \ll 1$) the absorption of electromagnetic field energy is insignificant. As the packet approaches the crest ($\eta \to 0$), the absorption rate increases substantially. It can thus be concluded that electromagnetic field energy can be absorbed in dense plasma layers near the crest of the density barrier. In the case $\eta_0 = 1$, for example—this case corresponds to excitation of the packet at the plasma-vacuum interface—the solution of (30) is

$$W(\eta) = W_0 \exp\{-\alpha q \omega_m^4 [{}^1/_3 (1-\eta^2)^{3/2} + (1-\eta^2)^{1/2} - \operatorname{arth}(1-\eta^2)^{1/2}]\}. \qquad (32)$$

We define the packet penetration depth η_d by $W(\eta_d) = W_0 e^{-1}$.

In particular, at values $\eta_d \ll 1$, which correspond to an absorption of electromagnetic field energy near the barrier crest, the field penetration depth found from (32) is

$$\eta_d = 2\exp\left(-\frac{1}{\alpha q \omega_m^4}\right).$$

This expression is valid if $\alpha q \omega_m^4 \gg 1$.

Let us specify the range of applicability of this analysis. It is legitimate to ignore the thermal losses in the reflection and transmission regimes if t_{abs} (the time scale of the absorption of electromagnetic field energy) is greater than the time taken by the packet frequency $\omega(t)$ to return to its initial value ω_0. The value of the packet energy at the time t_0 at which the frequency $\omega(t)$ returns to its initial value $\omega_0 = \omega(-t_0)$ is, according to (32),

$$W(t_0) = W(-t_0) \exp\left\{-\alpha^2 q \left[\int_{-t_0}^{0} \omega^2(t)\,dt + \int_{0}^{t} \omega^2(t)\,dt\right]\right\}. \qquad (33)$$

The time $t = 0$ corresponds to the time at which the center of the packet passes the barrier crest in the transmission regime or to the time at which the reflection point is passed in the reflection regime. Expression (33) can be put in the following form if we make the replacement $t \to -t$ in the first integral and make use of (24):

$$W(t_0) = W(-t_0)\exp\left\{-2\alpha^2 q\int_0^{t_0}\omega^2(t)\,dt\right\}. \qquad (34)$$

For the case in which packets pass through the density barrier with a fairly high velocity $V_m \gg (\omega_m)^{1/2}$, we find from (34) and (20)

$$W(\eta_0) = W(-\eta_0)\exp\left\{-\alpha^2 q\frac{\omega_m}{2V_m}[\eta_0(1-\eta_0^2)^{1/2}(5/2 - \eta_0^2)] + 3/4\arcsin\eta_0\right\}. \qquad (35)$$

We define the absorption time t_{abs} as the time over which the energy of the electromagnetic field decreases by a factor of e from its initial value. From (35) we thus find that the interval of initial coordinates η_0 of the center of the packet in which the time $2t_0$ over which the packet returns to its initial value η_0 is greater than the absorption time t_{abs} is

$$\frac{\alpha^2 q\omega_m}{2V_m}\{[\eta_0(1-\eta_0^2)^{1/2}(5/2-\eta_0^2)] + 3/4\arcsin\eta_0\} < 1. \qquad (36)$$

For motion of the packet out of the region with $\eta_0 \approx 1$ it is legitimate to ignore the thermal losses under the condition [here we are using (36)]

$$\frac{3\pi}{16}\alpha^2 q\frac{\omega_m}{V_m} < 1.$$

We note in conclusion that this analysis of the nonlinear dynamics of electromagnetic-field packets in inhomogeneous plasmas leads to the conclusion that intense electromagnetic fields can penetrate into and pass through dense plasma slabs.

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