# Theory of the suppression of the electron-phonon interaction in the strong field of a coherent optical pulse

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In the method of second quantization, by means of a standard decoupling procedure, a closed system of equations is derived that describes the time evolution of the nonequilibrium electron density and polarization (macroscopic dipole moment) that are induced in a semiconductor by a resonance optical pulse in direct interband transitions, with allowance for the interaction with the phonon subsystem. For ultrashort light pulses, when relaxation processes can be neglected, a coherent solution equivalent to the solution of the Bloch equations in two-level systems is obtained. An analytical method is developed for taking into account the influence of electron-phonon scattering processes on the coherent regime of interaction of a semiconductor with an intense optical field. It has been found that in this case the characteristic decay time of a coherent state of the electron subsystem depends in an essential way on the intensity of the external optical radiation, and this, in the final analysis, can lead to suppression of relaxation processes in a semiconductor by a strong external field.

## 1. INTRODUCTION

The appearance in recent years of powerful sources of ultrashort optical pulses (with lengths down to a few femtoseconds) makes it possible to place experimental studies in the area of the interaction of light with matter on a qualitatively new plane. Such short-duration and powerful influences rapidly bring the electron subsystem of a solid into a strongly nonequilibrium state, and subsequent probing of the relaxation of the electron-hole plasma by a series of less powerful ultrashort optical pulses gives direct information on the kinetic processes of decay or thermalization. Experimental investigation of coherent interactions of radiation with semiconductors and insulators, in which the relaxation times due to electron-electron and electron-phonon scattering lie in the sub-picosecond range, is now becoming entirely realistic. In support of this we may point to a number of recent experiments: 1) the observation of a quantum-size optical Stark effect in GaAs structures;[1–3] 2) spectral hole burning in semiconductors;[4–7] 3) transition oscillations in the transmission spectrum, which have been investigated in thin samples of CdSe and GaAs.[7–10]

The well known coherent optical phenomenon of photon echo,[11] which had been investigated previously in gases and metal vapors and at impurities in crystals, can be realized in a bulk intrinsic semiconductor. This was first predicted theoretically as long ago as 1973 (Ref. 12), and has recently been demonstrated experimentally with the use of femtosecond optical pulses that have induced direct interband transitions in a GaAs film.[13]

These experimental results prove convincingly that it is possible to generate a coherent electron state in semiconductors and insulators on a time scale shorter than or comparable to the characteristic times of the electron-electron and electron-phonon interactions. After the end of the action of the pulse on the medium, the nonequilibrium state of the electrons and holes that is induced by the field is rapidly thermalized, relaxing to a Fermi-Dirac distribution with its own Fermi quasilevels. For ultrashort pulses the coherence-destruction time (phase-memory time) is assumed in this case to be independent of the intensity of the previously acting external field. However, if the duration of the optical radiation acting on the semiconductor becomes comparable to the characteristic times of the relaxation processes, the optical field can have a direct influence on the collisions of electrons (holes) with each other and with phonons.

The dependence of the time of destruction of a coherent state created in a semiconductor by direct optical transitions of electrons from the valence band to the conduction band on the intensity of a strong external field has been investigated theoretically in a number of previous papers.[14–17] In these papers and in the present article, by a strong field we mean optical electromagnetic radiation for which the inequality $\Omega\tau > 1$ is satisfied, where $\Omega$ is the Rabi frequency ($\Omega = \mu\mathscr{E}/\hbar$, where $\mathscr{E}$ is the amplitude of the wave and $\mu$ is the matrix element of the dipole-moment operator for the direct interband transition) and $\tau$ is the characteristic relaxation time of the nonequilibrium current carriers. When this condition is fulfilled, the interband transitions induced by the field occur more rapidly than an individual act of electron collision or holes with each other or with phonons, and therefore we should expect the relaxation processes to have an appreciable dependence on the intensity of the external radiation.

Usually, either the electric-field intensity is assumed to be a monochromatic wave with constant amplitude, or the time-dependent amplitude of the field satisfies the slowness condition $\Omega t_{\rm pul} \gg 1$, where $t_{\rm pul}$ is the pulse length. Because of this, it is possible to introduce a quasiparticle representation and to obtain for the quasiparticles a corresponding renormalized system of equations.[18] Subsequently, this system was analyzed numerically with allowance for electron-phonon interaction processes, from the results of which it followed that the coherence time increased with the intensity of the external field.[15] On the basis of this, the anomalous transparency that had been observed when a powerful picosecond light pulse passed through a semiconductor in experiments performed by Dneprovskiĭ and co-workers[19,20] was explained as a coherence effect of the self-induced transpar-

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ency type. The phenomenon of the suppression of relaxation processes in a strong field was attributed to the presence in the quasiparticle-energy spectrum of a gap of magnitude proportional to the value of $\Omega$. For example, this ought to affect the electron-phonon scattering when the amplitude of the external field reaches values at which the inequality $\Omega > \omega_{\rm ph}$ is satisfied, where $\omega_{\rm ph}$ is the characteristic frequency of the phonons in the semiconductor.[15] However, it then remains unclear why the strong-field condition in the form $\Omega\tau > 1$ should be fulfilled. We note that in the analysis offered below this requirement arises in a natural manner.

The cause of the renewal of interest in the above-described problem of the suppression of relaxation processes in a semiconductor by a strong field has been the recent experiments of Belenov and Vasil'ev on the generation of a powerful picosecond light pulse by a semiconductor laser,[21,22] in which the condition $\Omega\tau > 1$ was certainly achieved. The authors proposed that the effect observed—the breaking up of the generated pulse into subpulses with an increase in the intensity of the radiation—is a consequence of the well known coherent generation[12] that develops over times shorter than the characteristic relaxation times. This implies that relaxation processes in a semiconductor should be suppressed by a strong field, as only in this case is a coherent regime of generation possible in the picosecond range. Our paper is devoted to an analysis of the physical mechanisms responsible for such coherent interactions.

The aim of the present paper is to develop a theory of the interaction of a resonance optical pulse in a semiconductor for direct interband transitions with allowance for electron-phonon scattering processes. The main problem consists in extracting in explicit form the dependence of the characteristic time of the loss of coherence on the intensity of the external field. We consider the case of an optical pulse of rectangular shape, for which the strong-field condition is fulfilled and the pulse length exceeds the characteristic relaxation time: $t_{\rm pul} > \tau$.

We shall show that the solution describing the coherent interaction of the field with the semiconductor is a superposition of two harmonics—the "zeroth" and the "oscillator" harmonic (the frequency of the oscillations is determined by the value of the Rabi frequency and by the magnitude of the detuning from resonance). This result is analogous to the well known coherent solution for the two-level model of an atom situated in a constant external field. The difference is that the resonance spectrum undergoes specific inhomogeneous broadening, determined by the dispersion law in the bands. Allowance for the electron-phonon interaction leads to decay of the coherent state induced by the external field. It is found that in this case the amplitudes of the coherent harmonics become time-dependent, and for the oscillator component of the solution the amplitude of the field appears explicitly in the characteristic time ($\tau \propto |\Omega|^2$). Therefore, as the intensity of the external field increases the process of the loss of coherence may be suppressed.

In Sec. 2 we give the Hamiltonian, and, by the method of second quantization, give a direct derivation of the equations of motion, for current carriers in a semiconductor that are interacting with a resonance external field and with phonons (in contrast to Ref. 15, in which a renormalized system of equations for the quasiparticles is used). In Sec. 3 we obtain a coherent solution, when relaxation processes can be neglected (the range of ultrashort times), and also develop an analytical method for taking into account the influence of electron-phonon scattering on the coherent regime of interaction of a semiconductor with a strong external field.

## 2. THE HAMILTONIAN AND EQUATIONS OF MOTION

The Hamiltonian describing the interaction of electrons in a semiconductor with an optical field and with phonons has the form

$$H = H_0 + H_{int} + H_{ph}, \tag{1}$$

where

$$H_0 = \sum_{\mathbf{k}} E_e(\mathbf{k}) a_{\mathbf{k}}^+ a_{\mathbf{k}} + E_h(\mathbf{k}) b_{-\mathbf{k}}^+ b_{-\mathbf{k}}, \tag{2a}$$

$$H_{int} = -\sum_{\mathbf{k}} \boldsymbol{\mu}_{\mathbf{k}} \mathbf{E}(t) a_{\mathbf{k}}^+ b_{-\mathbf{k}}^+ + \boldsymbol{\mu}_{\mathbf{k}}^* \mathbf{E}^*(t) b_{-\mathbf{k}} a_{\mathbf{k}}, \tag{2b}$$

$$H_{ph} = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} c_{\mathbf{q}}^+ c_{\mathbf{q}} + \sum_{\mathbf{k},\mathbf{q}} g(\mathbf{q}) (a_{\mathbf{k}}^+ a_{\mathbf{k}-\mathbf{q}} + b_{-\mathbf{k}} b_{-\mathbf{k}+\mathbf{q}}^+)(c_{\mathbf{q}} + c_{-\mathbf{q}}^+). \tag{2c}$$

Here, $E_c(\mathbf{k})$ and $E_h(\mathbf{k})$ are the dispersion laws for the electrons and holes; $a_{\mathbf{k}}^+$ ($a_{\mathbf{k}}$) and $b_{-\mathbf{k}}^+$ ($b_{-\mathbf{k}}$) are the creation (annihilation) operators for nonequilibrium electrons and holes with wave vector $\mathbf{k}$; $\mu_{\mathbf{k}}$ is the matrix element of the dipole-moment operator for interband transitions; $g(\mathbf{q})$ is the electron-phonon coupling constant; $c_{\mathbf{q}}^+$ ($c_{\mathbf{q}}$) is the creation (annihilation) operator for a phonon with wave vector $\mathbf{q}$, and $\omega_{\mathbf{q}}$ is the phonon frequency.

Let the intensity of the electric field of the optical wave be given by the expression

$$\mathbf{E}(t) = \mathbf{e}_E \mathscr{E}(t) \exp(-i\omega_0 t) + \text{c.c.}, \tag{3}$$

where $\omega_0$ is the frequency, $\mathscr{E}(t)$ is the slowly varying amplitude, and $\mathbf{e}_E$ is the unit polarization vector. We consider the case of a linearly polarized field, and therefore we replace $\boldsymbol{\mu}_{\mathbf{k}} \cdot \mathbf{E}(t)$ by $\mu_{\mathbf{k}}^e [\mathscr{E}(t) \exp(-i\omega_0 + \text{c.c.}]$, where $\mu_{\mathbf{k}}^e$ is the projection of the vector $\boldsymbol{\mu}_{\mathbf{k}}$ on to the direction of $\mathbf{e}_E$. In addition, henceforth we assume the external field (3) to be fixed, i.e., we neglect its changes resulting from the response of the medium.

The macroscopic characteristics of the system can be determined starting from the following quantum mechanical expectation values:

$$\begin{gathered} n_e(\mathbf{k}) = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle, \\ n_h(\mathbf{k}) = \langle b_{-\mathbf{k}}^+ b_{-\mathbf{k}} \rangle, \\ p^*(\mathbf{k}) \exp(i\omega_0 t) = \langle a_{\mathbf{k}}^+ b_{-\mathbf{k}}^+ \rangle, \\ N(\mathbf{q}) = \langle c_{\mathbf{q}}^+ c_{\mathbf{q}} \rangle. \end{gathered} \tag{4}$$

The first two expressions in (4) are the numbers of nonequilibrium electrons and holes in the $\mathbf{k}$ state; the third describes the field-induced transitions from the valence band to the conduction band and makes it possible to calculate the macroscopic polarization induced in the medium by the external radiation; $N(\mathbf{q})$ is the number of phonons with wave vector $\mathbf{q}$.

The time evolution of these quantum mechanical expectation values follows from the general equation of motion

$$i\hbar\frac{\partial}{\partial t}\langle A\rangle=\langle[AH]\rangle, \tag{5}$$

where $A$ is an arbitrary operator. Substituting the operators indicated above into (5) and using the Hamiltonian (1), we obtain

$$\frac{\partial}{\partial t}n_e(\mathbf{k})=-2\,\mathrm{Im}[\Omega p^*(\mathbf{k})]-i\hbar^{-1}\sum_{\mathbf{q}}g(\mathbf{q})[f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})+f_e^*(\mathbf{k}-\mathbf{q},\mathbf{k},-\mathbf{q})-f_e(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})-f_e^*(\mathbf{k},\mathbf{k}+\mathbf{q},-\mathbf{q})],$$

$$\frac{\partial}{\partial t}n_h(\mathbf{k})=-2\,\mathrm{Im}[\Omega p^*(\mathbf{k})]-i\hbar^{-1}\sum_{\mathbf{q}}g(\mathbf{q})[f_h(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})+f_h^*(\mathbf{k}+\mathbf{q},\mathbf{k},-\mathbf{q})-f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-f_h^*(\mathbf{k}-\mathbf{q},\mathbf{k},-\mathbf{q})],$$

$$\frac{\partial}{\partial t}p^*(\mathbf{k})=-i\Delta(\mathbf{k})p^*(\mathbf{k})+i\Omega^*[n_e(\mathbf{k})+n_h(\mathbf{k})-1]-i\hbar^{-1}\sum_{\mathbf{q}}g(\mathbf{q})[f_+(\mathbf{k},\mathbf{k}-\mathbf{q},-\mathbf{q})+f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-f_+(\mathbf{k}+\mathbf{q},\mathbf{k},-\mathbf{q})-f_-(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})], \tag{6}$$

$$\frac{\partial}{\partial t}N(\mathbf{q})=-i\hbar^{-1}\sum_{\mathbf{k}}g(\mathbf{q})[f_e^*(\mathbf{k}-\mathbf{q},\mathbf{k},-\mathbf{q})-f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})+f_h^*(\mathbf{k}-\mathbf{q},\mathbf{k},-\mathbf{q})-f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})],$$

where for the three-operator expectation values appearing on the right-hand sides of Eqs. (6) we use the following notation:

$$f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=\langle a_\mathbf{k}^+a_{\mathbf{k}-\mathbf{q}}c_\mathbf{q}\rangle,\quad f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=\langle b_{-\mathbf{k}}b^+_{-\mathbf{k}+\mathbf{q}}c_\mathbf{q}\rangle,$$
$$f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=\langle a_\mathbf{k}^+b^+_{-\mathbf{k}+\mathbf{q}}c_\mathbf{q}\rangle\exp(-i\omega_0t),$$
$$f_+(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q})=\langle a^+_{\mathbf{k}-\mathbf{q}}b_{-\mathbf{k}}{}^+c_\mathbf{q}{}^+\rangle\exp(-i\omega_0t),$$

and have also introduced the detuning from resonance, given by $\Delta(\mathbf{k})=\omega_0-\hbar^{-1}[E_e(\mathbf{k})+E_h(\mathbf{k})]$, and the Rabi frequency $\Omega=\mu_\mathbf{k}^e\mathscr{E}(t)/\hbar$, for which we neglect the dependence on the wave vector $\mathbf{k}$.

Equations for the expectation values appearing in Eqs. (6) can be obtained analogously. In this case, four-operator expectation values arise, the time evolution of which is described, in turn, by the general equation of motion (5). However, this also leads to an unclosed system of equations, since in the calculation of the commutation relations with the term in the Hamiltonian corresponding to the electron-phonon interaction expectation values of higher order appear in every case. The important point is that this contribution (which causes the system to be nonclosed) is proportional to the electron-phonon coupling constant $g(\mathbf{q})$ that appears in the expression (2c). Thus, if collision processes can be neglected $[g(\mathbf{q})=0]$, the system for the expectation values (4) becomes closed. Therefore, assuming henceforth that the electron-phonon interaction is weak, we replace the four-operator expectation values that arise by a product of the two-operator expectation values (4) introduced previously, and thus obtain, in second order, a closed system of equations. As will be seen below, these terms do indeed give a contribution proportional to $|g(\mathbf{q})|^2$ to the final system. According to the standard decoupling procedure described, the following relations should be fulfilled:

$$\langle a_\mathbf{k}{}^+a_{\mathbf{k}'}c_{\mathbf{q}'}{}^+c_\mathbf{q}\rangle\approx\delta_{\mathbf{k}\mathbf{k}'}\delta_{\mathbf{q}\mathbf{q}'}N(\mathbf{q})n_e(\mathbf{k}),$$
$$\langle a_\mathbf{k}{}^+a_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}'}{}^+a_{\mathbf{k}'+\mathbf{q}}\rangle\approx-\delta_{\mathbf{k}(\mathbf{k}'+\mathbf{q})}n_e(\mathbf{k})n_e(\mathbf{k}-\mathbf{q}), \tag{7}$$
$$\langle a_\mathbf{k}{}^+b_{-\mathbf{k}'}{}^+c_{\mathbf{q}'}{}^+c_\mathbf{q}\rangle\approx\delta_{\mathbf{k}\mathbf{k}'}\delta_{\mathbf{q}\mathbf{q}'}N(\mathbf{q})p^*(\mathbf{k})\exp(i\omega_0t)$$

etc. Here, we have noted that $\mathbf{q}\neq0$ and that all operator expectation values that contain $c_\mathbf{q},c_\mathbf{q}$ or $c_\mathbf{q}^+,c_\mathbf{q}^+$ vanish. In this approximation, it follows from the equations of motion (5) that

$$\frac{\partial}{\partial t}f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=-i\hbar^{-1}[E_e(\mathbf{k}-\mathbf{q})+\hbar\omega_\mathbf{q}-E_e(\mathbf{k})]f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\hbar^{-1}g(-\mathbf{q})N(\mathbf{q})[n_e(\mathbf{k})-n_e(\mathbf{k}-\mathbf{q})]+i\Omega f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\Omega^*f_+^*(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q}),$$

$$\frac{\partial}{\partial t}f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=-i\hbar^{-1}[E_h(\mathbf{k})+\hbar\omega_\mathbf{q}-E_h(\mathbf{k}-\mathbf{q})]f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\hbar^{-1}g(-\mathbf{q})N(\mathbf{q})[n_h(\mathbf{k}-\mathbf{q})-n_h(\mathbf{k})]-i\Omega f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})+i\Omega^*f_+^*(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q}),$$

$$\frac{\partial}{\partial t}f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})=-i\hbar^{-1}[\hbar\omega_0+\hbar\omega_\mathbf{q}-E_e(\mathbf{k})-E_h(\mathbf{k}-\mathbf{q})]f_-(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\hbar^{-1}g(-\mathbf{q})N(\mathbf{q})[p^*(\mathbf{k})-p^*(\mathbf{k}-\mathbf{q})]+i\Omega^*f_e(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\Omega^*f_h(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q}), \tag{8}$$

$$\frac{\partial}{\partial t}f_+(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q})=-i\hbar^{-1}[\hbar\omega_0-\hbar\omega_\mathbf{q}-E_h(\mathbf{k})-E_e(\mathbf{k}-\mathbf{q})]f_+(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q})-i\hbar^{-1}g(\mathbf{q})N(\mathbf{q})[p^*(\mathbf{k}-\mathbf{q})-p^*(\mathbf{k})]+i\Omega^*f_e^*(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-i\Omega^*f_h^*(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})$$

In deriving (8) we have assumed that the condition $N(\mathbf{q})\gg1$ is sufficient to satisfy an inequality of the form

$$N(\mathbf{q})[n_e(\mathbf{k})-n_e(\mathbf{k}-\mathbf{q})]\gg1 \tag{9}$$

and analogous relations with the functions $n_h(\mathbf{k})$ and $p^*(\mathbf{k})$ in place of $n_e(\mathbf{k})$. This requirement will be considered in more detail below. On the basis of (9), in the right-hand sides of Eqs. (8) we have discarded free terms that do not include the factor $N(\mathbf{q})$. In addition, henceforth we consider an optical pulse of rectangular shape, for which we have $\Omega(t)=\Omega$ for $0\leqslant t\leqslant t_{\mathrm{pul}}$ and $\Omega(t)=0$ otherwise. Then, after the introduction of the notation

$$\Omega p^*(\mathbf{k})=P(\mathbf{k})+i\sigma(\mathbf{k}),\quad F_e^{(1)}+iF_e^{(2)}=\hbar^{-1}g(\mathbf{q})f_e,$$
$$F_h^{(1)}+iF_h^{(2)}=\hbar^{-1}g(\mathbf{q})f_h,\quad F_-^{(1)}+iF_-^{(2)}=\Omega\hbar^{-1}g(\mathbf{q})f_-,$$
$$F_+^{(1)}+iF_+^{(2)}=\Omega\hbar^{-1}g(-\mathbf{q})f_+$$

the system (6) can be brought to the form

$$\frac{\partial}{\partial t}n_e(\mathbf{k})=-2\sigma(\mathbf{k})+\sum_\mathbf{q}2[F_e^{(2)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-F_e^{(2)}(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})], \tag{10a}$$

$$\frac{\partial}{\partial t}n_h(\mathbf{k})=-2\sigma(\mathbf{k})-\sum_\mathbf{q}2[F_h^{(2)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-F_h^{(2)}(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})], \tag{10b}$$

$$\frac{\partial}{\partial t}P(\mathbf{k})=\Delta(\mathbf{k})\sigma(\mathbf{k})+\sum_{\mathbf{q}}[F_-^{(2)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})-F_+^{(2)}(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q})$$
$$-F_-^{(2)}(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})+F_+^{(2)}(\mathbf{k},\mathbf{k}+\mathbf{q},\mathbf{q})], \qquad (10c)$$

$$\frac{\partial}{\partial t}\sigma(\mathbf{k})=-\Delta(\mathbf{k})P(\mathbf{k})+|\Omega|^2[n_e(\mathbf{k})+n_h(\mathbf{k})-1]$$
$$-\sum_{\mathbf{q}}[F_-^{(1)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})$$
$$-F_+^{(1)}(\mathbf{k}-\mathbf{q},\mathbf{k},\mathbf{q})-F_-^{(1)}(\mathbf{k}+\mathbf{q},\mathbf{k},\mathbf{q})+F_+^{(1)}(\mathbf{k},\mathbf{k}+\mathbf{q},\mathbf{q})], \qquad (10d)$$

$$\frac{\partial}{\partial t}N(\mathbf{q})=-\sum_{\mathbf{k}}2[F_e^{(2)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})+F_h^{(2)}(\mathbf{k},\mathbf{k}-\mathbf{q},\mathbf{q})] \qquad (10e)$$

with scattering amplitudes satisfying the equations

$$\frac{\partial}{\partial t}F_e^{(1)}=\Delta_e(\mathbf{k},\mathbf{q})F_e^{(2)}-F_-^{(2)}-F_+^{(2)},$$
$$\frac{\partial}{\partial t}F_e^{(2)}=-\Delta_e(\mathbf{k},\mathbf{q})F_e^{(1)}-\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})$$
$$\times[n_e(\mathbf{k})-n_e(\mathbf{k}-\mathbf{q})]+F_-^{(1)}-F_+^{(1)},$$
$$\frac{\partial}{\partial t}F_h^{(1)}=\Delta_h(\mathbf{k},\mathbf{q})F_h^{(2)}+F_-^{(2)}+F_+^{(2)},$$
$$\frac{\partial}{\partial t}F_h^{(2)}=-\Delta_h(\mathbf{k},\mathbf{q})F_h^{(1)}+\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})$$
$$\times[n_h(\mathbf{k})-n_h(\mathbf{k}-\mathbf{q})]-F_-^{(1)}+F_+^{(1)}, \qquad (11)$$
$$\frac{\partial}{\partial t}F_-^{(1)}=\Delta_-(\mathbf{k},\mathbf{q})F_-^{(2)}+\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})[\sigma(\mathbf{k})-\sigma(\mathbf{k}-\mathbf{q})]$$
$$-|\Omega|^2F_e^{(2)}+|\Omega|^2F_h^{(2)},$$
$$\frac{\partial}{\partial t}F_-^{(2)}=-\Delta_-(\mathbf{k},\mathbf{q})F_-^{(1)}-\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})[P(\mathbf{k})-P(\mathbf{k}-\mathbf{q})]$$
$$+|\Omega|^2F_e^{(1)}-|\Omega|^2F_h^{(1)},$$
$$\frac{\partial}{\partial t}F_+^{(1)}=\Delta_+(\mathbf{k},\mathbf{q})F_+^{(2)}-\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})[\sigma(\mathbf{k})-\sigma(\mathbf{k}-\mathbf{q})]$$
$$+|\Omega|^2F_e^{(2)}-|\Omega|^2F_h^{(2)},$$
$$\frac{\partial}{\partial t}F_+^{(2)}=-\Delta_+(\mathbf{k},\mathbf{q})F_+^{(1)}+\hbar^{-2}|g(\mathbf{q})|^2N(\mathbf{q})[P(\mathbf{k})-P(\mathbf{k}-\mathbf{q})]$$
$$+|\Omega|^2F_e^{(1)}-|\Omega|^2F_h^{(1)},$$

where, for brevity, we have also used

$$\Delta_e(\mathbf{k},\mathbf{q})=\hbar^{-1}[E_e(\mathbf{k}-\mathbf{q})+\hbar\omega_\mathbf{q}-E_e(\mathbf{k})],$$
$$\Delta_h(\mathbf{k},\mathbf{q})=\hbar^{-1}[E_h(\mathbf{k})+\hbar\omega_\mathbf{q}-E_h(\mathbf{k}-\mathbf{q})],$$
$$\Delta_-(\mathbf{k},\mathbf{q})=\hbar^{-1}[\hbar\omega_0+\hbar\omega_\mathbf{q}-E_e(\mathbf{k})-E_h(\mathbf{k}-\mathbf{q})],$$
$$\Delta_+(\mathbf{k},\mathbf{q})=\hbar^{-1}[\hbar\omega_0-\hbar\omega_\mathbf{q}-E_h(\mathbf{k})-E_e(\mathbf{k}-\mathbf{q})].$$

Equations (11) correspond to $F$-amplitudes whose arguments include the values of the wave vectors $\mathbf{k}$ and $\mathbf{k}-\mathbf{q}$. Analogous equations for the $F$-amplitudes that depend on the $\mathbf{k}+\mathbf{q}$ and $\mathbf{k}$ states are obtained by replacing $\mathbf{k}$ by $\mathbf{k}+\mathbf{q}$ in the arguments of all functions.

The resulting system of equations makes it possible to investigate the evolution of the nonequilibrium population and polarization induced by an external field in direct interband transitions in a semiconductor. Unlike the renormalized system for the quasiparticles that was used for these purposes in Ref. 15, Eqs. (10) and (11) are written directly for the nonequilibrium electrons and holes. We note that the approach based on the introduction of quasiparticles, when the problem of the interaction with an external field is solved rigorously by means of a unitary transformation, is equivalent to the analysis performed here. As will be shown below, this is because the present system of equations is solved exactly with neglect of electron-phonon scattering processes, which can be taken into account subsequently as a perturbation. However, a treatment in the framework of the usual electron-hole representation is, in our view, more convenient for physical analysis, and, as will be seen below, makes it possible, with the aid of a number of simplifying assumptions, to obtain analytical results.

## 3. INFLUENCE OF THE ELECTRON-PHONON INTERACTION ON THE COHERENT REGIME

If the times under consideration are shorter than the characteristic times of the electron-phonon interaction, in Eqs. (10) we can neglect the contribution of the scattering amplitudes and obtain a solution in the absence of collision processes. For this we introduce the population difference $n(\mathbf{k},t)=n_e(\mathbf{k},t)+n_h(\mathbf{k},t)-1$, and also assume that the dispersion law in the bands is quadratic, with $m_e^*\approx m_h^*=m$ ($m_e^*$ and $m_h^*$ are the effective masses of an electron and a hole). Then for the system (10a)–(10d), after the terms describing scattering have been discarded, the following solution holds:

$$n(\mathbf{k},t)=n(k,t)=\frac{1}{\{1+[(k^2-k_0^2)/a^2]^2\}}[1-\cos(\varepsilon_\mathbf{k}t)]-1, \qquad (12a)$$

$$P(\mathbf{k},t)=P(k,t)=\frac{|\Omega|}{2}\frac{(k^2-k_0^2)/a^2}{\{1+[(k^2-k_0^2)/a^2]^2\}}[1-\cos(\varepsilon_\mathbf{k}t)], \qquad (12b)$$

$$\sigma(\mathbf{k},t)=\sigma(k,t)=-\frac{|\Omega|}{2}\frac{1}{\{1+[(k^2-k_0^2)/a^2]^2\}^{1/2}}\sin(\varepsilon_\mathbf{k}t), \qquad (12c)$$

where

$$\varepsilon_\mathbf{k}^2=\Delta(\mathbf{k})^2+4|\Omega|^2,\quad k=|\mathbf{k}|,\quad a^2=2|\Omega|m/\hbar,$$

and $k_0$ is determined from the condition that the detuning from resonance is equal to zero:

$$\Delta(\mathbf{k})=\hbar k_0^2/m-\hbar k^2/m.$$

The result obtained is fully analogous to the well known coherent solution for the two-level model of an atom situated in a constant external field. The difference is that the resonance spectrum undergoes a specific inhomogeneous broadening, determined by the dispersion law in the bands. A solution of the type (12) was given in Ref. 12 with allowance for the mechanism responsible for the Franz-Keldysh effect. Henceforth we assume that the condition $k_0>a$ is fulfilled. We then define the characteristic width of the coherent spectrum in (12) as $\Delta k=k_{max}-k_{min}$, where $k_{max}$ and $k_{min}$ are the positive roots of the equation

$$[(k^2-k_0^2)/a^2]^2=1, \tag{13}$$

whence it follows that

$$\Delta k=(k_0^2+a^2)^{1/2}-(k_0^2-a^2)^{1/2}. \tag{14}$$

Thus, if $k_0 \gg a$ holds, then we have $\Delta k \approx a/k_0$, i.e., $k_0 \gg \Delta k$. But for $k_0 \sim a$ (but with $k_0 > a$), then, as can be seen from (14), $\Delta k$ and $k_0$ are also of the same order.

We now turn to investigate the question of the influence of electron-phonon scattering processes on the coherent regime described by the solution (12). For this we consider the initial system (10), (11). In the right-hand sides of Eqs. (11) we have omitted the nonlinear terms with no factor $N(\mathbf{q})$, since it is assumed that $N(\mathbf{q}) \gg 1$. For the final linearization we neglect also the change of the number of phonons, i.e., we assume that $N(\mathbf{q})$ is constant in time (the presence of a so-called heat reservoir). This implies that Eq. (10e) is eliminated from the system, and in the remaining equations it is necessary to substitute the equilibrium distribution:

$$N(\mathbf{q}) = \frac{1}{\exp[\hbar\omega(\mathbf{q})/T]-1} \approx \frac{T}{\hbar\omega(\mathbf{q})}, \tag{15}$$

where $T$ is the temperature of the phonons in energy units.

As a result of the approximations made, the system becomes linear, and, generally speaking, can be solved by means of a Fourier transformation. However, because of the presence in (10a)–(10d) of sums over $\mathbf{q}$ it does not appear to be possible to solve the algebraic system of equations that follows for the Fourier amplitudes. Therefore, we first simplify the right-hand sides in (11), by imposing restrictions on the electron-phonon interaction process.

For this we consider the elementary scattering event, which satisfies the energy-conservation law (ECL): $\Delta_e(\mathbf{k},\mathbf{q}) = 0$. The latter can be represented in the form

$$(\mathbf{k}-\mathbf{q})^2 + \frac{2mc}{\hbar} q = k^2, \tag{16}$$

where $q = |\mathbf{q}|$, $c$ is the velocity of sound in the semiconductor, and it is assumed also that only acoustic phonons take part in the scattering: $\omega(\mathbf{q}) \approx cq$. It follows from (16) that if the condition $k \gg mc/\hbar$ is fulfilled, the magnitude $k$ of the electron wave vector changes little in each elementary scattering event. We then replace (16) by the approximate relation

$$|\mathbf{k}-\mathbf{q}| \approx k \tag{17a}$$

or, taking the next order into account,

$$|\mathbf{k}-\mathbf{q}| \approx k - \frac{mc}{\hbar k} q, \tag{17b}$$

Suppose now that, in analogy with the case of the coherent solution, the unknown functions $n(\mathbf{k},t)$, $P(\mathbf{k},t)$, and $\sigma(\mathbf{k},t)$ depend on the magnitude $k$ of the wave vector. Then the differences of the form $R(k,t) - R(|\mathbf{k} - \mathbf{q}|,t)$ appearing in the right-hand sides of Eqs. (11), where $R$ is any of the functions $n(k,t)$, $P(k,t)$, and $\sigma(k,t)$, can be replaced by the derivatives of these functions: $\partial R(k,t)/\partial k$. Thus, taking (17b) into account, in first order we obtain

$$R(k,t) - R(|\mathbf{k}-\mathbf{q}|,t) \approx \frac{mc}{\hbar k} q \frac{\partial}{\partial k} R(k,t). \tag{18}$$

Here it has also been assumed that the function $R(k,t)$ changes little even in the case of scattering in which the wave vector $q$ is the maximum possible allowed by the ECL ($q_{\max} \sim k$). This implies that the condition $\Delta k \gg mc/\hbar$ should be fulfilled, where $\Delta k$ is the characteristic scale of the variation of the function $R(k,t)$ and can be estimated with the aid of (14). Further analysis shows that under the influence of the electron-phonon interaction the value of $\Delta k$ increases in comparison with that in the coherent case, and therefore this requirement is not violated. We note that when $\Delta_h(\mathbf{k},\mathbf{q}) = 0$ holds analogous results follow, and in the right-hand sides of (17b) and (18) $q$ should be replaced by $-q$.

These approximations enable us to simplify substantially the system (10a)–(10d), (11). It should be considered in two different cases: $\Delta_e(\mathbf{k},\mathbf{q}) = 0$ and $\Delta_h(k,q) = 0$. In the analogous system for the $F$-amplitudes in which $\mathbf{k} + \mathbf{q}$ appears in place of $\mathbf{k}$ in the arguments, the corresponding ECL have the form $\Delta_e(\mathbf{k} + \mathbf{q},\mathbf{q}) = 0$ and $\Delta_h(\mathbf{k} + \mathbf{q},\mathbf{q}) = 0$. Finally, after differentiation of Eqs. (10a) and (10d) with respect to the time, we can eliminate the $F$-amplitudes from the system and obtain

$$\begin{aligned}
&\frac{\partial^2}{\partial t^2} n(k,t) - 2|\Omega|^2 n(k,t) + 2\varkappa \frac{\partial}{\partial k} n(k,t) \\
&\quad + 2\Delta P(k,t) + 6\frac{\partial}{\partial t}\sigma(k,t) = 0, \\
&\Delta|\Omega|^2 n(k,t) + \frac{\partial^2}{\partial t^2} P(k,t) - \Delta^2 P(k,t) \\
&\quad + 4\varkappa \frac{\partial}{\partial k} P(k,t) - 2\Delta \frac{\partial}{\partial t}\sigma(k,t) = 0, \\
&-2|\Omega|^2 \frac{\partial}{\partial t} n(k,t) + 2\Delta \frac{\partial}{\partial t} P(k,t) \\
&\quad + \frac{\partial^2}{\partial t^2}\sigma(k,t) - (4|\Omega|^2 + \Delta^2)\sigma(k,t) + 4\varkappa \frac{\partial}{\partial k}\sigma(k,t) = 0.
\end{aligned} \tag{19}$$

In the derivation we have used the notation

$$\varkappa = \sum_{\mathbf{q}} \hbar^{-2} |g(\mathbf{q})|^2 N(\mathbf{q}) \frac{mc}{\hbar} \frac{q}{k} [\delta_{qq_1} - \delta_{qq_2}], \tag{20}$$

where

$$q_1 = 2k\cos\vartheta - 2mc/\hbar, \quad q_2 = -2k\cos\vartheta + 2mc/\hbar.$$

We have denoted the angle between the vectors $k$ and $q$ by $\vartheta$. The expressions for $q_1$ and $q_2$ are consequences of the corresponding ECL: $\Delta_e(\mathbf{k},\mathbf{q}) = 0$ and $\Delta_e(\mathbf{k} + \mathbf{q},\mathbf{q}) = 0$. Henceforth, for definiteness, we assume $\varkappa < 0$ (the final result does not depend on the sign of $\varkappa$). We have also used the fact that, if $m_e^* \approx m_h^*$, the ECL $\Delta_h(\mathbf{k} + \mathbf{q},\mathbf{q}) = 0$ after the replacement of $\mathbf{q}$ by $-\mathbf{q}$ is equivalent to the ECL $\Delta_e(\mathbf{k},\mathbf{q}) = 0$, and, analogously, the ECL $\Delta_h(\mathbf{k},\mathbf{q}) = 0$ after the replacement of $\mathbf{q}$ by $-\mathbf{q}$ coincides with the ECL $\Delta_e(\mathbf{k} + \mathbf{q},\mathbf{q}) = 0$. In addition, in $\Delta(\mathbf{k}) = \Delta(k)$ for brevity we have omitted the dependence on $k$, and, in accordance with (17a), in the derivation of the system (19) we have neglected the difference between $\Delta(k)$ and $\Delta(|\mathbf{k} - \mathbf{q}|)$.

In the resulting homogeneous linear system (19) the coefficients $\Delta$ and $\varkappa$ are functions of $k$. Suppose that this dependence is slow in comparison with the dependence on $k$ for the functions sought. This implies that the inequalities

$$\frac{\partial}{\partial k}R(k,\Delta,\varkappa,t) \gg \frac{\partial}{\partial\Delta}R(k,\Delta,\varkappa,t)\frac{\partial}{\partial k}\Delta(k), \qquad (21)$$

$$\frac{\partial}{\partial k}R(k,\Delta,\varkappa,t) \gg \frac{\partial}{\partial\varkappa}R(k,\Delta,\varkappa,t)\frac{\partial}{\partial k}\varkappa(k) \qquad (22)$$

should be fulfilled, where the functions $R(k,\Delta,\varkappa,t)$ are the exact solution of the system (19), if the quantities $\Delta$ and $\varkappa$ do not depend on $k$. The conditions (21) and (22) impose restrictions on the region of applicability of the final solution, and can be satisfied for sufficiently small values of $\varkappa$ and $t$. This is in accord with the assumption that the perturbation of the coherent regime by the electron-phonon interaction is weak. We note that the inequalities (21) and (22) are also violated in the limit $k\to 0$. With the restrictions indicated above, henceforth we shall assume the dependence on $k$ in $\Delta(k)$ and $\varkappa(k)$ to be parametric and solve the system (19) with the aid of a Fourier transformation:

$$R(k,t)=\int_{-\infty}^{\infty} d\omega\Big[\int_0^{\infty} dr\exp(-i\omega t+ikr)R_+(r,\omega)$$
$$+\int_0^{\infty} dr\exp(-i\omega t-ikr)R_-(r,\omega)\Big], \qquad (23)$$

where

$$R_+(r,\omega)=(2\pi)^{-2}\int_{-\infty}^{\infty}dt'\int_0^{\infty}dk'\exp(i\omega t'-ik'r)R(k',t'), \quad (24a)$$

$$R_-(r,\omega)=(2\pi)^{-2}\int_{-\infty}^{\infty}dt'\int_0^{\infty}dk'\exp(i\omega t'+ik'r)R(k',t'). \quad (24b)$$

After substitution of (23) into (19) we obtain two linear homogeneous algebraic systems for the Fourier amplitudes $n_+, P_+, \sigma_+$ and $n_-, P_-, \sigma_-$ [in (23), for brevity, they are denoted by $R_+(r,\omega)$ and $R_-(r,\omega)$]. In the case of the "+" amplitudes [the Fourier amplitudes of outgoing spherical waves $\exp(-i\omega t+ikr)$], from the condition for the existence of a nontrivial solution of the given system it follows that

$$\omega^6+[-2(\Delta^2+3|\Omega|^2)-{}^5/_2\delta]\omega^4+[\varepsilon_k{}^2(\Delta^2+2|\Omega|^2)$$
$$+\delta(\Delta^2+2|\Omega|^2)+2\delta^2]\omega^2+(-{}^1/_2\delta^3+\varepsilon_k{}^2\delta^2-2\varepsilon_k{}^4\delta)=0, \quad (25)$$

where $\delta=4i\varkappa r$ and $\varepsilon_k=\varepsilon_{\mathbf{k}}$. The analogous equation in the case of the "−" amplitudes [the Fourier amplitudes of incoming spherical waves $\exp(-i\omega t-ikr)$] are obtained from (25) by replacing $r$ by $-r$].

Equation (25) determines the frequency as a function of $r$: $\omega^2=\omega^2(r)$. If we neglect the scattering ($\varkappa=0$), from (25) with $\delta=0$ we can obtain the frequencies corresponding to the coherent solution: $\omega^2=0$ and $\omega^2=\varepsilon_k^2$. In order to take the electron-phonon interaction into account we assume that $\delta$ is small ($|\delta|\ll\varepsilon_k^2$) and seek the roots of Eq. (25) in the form

$$\omega^2=\alpha(r), \quad \omega^2=\varepsilon_k{}^2+\beta(r),$$

where $|\alpha(r)|$, $|\beta(r)|\ll\varepsilon_k^2$. Keeping only terms linear in the small corrections, we can obtain

$$\omega_+^{(0)}(r)=\pm4(|\varkappa|r)^{1/2}\left[\frac{1+(\Delta/2|\Omega|)^2}{1+2(\Delta/2|\Omega|)^2}\right]^{1/2}(1-i),$$
$$\omega_+^{(\pm)}(r)=\pm\left\{\varepsilon_k-\frac{4i|\varkappa|r}{|\Omega|}\left[1+\left(\frac{\Delta}{2|\Omega|}\right)^2\right]^{1/2}\right\}. \qquad (26)$$

Here, the subscript + indicates that this value of the frequency corresponds to the "+" amplitudes (24a), while the superscripts indicate to which frequency of the coherent solution the given correction applies [$(0)\to\omega=0$, $(\pm)\to\omega=\pm\varepsilon_k$]. The analogous expressions corresponding to the "−" amplitudes (24b) can be obtained from (26) by replacing $r$ by $-r$.

Henceforth, when substituting the values obtained for $\omega(r)$ into (23), we shall keep only those eigenfrequencies that lead to integrals that are convergent in $r$. We recall that the solutions (26) have been found in the approximation $|\delta|/|\Omega|^2\ll 1$. On the other hand, since $\delta=4i\varkappa r$, for $r\to\infty$ this condition is violated, i.e., the extra terms cease to be small. However, the integral (23) also contains the Fourier amplitude $R(r,\omega)$, which is bounded in $r$ and decays away as $r\to\infty$. In order to estimate the characteristic region in which this function is nonzero, we shall make use of the coherent solution with the quantity $\Delta k$ specified by (14). Then the characteristic width of the function $R(r,\omega)$ can be defined as $\Delta r\sim\Delta k^{-1}$. Thus, the condition that makes it possible to assume that the influence of the electron-phonon interaction on the coherent regime is small has the form

$$(|\delta|/|\Omega|^2)_{r=1/\Delta k}\ll 1, \quad \text{or} \quad 4(|\varkappa|/\Delta k|\Omega|^2)\ll 1. \qquad (27)$$

In this approximation the expressions obtained for $\omega(r)$ can be used over the entire range of the integration, since for large values of $r$, for which the true form of $\omega(r)$ is unknown, the integrand function $R(r,\omega)$ vanishes and does not make a contribution to the integral. Below it will be shown that the restriction (27) imposed on the magnitude of the electron-phonon interaction is the strong-field condition.

In order to obtain the solution of the system (19) in the form (23), the Fourier amplitudes corresponding to the eigenfrequencies $\omega(r)$ should be found. For this it is possible to make use of the known coherent solution (12), to which the desired expression (23) should go over as $\varkappa\to 0$ (absence of scattering). This implies that it is necessary to substitute our eigenfrequencies (26) into the Fourier integral (23) and equate the latter to the corresponding functions of the coherent solution in the limit $\varkappa\to 0$. From the relations obtained, expressions for the Fourier amplitudes (24) follow uniquely. Then, finally, we find the solution of the system (19) in the form

$$R(k,t)=R^{(0)}(k,t)+R^{(+)}(k,t)\exp(-i\varepsilon_k t)$$
$$+R^{(-)}(k,t)\exp(i\varepsilon_k t), \qquad (28)$$

where

$$R^{(0)}(k,t)=\int_0^{\infty}dk'R_{coh}^{(0)}(k')\pi^{-1}\int_0^{\infty}dr\exp[-(b_0 r)^{1/2}]$$
$$\times\cos[(k-k')r-(b_0 r)^{1/2}], \qquad (29)$$

$$R^{(\pm)}(k,t)=\int_0^{\infty}dk'R_{coh}^{(\pm)}(k')\pi^{-1}\int_0^{\infty}dr\exp(-br)\cos[(k-k')r],$$

in which all the functions in the coherent solution (12) are represented in the form

$$R_{coh}(k,t)=R_{coh}^{(0)}(k)+R_{coh}^{(+)}(k)\exp(-i\varepsilon_k t)+R_{coh}^{(-)}(k)\exp(i\varepsilon_k t). \tag{30}$$

In addition, we have introduced the notation

$$b_0=16|\varkappa|\frac{1+(\Delta/2|\Omega|)^2}{1+2(\Delta/2|\Omega|)^2}t^2,$$

$$b=\frac{4|\varkappa|}{|\Omega|}\left[1+\left(\frac{\Delta}{2|\Omega|}\right)^2\right]^{1/2}t.$$

Calculating the integrals over $r$, we can rewrite the expressions (29) in the form

$$R^{(0)}(k,t)=\frac{2}{\pi^{1/2}}\int_{x_0}^{\infty}dx\,e^{-x^2}R_{coh}^{(0)}\left(k\left[1-\left(\frac{x_0}{x}\right)^2\right]\right),$$

$$R^{(\pm)}(k,t)=\pi^{-1}\int_{-x_\pm}^{\infty}\frac{dx}{1+x^2}R_{coh}^{(\pm)}\left(k\left[1+\frac{x}{x_\pm}\right]\right), \tag{31}$$

where $x_0=(b_0/2k)^{1/2}$ and $x_+=x_-=k/b$.

The result (28) has a general form similar to the coherent solution (30). However, now the amplitudes of the zeroth harmonic and oscillator harmonics depend on the time, this being determined by the presence of the corresponding parameters $x_0$, $x_+$, and $x_-$ in the integrals (31). This dependence is characterized by the strength of the electron-phonon interaction, which is specified by the magnitude of the coefficient $\varkappa$. If $\varkappa\to 0$ ($x_0\to 0$, $x_+$, $x_-\to\infty$), then $R(k,t)\to R_{coh}(k,t)$, i.e., we obtain the coherent solution in the form (12). It is important to note that in the case of the oscillator components of the solution (28) the influence of the electron-phonon interaction on the coherent regime depends in an essential way on the intensity of the external field, since $x_+$, $x_-\propto|\Omega|$. This implies that the change of the coherent solution is smaller at a given time, the greater the Rabi frequency.

In order to obtain the characteristic time responsible for this effect, we substitute the coherent population difference (12a) into the expressions (31) for $R^{(\pm)}$ and find the form of the functions $R^{(\pm)}(k,t)=n^{(\pm)}(k,t)$. If the condition $b<k_0$ is fulfilled, the lower limit in the expression (31) for $R^{(\pm)}$ can be replaced by $-\infty$. After this, the integral can be evaluated analytically, and the result for $n^{(\pm)}(k,t)$ has the form

$$n^{(\pm)}(k,t)=-\frac{1}{2(1+2\eta b/a)^2}\times\left\{1+\frac{2\eta b/a}{2(1+4\eta^4)}\left[1+4\eta^2\left(\eta^2+\left(\frac{k}{a}\right)^2\right)+\left(2+2\eta\frac{b}{a}\right)^2\right]\right\}\times\left\{1+\left[k^2-k_0^2\left[1-\frac{(2\eta b/a)(2+2\eta b/a)}{(2\eta k_0/a)^2}\right]\right]^2\Big/\left[a^4\left(1+2\eta\frac{b}{a}\right)^2\right]\right\}^{-1}, \tag{32}$$

where

$$\eta=\{{}^1/{}_2[(k_0/a)^2+(1+(k_0/a)^4)^{1/2}]\}^{1/2}.$$

We consider (32) in two cases: $k_0\gg a$ and $k_0\sim a$ (but $k_0>a$). In the first approximation we have

$$n^{(\pm)}(k,t)|_{k_0\gg a}=-\frac{1}{2(1+t/\tau)}\Big/\left[1+\frac{(k^2-k_0^2)^2}{a^4(1+t/\tau)^2}\right], \tag{33}$$

where

$$\frac{t}{\tau}=b\left(|\Omega|\Big/\frac{\hbar}{m}k_0\right)^{-1}, \tag{34}$$

or

$$\tau^{-1}=4|\varkappa|\left\{1+\left[\frac{\Delta(k)}{2|\Omega|}\right]^2\right\}^{1/2}\left(|\Omega|^2\Big/\frac{\hbar}{m}k_0\right)^{-1}.$$

From comparison of (33) with the expression for $n_{coh}^{(\pm)}(k)$, which, according to (12a), has the form

$$n_{coh}^{(\pm)}(k)=-{}^1/{}_2\{1+[(k^2-k_0^2)/a^2]^2\}^{-1} \tag{35}$$

it follows that $\tau$ is the characteristic time over which, under the influence of the electron-phonon interaction, the broadening of the coherent distribution (35) occurs, and, in addition, the magnitude of its maximum decreases.

If the condition $k_0\sim a$ is fulfilled, (32) can be represented in the form

$$n^{(\pm)}(k,t)|_{k_0\sim a}=-\frac{1}{2(1+t/\tau)^2}\left\{1+\frac{t}{2\tau}\left[1+\left(\frac{k}{a}\right)^2+\left(1+\frac{t}{2\tau}\right)^2\right]\right\}\Big/\left\{1+\frac{\{k^2-k_0^2[1-(t/2\tau)(1+t/2\tau)]\}^2}{a^4(1+t/\tau)^2}\right\}, \tag{36}$$

where now the characteristic time $\tau$ is defined by the expression

$$\frac{t}{\tau}=b\left(|\Omega|\Big/\frac{\hbar}{m}k_{eff}\right)^{-1} \tag{37}$$

or

$$\tau^{-1}=4|\varkappa|\left[1+\left(\frac{\Delta(k)}{2|\Omega|}\right)^2\right]^{1/2}\left(|\Omega|^2\Big/\frac{\hbar}{m}k_{eff}\right)^{-1},$$

and

$$k_{eff}=\{{}^1/{}_2[k_0^2+(k_0^4+a^4)^{1/2}]\}^{1/2}.$$

It can be seen that in this case $\tau$ coincides in order of magnitude with the previously obtained value (34). Now, however, under the influence of the electron-phonon interaction, the maximum of the distribution (36) is also shifted to a lower energy. We emphasize that the time characterizing the decay of the oscillator components of the coherent solution depends on the amplitude of the external field ($\tau\propto|\Omega|^2$). This means that as the intensity of the field increases the relaxation processes may turn out to be suppressed to a considerable extent.

Now, using the expression (37) for $\tau$, we show that our previously obtained restriction (27) on the magnitude of the electron-phonon interaction is the strong-field condition. Since we are considering the case $k_0\sim a$, it follows from (14) that $\Delta k\sim k_0$, $a$, and, in turn, that $k_{eff}\sim k_0$, $a$, i.e., $\tau^{-1}\sim 4(|\varkappa|/|\Omega|\Delta k)$, where we have taken into account the relation $2|\Omega|(\hbar/m)^{-1}=a$, and have also discarded the region corresponding to considerable detunings from resonance: $\Delta(k)/2|\Omega|\gg 1$. Then the inequality (27) can be transformed as follows:

$$4(|\varkappa|/|\Omega|^2\Delta k)\sim\tau^{-1}/|\Omega|\ll 1, \quad \text{or} \quad |\Omega|\tau\gg 1. \qquad (38)$$

The requirement (38) is the strong-field condition, and describes the approximation in which the solution (36), (37) was obtained. The case $k_0 \gg a$ can be treated analogously, when (27) is also the strong-field condition with the corresponding characteristic time $\tau$.

To conclude this section, we note that the applicability of this solution is limited by the inequalities (21) and (22). In addition, there is the extra condition (9), related to the possibility of linearizing the initial system of equations for $N(\mathbf{q}) \gg 1$. It follows from (18) that for this the requirement

$$N(\mathbf{q})\frac{mc}{\hbar k}q\frac{\partial}{\partial k}n(k,t)\gg 1, \quad \text{or} \quad \hbar^{-2}\frac{mT}{k\Delta k}\gg 1, \qquad (39)$$

should be satisfied, from which it can be seen that this approximation also imposes a restriction on the times under consideration, since under the influence of the electron-phonon interaction the characteristic width $\Delta k$ of the distribution increases and the condition (39) can be violated.

## 4. CONCLUSION

The results obtained in this paper point to the influence of a strong external field on the processes of relaxation of nonequilibrium carriers in a semiconductor. The analysis is performed by the method of second quantization and takes into account the electron-phonon interaction. The external radiation is assumed to be fixed and satisfies the strong-field condition ($\Omega\tau \gg 1$). In deriving the equations describing the time evolution of the nonequilibrium population and polarization in a semiconductor we have used the standard procedure of decoupling four-operator expectation values in order to obtain a closed system. In addition, the number of phonons taking part in the scattering is assumed to vary little in time, and also to be sufficiently large to permit linearization of the system of equations obtained.

The principal limitation imposed on the collision integrals is the assumption that the magnitude of the electron wave vector changes little when the electron is scattered by a phonon. In the case of the acoustic-phonon branch this implies that the conditions $k \gg mc/\hbar$ and $\Delta k \gg mc/\hbar$ are necessary. Since $mc/\hbar \sim 10^{-7}$ m$^{-1}$ ($c \sim 10^3$ m/sec), for values of $k_0 \sim (0.5\text{–}1)\times 10^8$ m$^{-1}$ it is necessary to consider fields that ensure $\Omega \sim 10^{11}\text{–}10^{12}$ sec$^{-1}$. For lower values of the Rabi frequency the width of the distribution becomes too small and the condition $\Delta k \gg mc/\hbar$ is violated. If we also take into account the interaction of the nonequilibrium carriers with the optical phonons, the corresponding requirements become more stringent: $k$, $\Delta k \gg (m\omega_{\text{opt}}/\hbar)^{1/2}$, where $\omega_{\text{opt}}$ is the characteristic optical-phonon frequency in the semiconductor. In this case, for values $k_0 \sim (0.5\text{–}1)\times 10^9$ m$^{-1}$ the Rabi frequency should take values of the order of $(1\text{–}5)\times 10^{13}$ sec$^{-1}$. For lower magnitudes of the external field it is not possible to satisfy the condition $\Delta k \gg (m\omega_{\text{opt}}/\hbar)^{1/2}$ $\times[\omega_{\text{opt}} \sim (0.5\text{–}1)\times 10^{13}$ sec$^{-1}]$. In addition now, in order to satisfy the condition (39), the temperature should be considerably higher than room temperature.

Using these results we can perform quantitative estimates for the characteristic times of the electron-phonon interaction in the presence of a strong external field. For a bulk crystal ($V \sim L^3$, where $V$ is the volume and $L$ is the characteristic linear dimension of the sample) with a simple cubic lattice, and for values $k_0 \sim 5\times 10^{-7}$ m$^{-1}$, $\Omega \sim 10^{11}$ sec$^{-1}$, $L \sim 10^{-2}$ m and room temperatures, it is possible to obtain $\tau \sim 10^{-10}$ sec, i.e., the relaxation processes due to scattering by acoustic phonons are suppressed to a considerable extent. If these conditions for the optical phonons are satisfied, the results obtained in this paper can be generalized to the case of interaction with optical phonons. For the sample described above, and for values $k_0 \approx 5\times 10^8$ m$^{-1}$, $\Omega \sim 10^{13}$ sec$^{-1}$, and $T \sim 10^{13}$ K, we can obtain $\tau \sim 10^{-8}$ sec on the assumption that the characteristic optical-phonon frequency has a magnitude $\omega_{\text{opt}} \approx 5\times 10^{12}$ sec$^{-1}$.

The estimates given show that in the approximations considered the decay of a coherent nonequilibrium state in a semiconductor under the influence of the electron-phonon interaction is substantially suppressed by a strong external field. It is shown that when the restrictions imposed are fulfilled the time characterizing the decay of the coherent polarization in a semiconductor should increase algebraically with the amplitude of the external field: $\tau \propto |\Omega|^2$. We note that in this paper we have neglected the influence of electron-electron scattering. In view of the complexity of this problem, allowance for the *e-e* interaction should be considered separately. The point is that, as the amplitude of the external field increases, the number of carriers excited into the band increases, and, consequently, so too does the strength of the Coulomb interaction. Therefore, in our opinion, the question of the character of the dependence of the relaxation processes on the intensity of the external field in the case of electron-electron scattering remains to a considerable degree open and requires further investigation.

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[1] A. Mysyrowicz, D. Hulin, A. Antonetti, A. Migus, W. T. Masselink, and H. Morkoc, Phys. Rev. Lett. **56**, 2748 (1986).

[2] A. Von Lehmen, D. S. Chemla, J. E. Zucker, and J. P. Heritage, Opt. Lett. **11**, 609 (1986).

[3] S. Schmitt-Rink and D. S. Chemla, Phys. Rev. Lett. **57**, 2752 (1986).

[4] J. L. Oudar, D. Hulin, A. Migus, A. Antonetti, and F. Alexandre, Phys. Rev. Lett. **55**, 2074 (1985).

[5] W. H. Knox, R. L. Fork, M. C. Downer, D. A. B. Miller, D. S. Chemla, C. V. Shank, A. S. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 1306 (1985).

[6] W. Z. Lin, R. W. Schönlein, S. D. Brorson *et al.*, in *Proceedings of the International Quantum Electronics Conference IQEC'87, Baltimore, Maryland, 1987* (Optical Society of America, Washington, 1987), p. 82.

[7] N. Peyghambarian and S. W. Koch, Rev. Phys. Appl. **22**, 1711 (1986).

[8] B. Fluegel, N. Peyghambarian, G. Olbright, M. Lindberg, S. W. Koch, M. Joffre, D. Hulin, A. Migus, and A. Antonetti, Phys. Rev. Lett. **59**, 2588 (1987).

[9] M. Joffre, D. Hulin, A. Migus, A. Antonetti, C. B. A. Laguillaume, N. Peyghambarian, M. Lindberg, and S. W. Koch, Opt. Lett. **13**, 276 (1988).

[10] S. W. Koch, N. Peyghambarian, and M. Lindberg, J. Phys. C **21**, 5229 (1988).

[11] É. A. Manykin and V. V. Samartsev, *Optical Echo Spectroscopy* [in Russian], (Nauka, Moscow, 1984).

[12] É. A. Manykin and S. M. Zakharov, Izv. Akad. Nauk SSSR, Ser. Fiz., **37**, 2171 (1973).

[13] P. C. Becker, H. L. Fragnito, C. H. Brito Cruz, R. L. Fork, J. E. Cunningham, J. E. Henry, and C. V. Shank, Phys. Rev. Lett. **61**, 1647 (1988).

[14] V. F. Elesin, Zh. Eksp. Teor. Fiz. **69**, 572 (1975) [Sov. Phys. JETP **42**, 291 (1975)].

[15] A. S. Aleksandrov, V. F. Elesin, Yu. P. Lisovets, V. G. Mikhaĭlov, I. A. Poluektov, and Yu. M. Popov, Kvantovaya Elektron. **5**, 359 (1978) [Sov. J. Quantum Electron. **8**, 207 (1978)].

[16] Yu. P. Lisovets, I. A. Poluektov, and Yu. M. Popov, Kvantovaya Elektron. **6**, 120 (1979) [Sov. J. Quantum Elektron. **9**, 67 (1979)].
[17] V. M. Galitskiĭ and V. F. Elesin, *Resonance Interaction of Electromagnetic Fields with Semiconductors* [in Russian] (Energoatomizdat, Moscow, 1986), p. 95.
[18] V. M. Galitskiĭ, S. P. Goreslavskiĭ, and V. F. Elesin, Zh. Eksp. Teor. Fiz. **57**, 207 (1969) [Sov. Phys. JETP **30**, 117 (1970)].
[19] F. Brukner, V. S. Dneprovskiĭ, D. G. Koshchug, and V. U. Khattatov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 27 (1973) [JETP Lett. **18**, 14 (1973)].
[20] F. Brückner, Ya. T. Vasil'ev, V. S. Dneprovskiĭ, D. G. Koshchug, E. K. Silina, and V. U. Khattatov, Zh. Eksp. Teor. Fiz. **67**, 2219 (1974) [Sov. Phys. JETP **40**, 1101 (1975)].
[21] É. M. Belenov and P. P. Vasil'ev, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 416 (1988) [JETP Lett. **48**, 456 (1988)].
[22] É. M. Belenov and P. P. Vasil'ev, Zh. Eksp. Teor. Fiz. **96**, 1629 (1989) [Sov. Phys. JETP **69**, 922 (1989)].
[23] É. M. Belenov, A. N. Oraevskiĭ, and V. A. Shcheglov, Zh. Eksp. Teor. Fiz. **56**, 2143 (1969) [Sov. Phys. JETP **29**, 1153 (1969)].

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