Scattering of light by electrons in superconductors at finite temperature

S. Klyama[1)] and L. A. Fal'kovskiĭ

L. D. Landau Institute of Theoretical Physics of the Academy of Sciences of the USSR

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Electron Raman scattering at finite temperature in a normal metal and in a superconductor is studied. The anisotropic BCS model is used for the superconductor. Decreasing the temperature in the superconducting state results mainly in exponential suppression of subthreshold scattering.

INTRODUCTION

There is considerable research on the electron scattering of light (ESL). This interest is explained by the fact that theoretically[1] ESL in a superconductor at zero temperature should exhibit a threshold—scattering does not occur if the transferred energy $\omega = \omega_i - \omega_s$ (ω_i and ω_s are the frequencies of the incident and scattered light) is less than the width 2Δ of the gap in the spectrum of the superconductor. The first observations of ESL in Nb_3Sn and V_3Si (Refs. 2–4) indeed revealed a sharp rise in the intensity of the scattered light and the gap width determined from the position of the maximum agrees quite well with the value obtained from tunneling experiments.

Electron scattering of light in high-T_c superconductors, however, has a different frequency dependence. For low frequencies $\omega < 2\Delta$, where ESL should not occur, a linear dependence of the scattering cross section on ω is apparently observed.[5-14] Several possible explanations of this fact have been advanced. First, because of the low quality of high-T_c samples the gap width can vary appreciably in the volume of the sample. Second, the character of the electron pairing can be such that the gap vanishes at separate points or on lines on the Fermi surface. In both the first and second (if the gap vanishes on lines[15]) cases the intensity of ESL is a linear function of ω. Finally, because the thermal conductivity of the sample is low the temperature in the laser spot could be significantly higher than in the cooling bath. It should be noted that in some experimental works the magnitude of the heating in the spot was estimated. As far as we know, however, the temperature has never been measured directly. At the same time, there exists a relatively simple method for measuring the temperature based on data on ESL. For this it is sufficient to compare the intensity of the Stokes $I(\omega)$ and anti-Stokes $I(-\omega)$ components for an arbitrary transferred quantum ω. Their ratio is equal to

$$I(-\omega)/I(\omega) = e^{-\omega/T}. \tag{1}$$

The aim of this work is to calculate the effect of the temperature on the electron scattering of light by metals in the normal and superconducting states. We note that there is still no information about the observation of ESL in normal metals.

SCATTERING MATRIX

We write the Hamiltonian of an isotropic superconductor interacting with an electromagnetic field in the form

$$H = \frac{e^2}{mc^2}\mathbf{A}^{(i)}\mathbf{A}^{(s)}\psi^+\psi - \frac{g}{2}\psi^+(\psi^+\psi)\psi, \tag{2}$$

where $\mathbf{A}^{(i)}$ and $\mathbf{A}^{(s)}$ are the vector potentials of the incident and scattered light in the metal. The term that is linear in the field and describes scattering in second-order perturbation theory makes a contribution that is small in proportion to the ratio ω/ω_i, and in addition we have $\omega_i \sim \omega_s$ and these energies are comparable to the Fermi energy.

The scattering matrix element can be written in the form

$$S = -i\left[\frac{e^2}{mc^2}\mathbf{A}^{(i)}\mathbf{A}^{(s)}\psi^+\psi - CI(\psi\psi + \psi^+\psi^+)\right], \tag{3}$$

where

$$CI_{\alpha\beta} = \frac{g}{2}\langle T\psi_\alpha^+\psi_\beta^+\rangle \tag{4}$$

is (see Fig. 1) the sum of the perturbation series in the constant g, and each term of this series is taken in first order in the interaction $\mathbf{A}^{(i)}\mathbf{A}^{(s)}$ with the field.[2)] A detailed analysis of this series shows that, first, the equality

$$\langle T\psi_\alpha\psi_\beta\rangle = \langle T\psi_\alpha^+\psi_\beta^+\rangle,$$

which has already been employed in writing down Eq. (3), is satisfied and, second, diagrams of the form $(g/2)\langle T\psi_\alpha\psi_\beta^+\rangle$ can be neglected. The last assertion can be understood by studying the second diagram in Fig. 1. For small momentum transfer the bottom loop, containing two parallel Green's functions, in contrast to the corresponding loop for $\langle T\psi\psi^+\rangle$, can make a large logarithmic contribution, which together with the additional (compared with the first diagram in Fig. 1) small factor g is close to unity on account of the equation determining the gap width in the theory of superconductivity. As a result, the denominator in the progression shown in Fig. 1 differs little from unity; this is why the term with C must be retained in Eq. (3).

The scattering cross section is determined by the average value

$$d\sigma \sim \int d^4x\, d^4y \langle S^+(y)S(x)\rangle. \tag{5}$$

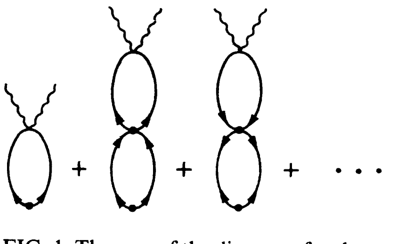

FIG. 1. The sum of the diagrams for the vertex function C.

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The brackets here denote both quantum-mechanical averaging and statistical averaging at finite temperature over the Gibbs distribution.

We go over to Fourier components $[x=(\mathbf{x}, t)\to q=(\mathbf{q},\omega)]$:

$$\mathbf{A}_x^{(i)}\mathbf{A}_x^{(s)}=\int \mathbf{A}^{(i)}\mathbf{A}_q^{(s)}\,e^{iqx}\frac{d^4q}{(2\pi)^4},$$
$$C_x=\int C_q e^{iqx}\frac{d^4q}{(2\pi)^4}.$$

Since the average of the product of ψ operators in Eq. (5) depends only on the difference $y-x$, we obtain

$$d\sigma\propto\int d^4q\int d^4(y-x)\,e^{iq(x-y)}\Big\langle\Big\{\frac{e^2}{mc^2}(\mathbf{A}^{(i)}\mathbf{A}_q^{(s)})^*\psi^+(y)\psi(y)$$
$$-C_q^*I[\psi(y)\psi(y)+\psi^+(y)\psi^+(y)]\Big\}\Big\{\frac{e^2}{mc^2}\mathbf{A}^{(i)}\mathbf{A}_q^{(s)}\psi^+(x)\psi(x)$$
$$-C_qI[\psi(x)\psi(x)+\psi^+(x)\psi^+(x)]\Big\}\Big\rangle. \qquad (6)$$

If the term in C were not present in Eq. (6), then everything would reduce to the Fourier component $\rho(\omega,q)$ of the density correlation function at finite temperature. The prescription for calculating it is well known (see Ref. 16, p. 205): It is necessary to calculate the two-particle temperature Green's function $\mathscr{K}(\tau-\tau')$, after which its Fourier component $\mathscr{K}(\omega_0)$ must be continued from discrete values $\omega_0=2\pi nT$ onto the imaginary axis $\omega_0=-i\omega$ so that the resulting Green's function $K^R(\omega)$ would not have singularities in the upper half of the ω plane. The required function $\rho(\omega,q)$ is obtained with the help of the relation

$$\rho(\omega,q)=-\frac{1}{\pi}\frac{\operatorname{Im}K^R(\omega)}{1-e^{-\omega/T}}. \qquad (7)$$

This prescription is also applicable in our case with C included in Eq. (6), since in the proof essentially only the Bose character of the commutation of the operators S^+ and S was employed.

We start by calculating $C(\tau)$, which depends on imaginary time. Summing the series shown in Fig. 1 leads to the equation

$$\langle T\bar\psi_\alpha^+(\tau)\bar\psi_\beta^+(\tau)\rangle=\frac{2e^2}{mc^2}\int d^4\tau'\,\mathbf{A}^{(i)}\mathbf{A}_{\tau'}^{(s)}I_{\alpha\beta}\mathscr{F}^+(\tau-\tau')\mathscr{G}(\tau'-\tau)$$
$$+\int d^4\tau'\,g[\mathscr{G}^2(\tau'-\tau)\langle T\bar\psi_\alpha^+(\tau')\bar\psi_\beta^+(\tau')\rangle$$
$$+\mathscr{F}^{+2}(\tau'-\tau)\langle T\bar\psi_\alpha(\tau')\bar\psi_\beta(\tau')\rangle], \qquad (8)$$

where the Fourier components of the temperature Green's functions are equal to

$$\mathscr{G}(p)=-\frac{i\omega+\xi}{\omega^2+\xi^2+\Delta^2},\quad \mathscr{F}^+(p)=\frac{\Delta}{\omega^2+\xi^2+\Delta^2}. \qquad (9)$$

Equation (8) confirms the matrix structure of Eq. (4) and is solved by Fourier transforming:

$$C_q=\frac{e^2}{mc^2}\mathbf{A}^{(i)}\mathbf{A}_q^{(s)}R_q,$$

where

$$R_q=gT\sum_{\omega'}\int\frac{d^3p}{(2\pi)^3}\mathscr{F}^+(p)\mathscr{G}(p-q)$$
$$\times\Big\{1-gT\sum_{\omega'}\int\frac{d^3p}{(2\pi)^3}[\mathscr{G}(-p)\mathscr{G}(p-q)$$
$$+\mathscr{F}^+(-p)\mathscr{F}^+(p-q)]\Big\}^{-1}. \qquad (10)$$

Substituting Eq. (10) into Eq. (6) we can see that the problem reduces to the calculation of the Fourier components of the two-particle function

$$\mathscr{K}(\tau-\tau')=-\alpha^2\langle T[\psi^+(\tau)\psi(\tau)-R_q^*I(\psi(\tau)\psi(\tau)$$
$$+\psi^+(\tau)\psi^+(\tau))]$$
$$\times[\psi^+(\tau')\psi(\tau')-R_qI(\psi(\tau')\psi(\tau')+\psi^+(\tau')\psi^+(\tau'))]\rangle.$$

Calculating the average we obtain

$$\mathscr{K}(\tau-\tau')=2\alpha^2\{[\mathscr{G}(\tau-\tau')\mathscr{G}(\tau'-\tau)-\mathscr{F}^+(\tau-\tau')\mathscr{F}^+(\tau'-\tau)]$$
$$+2(R_q+R_q^*)[\mathscr{G}(\tau-\tau')\mathscr{F}^+(\tau'-\tau)-\mathscr{F}^+(\tau-\tau')\mathscr{G}(\tau'-\tau)]$$
$$-2|R_q|^2[\mathscr{G}^2(\tau-\tau')+\mathscr{G}^2(\tau'-\tau)+\mathscr{F}^{+2}(\tau-\tau')+\mathscr{F}^{+2}(\tau'-\tau)]\}, \qquad (11)$$

where we retain the factor $\alpha=\mathbf{e}^{(i)}\mathbf{e}^{(s)}/m$, since below we go over to the anisotropic case; $\mathbf{e}^{(i)}$ and $\mathbf{e}^{(s)}$ are the unit polarization vectors of the incident and scattered light.

The expressions appearing in Eq. (11) are analogous to those present in Eqs. (8) and (10). We demonstrate how to calculate them for the example of the denominator in Eq. (10):

$$J(q)=T\sum_{\omega'}\int\frac{d^3p}{(2\pi)^3}[\mathscr{G}(-p)\mathscr{G}(p-q)+\mathscr{F}^+(-p)\mathscr{F}^+(p-q)]$$
$$=\frac{T}{(2\pi)^3}\int\frac{ds}{v}\int d\xi\sum_{\omega'}\frac{(i\omega'+\xi)(-i\omega_-'+\xi_-)+\Delta^2}{(\omega'^2+\xi^2+\Delta^2)(\omega_-'^2+\xi_-^2+\Delta^2)}. \qquad (12)$$

The components of the four-vector $p-q$ are $\mathbf{p}-\mathbf{q}$ and $\omega'_-=\omega'-\omega_0$. The integral over the momentum $\mathbf{p}$ reduces to an integral over the energy variable ξ, measured from the Fermi energy, and over the Fermi surface. Once again, we denote the latter integration by angular brackets $\int ds/v(2\pi)^3...=\langle...\rangle$, hoping that no confusion arises, since the quantum-mechanical and thermodynamic averaging will no longer appear. We designate by μ the angle between $\mathbf{p}$ and $\mathbf{q}$, so that $\xi_-=\xi(\mathbf{p}-\mathbf{q})=\xi(\mathbf{p})-vq\mu$; $q=|\mathbf{q}|$.

The integral over ξ and the sum over ω' in Eq. (12) diverge at the upper limit. We single out this divergence, subtracting and adding the corresponding expression for $q=\omega_0=0$:

$$T\Big\langle\int d\xi\sum_{\omega'}\frac{1}{\omega'^2+\xi^2+\Delta^2}\Big\rangle,$$

which, according to the equation determining the gap (see Ref. 16, p. 387), is equal to $1/g$. We integrate over ξ the remaining difference, making the substitution $\mathbf{p}\to-\mathbf{p}$, i.e., $\mu\to-\mu$, and the shift $\omega'-\omega_0\to\omega'$ in the intermediate calculations:

$$J(q)=\frac{1}{g}+T\left\langle\sum_{\omega'}\int d\xi\,\frac{1}{\omega'^2+\xi^2+\Delta^2}\left[\frac{(i\omega'+\xi)(-i\omega_-'+\xi_-)+\Delta^2}{\omega'^2+\xi_-^2+\Delta^2}-1\right]\right\rangle$$

$$=\frac{1}{g}+\pi T\left\langle\sum_{\omega'}\left\{\frac{\Delta^2+\omega'\omega_-'+(\omega'^2+\Delta^2)^{1/2}(\omega_-'^2+\Delta^2)^{1/2}}{(\omega'^2+\Delta^2)^{1/2}[(\omega_-'^2+\Delta^2)^{1/2}[(\omega'^2+\Delta^2)^{1/2}+(\omega_-'^2+\Delta^2)^{1/2}-ivq\mu]}-\frac{1}{(\omega'^2+\Delta^2)^{1/2}}\right\}\right\rangle$$

$$=\frac{1}{g}+\pi T\left\langle\sum_{\omega'}\frac{-\omega_0^2/2+ivq\mu(\omega_-'^2+\Delta^2)^{1/2}}{(\omega'^2+\Delta^2)^{1/2}(\omega_-'^2+\Delta^2)^{1/2}[(\omega'^2+\Delta^2)^{1/2}+(\omega_-'^2+\Delta^2)^{1/2}-ivq\mu}\right\rangle. \tag{13}$$

A similar sum was studied in Ref. 16, where the conductivity of a superconductor was calculated in the limit $vq \gg \max(T_0,\ \omega_0)$, but the scheme employed there is applicable in our case also. The summation over ω' is replaced by integration

$$T\sum_{\omega'}\ldots=\frac{i}{4\pi}\int d\omega'\,\mathrm{tg}\frac{\omega'}{2T}$$

over the contour enclosing the real axis, on which the poles of $\tan(\omega'/2T)$ lie. After deforming the contour and making a substitution of variables and the transformation $\omega_0 \to -i\omega$ we obtain an integral along the contour in Fig. 2. This integral gives the function $K^R(\omega)$ for the arrangement of cuts shown, which is analytic in the upper half of the ω plane and which we need for real values of ω; the signs of the imaginary part of the functions $(\omega'^2+\Delta^2)^{1/2}$ and $[(\omega'-\omega_0)^2+\Delta^2]^{1/2}$ on the edges of the cuts are circled.

We study the two limiting cases of small and large vq. If $vq \ll \max(T_c,\ \omega)$ holds, we can drop vq in Eq. (13) in the zeroth-order approximation. For $-\omega/2 < -\Delta$ the purely imaginary square roots in the denominator of Eq. (13) can have different signs, and at the point $\omega' = -i\omega/2$ in the segment of integration from $i(\Delta-\omega)$ up to $-i\Delta$ there arises a pole which determines the imaginary part of the integral

$$J(q)=\frac{1}{g}+\omega\left\langle i\pi\,\mathrm{th}\frac{\omega}{4T}\frac{\theta(\omega-2\Delta)}{2(\omega^2-4\Delta^2)^{1/2}}-\omega\int_{\Delta}^{\infty}\mathrm{th}\frac{\omega'}{2T}\,\frac{d\omega'}{(\omega^2-4\omega'^2)(\omega'^2-\Delta^2)^{1/2}}\right\rangle. \tag{14}$$

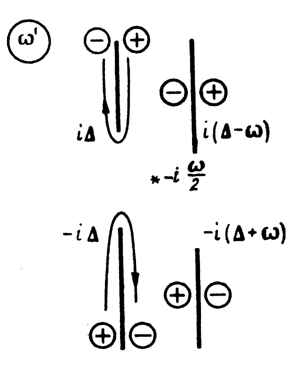

FIG. 2. The contour of integration (curves with the arrows) and the position of the cuts (heavy straight lines) and of the pole (*).

The number 1 in the denominator of Eq. (10) cancels with the first term in Eq. (14), after which the ratio R_q is found to be independent of the small constant g and, as one can easily see, is equal to $-\Delta/\omega$.

According to Eq. (7), we are interested in the imaginary part of the analytically continued Fourier component of the expression (11). In the limit of small q, studied here, the imaginary parts of the first, second, and third expressions in brackets in Eq. (11) are obtained from the imaginary part of Eq. (14) by multiplying by $-2\Delta^2/\omega^2$, $-\Delta/\omega$, and 1, respectively. Thanks to these relations Im K^R vanishes at $q=0$, and the next term in the expansion must be retained, which was done in Ref. 17 for the case $T=0$. The nonvanishing correction is a function of the ratio

$$u=vq/(\omega^2-4\Delta^2)^{1/2}$$

and it reaches a maximum value of order $(vq/\omega)^3$ for $u \sim 1$.

Exact cancellation in Im K^R at $q=0$ does not occur for the case of an anisotropic superconductor, which is of special interest, if we have in mind modern high-T_c superconductors. When anisotropy is taken into account, there the inverse mass tensor m_{jk}^{-1} appears in the interaction with the electromagnetic field, and the quantity α in Eq. (11) must be replaced by

$$\alpha\to e_j^{(i)}m_{jk}^{-1}e_k^{(s)}.$$

In addition, in the electronic interaction the kernel $V(\mathbf{n},\mathbf{n}')$ arises, which depends on the direction of the electron momenta. An equation of the form (8), which sums the loops, can also be easily solved when anisotropy is taken into account, if it is assumed that the kernel factors in terms of some functions $\varphi(\mathbf{n})$:

$$V(\mathbf{n},\mathbf{n}')=\varphi(\mathbf{n})\varphi(\mathbf{n}').$$

Repeating these calculations for the anisotropic case, we obtain for $vq \ll \max(T_c,\ \omega)$

$$-\frac{1}{\pi}\,\mathrm{Im}\,K^R(\omega)=\frac{2}{\omega}\,\mathrm{th}\frac{\omega}{4T}\left\langle\frac{\theta(\omega-2\Delta(\mathbf{n}'))}{[\omega^2-4\Delta(\mathbf{n}')^2]^{1/2}}\left|\alpha\Delta(\mathbf{n}')-\varphi(\mathbf{n}')\frac{\langle\alpha\Delta\varphi I(\mathbf{n},\omega,T)\rangle}{\langle\varphi^2 I(\mathbf{n},\omega,T)\rangle}\right|^2\right\rangle, \tag{15}$$

where $I(\mathbf{n},\omega,T)$ is the function appearing in the brackets in Eq. (14) and depends on the direction of $\mathbf{n}$, for which the gap assumes the value Δ.

We now study the limit of large $vq \gg \max(T_c,\ \omega)$. The integral term in Eq. (13) gives the large contribution $\ln(vq/\Delta)$, which does not appear in the numerator of R_q in

Eq. (10). Together with this contribution, in the neighborhood of the threshold $\omega\to2\Delta$ there arises in the numerator and denominator of R_q a singular term of order

$$\frac{\Delta}{vq}\ln|8\Delta/(\omega-2\Delta)|.$$

For this reason, R_q can be of order unity only in a very narrow neighborhood of the threshold, when

$$\ln\frac{vq}{\Delta}\ll\frac{\Delta}{vq}\ln|8\Delta/(\omega-2\Delta)|.$$

Elsewhere the terms with R_q can be neglected in Eq. (11), and after integrating over ξ we obtain

$$\mathcal{K}(\omega_0)=2\pi T\left\langle \alpha^2\sum_{\omega'}\frac{(\omega'^2+\Delta^2)^{1/2}(\omega_-'^2+\Delta^2)^{1/2}-\Delta^2-\omega'\omega_-'}{(\omega'^2+\Delta^2)^{1/2}(\omega_-'^2+\Delta^2)^{1/2}[(\omega'^2+\Delta^2)^{1/2}+(\omega_-'^2+\Delta^2)^{1/2}-ivq\mu]}\right\rangle.$$

For large vq the integration over μ can be extended to infinity, after symmetrizing $\mu\to-\mu$. This integral can be evaluated formally by passing to the δ-function

$$[(\omega'^2+\Delta^2)^{1/2}+(\omega_-'^2+\Delta^2)^{1/2}-ivq\mu]^{-1}\to\frac{\pi}{vq}\delta(\mu).$$

After performing the analytic continuation according to the prescription described above, we obtain

$$\begin{aligned}-\frac{1}{\pi}&\operatorname{Im}K^R(\omega)\\ &=\frac{2}{q}\left\langle\frac{\alpha^2}{v}\delta(\mu)\left[\int_\Delta^\infty d\omega'\frac{\omega'(\omega'+\omega)-\Delta^2}{\{(\omega'^2-\Delta^2)[(\omega'+\omega)^2-\Delta^2]\}^{1/2}}\right.\right.\\ &\qquad\times\left(\operatorname{th}\frac{\omega'+\omega}{2T}-\operatorname{th}\frac{\omega'}{2T}\right)+\theta(\omega-2\Delta)\\ &\qquad\left.\left.\times\int_\Delta^{\omega-\Delta}d\omega'\frac{\omega'(\omega-\omega')+\Delta^2}{\{(\omega'^2-\Delta^2)[(\omega-\omega')^2-\Delta^2]\}^{1/2}}\operatorname{th}\frac{\omega'}{2T}\right]\right\rangle.\end{aligned}\tag{16}$$

The two integral terms in Eq. (16) have a different physical meaning. The first term describes single-particle excitations on one side of the gap and the second term describes pair creation through the gap. The first term vanishes as $T\to0$, and it has no analog in the limit (15) of small q. This is explained by the fact that the law of conservation of energy for single-particle excitations $\varepsilon_p-\varepsilon_{p-q}=\omega$, in contrast to the law $\varepsilon_p+\varepsilon_{p-q}=\omega$ corresponding to pair creation, cannot be satisfied for $q=0$ and finite ω.

REFLECTION COEFFICIENT

The relative number of photons reflected into the solid angle $d\Omega$ and the frequency interval $d\omega$ can be written in the form[1]

$$d\sigma=\frac{4}{\pi}\left(\frac{e^2}{\hbar c^2}\right)^2\frac{I(\omega)}{[(n+1)^2+\varkappa^2]^2}d\Omega\,d\omega,\tag{17}$$

where the integral $I(\omega)$ takes into account the decay of the field in the metal at a depth δ:

$$I(\omega)=\frac{16}{\delta^2}\int_0^\infty dq\frac{\rho(\omega,q)}{(q^2+4\delta^{-2})^2},\tag{18}$$

n and $\varkappa$ are the refraction coefficient and the damping coefficient at the frequency of the incident light $\omega_i\approx\omega_s$, and $\delta=c/\omega_i$. We shall employ the simplified form of Eq. (18) in the limit $n\ll\varkappa$. In the general case the factor in front of $\rho(\omega,q)$ has the form

$$\frac{16(n^2+\varkappa^2)(\omega_i/c)^2}{[q^2-(2\omega_i/c)^2(n^2-\varkappa^2)]^2+2^6(\omega_i/c)^4n^2\varkappa^2}.$$

The Fourier component $\rho(\omega,q)$ can be expressed with the help of Eq. (7) in terms of $\operatorname{Im}K^R(\omega)$, which is given by Eqs. (15) and (16) in the limiting cases, where the transferred quantum ω was assumed to be positive. Direct calculation shows that $\operatorname{Im}K^R(-\omega)=-\operatorname{Im}K^R(\omega)$. Therefore

$$\rho(-\omega,q)=-\frac{1-e^{-\omega/T}}{1-e^{\omega/T}}\rho(\omega,q)=e^{-\omega/T}\rho(\omega,q),\tag{19}$$

which proves Eq. (1) given in the Introduction. Bearing in mind the relation (19), we confine our attention in what follows to values $\omega>0$.

We start with the case of a normal metal, when $\Delta=0$ holds and only the region $vq>\omega$ exists (v is the maximum value of the projection of the Fermi velocity on the normal to the surface of the sample), for which Eq. (16) is valid. Carrying out the integration in Eq. (16) we obtain

$$-\frac{1}{\pi}\operatorname{Im}K^R(\omega)=\frac{2\omega}{q}\left\langle\frac{\alpha^2}{v}\delta(\mu)\right\rangle,\tag{20}$$

where $\alpha=e_j^{(i)}m_{jk}^{-1}e_k^{(s)}$. We note that taking into account Debye screening, which must be done at low frequencies,[18] leads in Eq. (20) to the substitution $\alpha\to\alpha-\langle\alpha\rangle$.

The integral over q in Eq. (18) can be easily evaluated, and for the intensity we obtain

$$I(\omega)\propto\frac{\omega}{1-e^{-\omega/T}}\begin{cases}\delta^2\ln(2v/\omega\delta), & \omega\ll v/\delta, \quad (21)\\ 4\delta^{-2}(v/\omega)^4, & \omega\gg v/\delta. \quad (22)\end{cases}$$

The qualitative frequency dependence of the intensity given by Eqs. (21) and (22) is shown in Fig. 3.

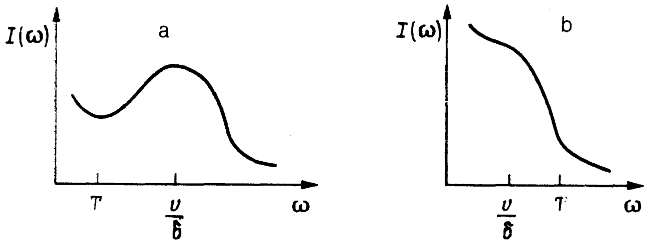

FIG. 3. The spectrum of radiation scattered by a normal metal at low temperatures $T\ll v/\delta$ (a) and at high temperatures $T\gg v/\delta$ (b). The typical values of the Fermi velocity $v\sim10^8$ cm/s and penetration depth $\delta\sim10^{-5}$ cm correspond to the frequency $v/\delta\sim50$ cm^{-1} (1 cm^{-1} = 1.44 K).

As the structure of the expressions (15) and (16) shows, the spectrum of light scattered by a superconductor consists of a background, given by the first integral in Eq. (16), and a maximum, which lies in the frequency range $\omega > 2\Delta_{\min}$, where $2\Delta_{\min}$ is the minimum gap width.

The background can be easily calculated at low temperatures $T \ll \Delta_{\min}$, when the corresponding integral in Eq. (16) over ω' gives $2T(1-e^{-\omega/T})e^{-\Delta/T}$ for $\omega \ll T$ and $(2\pi\omega T)^{1/2}(1-e^{-\omega/T})e^{-\Delta/T}$ for $\omega \gg T$. The remaining integral over the strip lying on the Fermi surface, where $\mu = 0$, contributes the additional factor $(\pi T/2\Delta''_{\min})^{1/2}$, if the gap varies sufficiently rapidly $\Delta'' \gg T$; the derivative is evaluated at the point on the strip where the gap width is minimum.

Thus for the background at temperatures $T \ll \Delta_{\min}$ the following interpolation formula is obtained:

$$I_q(\omega) \propto [T(\omega+T)]^{1/2} \exp\left(-\frac{\Delta_{min}}{T}\right) \begin{cases} \delta^2 \ln \dfrac{2}{q_{min}\delta}, & q_{min}\delta \ll 1, \\ 4\delta^{-2} q_{min}^{-4}, & q_{min}\delta \gg 1, \end{cases} \tag{23}$$

where $vq_{\min} = \max(T_c, \omega)$ and when the anisotropy of the gap width is large the additional factor $(T/\Delta''_{\min})^{1/2}$ must be taken into account.

The contribution due to pair creation in Eq. (16) has the following form near the threshold after integration over ω' (for $q \gg T_c/v$):

$$-\frac{1}{\pi} \operatorname{Im} K^R(\omega) = \frac{2\pi}{q} \left\langle \frac{\alpha^2}{v} \delta(\mu)\theta(\omega-2\Delta)\Delta \operatorname{th}\frac{\Delta}{2T} \right\rangle.$$

Expanding with respect to the angle φ near the minimum value $2\Delta = 2\Delta_{\min} + \Delta''\varphi^2$ and integrating, we find

$$-\frac{1}{\pi} \operatorname{Im} K^R(\omega) = \frac{1}{(2\pi)^2 q} \left(\frac{\omega-2\Delta_{min}}{\Delta''_{min}}\right)^{1/2} \left(\frac{\alpha^2 p^2}{v^2} \Delta \operatorname{th}\frac{\Delta}{2T}\right)_{min}. \tag{24}$$

Here all values are taken at the point on the strip of the Fermi surface $\mu = 0$ where the gap width is minimum.

For $q \ll T_c/v$ we find with the help of Eq. (15)

$$-\frac{1}{\pi} \operatorname{Im} K^R(\omega) = \frac{1}{(2\pi)^3} \left(\frac{\omega-2\Delta_{min}}{\Delta_{min}}\right)^{1/2} \times \left[\left(\alpha - \frac{\langle\alpha\Delta\rangle}{\Delta_{min}}\right) \frac{\Delta}{\Delta''} \frac{p_0^2}{v} \operatorname{th}\frac{\Delta}{2T}\right]_{min}, \tag{25}$$

where the minimum of the gap width is determined on the entire Fermi surface, and all values are taken at the corresponding point. From the expressions (24) and (25) it is evident that the intensity of the scattered light associated with pair creation

$$I_p(\omega) \propto (\omega-2\Delta_{min})^{1/2} \operatorname{th}\frac{\Delta_{min}}{2T} \Big/ \left[1 - \exp\left(-\frac{2\Delta_{min}}{T}\right)\right], \tag{26}$$

vanishes quite rapidly—according to a square-root law—near the threshold $\omega - 2\Delta_{\min} \ll \Delta_{\min}$ and, neglecting the temperature dependence of the gap width itself $2\Delta_{\min}(T)$, it is virtually independent of the temperature.

The behavior of the pair contribution far from threshold $\omega \gg \Delta_{\max}$ is evaluated with the help of the formulas (15) and (16), which lead to the expression

$$-\frac{1}{\pi} \operatorname{Im} K^R(\omega) = \frac{2}{\omega^2} \operatorname{th}\frac{\omega}{4T} \langle(\alpha\Delta - \langle\alpha\Delta\rangle)^2\rangle \tag{27}$$

for $q \ll \omega/v$ and to the expression (20), corresponding to a normal metal, for $q \gg \omega/v$. Thus, far from threshold we obtain the same result as in the case of a normal metal (21) and (22), and moreover in the region $\omega \gg v/\delta$ we obtain the superconducting "tail"

$$I(\omega) \propto \frac{\operatorname{th}(\omega/4T)}{1-e^{-\omega/T}} v\delta\Delta^2/\omega^2, \tag{28}$$

which decays more slowly than the normal term (22).

CONCLUSIONS

An increase in temperature affects electron Raman scattering both in a normal metal and in a superconductor. For a normal metal the most significant change is that for low transferred energy $\omega \ll T$ the cross section, instead of decreasing as the frequency increases approximately as $I \propto \omega \ln(2v/\omega\delta)$, starts to increase slowly as $I \propto T \ln(2v/\omega\delta)$ [see Eq. (21)]. In a superconductor subthreshold scattering arises for frequencies $\omega < 2\Delta_{\min}$ (if the minimum gap width $2\Delta_{\min}$ is not equal to zero). Here the cross section depends exponentially on the temperature (23), and the frequency dependence is described primarily by the factor $[T(\omega + T)]^{1/2}$. In the region above threshold the cross section increases somewhat at the transition to the superconducting state.

At the present time it is difficult to draw an unequivocal conclusion about the extent to which the experimentally observed picture can be explained by uncontrollable heating of the sample by a laser beam. The frequency dependence observed at low frequencies is closer to a linear dependence than to a square-root dependence (we note that $[T(\omega + T)]^{1/2} \sim (T\omega)^{1/2}$ holds for $\omega \gg T$). In order to clarify the situation it would be helpful to obtain the detailed frequency and temperature dependence of the scattering cross section with careful monitoring of the temperature, for example, with the help of the relation (1).

Finally, we make a remark concerning the role of interband transitions in scattering. They can be taken into account in the spirit of the k-p scheme. Aside from the dependence of the inverse-mass tensor on the point on the Fermi surface, which we have already discussed, a dependence of the inverse-mass tensor on the frequency of the incident and scattered light also appears:

$$m_{ij}^{-1} = \delta_{ij} m_0^{-1} + m_0^{-2} \sum_v \left(\frac{p_{cv}^i p_{vc}^j}{E_c(p) - E_v(p) + \omega_i} + \frac{p_{cv}^j p_{vc}^i}{E_c(p) - E_v(p) - \omega_s} \right),$$

where p_{vc} is the matrix element for a transition from the state c on the Fermi surface into other bands v; we neglect the change in the electron momentum in the energy denominators to the extent that the photon momentum is small compared with electron momentum. Thus, if the frequency of the incident light approaches some interband-absorption frequency, then the intensity of Raman scattering, on the one hand, increases owing to the resonance denominator in the amplitude while, on the other hand, it decreases owing to the decrease of the penetration depth of the light. As regards the operator structure of the expressions (2) and (3), it remains the same even when interband transitions are taken into account.

In Ref. 19 the cross section near threshold increased more slowly than in our case.[1] The reason for this discrepancy lies not in the more complicated form of the scattering matrix element, which as a result is still replaced by a constant, but rather in the error in the calculation of the coherence factor: In Eq. (5.8) of Ref. 19 $u_p v_{p+q} - v_p u_{p+q}$ should be replaced by $u_p v_{p+q} + v_p u_{p+q}$ [see Eq. (11) with $C_2 = 0$ from Ref. 17].

[1)]Institute of Molecular Physics, Polish Academy of Sciences, Poznon.
[2)]*I*—well known antisymmetric matrix in the theory of superconductivity.

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