Oscillation effects in metals in a magnetic field and macroscopic quantum interference

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The oscillatory part of the electron density of states of a metal in a magnetic field—the part of the density of states that determines the physical nature of the oscillations of the thermodynamic and kinetic characteristics of a metal in a magnetic field—is calculated using the coherent states of the electrons in the magnetic field. The physical reason for the substantial simplification, achieved in this approach, of the mathematical procedure lies in the fact that the coherent states employed in the calculations best describe quantum macroscopic phenomena, which the Shubnikov-de Haas and the de Haas-van Alphen effects in metals, semimetals, and degenerate semiconductors are. Oscillation effects in metals in a magnetic field are related to quantum macroscopic phenomena, such as superconductivity and weak-link superconductivity, and they are governed by a specific macroscopic quantum interference of elementary excitations of the boson type, with whose help the partial motion of electrons in a plane perpendicular to the magnetic field **H** is described.

1. INTRODUCTION

The foundations of the quantum physics of charged particles in a magnetic field were laid in Refs. 1–3, where oscillations of the kinetic coefficients of a metal (semimetal, degenerate semiconductor) in a magnetic field (the Shubnikov-de Haas effect (SdHE), Landau diamagnetism, and oscillations of the thermodynamic potentials of a metal and their derivatives (first and higher order) in a magnetic field [the de Haas-van Alphen effect (dHvAE)] were first studied.[4,5]

In the 1950's a relation (the Lifshitz-Onsager relation) was established between the period of the oscillations and the parameters of the Fermi surface, and the oscillation phenomena became the most accurate and reliable method for determining the parameters of the Fermi surface.[5]

After Glauber introduced in 1963 (Refs. 6 and 7) the concept of a coherent state $|\alpha\rangle$ as an eigenstate of a nonhermitian annihilation operator a of excitations of the boson type ($a|\alpha\rangle = \alpha|\alpha\rangle$) Malkin and Man'ko[8,9] were the first to construct a coherent state for a charge in a constant uniform magnetic field. As shown in Ref. 2, the corresponding Schrödinger equation for the eigenfunctions and eigenvalues reduces to the Schrödinger equation for a one-dimensional displaced harmonic oscillator. This equation, by Haken's definition,[10] is the "prototype of an elementary excitation in a solid" and an example of the simplest physical system for which a coherent state can be introduced in a natural manner. Later, Feldman and Kahn[11] formulated in terms of coherent states the theory of dimagnetism of free electrons, and Granovskiĭ and Dimashko[12] employed coherent states and the method of Green's functions for describing the de Haas-van Alphen effect in the case of a free-electron gas.

Dodonov and Man'ko[13,14] comparatively recently, in the context of their theory of quantum integrals of motion which are explicitly time-dependent, proposed their own approach to the use of coherent states for obtaining the de Haas-van Alphen effect for a free-electron gas. In Refs. 12 and 14 it was pointed out that in the proposed approaches the use of coherent states significantly simplifies the mathematical calculation of the oscillating part of the thermodynamic characteristics. In particular, Dodonov and Man'ko[14] point out that their method does not require the energy spectrum of the elementary excitations and it is also not necessary to perform the difficult summation of the energy levels using the Poisson summation formula, as is done in the usual approach.[4,5] At the same time, it is significant that in Refs. 12–14 only thermodynamic and not kinetic phenomena in a free-electron gas were studied. This is undoubtedly connected with the fact that coherent states, as eigenstates of a nonhermitian operator, are not orthogonal, i.e., transitions between different coherent states can occur spontaneously.

At the same time it is well known[4] on the basis of the traditional theory of oscillation phenomena in metals in a magnetic field that the Shubnikov-de Haas effect and the de Haas-van Alphen effect are related with one another and that the period of the oscillations—the main characteristic providing information about the parameters of the Fermi surface—is the same for both the thermodynamic and kinetic quantities.

These phenomena have another interesting characteristic feature: The Shubnikov-de Haas and de Haas-van Alphen effects are not only quantum effects. They are also macroscopic effects, and in these respects (the quantum character and macroscopic nature, simultaneously) they are related with phenomena such as superconductivity, weak-link superconductivity (Josephson effects), lasing, and von Klitzing's effect.[1)] It is natural that this sequence of definitions of the phenomena mentioned should end with quasiclassicity, i.e., quantum-macroscopic-quasiclassical.

Our aim in this paper is to demonstrate clearly, by using the method of coherent states combined with a universal approach for describing both thermodynamic and kinetic characteristics of a metal in a constant uniform magnetic field, not only the mathematical advantage of such a combination, but also to establish the physical reasons for why the mathematical description is adequate for the physics of the quantum oscillation effects studied.

The physical nature of oscillations of the kinetic coefficients of a metal in a magnetic field (Shubnikov-de Haas effect) as well as oscillations of the thermodynamic poten-

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tials and their derivatives (first and higher order) has been established on the basis of Landau's theory of diamagnetism. The oscillations are governed by two factors: the presence of the Fermi surface and the radical change in the density of states $\rho(\varepsilon)$ when the magnetic field is turned on (Ref. 4).[2)]

Turning on a constant uniform magnetic field $\mathbf{H}$ parallel to the z-axis makes the motion of a current-carrying particle quasi-one-dimensional and in the process changes the behavior of the density of states from $\rho_{3d}(\varepsilon) \propto \varepsilon^{1/2}$ to $\rho_{1d}(\varepsilon) \propto 1/\varepsilon^{1/2}$ (for the three- and one-dimensional situations, respectively). Because of Landau quantization of the electron energy spectrum this inverse square-root singularity of $\rho(\varepsilon)$ is repeated many times in the energy interval $0 \leqslant \varepsilon \leqslant \mu$ (μ is the chemical potential, which at zero temperature is equal to the Fermi energy) near the bottom ($p_z = 0$) of each Landau band located below the Fermi level. When the condition

$$\mu/\hbar\omega_H \gg 1$$

is satisfied, where $\omega_H = eH/mc$ is the cyclotron frequency, m is the effective mass of the current carrier, and c is the velocity of light in vacuum, for energies $\varepsilon \approx \mu$ near the Fermi surface the density of states $\rho(\varepsilon)$ is an almost-periodic function of the magnetic field. The presence of the characteristic energy μ and this almost-periodic dependence of $\rho(\varepsilon)$ on the magnetic field H are responsible for the oscillatory character of the magnetic field dependence of both the thermodynamic quantities ["linear" with respect to $\rho(\varepsilon)$] and the kinetic coefficients ["quadratic" with respect to $\rho(\varepsilon)$].[4] It is obvious that the period of the oscillations is the same for both types of quantities and is equal to the oscillation period of the function $\rho(\varepsilon)$.

We note that the above condition, for $\rho(\varepsilon)$ to be an almost-periodic function in a magnetic field, is identical to the condition of quasiclassicity. This fact in itself indicates that the method of coherent states can be used to describe these effects; it is well known[7,10] that this method is used successfully to describe quantum states that are as close as possible to classical states.

Hence the implementation of the above program reduces to establishing a relation between the density of states $\rho(\varepsilon)$ and the thermodynamic potential and kinetic coefficients, introducing coherent states characteristic for the problem of the motion of a charge in a magnetic field, and calculating $\rho(\varepsilon)$ with the help of these coherent states.

2. STARTING RELATIONS

The thermodynamic potential $\Omega_H = F_H - \mu N$ is defined by the expression[15]

$$\Omega_H = -T\sum_{\nu} \ln\{1+\exp[(\mu-\varepsilon_\nu)/T]\}, \tag{2.1}$$

which can be represented in the integral form

$$\Omega_H = -T\int_0^\infty d\varepsilon \rho(\varepsilon)\ln\{1+\exp[(\mu-\varepsilon)/T]\} \tag{2.2}$$

with the help of the density of states

$$\rho(\varepsilon) = \sum_{\nu} \delta(\varepsilon-\varepsilon_\nu) = \mathrm{Sp}\,\delta(\varepsilon-\hat{\mathscr{H}}). \tag{2.3}$$

Here F_H is the free energy, N is the total number of particles, T is the temperature (in energy units), ν is the collection of all quantum numbers characterizing a single-particle state, and $\hat{\mathscr{H}}$ is the single-particle Hamiltonian. Thus the theoretical description of oscillations of the thermodynamic characteristics of a metal in a magnetic field reduces to calculating the thermodynamic potential Ω_H and its derivatives

$$N = -\left(\frac{\partial\Omega_H}{\partial\mu}\right)_{T,V,H}, \quad \mathbf{M} = -\left(\frac{\partial\Omega_H}{\partial\mathbf{H}}\right)_{T,V,\mu}, \quad C = -T\left(\frac{\partial^2\Omega}{\partial T^2}\right)_{V,\mu,H} \tag{2.4}$$

etc., where $\mathbf{M}$ is the magnetic moment and C is the heat capacity.

As one can see from Eq. (2.2), the kernel of the integral defining Ω_H contains the product of two functions, one of which is a logarithm, which, obviously, is not responsible for the oscillatory character of the dependence $\Omega_H(H)$. Its presence in Eq. (2.2) does not present any fundamental difficulties for the calculation, which is performed with the help of an elegant technique, first proposed by Rumer,[16] and is based on the use of the two-sided Laplace transform of the function $\ln|1 \pm e^x|$.[12,14]

It seems to us, however, that this procedure largely obscures the decisive role of the use of coherent states in the simplification of the mathematical procedure and of the purely physical factors on which the simplification is based. For this reason, we shall calculate directly, starting from Eq. (2.2), the density of states, setting $T = 0$ for simplicity. In the process, $\rho(\varepsilon)$ transforms into $\rho(\mu)$ and is related with Ω_H by the simple relation

$$\rho(\mu) = -\left(\frac{\partial^2\Omega_H}{\partial\mu^2}\right)_{V,H,T=0}. \tag{2.5}$$

As one can easily see from Eq. (2.5), the density of states $\rho(\mu)$ at the Fermi surface is not only related with the observable quantities presented in Eq. (2.4), but, in a certain sense, it is itself an observable quantity (for example, through T_c from the BCS formula). Its oscillatory part $\tilde{\rho}(\mu)$ contains the period of the oscillations, which in turn through the Lifshitz-Onsager relation determines the area of the extremal section of the Fermi surface by a plane perpendicular to $\mathbf{H}$.

The solution of the problem of finding by a universal method the oscillatory part $\tilde{\rho}(\varepsilon)$ of the density of states also answers the question of the physical nature of the oscillations of the kinetic coefficients in a magnetic field. As is well known from the theory of the Shubnikov-de Haas effect,[4,17] the appearance of a nonzero current in the direction of the stretching electric field $\mathbf{E}\|x$ is attributable to the appearance of electron scattering, which under the conditions of the Shubnikov-de Haas effect can be assumed to be elastic. In this case, using Fermi's golden rule for the transition probability per unit time in elastic scattering processes at the Fermi surface, we have for the transverse electric conductivity

$$\sigma_{xx} \propto \overline{W} = \frac{2\pi}{\hbar}\sum_{\nu,\nu'} |\langle\nu'|V_{int}|\nu\rangle|^2 f_\nu(1-f_{\nu'})\delta(\varepsilon_\nu-\varepsilon_{\nu'}) \sim \int d\varepsilon\, d\varepsilon' \rho(\varepsilon)\rho(\varepsilon')G(\varepsilon,\varepsilon')\delta(\varepsilon-\varepsilon'), \tag{2.6}$$

where f_ν is the occupation number of the state ν and $G(\varepsilon,\varepsilon')$ is the kernel of the integral.

As one can see from Eq. (2.6), summing over the final states ν', averaging over the initial states ν, and going over to the integral representation with the help of the densities of states $\rho(\varepsilon)$ and $\rho(\varepsilon')$ gives an expression which is "quadratic" in $\rho(\varepsilon)$ and whose period of oscillations in a magnetic field is determined by the period of oscillations of $\tilde{\rho}(\varepsilon)$, as happened in the case of the thermodynamic potential (2.2).

Thus the oscillatory behavior of both the thermodynamic and kinetic quantities in a magnetic field is determined by the presence of the Fermi surface and the almost-periodic magnetic-field dependence of the density of states $\rho(\varepsilon)$. The fact that $\rho(\varepsilon)$ in Eq. (2.3) is represented in the form of a trace makes it possible to employ any complete set of wave functions in the computational procedure. But, obviously, the mathematical conciseness and simplicity depend decisively on the degree to which the the set of wave functions employed is appropriate for the physical nature of the phenomenon under study. The customary use of oscillator wave functions, which are essentially the eigenfunctions of the number operator of boson excitations of the corresponding problem (see below) $a^+|n\rangle = n|n\rangle$ seems obvious at first glance. It is immediately clear, however, that these wave functions do not carry any information about the presence of the Fermi surface, while for the coherent states $|\alpha\rangle$, which are eigenfunctions of the operator a ($a|\alpha\rangle = \alpha|\alpha\rangle$), the average of the particle-number operator is equal to

$$\langle\alpha|a^+a|\alpha\rangle = \bar{n}_\alpha \approx \mu/\hbar\omega_H.$$

In addition, the coherent states are characterized by a well-defined phase.[7,10] This is related with the existence of a phase characteristic (cyclotron period) of the oscillation phenomena under study. For this reason, we now introduce coherent states for the problem at hand.

3. COHERENT STATES OF A CHARGED PARTICLE IN A CONSTANT UNIFORM MAGNETIC FIELD

Landau was the first to show[2] that the Schrödinger equation for the eigenfunctions and eigenvalues for a charge in a constant uniform magnetic field has the form of the Schrödinger equation for the one-dimensional displaced oscillator. Forty years later, using this fact, Malkin and Man'ko[8,9] determined the two-dimensional coherent state corresponding to this problem. As we have already mentioned in the introduction, such coherent states have been used to recast in new terms the theory of Landau dimagnetism and the theory of the de Haas-van Alphen effect for a free-electron gas.[11,12,14] Up to now, however, the achievements of the physics of coherent states have not been extended to oscillation effects in metals with an arbitrary dispersion relation for electrons or to numerous other quantum physical phenomena observed in metals in a magnetic field.

In what follows a procedure is developed for introducing the Fock and coherent states for a charge in a constant uniform magnetic field $\mathbf{H}\|z$. The Hamiltonian for a particle with mass m and charge e (for convenience we take $e>0$) has the form[18]

$$\hat{\mathcal{H}} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + \hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_\perp + \hat{\mathcal{H}}_z + \hat{\mathcal{H}}_0,$$
$$\hat{\mathcal{H}}_z = \frac{\hat{p}_z^2}{2m}, \quad \hat{\mathcal{H}}_0 = -\frac{g^*}{2}\mu_B\sigma_z H, \quad \sigma_z = \pm 1. \tag{3.1}$$

If the only symmetry of the problem—the direction distinguished by the magnetic field (the z axis)—and the physical equivalence of the motion along the x and y axes are preserved, then it is natural to choose the vector potential $\mathbf{A}$ of the magnetic field in the form

$$\mathbf{A} = {}^1/_2[\mathbf{Hr}].$$

In this case the part $\hat{\mathcal{H}}_\perp$ of the Hamiltonian that is responsible for motion in the xy plane is a sum of the Hamiltonians of two coupled standard one-dimensional oscillators (along the x and y axes, respectively)

$$\hat{\mathcal{H}}_\perp = \left(\frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega_L^2 x^2\right) + \left(\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_L^2 y^2\right) + \omega_L(y\hat{p}_x - x\hat{p}_y). \tag{3.2}$$

Here $\hat{\mathbf{p}}$ is the momentum operator, μ_B is the Bohr magneton, m_0 is the mass of a free electron, g^* is the effective spectroscopic splitting factor, and $\omega_L = eH/2mc$ is the Larmor frequency.[19] It is interesting to note that the frequency characteristic of each of the two oscillators in Eq. (3.2) is the Larmor frequency—a characteristic classical quantity.[19] Going over to dimensionless coordinates

$$\tilde{\xi} = \frac{x}{l_L}, \quad \tilde{\eta} = \frac{y}{l_L}; \quad l_L = \left(\frac{\hbar}{m\omega_L}\right)^{1/2} = 2^{1/2}l_H, \quad l_H = \left(\frac{\hbar}{m\omega_H}\right)^{1/2}, \tag{3.3}$$

we introduce instead of the four starting operators of the coordinates and projections of the momenta for Bose operators $\tilde{a}$, $\tilde{a}^+$, $\tilde{b}$, and $\tilde{b}^+$ (Ref. 9):

$$\tilde{a} = \frac{1}{2^{1/2}}\left(\tilde{\eta} + \frac{\partial}{\partial\tilde{\eta}}\right), \quad \tilde{b} = \frac{1}{2^{1/2}}\left(\tilde{\xi} + \frac{\partial}{\partial\tilde{\xi}}\right),$$
$$[\tilde{a}, \tilde{a}^+] = [\tilde{b}, \tilde{b}^+] = 1,$$
$$\tilde{a}^+ = \frac{1}{2^{1/2}}\left(\tilde{\eta} - \frac{\partial}{\partial\tilde{\eta}}\right), \quad \tilde{b}^+ = \frac{1}{2^{1/2}}\left(\tilde{\xi} - \frac{\partial}{\partial\tilde{\xi}}\right), \tag{3.4}$$
$$[\tilde{a}, \tilde{b}] = \ldots = [\tilde{a}^+, \tilde{b}^+] = 0.$$

Then $\hat{\mathcal{H}}_\perp$ assumes the form

$$\hat{\mathcal{H}}_\perp = \hbar\omega_L(\tilde{a}^+\tilde{a} + {}^1/_2) + \hbar\omega_L(\tilde{b}^+\tilde{b} + {}^1/_2) + i\hbar\omega_L(\tilde{a}\tilde{b}^+ - \tilde{a}^+\tilde{b})$$
$$= 2\hbar\omega_L\left(\frac{\tilde{a}^+ + i\tilde{b}^+}{2^{1/2}}\frac{\tilde{a} - i\tilde{b}}{2^{1/2}} + \frac{1}{2}\right), \tag{3.5}$$

in which a pair of new operators has been distinguished in a natural manner:

$$a = \frac{\tilde{a} - i\tilde{b}}{2^{1/2}}, \quad a^+ = \frac{\tilde{a}^+ - i\tilde{b}^+}{2^{1/2}}, \quad [a, a^+] = 1. \tag{3.6}$$

In accordance with the Stone-von Neumann theorem, the complete set of operators contains $2s$ operators (s is the dimension of the system). For this reason, we introduce, in addition to the operators a and a^+, the operators b and b^+:

$$b = \frac{\tilde{b} - i\tilde{a}}{2^{1/2}}, \quad b^+ = \frac{\tilde{b}^+ + i\tilde{a}^+}{2^{1/2}}, \quad [b, b^+] = 1. \tag{3.7}$$

The operators b and b^+ are obtained from the operators a and a^+ (3.6) by the simple substitution $\tilde{a} \leftrightarrow \tilde{b}$ (or $x \leftrightarrow y$); this reflects the physical equivalence of motion along the x and y axes and the symmetry of the gauge chosen for A. It is easy to verify that[9]

$$a=-\frac{i}{2^{1/2}}\left(\xi+\frac{\partial}{\partial\xi^*}\right),\quad a^+=\frac{i}{2^{1/2}}\left(\xi^*-\frac{\partial}{\partial\xi}\right),$$
$$b=\frac{1}{2^{1/2}}\left(\xi^*+\frac{\partial}{\partial\xi}\right),\quad b^+=\frac{1}{2^{1/2}}\left(\xi-\frac{\partial}{\partial\xi^*}\right), \tag{3.8}$$

where the direct and inverse relations have been employed:

$$\xi=\frac{\tilde\xi+i\tilde\eta}{2^{1/2}}=\frac{x+iy}{2^{1/2}l_L},\quad \xi^*=\frac{\tilde\xi-i\tilde\eta}{2^{1/2}}=\frac{x-iy}{2^{1/2}l_L},$$
$$x=\frac{\xi+\xi^*}{2^{1/2}}l_L,\quad y=-i\frac{\xi-\xi^*}{2^{1/2}}l_L. \tag{3.9}$$

To complete the picture we introduce the relations between the operators a and b and the operators of the coordinates of the center of an orbit x_0 and y_0 and of the relative motion x and y:[9]

$$a=-\frac{i}{l_L}[(x-x_0)+i(y-y_0)],\quad b=\frac{x_0-iy_0}{l_L},$$
$$a^+=\frac{i}{l_L}[(x-x_0)-i(y-y_0)],\quad b^+=\frac{x_0+iy_0}{l_L}. \tag{3.10}$$

It is clear that the operators b and b^+ are expressed only in terms of the integrals of motion x_0 and y_0 and therefore are themselves integrals of motion. We note here also that $L_z=(\mathbf{r}\times\mathbf{p})_z$ is another integral of motion.

Thus after the operators a, a^+, b, and b^+ are introduced the operator $\hat{\mathscr{H}}_\perp$ assumes the form

$$\hat{\mathscr{H}}_\perp=2\hbar\omega_L(a^+a+{}^1/_2)=\hbar\omega_H(a^+a+{}^1/_2). \tag{3.11}$$

Note that the cyclotron frequency $\omega_H=2\omega_L$ naturally appeared in Eq. (3.11) when the problem of two coupled oscillators with frequency characteristics ω_L was reduced to the problem of a one-dimensional oscillator or, in other words, to the problem of two equivalent, but uncoupled, one-dimensional oscillators.

Since the partial motion of an electron in a magnetic field in the xy plane is described by the operator (3.11), which contains the operators a, a^+ (3.6) and b, b^+ (3.7), which satisfy the Bose commutation relations, the following "two-dimensional" states can be defined in the occupation number representation with respect to these operators:

a) the vacuum state $|00\rangle$ defined by the relations

$$a|00\rangle=0,\quad b|00\rangle=0; \tag{3.12}$$

b) the Fock (after V. A. Fock) state $|n_an_b\rangle$, which is an eigenstate of the number operators of boson excitations $\hat n_a=a^+a$ and $\hat n_b=b^+b$:

$$\hat n_a|n_an_b\rangle=n_a|n_an_b\rangle,\quad \hat n_b|n_an_b\rangle=n_b|n_an_b\rangle,$$
$$|n_an_b\rangle=\frac{(a^+)^{n_a}(b^+)^{n_b}}{(n_a!n_b!)^{1/2}}|00\rangle; \tag{3.13}$$

c) the two-dimensional coherent state $|\alpha\beta\rangle$, which is an eigenstate of the operators a and b:[9]

$$a|\alpha\beta\rangle=\alpha|\alpha\beta\rangle,\quad b|\alpha\beta\rangle=\beta|\alpha\beta\rangle. \tag{3.14}$$

The coherent state $|\alpha\beta\rangle$ can be obtained in an explicit form by the action of displacement operators on the vacuum:

$$|\alpha\beta\rangle=D(\alpha)D(\beta)|00\rangle, \tag{3.15}$$

where, for example,

$$D(\alpha)=\exp(\alpha a^+-\alpha^*a)$$
$$=\exp(-|\alpha|^2/2)\exp(\alpha a^+)\exp(-\alpha^*a),$$
$$D(\beta)=\exp(\beta b^+-\beta^*b). \tag{3.16}$$

The above method of introducing coherent states is standard.[7,9,10] As a result, we now have a complete normalized set of wave functions, which are the eigenfunctions of nonhermitian operators and for this reason are not orthogonal to one another. In addition, in accordance with Eqs. (3.15) and (3.16), they are a superposition of states with different occupation numbers, though they are characterized by a definite phase. It should be specially noted, however, that the partial motion of a fermion (electron) in the xy plane perpendicular to the magnetic field $\mathbf{H}$ is described with the help of a boson field and all the properties corresponding to it.

4. OSCILLATIONS OF THE ELECTRON DENSITY OF STATES OF A METAL IN A MAGNETIC FIELD

In order to calculate $\rho(\mu)$ (2.3) we employ the complete normalized set of wave functions

$$|\sigma_z,\ p_z;\ \alpha\beta\rangle=L_z^{-1/2}\exp(ip_zz/\hbar)\chi|\alpha\beta\rangle, \tag{4.1}$$

where

$$\hat\sigma_z\chi=\sigma_z\chi,\quad \sigma_z=\pm1, \tag{4.2}$$

L_z is the normalization length, and $\hat\sigma_z$ is the Pauli spin matrix along the z-axis.

Using Eq. (4.1) in taking the trace in Eq. (2.3), we obtain

$$\rho(\mu)=\sum_{p_z,\sigma_z}\int\frac{d^2\alpha}{\pi}\int\frac{d^2\beta}{\pi}\langle\alpha\beta;\ p_z,\sigma_z|\,\delta(\mu-\hat{\mathscr{H}})\,|\sigma_z,\ p_z;\ \alpha\beta\rangle$$
$$=\frac{L_z}{\pi^2(2\pi\hbar)^2}\sum_{\sigma_z}\int_{-\infty}^{\infty}dp_z\int d^2\alpha\int d^2\beta\int_{-\infty}^{\infty}dt\langle\alpha\beta;\ p_z,\sigma_z|$$
$$\times\exp[i(\mu-\hat{\mathscr{H}})t/\hbar]\,|\sigma_z,\ p_z;\ \alpha\beta\rangle, \tag{4.3}$$

where $d^2\beta=d(\mathrm{Re}\ \beta)d(\mathrm{Im}\ \beta)$. Keeping in mind the fact that all three terms appearing in the definition of the operator $\hat{\mathscr{H}}$ (3.1) commute with one another, we perform the calculations successively, using the following relations:

$$\sum_{\sigma_z=\pm1}\exp\left(\frac{it}{\hbar}\frac{g^*}{2}\mu_BH\right)=2\cos\left(\frac{g^*\mu_BH}{2\hbar}t\right), \tag{4.4}$$

$$\frac{1}{\pi}\int d^2\beta=\frac{1}{\pi}\int_0^{L_x}\frac{dx_0}{l_L}\int_0^{L_y}\frac{dy_0}{l_L}=\frac{\Phi}{\Phi_0},\quad \Phi=L_xL_yH,\quad \Phi_0=\frac{ch}{e} \tag{4.5}$$

[see Eq. (3.10)];

$$\int_{-\infty}^{\infty}dp_z\exp\left(-\frac{itp_z^2}{2m\hbar}\right)=\left(\frac{2\pi\hbar m}{|t|}\right)^{1/2}\exp\left(-i\frac{\pi}{4}\,\mathrm{sign}\,t\right), \tag{4.6}$$

$$\langle\alpha|\exp(-it\omega_Ha^+a)\,|\alpha\rangle=\sum_{n=0}^{\infty}\langle\alpha|\exp(-it\omega_Ha^+a)\,|n\rangle\langle n|\alpha\rangle$$
$$=\sum_{n=0}^{\infty}\exp(-it\omega_Hn)\,|\langle n|\alpha\rangle|^2=\exp\{-|\alpha|^2[1-\exp(-it\omega_H)]\}. \tag{4.7}$$

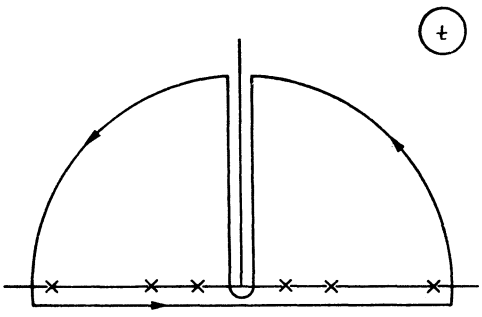

FIG. 1. The contour of integration in the complex t-plane for calculating the integral in Eq. (4.9).

In the calculation of Eq. (4.7) we have used the condition

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = 1, \qquad |n\rangle = \frac{(a^+)^n}{(n!)^{1/2}}|0\rangle$$

expressing the fact that the set of "one-dimensional" Fock states is complete, and the expression for the scalar product of the Fock and coherent states:

$$\langle n|\alpha\rangle = \langle n|\exp(-|\alpha|^2/2)\exp(\alpha a^+)|0\rangle = \frac{\alpha^n}{(n!)^{1/2}}\exp(-|\alpha|^2/2). \tag{4.8}$$

Using the relations (4.4)–(4.7) we obtain for the density of states $\rho(\mu)$ at the Fermi surface an expression in the form of a single integral;

$$\rho(\mu) = \frac{L_z\Phi m^{1/2}}{(2\pi\hbar)^{3/2}\Phi_0}\int_{-\infty}^{\infty} dt \exp\left[i\left(\frac{\mu t}{\hbar} - \frac{\pi}{4}\operatorname{sign} t\right)\right] \times \cos\left(\frac{g^*\mu_B H}{2\hbar}t\right) \Big/ i|t|^{1/2}\sin(t\omega_L), \tag{4.9}$$

which can be calculated with the help of the residue theorem by integrating along the contour shown in Fig. 1.

It is easy to see that the oscillating part of the density of states $\tilde{\rho}(\mu)$ is determined by the contribution to the integral of the poles located on the real axis at the points

$$t_k = \frac{\pi}{\omega_L}k, \qquad k = \pm 1, \pm 2 \ldots \tag{4.10}$$

and has the form

$$\tilde{\rho}(\mu) = \frac{mV}{\pi^2\hbar^2}\left(\frac{eH}{c\hbar}\right)^{1/2}\sum_{k=1}^{\infty} k^{-1/2}\cos\left(\frac{\pi g^* m}{2m_0}k\right) \times \cos\left(2\pi k\frac{\mu}{\hbar\omega_H} - \frac{\pi}{4}\right). \tag{4.11}$$

As one can see from Eq. (4.11), the period of the oscillations of $\tilde{\rho}(\mu)$ with respect to the inverse magnetic field is

$$\Delta(1/H) = \frac{e\hbar}{mc\mu}. \tag{4.12}$$

The expression (4.11) is, naturally, identical to the corresponding expression for $-(\partial^2\Omega_H/\partial\mu^2)_{T=0}$ from Ref. 4.

The extension to the case of current carriers with an arbitrary energy spectrum is made in the spirit of the Onsager-Lifshitz-Kosevich theory[4,5] by making the substitution $\mu \to S_s/2\pi m$, where S_s is the area of the sth extremal section of the Fermi surface by the plane $p_z^{(s)} = \text{const}$, and by introducing an additional summation over the numbers s of the extremal sections. Taking into account $T \neq 0$ does not change the period of the oscillations and is only reflected in the form of the oscillations, in particular, their amplitude.

5. DISCUSSION

Our approach to the analysis of oscillation effects in a metal in a magnetic field—calculation of the density of states $\rho(\varepsilon)$ using the wave functions (4.1), which include the coherent states (3.14)–(3.16), as the basis—makes it possible not only to substantially simplify the mathematical procedure as compared with the traditional method of analyzing these phenomena,[4,5] but it also to study in a universal manner both the thermodynamic and kinetic effects, to easily extend the analysis to the case of current carriers with an arbitrary energy spectrum and nonzero temperature, and to include the effect of scattering on the form of the oscillation dependence.

The physical reason for the simplification achieved in the mathematical procedure is that the coherent states employed in the calculations best describe quantum macroscopic and therefore also quasiclassical phenomena, which the Shubnikov-de Haas and de Haas-van Alphen effects in metals, semimetals, and degenerate semiconductors are.[20,21] On the basis of the indicators indicated these oscillation effects in metals are related with macroscopic quantum phenomena, such as superconductivity, weak-link superconductivity, lasing, von Klitzing's effect, etc.

Extending this analogy and keeping in mind the oscillatory character of the dependence of the observed quantities on H, it is logical to determine the Shubnikov-de Haas and de Haas-van Alphen effects as manifestations of the macroscopic quantum interference. In this case the macroscopic quantum interference has an entire series of specific features, the main one being that the elementary excitations initially responsible for the observed effects are fermions (electrons and holes in the metal). When a magnetic field **H** is turned on, however, the motion of the fermions in a plane perpendicular to **H** is described with the help of elementary excitations of the boson type, for which the possibility of coherence (matching with respect to phase characteristics) and macroscopic quantum interference resulting from it are also typical. The possibility of interference of the contributions to the oscillatory dependences of the observed quantities resulting from different extremal sections of the Fermi surface also contribute their own distinctive characteristics.

[1)] In 1913 Kamerling-Onnes prophetically ascribed superconductivity to quantum phenomena, while in 1950 F. London[15] emphatically introduced into the definition of superconductivity its macroscopic nature also, defining in this manner superconductivity as a "quantum phenomenon on a macroscopic scale." Thus London was the first one to identify the new class of physical phenomena.

[2)] It is interesting to compare this with the fact that the presence of a Fermi surface is also a necessary condition for the existence of the superconducting state, and the density of states at the Fermi surface $\rho(\mu)$ enters directly into the expression for the superconducting phase transition temperature T_c in the Bardeen-Cooper-Schrieffer theory.[4]

[1] L. V. Schubnikov and W. F. de Haas, Proc. Netherlands Roy. Acad. Sci. **33**, 130 (1930).

[2] L. Landau, Z. Phys. **64**, 629 (1930).

[3] W. F. de Haas and P. M. van Alphen, Proc. Netherlands Roy. Acad. Sci. **33**, 1106 (1930).

[4] A. A. Abrikosov, *Fundamentals of the Theory of Metals*, North-Hol-

land, Amsterdam, 1988 [Nauka, Moscow, 1987, p. 520].
[5] D. Shoenberg, *Magnetic Oscillations in Metals*, Cambridge University Press, New York, 1984.
[6] R. J. Glauber, Phys. Rev. Lett. **10**, 84 (1963).
[7] R. J. Glauber in *Quantum Optics and Electronics*, edited by C. DeWitt and A. Blandin, Gordon and Breach, New York, 1965.
[8] I. A. Malkin and V. I. Man'ko, Zh. Eksp. Teor. Fiz. **55**, 1014 (1968) [Sov. Phys. JETP **28**(3), 528 (1969).
[9] I. A. Malkin and V. I. Man'ko, *Dynamic Symmetries and Coherent States of Quantum Systems* [in Russian], Nauka, Moscow, 1979, p. 320.
[10] H. Haken, *Quantenfeld Theorie des Festkorpers*, B. G. Teubner, Stuttgart (1973).
[11] A. Feldman and A. Kahn, Phys. Rev. B **1**, 4584 (1970).
[12] Ya. I. Granovskiĭ and Yu. A. Dimashko, Ukr. Fiz. Zh. **19**, 1456 (1974).
[13] V. V. Dodonov and V. I. Man'ko, Tr. FIAN **183**, 71 (1987).
[14] V. V. Dodonov and V. I. Man'ko, Tr. FIAN **183**, 182 (1987).
[15] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1*, 2nd ed., Pergamon, Oxford, 1980 (Nauka, Moscow, 1964, p. 568).
[16] Yu. B. Rumer, Zh. Eksp. Teor. Fiz. **18**, 1081 (1948).
[17] E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Pergamon, Oxford, 1981, (Nauka, Moscow, 1979, p. 528).
[18] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed., Pergamon, Oxford, 1977 (Nauka, Moscow, 1989, p. 767).
[19] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed., Pergamon, Oxford, 1975 (Nauka, Moscow, 1988, p. 510).
[20] S. T. Pavlov and A. V. Prokhorov, Fiz. Tverd. Tela **32**, 3451 (1990) [Sov. Phys. Solid State **32**(11), 2001 (1990).
[21] S. T. Pavlov and I. S. Pivovarov, Fiz. Tverd. Tela, in press.

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