

# Stochastic model of high-pressure microwave discharge

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A method is proposed for calculating the distribution function of the plasma parameters of a high-pressure microwave discharge. The method is based on the assumption that the electromagnetic field is randomized in the discharge region. A differential equation describing the evolution of the distribution function of the plasma parameters is derived in the framework of the model of an electromagnetic field fully randomized in phase.

## INTRODUCTION

The most interesting type of experimentally observed microwave discharge is apparently the non-autonomous microwave discharge investigated in sufficient detail both experimentally<sup>1–3</sup> and theoretically.<sup>4–12</sup> A distinguishing feature of a discharge of this type is that the initially homogeneous plasma background produced by an external pre-ionizer evolves during the nonlinear stage into a strongly inhomogeneous spatial structure perceived as a three-dimensional stochastic spider web made up of brightly glowing plasma filaments separated by dark regions. This formation, having a strongly inhomogeneous spatial distribution of the plasma parameters and hence of the electron density, should scatter intensely the heating electromagnetic wave and lead to formation of a strongly inhomogeneous spatial structure of the microwave field in the discharge region.

The theoretical investigations of discharges of this type constitute a broad spectrum of problems, starting with the analysis of the linear stage of discharge evolution<sup>4–8</sup> and ending with various models of nonlinear plasma formations.<sup>9–12</sup> An investigation of the linear microwave-discharge stage has revealed the mechanism responsible for violation of the initial homogeneity of the plasma background through development of one of the types of plasma instability in a microwave field, namely, ionization–field<sup>4,7,8</sup> of ionization–superheat<sup>6</sup> instabilities. The nonlinear stage of the ionization–superheat instability was considered in an investigation<sup>9</sup> of the evolution of a single plasma filament in a heating microwave field of specified amplitude. It was observed that the evolution is explosive and leads to collapse of the filament.<sup>9</sup> Formation of screw solitons was investigated in Ref. 10. An evolution problem, whose formulation took quite adequately into account the specific features of a molecular-gas plasma in a microwave field, was considered in Ref. 11. Its formulation took quite adequate account of the peculiarities of heating a molecular-gas plasma in a microwave field, making it possible to distinguish between different characteristic stages of the plasma evolution. Self-similar solutions corresponding to vibrational-translational nonequilibrium plasma structures in a microwave field were investigated in Ref. 12 in a one-dimensional formulation.

The models proposed in the cited papers turned out, however, incapable of describing a steady-state microwave discharge, in which an important role is played by stochasticization of the electromagnetic field in the discharge region. Therefore, generally speaking, effects connected with stochasticization of the microwave field must be taken into ac-

count even when determining the evolution of a single plasma filament. This means consideration of the incidence, on a plasma filament, of a random microwave field (i.e., having an amplitude that varies randomly with time).

We discuss below the simplest stochastic model of a high-pressure microwave discharge.

## 1. FORMULATION OF PROBLEM

A correct description of the interaction of a microwave field with discharge-plasma inhomogeneities requires a simultaneous solution of the equations for the plasma and the equations for the field. Let  $\mathbf{X}$  be a certain state vector describing the evolution of a plasma in a microwave field. (The state vector can contain all sorts of parameters, such as the density, temperature, and directional velocity of the electrons and of other plasma components, describable by a system of hydrodynamic equations.  $\mathbf{X}$  can include if necessary also the populations of the internal vibrational and other degrees of freedom of the molecules, etc.) The system of equations defining the evolution of  $\mathbf{X}$  can in general be written in the form

$$\frac{\partial \mathbf{X}}{\partial t} = \nabla D \nabla \mathbf{X} + \mathbf{F}(\mathbf{X}, E_a), \quad (1)$$

where  $D$  is the tensor of the diffusion coefficients and of the thermal conductivity,  $\mathbf{F}(\mathbf{X}, E_a) = (F_1, \dots, F_n)$  is the nonlinear source term that depends both on the plasma parameters and on the local field amplitude  $E_a(r, t)$ :  $E(r, t) = E_a \exp(i\omega_0 t)$ . It is assumed that all the processes in the plasma are slow compared with the characteristic time  $\tau_\omega = 1/\omega_0$  of field variation. The processes taken into account in (1) are each defined by its own characteristic time scale  $\tau_i$  that is subject to a certain hierarchy

$$\tau_\omega \ll \tau_1 < \dots < \tau_i < \dots < \tau_n,$$

and a length  $L_i$ .

The system (1) must be supplemented by a field equation

$$\Delta \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla (\nabla \cdot \mathbf{E}) = \frac{4\pi}{c^2} \frac{\partial \mathbf{J}}{\partial t}, \quad \mathbf{J} = \sigma \mathbf{E}, \quad (2)$$

where  $\sigma$  is the electric conductivity of the plasma in the microwave field.<sup>8</sup>

The problem (1)–(2) as formulated is difficult to solve even in the steady state, since account must be taken of the scattering of the microwave radiation by the essentially inhomogeneous spatial structure. However, as shown below,

owing to the spatial disorder of the plasma filaments, this problem lends itself to a simple enough formulation in the context of the theory of stochastic processes.

1. Experiment<sup>1,2</sup> has shown that in a high-pressure microwave discharge the characteristic transverse dimension  $L_{pl}$  of the produced plasma formations is much larger than all the diffusion and heat-conduction lengths  $L_i$  ( $L_{pl} \gg L_i$ ), so that the corresponding processes are effective in a narrow boundary region  $G_3$  separating the strongly and weakly ionized  $G_2$  and  $G_1$  phases of the microwave discharge, i.e.,  $G_3 \ll G_2 < G_1$ . Recognizing that the main purpose of the investigation is to obtain the distribution function  $P(\mathbf{X})$ , we can neglect the region  $G_3$  in the zeroth approximation with respect to the small parameter  $\gamma = G_3/G_0$ , since

$$P(\mathbf{X}) = \sum_{i=1}^3 P_i(\mathbf{X}) \frac{G_i}{G_0}, \quad G_0 = \sum_{i=1}^3 G_i,$$

i.e., instead of investigating the evolution of the distribution of the system (1) we consider the evolution of a lumped system (with the diffusion and heat-conduction terms omitted). A valid neglect of the region  $G_3$  requires satisfaction of a condition imposed on the characteristic time  $\tau_f$  of displacement of the interphase boundary:  $\tau_x \ll \tau_f = L_f/v_f$ , (where  $\tau_x$  is the characteristic time of the problem and  $L_f$  is the characteristic scale of the front). In fact, a set of equations such as (1), while having several stationary homogeneous solutions, has also spatially inhomogeneous solutions in the form of traveling fronts of parameter switching from one phase to another (see, e.g., Ref. 12), and if the inverse condition  $\tau_x \gg \tau_f$  were met the plasma evolution would take place mainly in a thin interphase layer rather than in the regions  $G_1$  and  $G_2$ . In our case the velocity  $v_f$  is determined by slow heat-conduction processes<sup>10</sup> and neglect of  $G_3$  is therefore justified.

Note that neglect of the region  $G_3$  imposes certain restrictions on the possible characteristic time  $\tau_p$  of the problem. In fact, by neglecting the region  $G_3$  we exclude from consideration all the equations of motion of the plasma components. Two cases are possible then,  $\tau_p > \tau_{mi}$  or  $\tau_p < \tau_{mi}$ , where  $\tau_{mi}$  is the characteristic pressure relaxation time of the  $i$ -component.

2. As already mentioned, the presence of inhomogeneities with large plasma parameters leads to intense scattering of the microwave radiation and to formation of an inhomogeneous electromagnetic-field structure in the discharge region. From this standpoint we can liken the plasma of a high-pressure microwave discharge to a system of  $N$  spatially uncorrelated scatterers. The field at some selected point can then be written in the form

$$\mathbf{E}_a(\mathbf{r}, t) = \mathbf{E}_0 + \sum_{j=1}^N \mathbf{E}_j \exp(i\varphi_j),$$

where  $\mathbf{E}_0$  is the amplitude of the heating microwave in the absence of scattering. (The solution of Eq. (2) can be formally represented by a continual path integral, where  $\mathbf{E}_j$  and  $\varphi_j$  must be taken to mean the values of these parameters on the corresponding path, and  $N$  is the number of paths entering the chosen point of the volume. Since the scatterers are spatially uncorrelated,  $\mathbf{E}_j(r, t)$ , and most importantly  $\varphi_j(r, t)$ , will be random quantities. Owing to the random

character of the scattered field, new scatterers (plasma filaments) will be formed in certain regions where  $|\mathbf{E}| > \mathbf{E}_{cr}$  ( $\mathbf{E}_{cr}$  is the breakdown field), and conversely, some of the scatterers will relax, being located in the field "shadow" region. The general picture, however of the microwave discharge will consist in this case of random creation and annihilation of plasma filaments, accompanied by a global restructuring of the electromagnetic field in the discharge region.

3. The characteristic time scales of the correlations of the field-amplitude fluctuations, due to scattering by plasma inhomogeneities

$$\langle \mathbf{E}'_j(t_1) \mathbf{E}'_k(t_2) \rangle \propto \delta_{jk} E_j'^2(t_1) \exp(-|t_1 - t_2|/\tau_c),$$

where  $\mathbf{E}'_j = \mathbf{E}_j - \mathbf{E}_0$ , and  $\tau_c$  is the characteristic correlation time, will obviously correspond to the characteristic evolution time of the electronic plasma density, i.e., it can be roughly assumed that  $\tau_c \sim \tau_{ne}$ , where  $\tau_{ne}$  is a certain average evolution time of the electron density (the time  $\tau_{ne}$  is too short for the electron density, and hence for the field structure, to change in the microwave-discharge region).

We choose the characteristic time of the problem such that  $\tau_p > \tau_{ne}$ , in which case the random force  $E_a(r, t)$  can be regarded in the zeroth approximation in the small parameter  $\tau_c/\tau_p$  as a random process  $\delta$ -correlated in time.

4. For a correct formulation of the problem of determining the distribution function, we must know the statistical properties of the electromagnetic field in the discharge region. In the simplest case, a model that is fully randomized in phase and in field direction can be used. (This assumption is reasonable, since individual scatterers with  $L_{pl} \ll \lambda$  have a spherically symmetric scattering indicatrix, and as  $N \rightarrow \infty$  the field direction should be randomized, i.e., the field inside the charge will "forget" the side from which the heating field came.) In this approximation we obtain for the function  $G(|\mathbf{E}_a|)$ , with allowance for the symmetry

$$G(\mathbf{E}) = \prod_{i=1}^3 G(\mathbf{E}_i),$$

and also with allowance for the circumstance that  $G_2 \ll G_1$  (i.e., it can be assumed that the field penetrates unhindered into the region  $G_0$ , and consequently  $\langle |\mathbf{E}_i|^2 \rangle = \frac{1}{3} |\mathbf{E}_0|^2$ ), the expression

$$G(|\mathbf{E}_a|) = A |\mathbf{E}_a|^2 \exp(-3|\mathbf{E}_a|^2/2|\mathbf{E}_0|^2). \quad (3)$$

The problem is thus completely formulated: we must find the distribution function  $P_i(X)$  of the plasma parameters whose evolution obeys the dynamic equations (1). The plasma is acted upon in general by a non-Gaussian random force  $f$  having a distribution function that is compatible at any instant of time.

We confine ourselves below to the case of one parameter (generalization to any case is easy) for a system with a Joule heat release

$$\frac{\partial X}{\partial t} = F(X) + \sigma |\mathbf{E}_a|^2. \quad (1a)$$

It will be more convenient here to assume that  $f = |\mathbf{E}_a|^2$  and has a distribution function

$$G(f) = A f^h \exp(-3/2 f / \langle f \rangle), \quad \langle f \rangle = |\mathbf{E}_0|^2. \quad (3a)$$

**2. DERIVATION OF DIFFERENTIAL EQUATION FOR THE DISTRIBUTION FUNCTION  $P_t(X)$**

The differential equation for the function  $P_t(X)$  can be obtained by a method described in Klyatskin's book,<sup>13</sup> by differentiating  $P_t(X) \equiv \langle \delta(X - \xi(t)) \rangle$  with respect to time, using the properties of the  $\delta$ -function, and taking the non-random factor outside the averaging sign,

$$\frac{\partial P_t(X)}{\partial t} = -\frac{\partial}{\partial X} \{F(X)P_t(X)\} - \frac{\partial}{\partial X} \langle \sigma(X)f\delta(X-\xi(t)) \rangle. \quad (4)$$

To decouple correlations of the type  $\langle f(t)R[f] \rangle$  we introduce a certain functional of a random process

$$\theta_{t'}[v] = \left\langle f(t') \exp \left\{ i \int_0^{t'} d\tau f(\tau)v(\tau) \right\} \right\rangle / \Phi[v], \quad (5)$$

where  $\Phi[v]$  is a characteristic functional of the random process  $f$ , and  $\Phi[v] = \langle \exp\{i \int_0^t d\tau f(\tau)v(\tau)\} \rangle$ . In the case of the correlations of interest to us,  $\theta_{t'}$  can be represented as  $t' \rightarrow t$  in the form

$$\theta_{t'}[v] = \frac{1}{iv(t)} \theta_t[v], \quad (6)$$

where  $\theta = \ln(\Phi[v])$ ,  $\theta_t[v] = d\theta/dt$ .

With the aid of (5) and (6) we can rewrite the correlation from (4) in the form

$$\frac{\partial}{\partial X} \langle \sigma(X)f(t)\delta(\xi(t)-X) \rangle = \left\langle \theta_t \left[ \frac{\delta}{i\delta f(\tau)} \right] \delta(\xi(t)-X) \right\rangle. \quad (7)$$

It is recognized in the derivation of (7) that

$$\delta X/\delta f(t) = \sigma(X).$$

The differential equation for  $P_t(X)$

$$\frac{\partial P_t(X)}{\partial t} = -\frac{\partial}{\partial X} \{F(X)P_t(X)\} + \left\langle \theta \left[ \frac{\delta}{i\delta f(\tau)} \right] \delta(\xi(t)-X) \right\rangle, \quad (8)$$

written in operator form, is thus an exact consequence of the initial dynamic system (1a) with allowance for (3a). To make more specific the correlation-containing expression in the right-hand side of (8), we must determine the explicit forms of the functionals  $\theta_t[v]$  and  $\Phi[v]$ . The functional  $\Phi[v]$  can be represented by a functional expansion in the  $n$ th moments of the random process  $f$

$$\Phi[v] = \sum_{n=1}^{\infty} \frac{i^n}{n!} \int_0^t d\tau_1 \dots d\tau_n \langle f_1 \dots f_n \rangle v_1 \dots v_n, \quad f_i = f(\tau_i),$$

where  $\langle f_1 \dots f_n \rangle = (1/i^n) (\delta\Phi[v]/\delta v_1 \dots \delta v_n) |_{v=0}$ .  $\theta(v)$  can in turn be written in the form

$$\theta[v] = \sum_{n=1}^{\infty} \frac{i^n}{n!} \int_0^t d\tau_1 \dots d\tau_n K_n(\tau_1, \dots, \tau_n) v_1 \dots v_n, \quad (9)$$

where the cumulants  $K_n$  are given by

$$K_n = \frac{1}{i^n} \frac{\delta\theta}{\delta v_1 \dots \delta v_n} \Big|_{v=0}.$$

Recognizing that  $\theta = \ln(\Phi)$  and introducing  $f' = f - f_0$  and

$f_0 = |E_0|^2$ , we obtain (see also Ref. 14)

$$K_1 = \langle f \rangle = f_0, \quad K_n = \langle f_1' \dots f_n' \rangle. \quad (10)$$

Expressions for  $K_n$  can be derived also directly from the form of the functional  $\Phi[v]$ , by recognizing that

$$K_1 = \frac{1}{i\Phi} \frac{\delta\Phi}{\delta v_1} \Big|_{v=0} = f_0 + \frac{1}{i\Phi'} \frac{\delta\Phi'}{\delta v_1} \Big|_{v=0},$$

where  $\Phi'[v] = \langle \exp(i \int d\tau f'(\tau)v(\tau)) \rangle$  with  $\langle f' \rangle = 0$ . At the same time, the expression

$$\frac{\delta\theta'}{\delta v_1 \dots \delta v_n} \Big|_{v=0} = \frac{1}{\Phi} \frac{\delta\Phi'}{\delta v_1 \dots \delta v_n} \Big|_{v=0}$$

is valid for the random process  $f'$  with  $\langle f' \rangle = 0$ , and we arrive at (10).

To specify further expression (9) we must know the correlation characteristics of the process  $f$ , using for this purpose some model of the random process  $f$ . In our case, in the zeroth approximation in the small parameter  $\tau_c/\tau_p$ , we can regard  $f$  as a random process  $\delta$ -correlated in time. The correlations  $\langle f_1' \dots f_n' \rangle$  taken then the form

$$K_n'(\tau_1, \dots, \tau_n) = B_n(\tau_1) \delta(\tau_1 - \tau_2) \dots \delta(\tau_{n-1} - \tau_n), \quad (11)$$

where  $B_n(\tau)$  is determined from the conditions

$$B_n(\tau_1) = \int_0^t d\tau_1 \dots d\tau_n \langle f_1' \dots f_n' \rangle \propto \tau_c^{n-1} \langle f'(\tau_1) \rangle$$

( $\tau_c$  is the characteristic correlation time of the random process  $f'$ .)

Knowing  $\langle f^n \rangle = \int df G(f) f^n$  and  $\langle f'^n \rangle$  we can obtain  $K_n'$  and hence  $\theta'[v]$ . (In particular,  $B_1 = 0$ ,  $B_2 = \frac{2}{3} \tau_c f_0^2$ ,  $B_3 = -5(\frac{2}{3})^2 \tau_c^2 f_0^3$ , etc.) It is more convenient, however, to obtain an explicit form of the functional  $\theta'[v]$ . Substituting (11) in (9) and differentiating the results once with respect to time we get

$$\theta_t'[v] = \sum_{n=1}^{\infty} \frac{i^n}{n!} B_n(\tau) v^n(\tau), \quad B_n(\tau) = \frac{1}{i^n} \frac{d\theta_t'}{dv^n} \Big|_{v=0}. \quad (9a)$$

For a  $\delta$ -correlated process, generally speaking, it suffices to know in place of the characteristic functional  $\Phi[v]$  the characteristic function  $\psi = \langle \exp(if'v) \rangle = \int df' G(f') \exp(if'v)$ , which is easy to calculate

$$\psi(v) = (1 - \frac{2}{3} if_0 v)^{-3/2}.$$

Since the characteristic correlation time is finite, we have finally

$$\theta_t'(v) = \tau_c^{-1} \exp(-if_0 v \tau_c) (1 - \frac{2}{3} i \tau_c f_0 v)^{-3/2} \quad (12)$$

[Eqs. (12) yield for  $B_n$  the same values as determined from (10).] Using (12), we can rewrite (9) as

$$\frac{\partial P_t(X)}{\partial t} = -\frac{\partial}{\partial X} \{F(X) + \sigma(X)f_0\} P_t(X) + \theta_t' \left[ i \frac{\partial}{\partial X} \sigma(X) \right] P_t(X). \quad (13)$$

The operator notation in (13) must be understood in the

sense of a cumulant expansion of (9a) with allowance for (12).

We introduce a new independent variable  $Z = \int dX / \sigma(X)$  and a new function  $Q(Z) = P(X)\sigma(X)$ , and then the differential equation takes the form

$$L[Q] \equiv \frac{\partial Q(Z)}{\partial t} - \hat{\theta}_i' \left[ i \frac{\partial}{\partial Z} \right] Q(Z) = - \frac{\partial}{\partial X} \{ F'(Z) Q(X) \}, \quad (14)$$

$$F'(Z) = F(Z)/\sigma(Z) + f_0.$$

The solution of the new equation (14) for  $Q(Z)$  will also be a solution for (13), since the normalization is conserved. In fact,  $\int_Z dZ Q(Z) = \int_X dX P(X) = 1$ . The simplest stationary solution, under the condition that there exists at least one stationary point (1a) when  $|E_a| = |E_0|$ , we obtain in the quasiclassical approximation, confining ourselves to  $K_n$  with  $n \leq 3$ ,

$$P_i(X) = \frac{C'}{\sigma(X)} \exp \left\{ \int_0^X \frac{dX}{\sigma(X)} \varphi(X) \right\}, \quad (15)$$

where  $\varphi = - (3/K_3) \{ K_2/2 - [(K_2/2)^2 + \frac{4}{3} K_3 (F')^{1/2}] \}$ . For the long-wave approximation (15) to be valid we must have  $\dot{\varphi} \ll \varphi^2$  and in addition the contribution of the discarded terms with  $n > 3$  must be small enough. If the dynamic system has no stationary solutions we must consider the evolution problem with initial conditions  $t = 0$  and  $Q(Z) = \delta(Z - Z_0)$ .

### 3. MODEL OF VIBRATIONALLY TRANSLATIONAL NONEQUILIBRIUM STRUCTURES IN A STOCHASTIC MICROWAVE FIELD

We proposed in Ref. 12 for a high-pressure microwave discharge a very simple model of vibrationally translational nonequilibrium structures describable for times  $\tau_p \sim 1/K^{e-v} N_{eh} \sim 10^{-6} \text{ s}^{-1}$  (the subscripts  $h$  and  $l$  designate parameters corresponding to high and low ionizations of the microwave discharge, and  $K^{e-v}$  is the coefficient of the rate of excitation of the vibrational levels by electron impact) by the set of equations

$$\sigma E_a^2 - Q^{e-v} - Q^{e-x} = 0, \quad \sigma = \frac{\nu_e}{2M_e} \frac{N_e e^2}{\nu_e^2 + \omega_0^2},$$

$$Q^{e-x} = \hbar \omega_x N_g N_e K^{e-x},$$

$$N_e (\nu^i - \nu^r) - \beta N_e^2 + Y = 0, \quad (16)$$

$$C^v N_g \frac{\partial T_v}{\partial t} = Q^{e-v} - Q_0^{e-v} \equiv F_v(T_v),$$

where  $Q^{e-v}$  and  $Q^{e-x}$  stand for the energy dumped by the electrons in the vibrational and internal degrees of freedom of the molecules, respectively;  $\hbar \omega_x$  and  $\hbar \omega_e$  are the excitation energies of the corresponding levels;  $N_g$  and  $N_e$  are the densities of the gas molecules and of the electrons;  $\nu^i$  and  $\nu^r$  are the ionization and recombination frequencies;  $\beta$  is the dissociative-electron-sticking coefficient;  $Y$  is the intensity of the external preionization source,  $C^v$  is the vibrational heat capacity of the gas, and  $Q_0^{e-v}$  is the fraction of the energy, determined for the background plasma parameters, lost by the electrons in collisions with molecules and excitation of vibrational levels.<sup>12</sup>

The system (16) has one, two, or three homogeneous

stationary solutions, depending on the value of the bifurcation parameters  $E_0^2$  of the problem.<sup>12</sup> If the amplitude of the heating field is in the range corresponding to the presence of several stationary states of the plasma, it is natural to expect stratification of the microwave-discharge volume into regions occupied by phases with different parameters.

The described approach can yield the distribution function  $P_i(T_v)$  of the vibrational temperature. For the stationary case we obtain in the long-wave approximation

$$P_i(T_v) = \frac{C'}{\sigma(T_v)} \exp \left\{ \int_{T_{vi}}^{T_v} \frac{dT_v}{\sigma(T_v)} \varphi(T_v) \right\},$$

$$\varphi(T_v) = \frac{9}{20} (\tau_e f_0)^{-1} \left\{ 1 - \left[ 1 + \frac{40}{3} \frac{F'(T_v)}{f_0} \right]^{1/2} \right\}.$$

where  $\tau_e \sim (1/\nu^i)_{Te=Tel}$ ;  $T_{vi}$  is the vibrational parameter in the weakly ionized phase;  $C'$  is a normalization factor.

Knowing the distribution functions  $P_i(T_v)$  we can obtain in the usual manner the average microwave-discharge characteristics, including the average microwave power  $W = \langle \sigma |E_a|^2 \rangle_{T_v, E_a}$  absorbed per unit volume

$$W = |E_0|^2 C' \int_{T_{vi}}^{\infty} dT_v \exp \left\{ \int_{T_{vi}}^{T_v} \frac{dT_v}{\sigma(T_v)} \varphi(T_v) \right\}. \quad (17)$$

Results of a numerical calculation, using (17), for experimental conditions,<sup>1</sup> i.e., for a volume non-autonomous microwave discharge in molecular nitrogen at atmospheric pressure, are shown in Figs. 1 and 2. It follows from (16) and (17) that the average power  $W$  absorbed per unit volume, and also  $W/W_0$  (where  $W_0 = \sigma_1 E_0^2$  is the power determined from the plasma background parameters), depend parametrically on the intensity  $Y$  of the external ionizer and on the characteristic correlation time  $\tau_c$  of the field. Calculation shows that the dependences of  $W$  and  $W/W_0$  on  $\tau_c$  are weak, and at  $10^{-7} < \tau_c < 10^{-9}$  s the ratio  $\Delta W/W$  does not exceed 10%.

As seen from Fig. 2, at  $Y \ll 10^{15} \text{ cm}^{-3} \text{ s}^{-1}$  the stochastic character of the field in the discharge region has practically no effect on the plasma absorptivity, i.e., the background remains homogeneous. At  $Y \gg 10^{15} \text{ cm}^{-3} \text{ s}^{-1}$ , conversely, the ratio  $W/W_0$  exceeds unity substantially. In the parameter ranges  $10^3 < E_0 < 10^4$  [V/cm] and  $10^{14} < Y < 10^{17} \text{ cm}^{-3} \text{ s}^{-1}$  the average absorbed power is well approximated by the relation

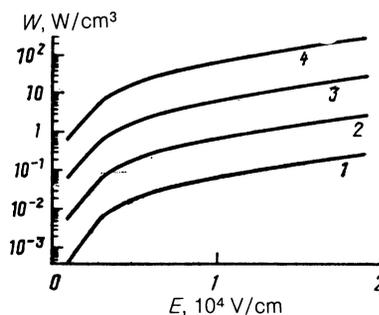


FIG. 1. Volume-averaged specific heat power absorbed by the plasma in a microwave discharge vs the heating-wave amplitude. 1- $Y = 10^{14} \text{ cm}^{-3} \text{ s}^{-1}$ ; 2- $10^{15}$ ; 3- $10^{16}$ ; 4- $10^{17}$ .

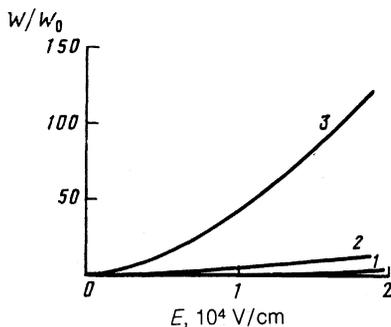


FIG. 2. Influence of field stochasticization on the absorptivity of a microwave discharge plasma. 1- $Y = 10^{15} \text{ cm}^{-3} \text{ s}^{-1}$ ; 2- $10^{16}$ ; 3- $10^{17}$ .

$$W = 4.33 \cdot 10^{-11} E_0^{3.32} Y^{0.2},$$

where  $W$  is in  $\text{W/cm}^3$ ,  $E_0$  in  $\text{V/cm}$ , and  $Y$  in  $\text{cm}^{-3} \text{ s}^{-1}$ .

We conclude by considering a question connected with the spatial correlations of the plasma parameters in the volume of a microwave discharge. Obviously, the analysis above yields no information whatever about the structure of the produced plasma formations. To estimate the characteristic dimensions of the plasma inhomogeneities it is necessary therefore to resort to additional considerations. Generally speaking, the spatial correlations of the plasma parameters are in fact determined by the diffusion and thermal-conductivity terms omitted from (1), so that it is possible to estimate the minimum transverse dimension  $L_{\min} \propto (\lambda_g T_v / \sigma_h E_0^2)^{1/2}$ , of the "seed" of the highly ionized phase, where  $\lambda_g \propto N_g T_g / M_g \nu_g$ ;  $\nu_g$  is the thermal-conductivity of the gas,  $\nu_g$  is the frequency of momentum loss by the

gas molecules, and  $T_g$  is the translational temperature of the gas.<sup>12</sup> On the other hand, the maximum transverse dimension of the gas will obviously be determined by the skin effect, i.e.,  $L_{\max} \propto \text{Im}(\omega_0 \epsilon_h^{1/2} / C)$ , where  $\epsilon_h$  is the dielectric constant of the highly ionized phase of the plasma and  $\epsilon_h = 1 - 4\pi\sigma_h / \omega_0$ .

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