Spin polarization of electrons by an electric current

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A theory is derived for the polarization of electron spins upon the flow of an electric current through a heterostructure or a crystal with a spin splitting of the spectrum which is linear in the momentum. As the current flows, the spins become oriented in the course of spin relaxation of thermalized carriers. Two mechanisms for spin polarization are examined. The first is a precession mechanism, involving an orientation of the spin in the effective magnetic field induced by the current. The second mechanism involves spin-dependent scattering: A deviation of the electron momentum distribution from equilibrium in the course of scattering accompanied by spin flip leads to nonequilibrium filling of spin states. For spin–orbit scattering, a leading role is played by interference of transitions which are and are not accompanied by spin flip. The interference contribution can be comparable to the precession contribution even if the spin–orbit scattering does not contribute significantly to the spin relaxation rate. The magnitude of the spin polarization depends strongly on the scattering mechanism. Experiments to detect the effect of a current on the spin polarization are discussed.

1. INTRODUCTION

In this paper we examine the spin polarization of electrons caused during current flow through quantum wells and superlattices and also in deformed crystals of class T_d. In these systems there is a spin splitting of the conduction band which is linear in the wave vector **k**. This property is a necessary condition for the occurrence of the effect.

That carriers could be oriented by a current by virtue of spin splitting of bands was first pointed out by Ivchenko and Pikus.[1] Vorob'ev *et al.*[2] observed this effect experimentally in tellurium, from a change in optical activity upon the flow of a current. Aronov *et al.*[3,4] showed that this effect could be observed in quantum wells and deformed crystals of III–V compounds. The mechanism for the orientation of spins by an electric current in these entities is distinguished fundamentally by the circumstance that the orientation occurs only in the course of a spin relaxation. In tellurium, in contrast, the shift of the distribution function in **k** space directly causes a change in the average moment of the holes also.

The effect which we will be discussing here is the inverse of the appearance of a current in a system of thermalized, spin-oriented carriers. The possibility of this effect was pointed out in Ref. 5, and a detailed theory was derived in Refs. 6–8. A fundamental condition for the occurrence of each of these effects—in addition to the spin splitting linear in **k**—is that spin relaxation or spin precession occur in a magnetic field. Kalevich and Korenev[9] have predicted and experimentally observed yet another effect which arises in a system of spin-oriented thermalized carriers: a current-induced spin precession, i.e., a Hanle effect in an electric field.

The spin polarization caused by an electric current in the course of a scattering by point defects was discussed in Ref. 3. It was suggested there that the primary mechanism for the spin relaxation is the D'yakonov–Perel' precession mechanism (see the review of Pikus and Titov[10]). In the present paper we examine this effect for various spin-relaxation mechanisms and various scattering mechanisms.

The precession mechanism for spin polarization caused by a current which was proposed in Ref. 4 may be thought of as the orientation of the spin in the effective magnetic field which arises during the flow of the current, as a result of spin splitting of the conduction band. We will see that the degree of polarization of the carriers in the case of the precession mechanism depends strongly on the scattering mechanism. For $2D$ systems, with scattering by Coulomb impurity centers, for example, the degree of orientation is nearly an order of magnitude higher than that in the case of scattering by point impurity centers.

In addition to this precession mechanism for the spin polarization, discussed in Refs. 3 and 4, it has been shown that there is yet another mechanism for a polarization of spins. This other mechanism involves a spin-dependent scattering: A deviation of the electron momentum distribution from equilibrium caused by the current, in the course of a scattering accompanied by a spin flip, leads to nonequilibrium filling of spin states. In the Elliott–Yafet (EY) spin relaxation mechanism,[10] which involves **k–p** mixing of states of the conduction and valence bands, a leading role is played by interference between transitions accompanied by and not accompanied by spin flip, as in Refs. 7 and 8. At temperatures which are not too low, this contribution can be comparable to the precession contribution, even if the EY mechanism itself does not contribute significantly to the spin relaxation rate.

For other spin relaxation mechanisms—scattering by holes (the Bir–Aronov–Pikus mechanism[10]) or scattering by paramagnetic ions—the contribution of spin-dependent scattering to the polarization of electrons would be dominant only if these mechanisms dominated the spin relaxation rate.

2. PRECESSION MECHANISM FOR SPIN POLARIZATION

The spin density matrix ρ of the electrons is determined by the kinetic equation

$$\frac{\partial\rho}{\partial t}+\frac{\rho}{\tau_0}+\frac{i}{\hbar}[\mathscr{H},\rho]+\frac{e\boldsymbol{\varepsilon}}{\hbar}\frac{\partial\rho}{\partial\mathbf{k}}=G+\mathrm{St}\,\rho. \tag{1}$$

Here G is the optical generation matrix, St ρ is a collision integral, τ is the lifetime, $\boldsymbol{\varepsilon}$ is the electric field, and **k** is the carrier quasimomentum. The Hamiltonian $\mathscr{H}(\mathbf{k})$ has, in addition to the usual terms involving $\mathbf{k}$,

$$E_{\mathbf{k}}=\frac{\hbar^2k^2}{2m} \tag{2}$$

(m is the effective mass), a term linear in $\mathbf{k}$:

$$\mathscr{H}_{\mathbf{k}}^{(1)}=\frac{\hbar}{2}(\boldsymbol{\sigma}\boldsymbol{\Omega}_{\mathbf{k}}^{(1)})=\beta_{ij}\sigma_i k_j, \tag{3}$$

where σ_i are the Pauli matrices, and the tensor β_{ij} determines the spin splitting of the band states.

The commutator in (1) contains only $\mathscr{H}^{(1)}(\mathbf{k})$. When this contribution is taken into account, the collision integral becomes[7]

$$\begin{aligned}\mathrm{St}\,\rho=&\frac{2\pi}{\hbar}\sum_{\mathbf{k}'}\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})\Big[M_{\mathbf{k}\mathbf{k}'}\rho(\mathbf{k}')M_{\mathbf{k}'\mathbf{k}}\\&-\{M_{\mathbf{k}\mathbf{k}'}M_{\mathbf{k}'\mathbf{k}},\rho(\mathbf{k})\}_{\text{symm}}\\&+\frac{2\pi}{\hbar}\sum_{\mathbf{k}'}\Big[\frac{\partial}{\partial\Delta}\delta(E_{\mathbf{k}}-E_{\mathbf{k}'}+\Delta)\Big]_{\Delta\to0}\\&\times[\{M_{\mathbf{k}\mathbf{k}'}\mathscr{H}_{\mathbf{k}'}^{(1)}M_{\mathbf{k}'\mathbf{k}},\rho(\mathbf{k})\}_{\text{symm}}\\&+\{M_{\mathbf{k}\mathbf{k}'}\rho(\mathbf{k}')M_{\mathbf{k}'\mathbf{k}},\mathscr{H}_{\mathbf{k}}^{(1)}\}_{\text{symm}}-M_{\mathbf{k}\mathbf{k}'}\{\mathscr{H}_{\mathbf{k}'}^{(1)},\rho(\mathbf{k}')\}_{\text{symm}}M_{\mathbf{k}'\mathbf{k}}\\&-\frac{1}{2}(M_{\mathbf{k}\mathbf{k}'}M_{\mathbf{k}'\mathbf{k}}\rho(\mathbf{k})\mathscr{H}_{\mathbf{k}}^{(1)}+\mathscr{H}_{\mathbf{k}}^{(1)}\rho(\mathbf{k})M_{\mathbf{k}\mathbf{k}'}M_{\mathbf{k}'\mathbf{k}})\Big].\end{aligned} \tag{4}$$

Here $\{AB\}_{\text{symm}}=(AB+BA)/2$. Terms of zeroth and first orders in $\mathscr{H}_{\mathbf{k}}^{(1)}$ have been retained in (4), and the scattering has been assumed to be elastic.

In general, the scattering matrix element can be written in the form[10]

$$M_{\mathbf{k}\mathbf{k}'}=A_{\mathbf{k}\mathbf{k}'}I+i\mathbf{B}_{\mathbf{k}\mathbf{k}'}\boldsymbol{\sigma}, \tag{5}$$

where I is the unit matrix. The second term in (5) describes scattering accompanied by a spin flip.

We will ignore these processes in this section of the paper. We correspondingly write the collision term St ρ, which determines the momentum relaxation, as

$$\begin{aligned}\mathrm{St}\,\rho=\sum_{\mathbf{k}'}W_{\mathbf{k}\mathbf{k}'}\{&\delta(E_{\mathbf{k}}-E_{\mathbf{k}'}+\mathscr{H}_{\mathbf{k}}^{(1)}-\mathscr{H}_{\mathbf{k}'}^{(1)}),\\&\rho(\mathbf{k})-\rho(\mathbf{k}')\}_{\text{symm}},\end{aligned} \tag{6}$$

where

$$W_{\mathbf{k}\mathbf{k}'}=\frac{2\pi}{\hbar}|A_{\mathbf{k}\mathbf{k}'}|^2.$$

The matrix

$$\rho_0=n\{f_0(E_{\mathbf{k}}+\mathscr{H}_{\mathbf{k}}^{(1)}),\ (^1/_2+\boldsymbol{\sigma}\mathbf{S})\} \tag{7}$$

describes the state of electrons which are in equilibrium in terms of energy and momentum but which have a nonequilibrium spin $\mathbf{S}$. The quantity n in (7) is the electron density, and f_0 is a Maxwellian distribution function. A matrix ρ_0 of this sort causes the collision integral (6) to vanish.

The generation matrix G can be written in a form similar to (7):

$$G=g\{f_0(E_{\mathbf{k}}+\mathscr{H}_{\mathbf{k}}^{(1)}),\ (^1/_2+\boldsymbol{\sigma}\mathbf{S}_0)\}. \tag{8}$$

Here it is being assumed that the electrons generated by the light quickly undergo an energy and momentum relaxation, while retaining the spin $\mathbf{S}_0$ which they had when they appeared. In the steady state we would have $g=n/\tau_0$.

Our goal in these calculations is to determine $\mathbf{S}$ in (7). To reach this goal, we solve Eq. (1) by a method of successive iterations in the parameters $\Omega\tau_i$ and $e\boldsymbol{\varepsilon}\mathbf{v}\tau_i/E_{\mathbf{k}}$, where τ_i is the relaxation time of the corresponding Legendre polynomial $P_i(\mu)$ in the electron momentum distribution. This time is determined by

$$\begin{aligned}&\frac{1}{\tau_i}=\sum_{\mathbf{k}'}W_{\mathbf{k}\mathbf{k}'}\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})[1-P_i(\mu)],\\&\mu=(\mathbf{k}\mathbf{k}')/kk'.\end{aligned} \tag{9}$$

We will discuss only effects which are linear in ε.

The correction of first order in $e\boldsymbol{\varepsilon}\cdot\mathbf{v}\tau/E_{\mathbf{k}}$ is found from the equation

$$n\Big(F_1\frac{\partial f_0}{\partial E_{\mathbf{k}}}+F_2\frac{\partial^2 f_0}{\partial E_{\mathbf{k}}^2}+F_0\Big)=\mathrm{St}\,\rho_1, \tag{10}$$

where

$$F_1=e\boldsymbol{\varepsilon}\mathbf{v}\Big(\frac{1}{2}+\boldsymbol{\sigma}\mathbf{S}\Big),$$

$$F_2=\frac{\hbar}{2}\sum_{ij}e\varepsilon_j(\Omega_i v_j-\overline{\Omega_i v_j})\Big(\frac{\sigma_i}{2}+S_i\Big),$$

$$F_0=\frac{\hbar}{2}\sum_{ij}e\varepsilon_j\Big(\overline{\Omega_i v_j}\frac{\partial^2 f_0}{\partial E_{\mathbf{k}}^2}+\frac{1}{\hbar}\frac{\partial\Omega_i}{\partial k_j}\frac{\partial f_0}{\partial E_{\mathbf{k}}}\Big)\Big(\frac{\sigma_i}{2}+S_i\Big).$$

Here $\mathbf{v}=\mathbf{k}/m$, and the superior bar means an average over the momentum direction. The quantity F_1 in (10) contains a Legendre polynomial of index 1; F_2 contains one of index 2; and F_0 contains one of index 0. A solution of Eq. (10) is

$$\rho_1=-n\Big[F_1\frac{\partial f_0}{\partial E_{\mathbf{k}}}\tau_1+F_2\chi(E_{\mathbf{k}})\Big], \tag{11}$$

where

$$\begin{aligned}\chi(E_{\mathbf{k}})=&\frac{\partial^2 f_0}{\partial E_{\mathbf{k}}^2}\tau_1+\frac{1}{2}\frac{\partial f_0}{\partial E_{\mathbf{k}}}\Big[\frac{\partial\tau_1}{\partial E_{\mathbf{k}}}+\frac{\tau_1}{E_{\mathbf{k}}}\Big(1+\frac{d}{2}\Big)\\&-\frac{d}{2}\frac{\tau_2}{E_{\mathbf{k}}}\Big].\end{aligned}$$

Here d is the dimensionality of the space.

Note that in substituting (11) into the collision term in (6) we find terms containing $P_0(\mu)$, which cancel with the term containing F_0 on the left side of the kinetic equation. In the next iteration in $\Omega\tau_i$, it is sufficient to substitute the second term in (11) into the left-hand side of the kinetic equation (in the commutator $[\mathscr{H},\rho]$) in place of ρ_0. Solving the corresponding equation, we then find ρ_2:

$$\rho_2=-n\tau_1(\boldsymbol{\sigma}[\boldsymbol{\Omega}\mathbf{S}])f_0(E_{\mathbf{k}})+\frac{\hbar}{4}n\tau_1 e\varepsilon_j[\boldsymbol{\sigma}\boldsymbol{\Omega}]_i\overline{\Omega_i v_j}\chi(E_{\mathbf{k}}). \tag{12}$$

Substituting ρ_0 and G from (7) and (8), ρ_2 from (12), and the first term in ρ_1 from (11) into (1), and multiplying the kinetic equation by $\sigma_i/2$, we can calculate the trace of this equation and integrate over momentum and energy. As a result we find an equation for the spin $\mathbf{S}$:

$$\sum_j S_j\Big(\frac{1}{\tilde{\tau}_S}\Big)_{ij}+\frac{S_i}{\tau_0}+[\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}\mathbf{S}]_i=\frac{S_{0i}}{\tau_0}+(\dot{S}_{\boldsymbol{\varepsilon}})_i. \tag{13}$$

Here $(1/\tau_S)_{ij}$ is the spin relaxation time for the D'yakonov–Perel' mechanism:[10]

$$\left(\frac{1}{\tau_S}\right)_{ij}=\langle(\overline{\Omega^2}\delta_{ij}-\overline{\Omega_i\Omega_j})\tau_1\rangle. \tag{14}$$

The angles brackets here mean an average over a Maxwellian distribution:

$$\langle\varphi\rangle=\int_0^\infty \varphi(E)E^{d/2-1}f_0(E)\,dE\Big/\int_0^\infty E^{d/2-1}f_0(E)\,dE,$$
$$(\boldsymbol{\Omega}_\varepsilon)_i=\frac{1}{\hbar}\,e\varepsilon_j\tau_p\,\frac{\partial}{\partial k_j}\,\Omega_i, \tag{15}$$

where, as usual,

$$\tau_p=\langle\tau_1E\rangle/\langle E\rangle,$$

and, finally,

$$(S_\varepsilon)_i=\frac{\hbar}{4}e\sum_{j,m,n,l}\varepsilon_j(\delta_{ij}\delta_{nm}-\delta_{im}\delta_{nl})$$
$$\times\int_0^\infty dE\,\overline{\Omega_m v_j}\,\overline{\Omega_n\Omega_l}\chi(E)\tau_1E^{d/2-1}\Big/\int_0^\infty dE\,E^{d/2-1}. \tag{16}$$

The term $[\boldsymbol{\Omega}_\varepsilon\times\mathbf{S}]$ in (13) describes spin precession in the electric field. Its effect is equivalent to that of an effective magnetic field

$$\mathbf{B}_{eff}=\frac{\hbar}{g\mu_B}e\tau_p(\boldsymbol{\varepsilon}\nabla_{\mathbf{k}})\boldsymbol{\Omega}. \tag{17}$$

As we mentioned above, the precession in an electric field was predicted and experimentally observed in Ref. 9 during optical orientation of carriers, i.e., with $\mathbf{S}_0\neq0$. The term $\mathbf{S}_\varepsilon$ describes the generation of a spin by the electric field. As a result we find an average spin

$$S_j=\sum_i T_{ji}(\dot{S}_{\boldsymbol{\varepsilon}})_i, \tag{18}$$

where

$$T^{-1}_{S,ij}=\tau^{-1}_{S,ij}+\tau^{-1}.$$

Here τ_S is the total spin relaxation time, which is determined by all the relaxation mechanisms. It follows from (18) that a spin orientation arises in the course of a spin relaxation, and the actual change in the distribution function in the electric field determined by (11) does not alter the average spin of the carriers. In this regard, the effect which we are discussing here differs from the effect observed in tellurium in Ref. 2. In that other case, because the degeneracy in the valence band is completely lifted, the shift of the center of gravity of the hole distribution in **k** space led to the simultaneous appearance of an average moment.

In quantum wells and (001) superlattices of III–V crystals, we have a pseudovector $\boldsymbol{\Omega}=\beta(-k_x,k_y,0)$, where the x axis runs parallel to the [100] direction, and the y axis runs parallel to the [010] direction. In (111) superlattices and wells we have $\boldsymbol{\Omega}=\beta[\mathbf{l}\times\mathbf{k}]$, where $\mathbf{l}$ is a unit vector parallel to the [111] direction.[11,12] In III–V crystals deformed along the [111] direction we again have[13] $\boldsymbol{\Omega}=\beta[\mathbf{l}\times\mathbf{k}]$.

In these cases, according to (16), we have

$$\mathbf{S}=e(\boldsymbol{\varepsilon}\nabla_{\mathbf{k}})\Omega\frac{T_S}{\tilde{\tau}_S}\frac{\tau_p}{k_BT}Q. \tag{19}$$

If the dimensionality of the space is $d=2$, we find

$$Q_2=\frac{\frac{1}{8}\left\langle E\tau_1\left(\tau_1+\tau_2+\frac{3}{2}\tau_1\frac{\partial\ln\tau}{\partial\ln E}\right)\right\rangle}{\tau_p^2k_BT}. \tag{20}$$

For $d=3$

$$Q_3=\frac{\frac{1}{36}\left\langle E\tau_1\left(\frac{5}{2}\tau_1+3\tau_2+3\tau_1\frac{\partial\ln\tau}{\partial\ln E}\right)\right\rangle}{\tau_p^2k_BT}. \tag{21}$$

In each case, according to (14), we have

$$\tilde{\tau}_S^{-1}=(\tau_S^{\mathrm{DP}})^{-1}=\frac{2m}{\hbar^2}\beta^2k_BT\tau_p. \tag{22}$$

Table I shows values of the coefficients Q_1 and Q_2 for the various scattering mechanisms:

1) deformation scattering by acoustic phonons and impurity centers with a short-range potential, with $W(E_{\mathbf{k}},\mu)=\mathrm{const}$;

2) piezoelectric scattering by acoustic phonons or a scattering by polar optical phonons, with1) $W(E_{\mathbf{k}},\mu)=E^{-1}/(1-\mu)$;1)

3) a scattering by Coulomb impurity centers, with $W(E_{\mathbf{k}},\mu)=E^{-2}/[1-\mu+\varkappa]^2$, where $\varkappa=(L_sk)^{-2}$, and L_s is the screening length. In the latter case we assume $\varkappa\ll1$, so small-angle scattering with $1-\mu\sim\varkappa\ll1$ dominates the situation. We also ignore the E dependence of $\varkappa$. Table I shows values of τ_2/τ_1 and $\nu=\partial\ln\tau/\partial\ln E$ for these scattering mechanisms.

If the majority carriers are oriented in the case $\tau_0\to\infty$, and the D'yakonov–Perel' mechanism for spin relaxation is dominant, the degree of orientation $\mathscr{P}=2S$ is equal in order of magnitude to the ratio of the spin splitting $\hbar\Omega(\mathbf{k}_\varepsilon)$ to the average electron energy:

$$\mathscr{P}=2Q\hbar\Omega(\mathbf{k}_\varepsilon)/k_BT, \tag{23}$$

where

$$\mathbf{k}_{\boldsymbol{\varepsilon}}=e\boldsymbol{\varepsilon}\tau_p/\hbar. \tag{24}$$

Expression (23) was derived by a different method in Ref. 3 for the case of scattering by impurity centers with a short-range potential in the $d=2$ case, with $Q_2=1/4$. It was also shown there that in the case of Fermi degeneracy the quantity k_BT in (24) is replaced by $2E_F/3$ in the $d=2$ case.

In deformed III–V crystals, if the deformation is not too large, the splitting which is cubic in **k** may be greater than the linear splitting. In such a case the expression for S will differ from that in (23) by an additional factor of 3/4 (Ref. 3).

We see from (19) and (23) that depolarization in an electric field can be thought of as an orientation of the spins in an effective magnetic field. This field is not, however, the same as $\mathbf{B}_{\mathrm{eff}}$ in (17), which determines the spin precession in an electric field. The difference between these effective fields arises because the polarization, like the precession, is a kinetic effect and depends on the scattering mechanisms. It can be seen from Table I that the strength of the effective field which determines the polarization depends on the scattering mechanism and may vary by a factor of several units.

TABLE. I.

Scattering mechanism	ν	τ_2/τ_1	Q	ν	τ_2/τ_1	Q
	$d=2$			$d=3$		
Acoustic phonons: deformation scattering, point impurity centers	0	1	0,25	$-1/2$	1	0,20
Acoustic phonons: piezoelectric scattering, polar optical phonons	1	$1/2$	0,56	$1/2$	$2/3$	0,28
Coulomb impurity centers	2	$1/4$	1,77	$3/2$	$1/3$	0,64

3. SPIN POLARIZATION CAUSED BY A SPIN-DEPENDENT SCATTERING

In order to take account of the contribution to spin polarization from processes involving a spin flip in the collision term in (4), we need to retain the second term in expression (5) for $M_{\mathbf{kk}'}$, and as $\rho(\mathbf{k})$ we should use the component ρ_1, linear in the electric field, which is given in (11). Upon the substitution $\rho_1^{(2)} = -nF_2\chi(E)$, the terms containing $\mathscr{H}_{\mathbf{k}}^{(1)}$ in (4) need not be considered, since they make a contribution proportional to $(\Omega/T)^2$, while we are retaining only contributions proportional to the product $\Omega\tau_p\cdot(\Omega/T)$. A calculation shows that incorporating $\rho_1^{(2)}$ in St ρ in this approximation does not contribute to the spin polarization. When we substitute the component

$$\rho_1^{(1)} = -nF_1\frac{\partial f_0}{\partial E}\tau_1$$

into St ρ, we need to retain only those terms which contain $\Omega_{\mathbf{k}}$. When the terms quadratic in $B_{\mathbf{kk}'}$ are taken into account in (4), as in the case of the spin relaxation caused by a scattering by holes (the Bir–Aronov–Pikus mechanism) or a scattering by paramagnetic ions, as in the case of the spin relaxation due to the EY mechanism, the corresponding contribution to S, which is determined only by the incoming terms in (4), is

$$\mathbf{S} = -\frac{\hbar\Omega(\varepsilon)}{4k_BT}\frac{T_S}{\tau_p}\left\langle\frac{\tau_1}{\tau_S^*}\frac{\partial\ln\tau_S^*}{\partial\ln E}\right\rangle, \quad d=2, \tag{25}$$

$$\mathbf{S} = -\frac{\hbar\Omega(\varepsilon)}{6k_BT}\frac{T_S}{\tau_p}\left\langle\frac{\tau_1}{\tau_S^*}\left(\frac{\partial\ln\tau_S^*}{\partial\ln E}-\frac{1}{2}\right)\right\rangle, \quad d=3. \tag{26}$$

Here $1/\tau_S^*$ is the contribution to the spin relaxation rate from the corresponding mechanism:

$$\left(\frac{1}{\tau_S^*}\right)_{ij} = \frac{4\pi}{\hbar}\sum_{\mathbf{k}'}(|B_{\mathbf{kk}'}|^2-|B_{i\mathbf{kk}'}|^2)\delta(E_{\mathbf{k}}-E_{\mathbf{k}'}). \tag{27}$$

For the Bir–Aronov–Pikus mechanism (or for a scattering by paramagnetic ions), expression (27) must be summed over all the scattering particles and averaged over their spin states. In this case we find $\tau_S^* \propto E^{-1/2}$ for nondegenerate holes, as in the case of scattering by paramagnetic ions. For the EY mechanism, with a scattering by piezoelectric acoustic vibrations, polar optical phonons, and impurity centers, we have

$$\mathbf{B}_{\mathbf{kk}'} = A_{\mathbf{kk}'}\mathbf{b}_{\mathbf{kk}'}, \tag{28}$$

$$\mathbf{b}_{\mathbf{kk}'} = -i[\mathbf{kk}']\frac{\hbar}{2mE_g}\eta\delta, \qquad \delta = \frac{\Delta_{\text{so}}}{E_g+\Delta_{\text{so}}},$$

Δ_{so} is the spin–orbit splitting of the valence band. For III–V crystals we would have $\eta = (2-\delta)/(3-\delta)$. We do not need to retain the terms linear in $\Omega_{\mathbf{k}}$ in $\mathbf{b}_{\mathbf{kk}'}$, since they make a small contribution of the next higher order in $\hbar\Omega_{\mathbf{k}}/k_BT$. In the case of the EY mechanism the spin relaxation rate is given by

$$\frac{1}{\tau_S^*} = C\frac{1}{\tau_p}\left(\frac{E}{E_g}\right)^2(\delta\eta)^2. \tag{29}$$

The numerical factor C in (29) depends on the particular scattering mechanism.

In the case of the EY mechanism, the leading role is played by interference between transitions which are and are not accompanied by spin flip. This interference becomes possible thanks to the spin splitting of $\mathbf{\Omega}(\mathbf{k})$, which mixes states differing in spin. The same effect occurs upon the appearance of a current of polarized electrons.[7,8] This interference is described by terms in St $\rho_1^{(1)}$ which contain the products $A_{\mathbf{kk}'}$ and $\mathbf{B}_{\mathbf{kk}'}$. While the interference contribution in Ref. 7 was due to incoming terms in the collision integral (4), in this case the interference stems from the outgoing terms, i.e., from the incorporation of the component $\rho_1^{(1)}(\mathbf{k})$, while the incorporation of $\rho_1^{(1)}(\mathbf{k}')$ contributes nothing.

A calculation leads to the following expression for S as a result of the interference mechanism:

$$\mathbf{S} = -\frac{1}{d}e\,\mathrm{rot}_{\mathbf{k}}[\boldsymbol{\varepsilon}\mathbf{\Omega}]\frac{\delta\eta}{E_g}T_S\left\langle E\frac{\tau_1}{\tau_2}\left(1-\frac{\partial\ln\tau}{\partial\ln E}\right)\right\rangle\Big/k_BT. \tag{30}$$

We see that if the contribution in (25) and (26) is proportional to τ_1/τ_S^* then the contribution in (29) will be proportional to $(\tau_1/\tau_S^*)^{1/2}$ and can be written

$$\mathbf{S} = -Qe\,\mathrm{rot}_{\mathbf{k}}[\boldsymbol{\varepsilon}\mathbf{\Omega}]\frac{T_S}{k_BT}\left(\frac{\tau_p}{\tau_S^*}\right)^{1/2} \times\left\langle E\frac{\tau_1}{\tau_2}\left(1-\frac{\partial\ln\tau}{\partial\ln E}\right)\right\rangle\Big/k_BT, \tag{31}$$

where we have $Q = C^{-1/2}/(2)^{3/2}$ for $d=2$ and $Q=(15C)^{-1/2}/2$ for $d=3$. We thus see that the contribution of (30), (31) dominates by a factor of order $(\tau_1/\tau_S^*)^{1/2}$ for the EY mechanism.

With regard to the relation between the contributions (30) and (24), we note that their ratio in the case $T_S \sim \tau_S^{\text{DP}}$ is on the order of $(k_B T/E_g)\cdot(T_S/\tau_p)$, so the precession contribution, (23), will always be dominant at low temperatures. At room temperature, estimates show that these contributions to the spin polarization are comparable in magnitude for GaAs, even if the EY contribution to the spin relaxation rate is negligible.

The contributions of scattering of electrons by holes or paramagnetic ions [(25) and (26)] to the polarization of the electron spins can become important only if the scattering by holes becomes the dominant mechanism for spin relaxation. This can be the case in doped p-type samples at sufficiently low temperatures or in stressed superlattices whose upper level contains light holes with $j_z = \pm 1/2$. In the scattering of electrons by heavy holes, the exchange interaction causes a spin flip of the electrons only to the extent that the states of heavy and light holes are mixed at $k = 0$. In addition, this contribution can be dominant in samples with a special paramagnetic dopant.

With regard to the absolute value of the effect, we note that at $T_S \simeq \tau_S^{\rm DP}$ for GaAs/$\rm Al_{0.35}Ga_{0.65}As$ quantum wells (with a barrier and well thickness of 15 Å), with $\beta = 10^{-2}$ eV·Å (Ref. 14), $\tau_p = 8\cdot 10^{-12}$ s, and $T = 5$ K in a field of 10 V/cm, the degree of spin orientation, $\mathscr{P} = 2S$, can range from 1% to 10%, depending on the particular scattering mechanism. When the orientation is caused by a current of minority carriers, the polarization of these carriers can be detected by detecting a polarization of the luminescence. In the case of the orientation of majority carriers, their polarization can be detected either from the luminescence accompanying the excitation of minority carriers or from the change in the impedance at the NMR or ESR frequency (a nuclear electric resonance or a spin electric resonance). This method for observing a resonance induced by an electric current at the NMR or ESR frequency was proposed in Refs. 3 and 4. One should bear in mind, however, that there are two circumstances which would interfere with observation of this effect.

1. The intensity of the nuclear electric resonance (or that of the spin electric resonance) is proportional to $\langle \mathbf{S}^2(\mathbf{r})\rangle$, where the angle brackets mean an average over the positions of the impurity centers. As a result of a local mesoscopic effect accompanying the current flow through the sample, there will be a local spin polarization $\langle \mathbf{S}^2(\mathbf{r})\rangle \neq 0$ (Ref. 15), although the corresponding average polarization will be zero. Consequently, in addition to the effect in the intensity of the resonances, described above, there will be a contribution from mesoscopic spatial fluctuations. Estimates show that the latter will be dominant under the same conditions. Incidentally, these effects could easily be distinguished from each other, since the random orientation of the spins in space would make the mesoscopic effect isotropic, while the effect associated with the average spin would have a pronounced anisotropy.

2. It should also be kept in mind that a resonance could be induced directly by the magnetic field of the current. Estimates show that the thickness of the sample should not exceed 1 μm if we wish to avoid the effect of the magnetic field of the current. Correspondingly, the number of layers in a quantum-well structure or in a superlattice should be such that the overall thickness of the structure does not exceed this value. In the observation of an ESR or NMR induced by an electric current, the magnetic field should not strongly influence the orbital motion of the electrons. Correspondingly, the cyclotron frequency should be small in comparison with τ_p^{-1} in the $3D$ case.

1) It is being assumed here that in the $d = 2$ case the phonons are also two-dimensional. For three-dimensional phonons in the case $L \ll k^{-1}$, where L is the size of the well in the $d = 2$ case, Q has the same value as in the case of scattering by point impurity centers.

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