Stabilization of the classical atom in a strong alternating field

A. L. Nefedov

Moscow Engineering Physics Institute
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It is shown that the suppression of the ionization of an atom, known as stabilization, as the intensity of a strong external alternating field is increased can be explained on the basis of classical ideas about the motion of an electron in an atom. The conditions under which stabilization arises are determined. These conditions describe well the results of other quantum and classical calculations.

1. INTRODUCTION

It is well known that the quasiclassical or classical description is applicable for highly excited atomic states. In a number of papers[1-3] the behavior of highly excited atoms in an electromagnetic field was investigated on the basis of classical dynamics of the motion of an electron in two fields—an external field and the atomic field. For the external field characteristic frequencies $\omega \sim n^{-3}$, where n is the principal quantum number of the atom, and intensities less than the intensities of the atomic field $\mathscr{E} \lesssim \mathscr{E}_a \sim 0.1 n^{-4}$ were studied (here and below the atomic system of units $e = \hbar = m = 1$ is employed). New qualitative phenomena were discovered: stochastization and diffusion of electron motion along the energy levels, overlapping of resonances, etc.

The range of parameters $\omega > n^{-2} \gg n^{-3}$, $\mathscr{E} > \mathscr{E}_a$ is, however, no less interesting. Under these conditions the quantum calculations[4-20] revealed an unexpected effect, termed stabilization of an atom in a strong field. It consists of an increase in the photoionization lifetime of an atom at a constant frequency ω as the field strength $\mathscr{E}$ increases. It is remarkable that stabilization arises in alternating fields which are much stronger than the atomic fields, while in a constant field of the same strength ionization occurs over shorter times, of the order of the Kepler period $T_n = 2\pi n^3$ of revolution of an electron in a classical orbit.[21,22]

Although the fact of stabilization itself is confirmed by independent calculations, the mechanism and conditions under which this phenomenon appears have not been adequately studied. In particular, the following fundamental questions have thus far not been answered: 1) is the stabilization purely a quantum effect or do simpler "classical" analogs exist for it? 2) If "classical" stabilization is indeed possible, then for what values of the parameters of the ionizing field can it be expected to arise? The last question is of direct practical value, since no work on the experimental observation of stabilization has thus far been performed. It is obvious that in order for such experiments to be successful the correct regimes must be chosen.

There are a few reports in which the problem of stabilization is studied on the basis of the classical description.[23-26] The main result of Refs. 23 and 24 is that suppression of ionization is possible only for highly excited states, if real fields, which can be achieved using modern sources of radiation, are employed. These results will be discussed in greater detail below (see Sec. 4). An extension of the results of Refs. 25 and 26 and new data are presented in the present paper.

In Sec. 2 the problem of photoionization of a classical atom is formulated and a method for solving it is given. The computational results and possible regimes in which stabilization appears are discussed in Sec. 3. In Sec. 4 the results of this work are compared with the results of other work.

2. FORMULATION OF THE PROBLEM AND METHOD OF CALCULATION

The starting point of the classical analysis of photoionization of an atom is the determination of the trajectory of an electron by numerically solving Newton's equation of motion:

$$d^2\mathbf{r}/dt^2 = -\mathbf{r}/r^3 + \vec{\mathscr{E}} \cos(\omega t + \varphi). \qquad (1)$$

In this equation $\mathbf{r}$ is the radius vector of an electron (the origin is located at the nucleus); $\mathscr{E}$, ω, and φ are the intensity, frequency, and initial phase of the external field. The first term on the right-hand side describes the Coulomb interaction with the nucleus; the second term describes the interaction of the electron with the electromagnetic field. It is assumed that the characteristic velocities of the electron are small compared with the velocity of light, so that the effect of the magnetic component of the field on the motion of the electron can be neglected.

As the initial conditions for Eq. (1) we chose the coordinate $\mathbf{r}_0$ and velocity $\mathbf{v}_0$ of an electron at the time $t = 0$, when the field is turned on (it is assumed that at times $t < 0$ there is no external field and that it is turned on instantaneously at $t = 0$). Only s-states are considered.

Here it is necessary to take into account the fact that at the moment the field is turned on the electron can be located at different points of its classical orbit, and the orbit itself can have various orientations in space. For this reason, the initial conditions are prescribed in accordance with the method of Monte Carlo trajectories.[3,27] The semi-major axis and the position of the plane of the orbit were chosen to be uniformly distributed in space, and the starting points on the trajectory were chosen to be uniformly distributed over the time of motion along the trajectory. The most complete characteristics of the process in this approach are stochastic and must be obtained by means of statistical analysis of the results of calculations performed with different initial conditions.

The equation (1) was solved numerically by the method of "splitting of physical processes," when the increments to the coordinates and velocities over some time interval τ are determined for the Coulomb and external fields separately, after which the results are added and the necessary corrections which take into account the nonlinearity of the problem are then introduced. In other words, at each

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time $t_n + \tau$ the solution is sought in the form

$$\mathbf{r}(t_n+\tau)=\mathbf{r}_1(t_n+\tau)+\mathbf{r}_2(t_n+\tau)+\mathbf{r}_3(t_n+\tau),$$

where $\mathbf{r}_1(t)$, $\mathbf{r}_2(t)$, and $\mathbf{r}_3(t)$ are determined by the following equations:

$$\begin{aligned} &d^2\mathbf{r}_1/dt^2=-\mathbf{r}_1/r_1^3,\\ &\mathbf{r}_1(t_n)=\mathbf{r}(t_n),\\ &\left.\frac{d\mathbf{r}_1}{dt}\right|_{t_n}=\left.\frac{d\mathbf{r}}{dt}\right|_{t_n}; \end{aligned} \tag{2}$$

$$\begin{aligned} &d^2\mathbf{r}_2/dt^2=\vec{\mathscr{E}}\cos(\omega t+\varphi),\\ &\mathbf{r}_2(t_n)=0,\\ &d\mathbf{r}_2/dt|_{t_n}=0; \end{aligned} \tag{3}$$

$$\begin{aligned} &\frac{d^2\mathbf{r}_3}{dt^2}=-\frac{(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)}{|\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3|^3}+\frac{\mathbf{r}_1}{r_1^3},\\ &\mathbf{r}_3(t_n)=0,\\ &d\mathbf{r}_3/dt|_{t_n}=0. \end{aligned} \tag{4}$$

It is easy to see that adding Eqs. (2)–(4) indeed gives Eq. (1) with the correct initial conditions for each time t_n.

The well known solution of Eqs. (2) is given in Ref. 28, and the solution of the system (3) is obtained by elementary integration. To find $\mathbf{r}_3(t_n+\tau)$ from Eqs. (4) it is convenient to employ Pekar's method, in accordance with which an expansion in a Taylor series in the form

$$\mathbf{r}_3(t_n+\tau)=\sum_{k=0}^{\infty}\mathbf{a}_k\tau^k$$

is employed.

From Eqs. (4) and the initial conditions to Eqs. (2) and (3) it follows that $\mathbf{a}_0=\ldots=\mathbf{a}_3=0$, and the ith component of the vector $\mathbf{a}_4$ is determined by the relation

$$a_{4i}=\frac{1}{24}\left(-\frac{1}{r^3}\frac{d^2x_2}{dt^2}\delta_{ix}+\frac{3r_{1i}x_1}{r^5}\frac{d^2x_2}{dt^2}\right)\Bigg|_{t_n}.$$

Here the external field $\vec{\mathscr{E}}$ is directed along the x axis, δ_{ix} is a delta Kronecker, x_1 and x_2 are the x-components of the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, and r_{1i} is the ith component of the vector $\mathbf{r}_1$.

In the calculations the series was truncated at the term $\mathbf{a}_4\tau^4$, though the subsequent terms of the expansion can in principle be obtained by differentiating Eq. (4) with respect to time. The method employed for the calculation gives, on the one hand, fourth-order accuracy in the step τ and, on the other hand, an exact solution for purely Coulomb or purely electromagnetic field. In addition, problems associated with the singularity of the Coulomb field at the origin do not arise in the computational algorithm, since this singularity is automatically taken into account when analytical formulas are employed for the purely Coulomb motion.

The computational accuracy was monitored by comparing the trajectories calculated forward and backward in time. The step τ was chosen so that the error accumulated over times of the order of the Kepler period did not exceed 0.1%.

For each trajectory the moment of ionization was determined from the condition that the total energy of an electron, averaged over a period of the external field, minus the average oscillatory energy $\mathscr{E}^2/4\omega^2$ be positive.

In the numerical calculations the range of frequencies of the external field $0 \lesssim \omega \lesssim 1/n^2$ was scanned. For each fixed frequency in this range we sought relations between $\mathscr{E}$, ω, and n for which there appeared electron trajectories with lifetime until ionization exceeding the Kepler period T_n. Stabilization was regarded as arising if more than 50% of the trajectories from the statistical sample had lifetimes exceeding T_n.

3. COMPUTATIONAL RESULTS

3.1. Ionization in a constant field

The intensity of the Coulomb field decreases with distance r according to the law $\mathscr{E}_c = -1/r^2$. The ionizing field has the constant value $\mathscr{E}$. From here it is easy to estimate the size r_c of the "Coulomb" region in which the field of the nucleus is always greater than the external field: $r_c = \mathscr{E}^{-1/2}$. If the electron at the moment the external constant field is turned on is located at a point of the orbit at a distance $r_0 > r_c$ from the nucleus, then at this point the external field is immediately stronger than the field of the nucleus from the moment of turn-on. For this reason the electron is trapped by the external field and escapes from the atom. We can say that for $r_0 > r_c$ ionization occurs instantaneously. The case when the external field pushes the electron in the direction toward the Coulomb region and the electron can approach to a distance $r < r_c$ from the nucleus is an exception. Then the motion of the electron is similar to the motion arising if the external field is turned on at a moment when the electron is located at a distance $r_0 < r_c$ from the nucleus. In this case the electron can at first complete one revolution around the nucleus and only then will the electron, moving on a Kepler trajectory distorted by the external field, approach the boundary of the Coulomb region after a time of order T_n. After this the electron is trapped by the external field and the atom is ionized.

The characteristic size of the orbit is equal to $2n^2$, which corresponds to the maximum distance of the electron from the nucleus in the s state in the Bohr-Sommerfeld model of the atom. From this qualitative picture of ionization and from the results of numerical calculations it follows that for $r_c < 2n^2$ ionization of the atom occurs over a time of the order of the Kepler period T_n of revolution of the electron on an undisturbed orbit. Since $r_c = \mathscr{E}^{-1/2}$, we obtain from here the condition for the strength of the field for which an ionization time T_n is realized:

$$\mathscr{E} > 1/4n^4.$$

We shall call such a field strong. It agrees well in order of magnitude with the known quantum field [$\mathscr{E} \gtrsim 0.1n^{-4}$ (Ref. 21)] and the classical field [$\mathscr{E} \gtrsim (0.13\text{–}0.38)n^{-4}$ (Refs. 21 and 22)] corresponding to ionization within a time T_n.

From what we have said above it follows that stabilization is impossible in a strong constant field.

3.2. Classification of stable trajectories in an alternating field

Numerical calculations showed that in alternating fields there can exist two qualitatively different types of stable trajectories.

Trajectories of the Kepler type are shown in Fig. 1. If the frequency of the external field is significantly lower than the Kepler frequency, then the trajectory is an open curve, which between successive passes near the nucleus looks like a slightly distorted elliptic orbit (Fig. 1a). When the frequency of the external field is comparable to the Kepler frequency, the trajectory consists of appreciably distorted Kepler orbits which have irregularly varying sizes and orientation in space and which transform into one another (Fig. 1b). If, however, the frequency of the external field is appreciably higher than the Kepler frequency, then small oscillations in the external field are superposed on the Keplerian motion of the electron (Fig. 1c). The position of the plane of the orbit usually changes after passage through the perihelion, so that the trajectory is also open. On trajectories of the Keplerian type the average total energy of the electron $\langle E \rangle$ is typically negative and equal in order of magnitude approximately to the initial total energy in the undisturbed state:

$$\langle E \rangle \sim -1/2n^2 < 0.$$

We can say that in the case of stabilization on Keplerian trajectories the external field plays the role of a small perturbation for the initial atomic state. Ionization occurs because the electron absorbs energy of the order of the binding energy $1/2n^2$. This happens within one or several passes around the nucleus. After ionization the electron moves along a hyperbola distorted by oscillations in the external field.

Stable trajectories of the oscillatory type, one of which is shown in Fig. 2, arise only in strong fields. One can see that the motion of the electron has the character of sharp oscillations around the nucleus whose excursion is of the order of $\mathscr{E}/\omega^2$ and on which slow irregular drift in the Coulomb field is superposed. The Keplerian orbits are completely destroyed. The average total energy is positive and approximately corresponds to the average oscillatory energy:

$$\langle E \rangle \sim \mathscr{E}^2/4\omega^2 > 0,$$

and in addition for $\mathscr{E} > \omega$ the energy can reach large values $\langle E \rangle \gg 1$, significantly higher than the binding energy of the undisturbed initial state. The motion is nonetheless finite. Ionization on trajectories of the oscillatory type occurs if the electron enters the Coulomb region $r < \mathscr{E}^{-1/2}$.

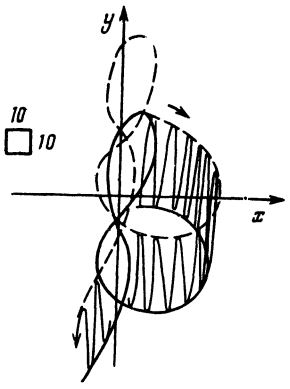

FIG. 2. An example of a stable trajectory of the oscillatory type in the case $n = 3$, $\mathscr{E} = 0.4$, and $\omega = 0.15$, corresponding to the conditions $\mathscr{E} \gg \omega^{4/3}$ and $\omega \gg n^{-3}$. The upper and lower envelopes are shown by the dashed and solid lines. The field is turned on adiabatically. The edges of the square show the scale in atomic units. More than 20% of the trajectories had lifetimes exceeding ten Keplerian periods.

The reasons why and the conditions under which Keplerian and oscillatory stable trajectories arise are discussed below.

3.3. Stabilization on Keplerian trajectories

Analysis of the results of numerical calculations showed that in order for stabilization to appear on trajectories of the Keplerian type the following relation must be satisfied:

$$\mathscr{E} < \omega^{2/3}/n^2. \tag{5}$$

We shall discuss this result in greater detail. If the frequency ω is low compared with the Kepler frequency n^{-3}, then the external field changes very little over times of the order of the Kepler period. For this reason, in order for stable trajectories with lifetimes much longer than T_n to exist the external field must reach values within the first Kepler period from the start-up time which are much smaller than the atomic values $1/4n^4$. For this the condition

$$\mathscr{E} T_n/T \ll 1/4n^4, \quad T = 2\pi/\omega,$$

or

$$\mathscr{E} \ll \omega^{-1} n^{-7}$$

must be satisfied.

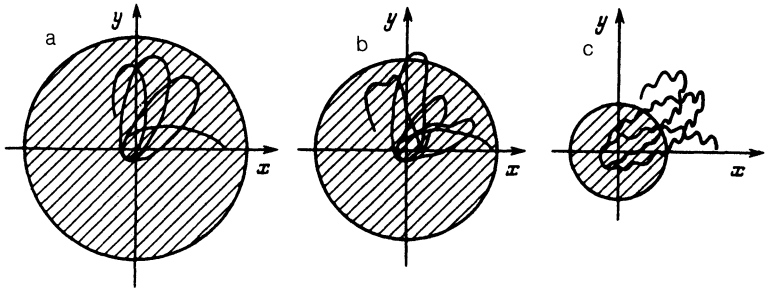

FIG. 1. Examples of stable Keplerian trajectories in a field $\mathscr{E} < \omega^{2/3}/n^2$: $\omega \ll n^{-3}$ (a), $\omega \sim n^{-3}$ (b), and $\omega \gg n^{-3}$ (c). The Coulomb region is hatched.

We note that the amplitude $\mathscr{E}$ can be much larger than $1/4n^4$. This value of the amplitude, however, is reached over times of the order of one-fourth the period T of the external field. For this reason, it is of great interest to estimate the external-field intensity for which stable trajectories exist over times of the order of T. This is easiest to illustrate for the example of one-dimensional motion (Fig. 3). An electron cannot pick up enough energy in one impact against the nucleus to escape from the nucleus, but the electron can do so over several passes, while the external field is oriented away from the nucleus. As a result, over each Kepler period the electron is displaced from the nucleus by a small distance

$$\Delta x \sim \mathscr{E} T_n^2/2$$

in the direction of the field. The characteristic displacement over one-half the period T of the external field, when the external field is directed away from the nucleus, is

$$x \sim \Delta x T 2 T_n.$$

Under conditions of stabilization the displacement x should not exceed the size $\mathscr{E}^{-1/2}$ of the Coulomb region, when the estimate (5) is obtained.

We note that for $\omega \ll n^{-3}$ condition (5) implies $\mathscr{E} \ll n^{-4}$, i.e., the field is not strong. Nonetheless, the excursion $\mathscr{E}/\omega^2$ of the oscillations of the electron in a purely external field under the conditions studied could be much greater than the size of a Keplerian orbit. This does not happen because of the stabilizing role of the Coulomb field. The characteristic trajectory in the three-dimensional case is shown in Fig. 1a. As one can see it lies wholly inside the Coulomb region.

If the frequency ω is comparable to the Kepler frequency n^{-3}, then in order for stable trajectories to arise the energy picked up by the electron in one pass near the nucleus must be less than the binding energy $1/2n^2$. The acquired energy can be estimated as follows:

$$\Delta E \sim \int \mathscr{E} \cos(\omega t) v(t) dt \sim \mathscr{E} \int v(t) dt \sim \mathscr{E} n^2 \sim \mathscr{E}/\omega^{2/3}. \tag{6}$$

From the condition $\Delta E < 1/2n^2$ and from Eq. (6) we obtain the relation (5).

We note that for $\omega \sim n^{-3}$ it follows from Eq. (5) that this regime of stable trajectories corresponds to the condition $\mathscr{E} < n^{-4}$, i.e., the field is not strong, and the trajectory lies in the Coulomb region (see Fig. 1b).

If the frequency ω of the external field is significantly higher than the Kepler frequency n^{-3}, then, as in the case $\omega \sim n^{-3}$, the relation (5) is obtained, if the condition that the energy absorbed by the electron from the external field and calculated from the relation (6) is significantly smaller than the binding energy is taken into account. However there is a difference in the mechanisms of energy absorption.

In the case of average frequencies $\omega \sim n^{-3}$ the contribution to the integral in Eq. (6) accumulates over times of the order of the Kepler period, i.e., energy is efficiently absorbed along the entire Keplerian orbit.

In the high-frequency case $\omega \gg n^{-3}$ the function $v(t)\cos\omega t$ in the integral (6) is rapidly oscillating, and the point $t = 0$ (the moment of passage through the perihelion of the orbit) is a point of stationary phase. Near this point the electron moves along a Keplerian orbit, and in addition we have $r(t) \sim t^{2/3}$ and $v(t) \sim t^{-1/3} \sim r^{-1/2}$. The effective region of integration in time is $t \sim 1/\omega$, and absorption of energy $\Delta E \sim \mathscr{E}\omega^{-2/3}$ occurs in a small region $r_R \sim \omega^{-2/3} \ll n^2$ near the nucleus.

We note that although Eq. (5) gives an upper limit on the field strength the field can be strong, since for $\omega \gg n^{-3}$ the condition $\mathscr{E} < \omega^{2/3}/n^2$ is not inconsistent with the condition $\mathscr{E} \gg 1/4n^4$.

The electron moves mainly in the region $r \sim n^2 \gg \mathscr{E}^{-1/2}$, where the external field predominates. One can see from the condition (5), however, that for $\omega \gg n^{-3}$ the excusion of the oscillations in the external field is much smaller than the dimensions of the Keplerian orbit:

$$\mathscr{E}/\omega^2 \ll \omega^{-4/3} n^{-2} \ll n^2.$$

For this reason and owing to the long-range nature of the Coulomb field Keplerian motion is preserved even far from the nucleus (see Fig. 1c). The average energy of the oscillatory motion on a Keplerian orbit is much smaller than the binding energy of an electron in an undisturbed atom:

$$\mathscr{E}^2/\omega^2 \ll (n^2\omega^{1/3})^{-2} \ll 1/n^2.$$

The numerical calculations showed that in order for stabilization to exist in a strong high-frequency field (which is possible for $\mathscr{E} \gg 1/4n^4$) under the conditions (5) the external field must be turned on with phase $\varphi \approx 0$ [see Eq. (1)]. The reasons for this will be discussed below.

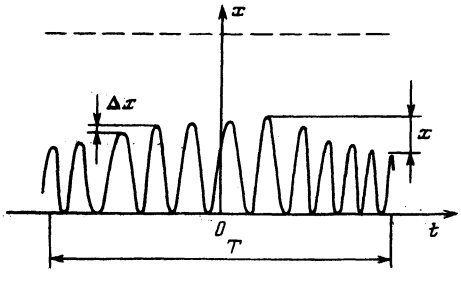

FIG. 3. An example of a one-dimensional stable motion in the case $\mathscr{E} < \omega^{2/3}/n^2$ and $\omega \ll n^{-3}$. The coordinate of the nucleus is $x = 0$. The dashed lines show the boundary of the Coulomb region.

3.4. Stabilization on oscillatory trajectories

Analysis of the results of numerical calculations showed that in order for stable trajectories of the oscillatory type to appear (see Fig. 2) several conditions must be satisfied. First, the external field must be strong, $\mathscr{E} \gg 1/4n^4$. In addition, the field must reach its maximum value within times much shorter than the Kepler period of the motion of the electron on the initial orbit. This is necessary in order for the Keplerian motion to be destroyed immediately when the external field is turned on and in order for the electron to begin oscillating while gradually drifting toward the nucleus. Second, the amplitude $\mathscr{E}/\omega^2$ of the oscillations of the electron in the external field must be much greater than the size $\mathscr{E}^{-1/2}$ of the Coulomb region. This gives the condition $\mathscr{E} \ll \omega^{4/3}$

Only is this case can the electron, because of the three-dimensional nature of its motion and the large amplitude of its oscillations in the external field, go around the Coulomb region as it approaches the nucleus. This is evident from Fig. 2. In this case, ionization may not occur for a long time. But when the electron sooner or later nonetheless randomly enters the Coulomb region, in the overwhelming majority of the cases this results in ionization. Finally, the manner in which the external field is turned on is fundamentally important for the appearance of stabilization on oscillatory trajectories. In the numerical calculations stable trajectories arise only when the field is turned on with the initial phase $\varphi \approx 0$ [see Eq. (1)]. This effect is discussed in the next section.

We note that on oscillatory trajectories with $\mathscr{E} \gg \omega^{4/3}$ and $\omega \gtrsim n^{-3}$ the average energy of the electron is determined primarily by its oscillatory energy, which is significantly higher than the binding energy of the electron in the unperturbed atom:

3.5. Effect of the introduction of the external field on the photoionization process

We shall first illustrate how the manner in which the ionizing field is turned on is manifested in the dynamics of the electron. We shall do this for the simplest case, when at the moment the field is switched on the electron is in a highly excited state far away from the nucleus. Under such conditions the effect of the Coulomb field of the nucleus can be neglected to a first approximation and the electron velocity as a function of time can be easily determined from Eq. (1):

$$\mathbf{v}=\mathbf{v}_0+\frac{\vec{\mathscr{E}}}{\omega}\sin(\omega t+\varphi)-\frac{\vec{\mathscr{E}}}{\omega}\sin\varphi.$$

It is obvious that together with oscillations of the electron with velocity amplitude $\mathscr{E}/\omega$ there is also an "outbound" motion with velocity $u = \mathscr{E}\sin\varphi/\omega$. In other words, at the moment the external field is turned on the electron is given an impulse, which ultimately results in ejection of the electron from the atom. In order for this not to happen, the field must be turned on in a special manner. For example, in the numerical calculations of trajectories in a strong field the electron did not escape when the field was turned on instantaneously with the phase $\varphi \approx 0$, when the outbound velocity was also equal to zero.

The results of the calculations and the conditions of stabilization, discussed above, were obtained assuming instantaneous turn-on. For this reason there arises the question of whether or not these results are applicable for real turn-on regimes. We shall now discuss this question in greater detail.

The real turn-on process is smooth and is determined by some envelope. By adjusting the envelope it is possible to eliminate the impulse to the electron associated with the turning on of the external field. This will be analogous to using the condition of adiabatic switch-on, employed in quantum-mechanical problems. We shall give a classical formulation of the condition of adiabaticity. We assume that the envelope $f(t)$ of the intensity of the external field reaches a constant value equal to unity within a time τ. Then in order for the Coulomb field of the nucleus to prevent the electron from escaping, the kinetic energy $u^2/2$ associated with the escape velocity u must be less than the absolute value of the characteristic potential energy of the electron $1/2n^2$ on the initial Keplerian orbit. This condition can be written in the following form:

$$u=\frac{\omega}{2\pi}\int\limits_{t>\tau}^{t+2\pi/\omega}dt\int\limits_{0}^{t}\mathscr{E}f(t')\cos(\omega t'+\varphi)\,dt'<\frac{1}{n}.$$

For instantaneous switch-on we must have $f(t) = 1$ and $u = \mathscr{E}\sin\varphi/\omega$. The condition of adiabaticity is satisfied if $\mathscr{E}/\omega < n^{-1}$ or if $\varphi \sim 0$. For linear switching-on the envelope has the form $f(t) = t/\tau$ for $t < \tau$ and $f(t) = 1$ for $t \geqslant \tau$. The escape velocity is determined by the relation

$$u=\frac{\mathscr{E}}{\tau\omega^2}[\cos(\omega\tau+\varphi)-\cos\varphi].$$

The condition of adiabaticity is satisfied for $\mathscr{E}/\tau\omega^2 \ll n^{-1}$ or when the turn-on time equals an integer number k of periods of the external field: $\tau = 2\pi k/\omega$.

If the amplitude of the external field reaches the value $\mathscr{E}$ within a time τ much shorter than the Kepler period of revolution of the electron along the orbit in the initial state, then for stabilization in a strong field $\mathscr{E} \gg 1/4n^4$ it is sufficient that the adiabaticity condition be satisfied. If, however, the amplitude of the field reaches maximum values over times shorter than the Kepler period T_n, then in order to achieve stabilization the field must become strong over times of the order of T_n. For this the relation

$$\mathscr{E} \gg \tau/4n^4T_n$$

must be satisfied. We note that here τ is the time over which the maximum value of the field is established. For instantaneous switch-on we have $0 \leqslant \tau \leqslant \pi/2\omega$.

We emphasize that for $\omega \ll n^{-3}$ and $\omega \sim n^{-3}$ under the weak-field conditions (5) the stabilized motion of the electron occurs in the Coulomb region $r \lesssim r_c$ (see Figs. 1a and b), where the field of the nucleus is stronger than the external field. For this reason the problem of the impulse to the electron at switch-on does not arise, since the field of the nucleus smooths out this impulse. The numerical calculations confirmed that stable trajectories appear for $\mathscr{E} < \omega^{2/3}/n^2$ and $\omega \lesssim n^{-3}$, irrespective of the choice of phase φ.

A brief discussion of the role of the width of the external-field pulse is in order. In this work we studied only the case when the external field is present for a time much longer than the Kepler period. For highly excited atoms, however, conditions under which the external field will be present for a time significantly shorter than the Coulomb period can be realized. If the field is turned on and off adiabatically, then the impulses to the electron which are associated with turning the field on and off will be suppressed. Since most of the time the electron is located far away from the nucleus, it is most likely to be far away from the nucleus when the external field is on. The Keplerian trajectory will be distorted on a short section when the external field is on, but a finite orbit is reestablished after the field is turned off. Thus photoionization will be suppressed in a field that is turned on and off adiabatically for a time interval much shorter than the Kepler period.

4. COMPARISON WITH THE RESULTS OF OTHER WORK

From the numerical calculations and their analysis it can be concluded that stabilization of an atom with respect to photoionization occurs not only in the quantum case but also in the model of classical dynamics of an electron. Stabilization in an external field acting for a time much longer than the Kepler period is possible in two regimes:

1) $0<\omega\leqslant n^{-2}$, $\mathscr{E}\ll\omega^{2/3}/n^2$,

and adiabatic switching-on with $\omega\gg n^{-3}$;

2) $0<\omega\leqslant n^{-2}$, $\mathscr{E}\gg\omega^{4/3}$,

$$\mathscr{E}\gg\begin{cases}1/4n^4, & \tau\leqslant T_n=2\pi n^3\\ \dfrac{\tau}{T_n}\dfrac{1}{4n^4}, & \tau>T_n\end{cases},$$

and adiabatic switching-on. Here τ is the time over which the amplitude of the external field is established.

We shall discuss how these regimes agree with the results of other work.

Classical calculations[3] of the total probability of ionization of an arbitrary state n when the external field is on for 150 Keplerian periods ($t=150T_n$) were performed for weak fields with intensity $0.06n^{-4}\leqslant\mathscr{E}\leqslant0.2n^{-4}$ and frequency $0.01n^{-3}\leqslant\omega\leqslant10n^{-3}$. For $\omega=0.01n^{-3}$ total ionization over the time t occurred with $\mathscr{E}=0.2n^{-4}$. We note that at this frequency ω the time t corresponds to one and a half periods of the external field. The ionization probability decreased as the field strength decreased and for $\mathscr{E}\sim0.1n^{-4}$ the probability was close to zero. In a field with frequency $\omega\sim n^{-3}$ ionization was suppressed for $\mathscr{E}\sim0.06n^{-4}$, while in a field with frequency $\omega\sim10n^{-3}$ ionization was already suppressed for $\mathscr{E}\sim0.2n^{-4}$. It is easy to see that these conditions of ionization suppression correspond to the regime $\mathscr{E}\ll\omega^{2/3}/n^2$.

Numerical calculations[23,24] based on classical electron dynamics were performed for the ground state of the hydrogen atom ($n=1$) in a field with fixed strength $\mathscr{E}=2$ and variable frequency $\omega=1$–40. Stabilization was observed only for $\omega\sim40$ (all trajectories had lifetimes longer than one-half the Keplerian period). It is obvious that this case satisfies the condition $\mathscr{E}\ll\omega^{2/3}/n^2$. Stabilization in a field with $\mathscr{E}\gg\omega^{4/3}$ could not be observed, since this requires frequencies $\omega\ll1$.

Quantum calculations[20] of the lifetime of the ground state ($n=1$) of a hydrogen atom in a strong field were performed in connection with the problem of determining the quasi-energy spectrum of the electrons. This approach obviously corresponds to adiabatic switching-on of the external field. The frequency of the field was equal to $\omega=0.65$ and the field strength $\mathscr{E}$ ranged from zero to 1.5. The minimum lifetime, equal to approximately the Kepler period, was achieved for $\mathscr{E}\sim0.6$. As the field strength was further increased the lifetime started to increase and at $\mathscr{E}\sim1.5$ it was already equal to approximately three Kepler periods. Stabilization at $\mathscr{E}\sim1.5$ corresponds to the condition $\mathscr{E}\gg\omega^{4/3}$. The shortness of the lifetime could be connected with the fact that the conditions $\mathscr{E}\gg1/4n^4$ and $\mathscr{E}\gg\omega^{4/3}$ are satisfied only weakly.

Quantum calculations[29] of the photoionization of a hydrogen atom in the ground state ($n=1$) were performed for a field with frequency $\omega=1$ and strengths $\mathscr{E}=3.4$, 6.8, and 13.5. The switching-on regime was set by an envelope which varied linearly over five optical cycles and then remained constant for ten cycles. This corresponds in the classical approach to adiabatic switching-on. The ionization probability after 15 periods of the external field was equal to 0.918. 0.69, and 0.485, respectively, for the three values of the field strength. It is obvious that stabilization at $\mathscr{E}=13.5$ is satisfactorily described by the condition $\mathscr{E}\gg\omega^{4/3}$. It is also interesting that stabilization was not observed when the field was turned on instantaneously and nonadiabatically according to the law

$$\mathscr{E}(t)=\mathscr{E}\sin\omega t.$$

Finally, we shall analyze the quantum calculations of Ref. 19, which were performed by numerical methods in a one-dimensional formulation of the problem with the potential

$$U(x)=-(x^2+1)^{-1/2}.$$

The fundamental property of this potential is that it is "transparent" in the sense that an electron incident on the nucleus freely passes through it and is not reflected backwards, as should happen in the field $U(x)=-1/|x|$. In order to determine the reasons for stabilization in the "transparent" potential classical calculations of the electron trajectories in the field $U(x)=-1/|x|$ were performed in one case with a boundary condition corresponding to reflection from the nucleus (i.e., with the true potential) and in another case when the electron passed through the nucleus (i.e., with a "transparent" potential).

The results of the calculations are presented in Fig. 4. One can see that in the second case the trajectories are indeed stable, though for the true potential ionization occurs within one Kepler period. Thus the use of the "transparent" potential is the same as introducing stabilization into the model artificially. In addition, in one-dimensional models the additional channel for electrons to escape in lateral directions is neglected while numerical calculations have shown that this is the main channel for ionization of the atom.

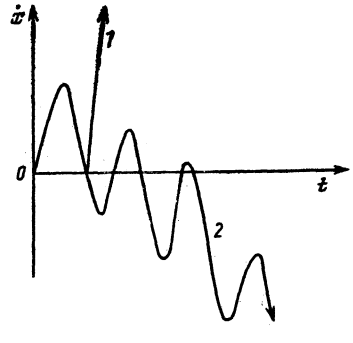

FIG. 4. Typical electron motion in models with the one-dimensional potential $U(x)=-1/|x|$: *1*) boundary condition corresponding to reflection at the nucleus and *2*) boundary condition corresponding to passage through the nucleus. The coordinate of the nucleus is $x=0$.

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