Electron localization induced in *n*-InSb by a strong magnetic field

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A comprehensive experimental investigation of the conductivity tensor of *n*-InSb single crystals at liquid-helium temperatures was performed in a wide range of carrier densities (from $1.5 \cdot 10^{15}$ to $1.5 \cdot 10^{16}$ cm^{-3}) and magnetic fields up to 72 kOe. It was found that $\sigma_{\parallel} \propto H^{-\alpha}$ and $\sigma_{\perp} \propto H^{\beta}$, and as the carrier density increases α decreases from 3 to 2.3 and β increases from 1 to 2 and higher at the beginning of the region of the ultraquantum limit (UQL). As H increases further β decreases. It is shown that the region of quantum screening (QS) occupies the most significant part of the region of the UQL. Longitudinal and transverse diffusion were studied theoretically for the region of QS on the basis of the single-electron model [previously used for the case of classical screening (CS)], taking into account the quasi-1D character of the electron dynamics and the possibility of Anderson localization of electrons in the random potential of chaotically distributed charged impurities. It is shown that in contrast to the CS region, where $\sigma_{\parallel} \propto H^{-6}$ and $\sigma_{\perp} \propto H^{8/3}$, in the quantum region these dependences should be significantly different, namely, $\sigma_{\parallel} \propto H^{2}$ and $\sigma_{\perp} \propto H^{0}$. In the CS region the theoretical results are not in complete agreement with the experimental behavior, while in the quantum region there is complete disagreement. Possible reasons for the observed discrepancies are discussed.

1. INTRODUCTION

In the last few years there has been a surge of interest in new and more careful experimental and theoretical investigations of the conductivity and mechanisms of localization of current carriers in extremely strong magnetic fields, i.e., stronger than the field of the transition to the ultraquantum limit (UQL). As noted previously in Refs. 1 and 2, the field dependence of the different components of the conductivity tensor $\sigma_{\alpha\beta}(H)$ of an entire series of narrow-gap semiconductors (CdHgTe, InSb, and others) which are relatively strongly doped, sufficiently strongly to obtain semimetallic-type conductivity, are observed to have a number of common characteristics. As the magnetic field **H** increases the Hall component of the conductivity σ_{xy} at first increases in proportion to H up to the field H_1, where σ_{xy} becomes equal in magnitude to the transverse component σ_{xx} which remains virtually constant in these fields. Then, in fields $H > H_1$, the region of classically strong fields starts. Here the cyclotron frequency ω_c, which increases linearly with H, now exceeds the frequency $1/\tau_e$ of electron scattering processes and as H increases further both components σ_{xy} and σ_{xx} decrease (as H^{-1} and H^{-2}, respectively).

As the field H increases still further quantization effects (Shubnikov oscillations) appear, and finally the region of the UQL starts in fields $H > H_2$. Here σ_{xy} continues to decrease as H^{-1}, while σ_{xx}, on the contrary, starts to increase. For sufficiently strong fields σ_{xx} again becomes equal to σ_{xy} and their field dependences can intersect once again. Then, at some critical field H_c the values of all three components of the conductivity tensor, including σ_{zz}, drop sharply; this indicates the presence of strong localization, which is usually attributed to carrier freeze-out on isolated impurity centers or to the formation of large electron drops.[1] To within a logarithmic factor the condition of the metal-to-insulator transition can be represented in the form

$$n\alpha_0 l_{H_c}^2 \sim 1,$$

where n is the electron density, α_0 is the Bohr radius, and $l_h = (\hbar c/eH)^{1/2}$ is the magnetic length, which determines the transverse size of the wave function of a bound impurity state. Collective electronic effects such as Wigner crystallization or charge-density waves can appear in this same region of magnetic fields.[3] Since the fact that the motion of electrons becomes one-dimensional in the UQL gives rise to a tendency toward different forms of localization, it is of interest to investigate the conductivity tensor in the entire region of the UQL, including also in the region close to the critical field H_c, as well as to clarify the possible mechanisms of localization.

The region where the components σ_{xx} and σ_{xy} become equal to one another for the second time or even intersect also lies in the same neighborhood of H_c. In this region, as the theoretical estimates presented below show, the condition $\omega_c \tau_e \gg 1$ continues to hold, while the Fermi energy ε_F, measured from the bottom Landau level, and the collisional broadening $\hbar/\tau_e$ become comparable, indicating that there is an additional form of localization.

In addition, the mechanisms of longitudinal and transverse diffusion in the UQL, as well as the field and temperature dependence of all components of the conductivity tensor, are not completely understood in the entire range of temperatures and magnetic fields. In a direction transverse to **H**, in the UQL the current carriers theoretically become localized on a scale l_H on account of gyration around the magnetic lines of force. Under these conditions electron motion along the magnetic field becomes quasi-one-dimensional, as a result of which Anderson localization, which arises in the single-electron problem of quantum interference for an electron moving in the field of the random potential of chaotically distributed impurities, becomes possible.

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Abrikosov and Ryzhkin[4] and Murzin[5] studied the problem of longitudinal and transverse diffusion under conditions when the lifetime τ_v of the state of Anderson localization, which decays on account of transverse motion, is finite. They obtained the following behavior: $\sigma_\parallel \propto H^{-6}$ (Ref. 4) and $\sigma_\perp \propto H^{8/3}$ (Ref. 5) for the longitudinal and transverse components of the conductivity tensor, respectively. The latter result follows from the relation

$$\sigma_\perp = \frac{(e^4N)^{2/3}}{n\alpha_0 l_H^{4/3}}\sigma_\parallel^{-1/3} \quad (1)$$

derived in Ref. 5.

This entire analysis was performed, however, only for the region of classical screening (CS), i.e., in the region where $\lambda \ll r_s$ holds (here λ is the de Broglie wavelength for the longitudinal motion of electrons and r_s is the screening radius). If H_3 is defined as the field in which a transition occurs into the region of quantum screening (QS), for which $\lambda \gg r_s$ holds, then it is not difficult to show that in a real experimental situation $H_2 \sim H_3$ and the CS region is of insignificant size. In reality, in the region of the UQL, i.e., in fields $H > H_2$, λ increases linearly with H, while

$$r_s = \left(\frac{4\pi e^2}{\chi_0}\frac{\partial n}{\partial \varepsilon_F}\right)^{-1/2} = (\pi^3 n\alpha_0)^{1/2} l_H^2 \propto H^{-1},$$

i.e., r_s decreases quite rapidly (here χ_0 is the static permittivity). Since the field H_2, taking into account spin splitting, is determined by the condition $\varepsilon_F = g\mu_0 H_2$, while the field H_3 is determined by the condition $r_s \sim \lambda = (2\pi^2 n l_{H_3}^2)^{-1}$, for the magnetic lengths corresponding to these conditions we obtain

$$l_{H_2} = [(m^* g)^{1/2}/2\pi^2 n]^{1/3}, \quad 1/l_{H_3} = 2^{1/4}(\pi^7 \alpha_0 n^3)^{1/8},$$

where m^* is the effective mass in units of m_0 and g is the spin-splitting factor. The ratio H_3/H_2 is then found to be equal to $(m^*g)^{1/3} \cdot (n\alpha_0^3\pi^5/4)^{1/12}$.

At the same time it is not difficult to show $H_c/H_2 \sim (n\alpha_0^3)^{1/3}$, and therefore the observed region of the UQL grows with n significantly more rapidly than does the CS region, and for sufficiently large n the QS region should be much larger than the CS region. In practice, however, the chosen electron density n is limited by the experimental possibilities of reaching the region of the UQL. Thus for the four samples we investigated the ratio H_3/H_2 should range from 1.15 to 1.4, while H_c/H_2 ranges from 1.7 to 3. For sample No. 5, which has the highest charge-carrier density $n = 5.6 \cdot 10^{16}$ cm^{-3} among the samples which have been studied, the ratio H_3/H_2 is equal to 1.54 and $H_c/H_2 \sim 4$.

Hence it follows that the previously proposed theoretical model must be extended to the case of quantum screening ($\lambda > r_s$), which, in contrast to the classical case, is significantly anisotropic. We study this case below. In determining the scattering cross sections it is found that here only the long-wavelength harmonics of the Fourier transform of the screened Coulomb potential are important; for these harmonics the transverse screening radius is equal to the Debye radius r_D and the longitudinal screening radius is much greater than r_D and increases in proportion to λ. It is found that to within a logarithmic factor the Anderson localization radius, determined by the mean free path for backscattering, is equal to $n\alpha_0/N$ (here N is the concentration of impurities), i.e., it does not depend on H. Although the lifetime of a localized state $\tau_v = \hbar n/N\varepsilon_0$ (here ε_0 is the energy of a bound state on an impurity center) is found to be identical to that in the region $\lambda \ll r_s$, the field dependences of $\sigma_\parallel$ and $\sigma_\perp$ are substantially different: $\propto H^2$ and $\propto H^0$, respectively. The behavior predicted for the QS region then agrees with the measurements.

2. RESULTS OF MEASUREMENTS

Since, as is evident from the Introduction, the problem of electron localization in the UQL requires a detailed and careful analysis, we made a comprehensive experimental investigation of the conductivity tensor $\sigma_{\alpha\beta}$ in strong quantizing magnetic fields for indium antimonide crystals with electron-type conductivity. This semiconductor compound was chosen for investigation because it has an isotropic energy spectrum which has been studied in detail, and this spectrum can be used as a model spectrum. In order to supplement the published data[6-9] in the best possible manner we investigated four samples, whose lowest and highest carrier densities differ by a factor of 10. Samples with dimensions of $2.5 \times 2.5 \times 18$ mm^3 were cut on an electroerosion machine tool from a single-crystalline ingot and then treated with the standard SR-4 etchant. Data on the samples are presented in Table I. The high mobility of these samples indicates that their compensation is low, and a comparison of the measurements of the Hall coefficient R on different pairs of contact probes together with the sharp quantum oscillations indicates that the samples are highly uniform. Aside from the Hall coefficient, the resistivities in transverse (ρ_{xx}) and longitudinal (ρ_{zz}) magnetic fields were also measured (H is assumed to be oriented parallel to the z axis). From the measurements of the resistivity ρ we calculated and constructed the field dependence of all three independent components of the conductivity tensor $\sigma_{\alpha\beta}(\mathbf{H})$ simultaneously for each

TABLE I.

No.	$n \cdot 10^{-15}$, cm^{-3}	ε_F, meV	$R\sigma$, cm^2/V·s	H_s, kOe		H_3, kOe		H_{c1}, kOe	H_{c2}, kOe
				Theory	Experiment	Theory	Experiment		
1	1,49	3,61	$1,05 \cdot 10^5$	6,9	6,5	8	—	12	23
2	3,3	6,06	$1,2 \cdot 10^5$	11,7	11,3	15	—	25	54
3	7,0	9,9	$1,07 \cdot 10^5$	19,4	18,8	28	33	45	—
4	14,9	15,9	$9,6 \cdot 10^4$	32,2	31	45	48	≈90	—
5*	56	38,4	$8,0 \cdot 10^4$	77	76	116	105	—	—

*The data for sample No. 5 were obtained by analyzing the results of Ref. 23.

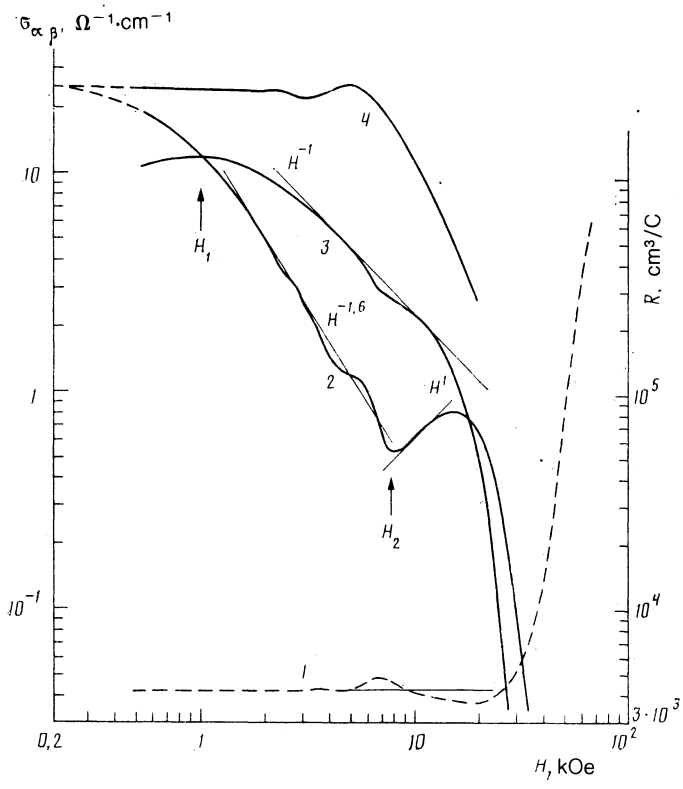

FIG. 1. The conductivities $\sigma_{\alpha\beta}$ and the Hall coefficient R as a function of the magnetic field H for sample No. 1 n-InSb ($n = 1.5\cdot 10^{15}$ cm^{-3}). Curve 1—$R(H)$; 2—$\sigma_{xx}(H)$; 3—$\sigma_{xy}(H)$ with $\mathbf{j}\|x$ and $\mathbf{H}\|z$; 4—$\sigma_{zz}(H)$ with $\mathbf{j}\|z$ and $\mathbf{H}\|z$.

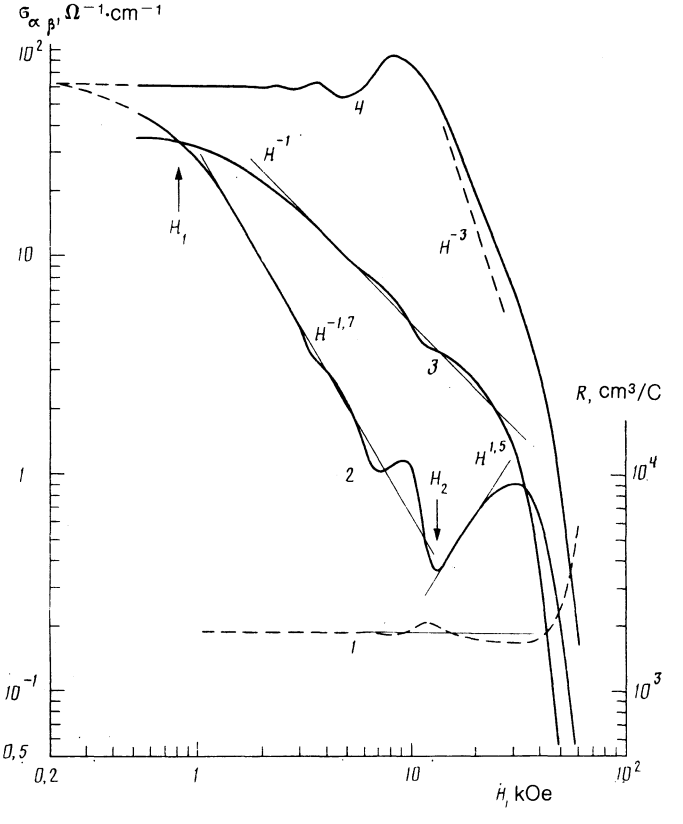

FIG. 2. The conductivities $\sigma_{\alpha\beta}$ and the Hall coefficient R as a function of the magnetic field H for sample No. 2 n-InSb ($n = 3.3\cdot 10^{15}$ cm^{-3}). Curve 1—$R(H)$; 2—$\sigma_{xx}(H)$; 3—$\sigma_{xy}(H)$ with $\mathbf{j}\|x$ and $\mathbf{H}\|z$; 4—$\sigma_{zz}(H)$ with $\mathbf{j}\|z$ and $\mathbf{H}\|z$. The dashed straight line $\sigma_{zz}(H) \propto H^{-3}$ was found by linearly extrapolating the longitudinal conductivity to zero temperature.

sample. These data are presented in Figs. 1–4.

The field H_1 in which a transition occurs to the region of classically strong fields was found to be approximately the same (~ 1 kOe) for all samples. In the subultraquantum region $H < H_2$ characteristic Shubnikov oscillations are observed in the field dependence of the longitudinal and transverse components of the conductivity. These oscillations agree well with the standard theory taking spin splitting into account. In fields corresponding to the passage of the upper of the two spin-split sublevels of the bottom (zero) Landau level through the Fermi level a dip is observed in the plot of the Hall component of the conductivity. This dip corresponds to a peak in the field dependence of the Hall component. In fields $H > H_2$ decreasing behavior $\sigma_{zz}(H) \propto H^{-\alpha}$ and increasing behavior $\sigma_{xx}(H) \propto H^{\beta}$, where α varies from 3 to 2.3 and β from 1 to 2, depending on the carrier density n, are observed on the initial section of the UQL.

Figures 3b and 4b as well as Fig. 5 show the temperature dependence $\sigma_{zz}(T)$ which we measured in the region of liquid-helium temperatures $1.6 < T < 4.2$ K for different magnetic fields. In Fig. 5 $\sigma_{zz}(T)$ for sample No. 2 is compared with the function $\sigma_{zz}(T)$ which we constructed from the data of Ref. 8, where such measurements were performed at lower temperatures. For all samples studied in the region of fields $H_1 < H < H_c$ the $\sigma_{zz}(T)$ dependence is linear right up to $T = 0.4$ K. Below this temperature, according to the data of Ref. 9, a nearly square-root dependence $\sigma_{zz}(T)$ is observed: $\sigma_{zz}(T) = \sigma_{zz}(0) + AT^{1/2}$.

The temperature dependence $\sigma_{xx}(T)$ and $\sigma_{xy}(T)$ at liquid-helium temperatures was found to be weaker and is manifested only close to H_c. The strong difference in the

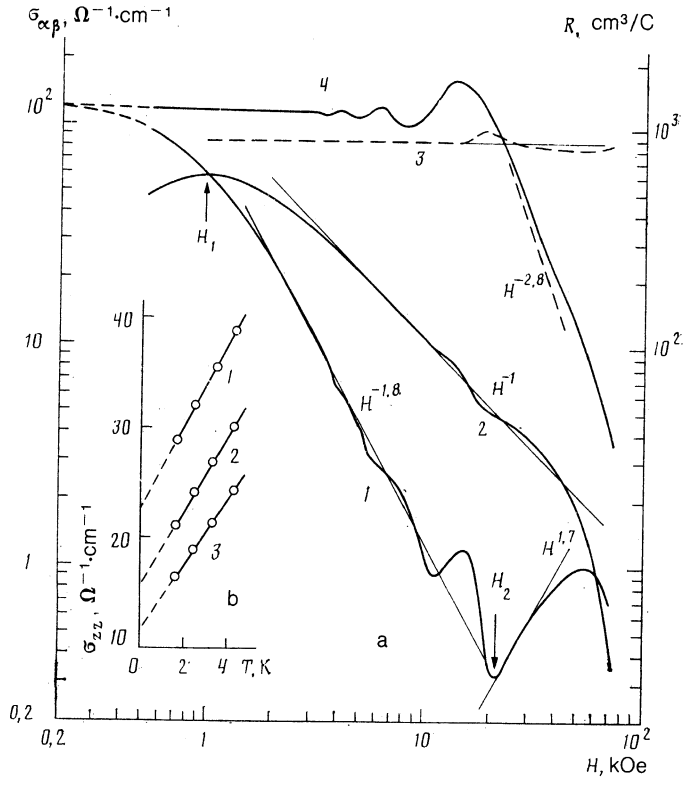

FIG. 3. a) The conductivities $\sigma_{\alpha\beta}$ and the Hall coefficient R as a function of the magnetic field H for sample No. 3 n-InSb ($n = 7\cdot 10^{15}$ cm^{-3}). Curve 1—$\sigma_{xx}(H)$; 2—$\sigma_{xy}(H)$; 3—$R(H)$ with $\mathbf{j}\|x$ and $\mathbf{H}\|z$; 4—$\sigma_{zz}(H)$ with $\mathbf{j}\|z$ and $\mathbf{H}\|z$. The dashed straight line $\sigma_{zz}(H) \propto H^{-2.8}$ was found by linearly extrapolating the longitudinal dependence to zero temperature. b) The temperature dependence of the conductivity $\sigma_{zz}(T)$ for different magnetic fields: 1) 35, 2) 40, 3) 45 kOe.

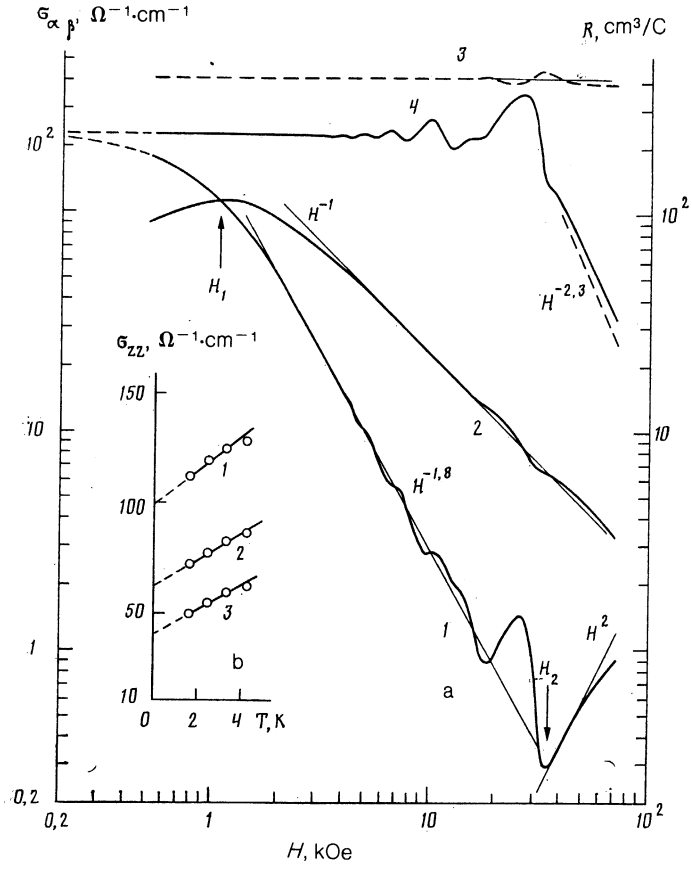

FIG. 4. a) The conductivities $\sigma_{\alpha\beta}$ and the Hall coefficient R as functions of the magnetic field H for sample No. 4 n-InSb ($n = 1.5\cdot 10^{16}$ cm^{-3}). Curve 1—$\sigma_{xx}(h)$; 2—$\sigma_{xy}(H)$; 3—$R(H)$ with $\mathbf{j}\|x$ and $\mathbf{H}\|z$; 4—$\sigma_{zz}(H)$ with $\mathbf{j}\|z$ and $\mathbf{H}\|z$. The dashed straight line $\sigma_{zz}(H) \propto H^{-2.3}$ was found by linearly extrapolating the longitudinal conductivity to zero temperature. b) The temperature dependence of the conductivity $\sigma_{zz}(T)$ for different magnetic fields: 1) 40, 2) 50, and 3) 60 kOe.

character of the temperature dependence of the transverse $\sigma_{xx}(T)$ and Hall $\sigma_{xy}(T)$ conductivities on the one hand and the longitudinal conductivity $\sigma_{zz}(T)$ on the other confirms the existence in the region of fields $H_1 < H < H_c$ of quasi-1D localization phenomenon of the Anderson localization type, gradually vanishing as the temperature increases. It is also interesting that the relation (1) is not satisfied for $\sigma_\perp$ and $\sigma_\parallel$ at liquid-helium temperatures.

A distinct break is observed in $\sigma_\perp(H)$ for the quite strongly doped samples Nos. 3, 4, and 5 in the neighborhood of the theoretically computed field H_3, corresponding to the transition from classical to quantum screening. The fields H_3, determined for these samples from the break point in the experimental field dependence $\sigma_\perp(H)$ and computed theoretically, are presented in Table I. For samples Nos. 1 and 2, however, the computed values of the fields H_2 and H_3 are found to be so close to one another that the regions of classical and quantum screening are not distinctly separated in the experimental curves. The computed fields H_2 and H_3 presented in Table I agree quite well with the experimental data for all strongly doped samples. Since the transition to the UQL is not sharp, the field H_2 was determined as the average of the fields corresponding to the maximum of the last oscillation and the minimum value of the transverse conductivity in the region of the UQL. In the region of classical screening ($H_2 < H < H_3$) the conductivity depends on H significantly more strongly than linearly, and as the carrier density n in-

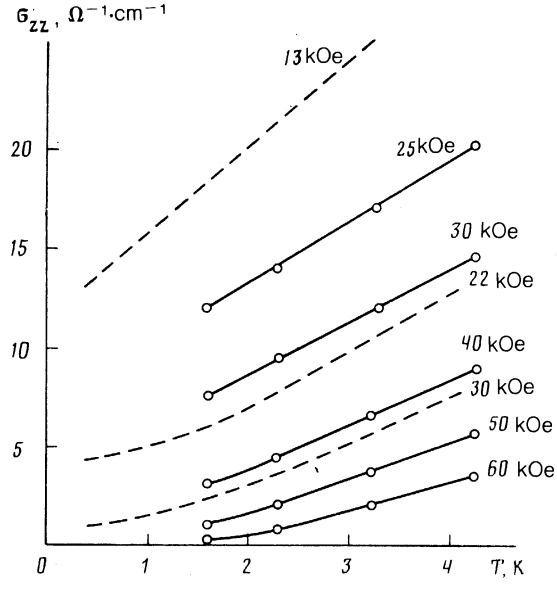

FIG. 5. The temperature dependence $\sigma_{zz}(T)$ for n-InSb with $\mathbf{j}\|z$ and $\mathbf{H}\|z$. The solid lines are our data for sample No. 2 with $n = 3.3\cdot 10^{15}$ cm^{-3} and the dashed lines are the data of Ref. 8 for n-InSb with $n = 2.2\cdot 10^{15}$ cm^{-3}.

creases it approaches the predicted dependence $\sigma \propto H^{8/3}$. The region of quantum screening requires, however, a special theoretical analysis, to which the next section is devoted.

3. ELECTRON DIFFUSION ALONG AND ACROSS A MAGNETIC FIELD IN THE ULTRAQUANTUM LIMIT

In the first theoretical treatments electron diffusion across a magnetic field **H** was studied as a sequence of discrete scattering-induced changes in the position of the center of an orbit. The possibility of Anderson localization and hopping along **H** as well as the correlation of such hops with transverse diffusion were completely deglected. It was also assumed that longitudinal diffusion can be described by the standard kinetic equation. Under these assumptions the longitudinal diffusion coefficient $D_\parallel$ is determined by the backscattering mean free path l_{b0}, i.e., $D \propto l_{b0} v_\varepsilon$, where $v_\varepsilon = \partial\varepsilon/\partial(\hbar p_z)$ is the velocity of an electron with energy ε and momentum $\hbar p_z$ in the direction $\mathbf{H} \| z$. For transverse diffusion Titeica's semiqualitative formula was later proved with the help of the theory of transport in quantum systems.[10,11] In the case of scattering of electrons by impurities the result can be represented in the following form:

$$D_\parallel \sim l_{b0}^2/\tau_{b0}, \tag{2a}$$

$$D_\perp \sim l_H^2/\tau_1, \tag{2b}$$

$$\hbar/\tau_{b0} = \hbar v_\varepsilon/l_{b0} \sim \int |V_{q_x, q_y, -2p_z}|^2 \exp(-q^2 l_H^2/2) \frac{dq_x\, dq_y}{v_\varepsilon}, \tag{3a}$$

$$\hbar/\tau_1 \sim \int \{|V_{q_x, q_y, 0}|^2 + |V_{q_x, q_y, -2p_z}|^2\} (l_H^2 q^2) \times \exp(-q^2 l_H^2/2) \frac{dq_x\, dq_y}{v_\varepsilon}, \tag{3b}$$

where $q^2 = q_x^2 + q_y^2$; $V_{qx,qy,0}$ and $V_{qx,qy,-2p_z}$ are the matrix elements of forward and backward scattering by the random potential generated by chaotically distributed impurities. As follows from Eq. (2b), on the average an electron diffuses across the magnetic field **H** over a distance l_H within a time τ_1.

In the case when the range of the potential of a separate impurity is much shorter than l_H, the potential can be assumed to be short-ranged and can be replaced by a delta function. In this case the matrix elements of $V(q)$ do not depend on q and the exponentials in Eq. (3) cut off the integrands for $q \gtrsim l_H^{-1}$. As a result, here we have $\tau_{b0} \approx \tau_1$. For the long-range Coulomb potential of an impurity the screening radius r_s in the UQL is greater than l_H, and the exponentials in Eq. (3) can be neglected, since the matrix elements cut off both integrals for $q \gtrsim r_s^{-1}$. Introducing the parameter $\gamma = \tau_{b0}/\tau_1$ we obtain

$$\gamma = \{\ln(r_s^2/l_H^2) - \ln[l_H^2(r_s^{-2} + 4p_z^2)]\}(r_s^{-2} + 4p_z^2) l_H^2 \ll 1. \tag{4}$$

It should be noted that $r_s \propto H^{-1}$, i.e., it decreases as H increases much more rapidly than does $l_H \propto H^{-1/2}$. However it is not difficult to show that in the UQL the relation $r_s > l_H$ should be satisfied right up to the critical field H_c at which electrons freeze out on separate impurity centers, corresponding to the condition $n\alpha_0 l_{H_c}^2 \sim 1$.

Thus for a long-range impurity potential an electron can be reflected many times parallel to the magnetic field before being displaced laterally by a distance l_H, and in addition in this case γ characterizes the degree to which the motion of the electron in the magnetic field is one-dimensional.

a) Anderson localization

Abrikosov and Ryzhkin[4] found, using the diagramatic technique for Green's functions, that for a short-range potential (i.e., for $\gamma \approx 1$) the longitudinal diffusion coefficient, as expected, differs from Eq. (2a) only by a numerical factor. For a long-range impurity potential, however, when $\gamma < 1$ holds, the motion of the electron can be considered to be virtually one-dimensional on the time scale $t < \tau_1$, and Anderson localization should arise on account of quantum interference accompanying scattering by the total potential of the impurities,[12] and in addition the radius of such localization is of the order of l_{b0}. In this case, starting from the hypothesis of scale invariance Abrikosov and Ryzhkin proposed that if for $\gamma \approx 1$ the diffusion coefficient $D_\parallel$ is determined by the expression (2a), then for $\gamma \ll 1$, on account of partial Anderson localization, $D_\parallel$ should differ from it by the factor γ, i.e.,

$$D_\parallel \sim \gamma l_{b0}^2/\tau_{b0} = l_{b0}^2/\tau_1.$$

Abrikosov and Ryzhkin, however, failed to note that over short time intervals the transverse displacement of such a quasilocalized state (QLS) is not diffusive, but rather of a drift nature.[13] For this reason the agreement between their result and those obtained more rigorously later must be accidental. This relation can be proved with the help of the expression for the diffusion coefficient of a one-dimensional disordered system, in which for nonzero T and ω the QLS have a finite lifetime τ_v (Ref. 14):

$$D_\parallel(\omega) = \frac{8}{\pi} \zeta(3) l_{b0}^2(\varepsilon) [-i\omega + 1/\tau_v(\varepsilon)] \tag{5}$$

[in the derivation of Eq. (5) the inequalities $\omega\tau_{b0} \ll 1$ and $\tau_{b0} \ll \tau_v$ were taken into account]. This result agrees for small values of ω with the result of Abrikosov and Ryshkin, if in Eq. (5) the time τ_1 for transverse displacement by l_H is substituted for the lifetime of the quasi-1D state τ_v. This substitution requires, however, additional justification. Up to now no such justification has been published, but Polyakov[13] studied a similar problem by using the classical method of the drift approximation, which is valid for $l_H \ll r_s$ (it is precisely under this condition that in the UQL we have $\gamma \ll 1$). In Ref. 13 it is shown that the transverse displacement ρ of an electron in this case occurs not as a result of diffusion, for which $\rho(t) \propto t^{1/2}$ holds, but rather as a result of electric drift in the random total field $E(r)$ of the impurities, i.e., much more rapidly, $\rho \propto t$, with the velocity

$$v_d = [\mathbf{E}(\mathbf{r}), \mathbf{H}]/cH^2, \tag{6}$$

over times t shorter than some time t_0 (c is the velocity of light).

The mean-square transverse displacement in this case is equal to

$$\langle \rho^2(t) \rangle = \int_0^t dt_1 \int_0^t dt_2 \langle\langle v_d(\mathbf{r}(t_1)) v_d(\mathbf{r}(t_2)) \rangle\rangle \tag{7}$$

(here the velocity correlation function is averaged twice:

once over the random potential of the impurities and then over the "spread" of the electron over the localization length l_{b0} with the probability density $P \sim 1/l_{b0}$). It is easy to calculate the correlation function of the random field, generated by randomly distributed screened Coulomb impurities with concentration N:

$$\langle \mathbf{E}(0)\mathbf{E}(\mathbf{r})\rangle = \frac{4\pi e^2 N}{\chi_0^2 r}(1-r/2r_s)\exp(-r/r_s) \qquad (8)$$

(χ_0 is the static permittivity).

After substituting into Eq. (7) the expressions (6) and (8) it is not difficult to derive an equation for $\rho(t)$ similar to that derived in Ref. 5:

$$\langle \rho^2(t)\rangle = t^2 \int_0^{l_{b0}} \frac{dz}{l_{b0}} \langle v_d(0,0,0) v_d(\langle \rho^2(t)\rangle^{1/2}, \langle \rho^2(t)\rangle^{1/2}, z)\rangle$$

$$\approx \frac{Ne^4}{\hbar^2 \chi_0^2 l_{b0}} l_H^4 t^2 \ln[r_s/\langle \rho^2(t)\rangle^{1/2}]. \qquad (9)$$

Equation (9) can be easily solved, neglecting, in a first approximation, the logarithmic factor. The result, to within this factor, can also be obtained by a different method from qualitative physical considerations. Consider a cylindrical region of length l_{b0} and transverse radius r_s. The average number of particles in this region is $\sim N r_s^2 l_{b0}$, and the fluctuation from this value is equal to the square-root of this number. The concentration of excess impurities is $(N r_s^2 l_{b0})^{1/2}/l_{b0} r_s^2$. Noting that only the impurities contained in the volume r_s^3 contribute to the fluctuation of the electric field and each such impurity creates the excess field $e/\chi_0 r_s^2$, we obtain for the rms fluctuation of the electric field

$$\langle E^2\rangle^{1/2} = \frac{e}{\chi_0 r_s^2} \frac{(N r_s^2 l_{b0})^{1/2}}{r_s^2 l_{b0}} r_s^3 = \frac{e}{\chi_0} (N/l_{b0})^{1/2}. \qquad (10)$$

This field, according to Eq. (6), gives rise to drift of a localized wave packet with the velocity

$$v_d = \frac{ec}{H\chi_0}(N/l_{b0})^{1/2} = \frac{e^2 l_H^2}{\chi_0 \hbar}(N/l_{b0})^{1/2}. \qquad (11)$$

As a result the mean displacement of the packet $\overline{\rho(t)} \approx v_d t$ agrees, to within a logarithm, with Eq. (9). As Murzin showed,[5] in order to obtain the result of Abrikosov and Ryshkin[4] for $p_F r_s \gg 1$ it must be assumed that the Anderson-localized state decays for lateral displacement by an amount equal to the wavelength of Fermi electrons $\lambda = p_F^{-1}$, which in the UQL exceeds l_H. Indeed, it then follows from Eq. (9) that the lifetime of a localized state is equal to

$$\tau_v \approx \frac{\chi_0 (l_{b0}/N)^{1/2}}{e^2 p_z l_H^2} \approx \frac{n\hbar}{N\varepsilon_0} \qquad (12)$$

and, to within a logarithmic factor, it is equal to τ_1 [here we have used the facts that $p_z \approx l_H^2 n$ and that Eq. (3) implies $l_{b0} \approx \alpha_0^2 p_z^4/N$ and also that α_0 and $\varepsilon_0 = e^2/\chi_0\alpha_0$ are, respectively, the Bohr radius and energy of the Bohr state of an electron on an impurity center].

b) Delocalization induced by transverse drift of the wave packet

Since the electron motion along **H** is not strictly one-dimensional, localization is possible only in the case when the displacement of an electron transverse to the field over the traveling time between backward reflections is significantly smaller than the transverse size l_H of the wave function. In the case at hand, $\gamma \ll 1$, this condition is satisfied, and we can say that on some time scale and corresponding length scale ρ_1 QLS drifts occur in the direction transverse to **H**, where ρ_1 is determined by the condition of decay of the QLS.

The condition $\rho_1 \sim \lambda$, adopted by Murzin[5] without proof, can be justified as follows. Electron scattering by a Coulomb center in a quantizing magnetic field depends significantly on the impact parameter ρ_0. For $\rho \gg l_H$ an electron passing a Coulomb center drifts around the center with velocity v_d and virtually no change in $\rho = (r^2 - z^2)^{1/2}$, i.e., ρ remains equal to ρ_0. For this reason, the three-dimensional scattering problem reduces here to a one-dimensional scattering problem with the effective potential $\overline{V(r)} = V((\rho^2 + z^2)^{1/2})$ with $\rho = \text{const}$.[15] For the Coulomb potential the width Δz of such a one-dimensional well for $\rho < r_s$ is also equal to ρ. As a qualitative model of this well we can use the potential $V_0 \cosh^{-2}(z/\varkappa)$ with width $\varkappa = \rho$ and $V_0 = e/\chi_0\rho$, for which the scattering problem can be solved exactly. According to Eq. (16), the corresponding reflection coefficient is equal to

$$R(\varkappa) = \frac{\Sigma^2}{\mathrm{sh}^2(\pi p_z \varkappa) + \Sigma^2}, \qquad \Sigma = \cos\left[\frac{\pi}{2}(1 - 8m\varkappa^2 V_0)^{1/2}\right]. \qquad (13)$$

The conditions $N\alpha_0^3 \gg 1$ and $r_s \ll \alpha_0$ imply that in semimetallic semiconductors this well is shallow, since in our case we have $\varkappa \sim \rho$ and as a result of screening only $\rho < r_s$ is important. Therefore we have

$$\frac{1}{\hbar^2} m\rho^2 V_0 = \rho/\alpha_0 \ll 1.$$

In this case $\Sigma = 2\pi\rho/\alpha_0$ holds, and the reflection coefficient satisfies $R(\rho) \propto (p_z\alpha_0)^{-2} \ll 1$ for $\rho < \lambda = p_z^{-1}$ and does not depend on ρ, while for $\rho > \lambda$ it decreases exponentially as ρ increases. For this reason the displacement of the QLS by an amount λ transverse to **H** should lead to a change in the reflection coefficient and, as a consequence, to a change in the conditions of interference and to delocalization. A new QLS arises at a different location, displaced from the previous location by a distance of the order of l_{b0}, so that the process of delocalization and formation of a new QLS can be thought of as hopping of the state over the distance l_{b0}.

It should be noted that the resulting lifetime τ_v does not depend on H, and the delocalization parameter $\gamma = \tau_{b0}/\tau_1 \sim \tau_{b0}/\tau_v \sim (p_z l_H)^2 \propto H^{-3}$ decreases rapidly as H increases. This leads to the following strong power-law field dependence:

$$D_\parallel \sim l_{b0}^2/\tau_v \propto H^{-8}, \qquad \sigma_\parallel = \frac{e^2}{l_H^2 v_e} D_\parallel \propto H^{-6}. \qquad (14)$$

The Einstein relation

$$\sigma_{\alpha\beta} = e^2 D_{\alpha\beta} \rho(\varepsilon_F), \qquad (15)$$

where $\rho(\varepsilon) \sim (l_H^2 V_\varepsilon)^{-1} = m(nl_H^4)^{-1}$ is the density of states at the Fermi level ε_F, was employed in the second expression in Eqs. (14).

Since at $T = 0$ K delocalization occurs with transverse

displacements by an amount λ and since in the case of classical screening studied here $\lambda \ll r_s$ holds, before transverse diffusion is established the QLS can return repeatedly to the same region in space because diffusion is significantly more rapid along **H** than across **H**. As a result of longitudinal diffusion the QLS is "spread out" now not over the quantum localization length l_{b0} but rather in a purely classical manner over the diffusion length $(D_{\parallel} t)^{1/2}$. In this case, as Polyakov showed,[13] the transverse displacement satisfies $\rho(t) \propto t^{3/4}$, and the transverse conductivity, as follows from Refs. 5 and 9, must be related with the longitudinal conductivity by the relation (1). Hence it follows, taking into account Eq. (14), that $\sigma_{\perp} \propto H^{8/3}$. At temperatures $T > 0$ K the conditions under which the relation (1) is valid should be satisfied even better.

c) The case of quantum screening

In the general case for the UQL the dielectric response function, taking into account the spatial dispersion in the random-phase approximation,[17,18] has the following form:

$$\chi(\mathbf{q}) = \chi_0 (1 + k_s^2(\mathbf{q})/q^2), \tag{16}$$

where

$$k_s^2(\mathbf{q}) = \frac{p_F \exp(-q_\perp l_H^2/2)}{r_s^2 |q_z|} \ln \left| \frac{2p_{zF} + q_z}{2p_{zF} - q_z} \right|.$$

If the low-frequency harmonics of this potential are studied by smoothing it over distances $\Delta z > \lambda$ and $\Delta\rho > l_H$, then in this case only $q_\perp < l_H^{-1}$ and $q_z < p_F$ are important. Expanding the logarithm and the exponential in Eq. (16) in powers of the small parameters q_z/p_{zF} and $(q_\perp l_H)$, we obtain the following expression for the Fourier transform of the screened Coulomb potential of an impurity center:

$$V(\mathbf{q}) = \frac{4\pi e}{\chi_0 \{r_s^{-2} + q_\perp^2 + q_z^2 [1 + (2 \cdot 3^{1/2} p_{zF} r_s)^{-2}]\}}. \tag{17}$$

Hence it follows that in the classical region ($p_{zF} r_s \gg 1$) the potential is screened isotropically with radius r_s, while in the quantum case only the transverse screening radius is equal to r_s and the longitudinal screening radius is equal to $\lambda/2 \cdot 3^{1/2}$, i.e., it is significantly greater than r_s. This result agrees with the results of Refs. 19 and 20 and is explained by saying that the free electrons can screen the potential relief along **H** only on a scale greater than the de Broglie wavelength λ, which determines the quantum uncertainty of the position of an electron.

In Ref. 17 Shklovskiĭ and Éfros showed that at $z = 0$ the contribution of large $q_z \sim (P_{zF} r_s)^{1/2} \equiv r^{-1}$ to the Fourier integral, which determines the resulting screened potential, gives exponential decay only at distances $r > r_\perp$, and in addition the radius satisfies $r_\perp \gg r_s$ and does not depend on H. It can be shown, however, that this contribution oscillates as a function of z with period $r_\perp$. It vanishes (it is cancelled) for the potential of a charged string; this potential is screened in the transverse direction once again at r_s and not $r_\perp$. The oscillating parts also do not contribute to electron scattering, since the oscillating parts are not in resonance with the Fermi electrons, for which in our case $\lambda > r_\perp$ holds. As a result the electron forward and backward scattering amplitudes remain the standard Born amplitudes, i.e., they are equal to, respectively,

$$|V(0)|^2 = 16\pi^2 e^4/\chi_0^2 (q_\perp^2 + r_s^{-2})^2,$$
$$|V(2p_{zF})|^2 = \frac{16\pi^2 e^4}{\chi_0^2 [q_\perp^2 + (2p_{zF})^2 + [\ln(8\hbar^{-1}\varepsilon_F \tau_e)]/2r_s^2]^2}. \tag{18}$$

In the second expression the spread of the logarithmic Kohn singularity at $q_z = 2p_F$ on account of the broadening of the electronic states by the amount $\hbar\tau_e^{-1} = \hbar\tau_{f0}^{-1} + \hbar\tau_{b0}^{-1}$ (Ref. 21), where τ_{f0} and τ_{b0} are the relaxation times for forward and backward scattering, is taken into account. Taking this spread into account results in an integral equation for τ_e. This equation can be solved, neglecting, as always, to lowest order, the logarithmic term. Then from Eq. (3a) for $p_{zF} \leqslant r_s^{-1}$ we obtain

$$\tau_{f0} \approx \frac{n\hbar}{N\varepsilon_0} l_H^2 r_s^{-2}, \quad \tau_{b0} \approx \tau_{f0} \ln^2(8\hbar^{-1}\varepsilon_F \tau_e),$$
$$l_{b0} = \tau_{b0} p_{zF} \hbar/m \approx \frac{n}{N} \alpha_0 \ln^2(8\hbar^{-1}\varepsilon_F \tau_e). \tag{19}$$

Thus in the QS region the forward-scattering time is equal to the backward-scattering time, to within a logarithmic factor, while the mean free path for backward scattering or the Anderson-localization radius for weakly compensated semiconductors are equal, in the same approximation, to the Bohr radius. The same result can be obtained by expressing l_{b0} in terms of the backscattering cross section, defined as the integral

$$S_b = \pi \int_0^\infty R(\rho) \rho \, d\rho, \tag{20}$$

where the reflectance $R(\rho)$ can be calculated using Eq. (13). As shown above, $R(\rho) \approx (p_z \alpha_0)^{-2}$ holds in the region $\rho \leqslant r_s$, while for $\rho > r_s$ both the impurity potential and $R(\rho)$ are exponentially small. Hence $S_b \approx r_s^2/(p_{zF}\alpha_0)^2 \approx 1/n\alpha_0$ and $l_{b0} = (NS_b)^{-1}$. Thus the cross section for electron scattering by a screened Coulomb center in the QS region is limited by the transverse screening radius and is independent of the longitudinal screening radius.

d) Fluctuations in the region of quantum screening

It follows from Eq. (19) that under conditions of quantum screening $\tau_{b0} \propto H$, in contrast to the classical region, where $\tau_{b0} \propto H^{-3}$ holds. However before analyzing the consequences of this radical change in the field dependence $\tau_{b0}(H)$, we study the question of the large-scale fluctuations of the potential relief of the bottom of the band under conditions of quantum screening. This question is of interest in itself.

The probability that a definite number of impurities are contained in a prescribed volume is determined by the Poisson distribution function, which near its maximum is close to a Gaussian distribution. According to Ref. 22 the logarithm of the probability for the appearance of an excess in the number of impurities Z in the volume S_0 is equal to

$$\ln W = -\frac{Z^2}{NS_0}. \tag{21}$$

Since quantum screening is anisotopic, we separate an arbitrary cylindrical region with radius r_s and length L and volume $S_0 \approx r_s^2 L$, and find the potential generated by the excess

impurities in this volume. If $L < r_s$ holds, then the excess potential U is equal to

$$U \approx \frac{e^2 Z}{\chi_0 r_s}, \quad Z \approx U\chi_0 r_s/e^2. \tag{22}$$

If, however, we have $L > r_s$, then

$$U \approx \frac{e^2 Z}{\chi_0 L}, \quad Z \approx U\chi_0 L/e^2. \tag{23}$$

Substituting Eqs. (23) and (22) into Eq. (21), we find that the function ln $W(L)$ for $L < r_s$ and constant U is negative and increases as L increases, while for $L > r_s$ it decreases. This means that $W(L)$ has a maximum at $L \approx r_s$ and therefore the most probable fluctuations are isotropic, i.e., they have a three-dimensional scale $\sim r_s$; this corresponds to a fluctuation depth $U \sim e^2(Nr_s)^{1/2}/\chi_0$. In addition to the most likely spherical fluctuations, cylindrical fluctuations of length L are also present. Among these fluctuations the fluctuations with $L \sim \lambda_F$ have the longest wavelength, since all fluctuations with longer wavelengths are effectively screened. The depth of such fluctuations, equal to $e^2(N/L)^{1/2} r_s/\chi_0$, decreases as L increases.

e) Electron scattering by large-scale fluctuations

The reflection coefficient R_0 and the reflection cross section S_b of cylindrical fluctuations can be estimated by constructing, as done above, the model potential $V_0 \cosh^{-2}(z/L)$, where $V_0 = e^2(N/L)^{1/2} r_s/\chi_0$. As long as this potential can be considered to be shallow, it follows from Eqs. (13) and (20) that

$$R_0 \sim Nr_s^2 L/\alpha_0^2 p_{zF}^2, \quad S_b \sim R_0 r_s^2 \sim Nr_s^4 L/\alpha_0^2 p_{zF}^2. \tag{24}$$

If the fact that the density of such fluctuations $\sim r_s^{-2} L^{-1}$ is taken into account, then the mean free path for scattering by them does not depend on L and is equal to Eq. (19), to within a logarithmic factor. The fact that the results of scattering by Coulomb centers are the same as the results of scattering by a smoothed fluctuation potential is explained by the significance of the contribution to scattering made by regions far from the center of the long-range Coulomb potential of the impurity.

The following relations follow from Eqs. (19) and (24):

$$\omega_c \tau_e \sim \alpha_0/N l_H^4 \propto H^2, \quad \varepsilon_F \tau_e = \omega_c \tau_e k_F^2 l_H^2 \propto H^{-1},$$

whence it follows that in the region of quantum screening the product $\omega_c \tau_e$ should increase while the parameter $\varepsilon_F \tau_e$ decreases, in the process, inversely as the magnetic field and can ultimately approach unity when $n^2 \alpha_0 l_H^2/N \sim 1$. According to Ref. 17, this condition is identical to the condition of localization in large-scale fluctuations with $L = \lambda_F$. This agreement is to a certain extent accidental, since near the critical field for localization the scattering potential can no longer be regarded as shallow and the expansion in powers of the small parameter in Eq. (13) cannot be employed. It follows from Eqs. (13), however, that when ε_F becomes comparable to the depth of fluctuations of the localization length $L \sim \lambda_F$, the reflectance becomes close to unity. Then the electron mean-free path becomes equal to λ_F, i.e., the Ioffe-Regel' localization condition $l_{b0} \sim \lambda_F$ is satisfied.

For samples of a weakly compensated semiconductor the critical field H_{c1} for such localization and the field H_{c2} of magnetic freeze-out on separate impurities are close parametrically, i.e., to within a numerical factor. For this reason, even for $N = n$ the fields H_{c1} and H_{c2} may be different. In this case, when $H_{c1} < H < H_{c2}$ holds the electrons are localized in valleys between "humps" extending above the Fermi level. In addition, the localization radius L increases as H increases together with the wavelength $\lambda_F \propto H$, starting from $L \sim \lambda_F \sim n\alpha_0/N$ at $H = H_{c1}$ and up to $L \sim \lambda_F \sim \alpha_0$ at $H = H_{c2}$. On the humps themselves, however, the electrons are localized on separate impurities, as a result of which insulating islands arise in the bulk of a semimetallic sample. The number of free electrons in the semimetallic part decreases correspondingly as the volume of this part decreases, but their density in these parts remains constant. In the process, the conductivity should remain finite at $T = 0$ K until all insulating parts overlap. The problem of calculating the conductivity reduces here to the percolation problem of diffusion of electrons trapped between insulating regions. This problem requires a separate analysis, which falls outside the scope of this paper.

f) Lifetime of a localized state and diffusion coefficients

Thus the transition to quantum screening is accompanied by a sharp change in the dependence $\tau_{b0}(H)$: rapid decay as H^{-3} is replaced by growth as H. In addition, the mean free path no longer varies (for a weakly compensated semiconductor, $l_{b0} \sim \alpha_0$). Since in the QS region we have $r_s \ll \lambda$, the condition of delocalization becomes displacement of the packet by a distance r_s transverse to **H**. Using this condition, together with Eq. (11), it is easy to obtain as a result that τ_v is equal to (12) in this case also and therefore it does not depend on H. Then for the diffusion coefficient and the conductivity along the magnetic field **H** we have

$$D_\parallel \approx \frac{l_{b0}^2}{\tau_v} = \frac{n\alpha_0^2}{N\hbar}\varepsilon_0 = \text{const}, \quad \sigma_\parallel \propto H^2. \tag{25}$$

In the absence of inelastic processes which would decrease τ_v for $T > 0$ K, the relation (15) should not be satisfied. Here

$$D_\perp \approx r_s^2/\tau_v = \frac{\alpha_0 n^2 l_H^4}{N\hbar}\varepsilon_0 \propto H^{-2}, \quad \sigma_\perp = \text{const}. \tag{26}$$

Although, as indicated above, the relation (1) is not satisfied here, the product $\sigma_\perp \sigma_\parallel^{1/2} \propto H^{2/3}$ has same field dependence as in the case of classical screening.

Thus in the region of quantum screening the $\sigma_{\alpha\beta}(H)$ dependence should differ strongly from the analogous dependence in the case of classical screening. In the experimental situation at hand, however, such a sharp transition should be significantly smeared primarily because the backscattering matrix element contains not r_s and not λ, but rather a parameter of the form

$$\xi^{-2} = (2p_{zF})^2 + [\ln(8\hbar^{-1}\varepsilon_F\tau_0)]/2r_s^2, \tag{27}$$

which, for this reason, should actually play the role of a critical transverse length; a displacement by this distance leads to delocalization. If it is assumed that in the transitional region we have $\xi \approx$ const, then we obtain from Eq. (20)

$$S_b \sim \text{const}/(p_{zF}\alpha_0)^2 \propto H^2, \quad l_{b0} \sim H^{-2}. \tag{28}$$

In the process τ_v does not change, and for the diffusion coefficient and the components of the conductivity tensor we have, taking into account Eq. (1),

$$D_{\parallel} \approx \frac{l_{b0}^2}{\tau_v} \propto H^{-4}, \quad \sigma_{\parallel} \propto H^{-2}, \quad \sigma_{\perp} \propto H^{4/3}. \tag{29}$$

Since the field dependence for the transverse conductivity σ_{xx} is increasing or, at least, not decreasing, as follows from Eqs. (26) and (29), there arises the possibility of a second intersection of the components σ_{xx} and σ_{xy} of the conductivity tensor. Although the first intersection indicated that the condition $\omega_c \tau_e \sim 1$ is satisfied, here, as was shown above, the strong-field condition should be satisfied well, in spite of the possible convergence and intersection of the dependences $\sigma_{xx}(H)$ and $\sigma_{xy}(H)$. If Eq. (26) and the Einstein relation (15) are employed, then $\sigma_{xy}/\sigma_{xx} \sim n^2 \alpha_0 l_H^2/N$. Thus the second intersection of the components σ_{xy} and σ_{xx} also occurs in the region of fields H_{c1} of large-scale localization.

4. DISCUSSION

In order to compare experiment with the theoretical field dependences $\sigma_{\alpha\beta}(H)$ for the case of Fermi degeneracy it is necessary to have an estimate of the values of the components of the conductivity tensor at $T = 0$ K. Such an estimate can be obtained by extrapolating the $\sigma_{\alpha\beta}(T)$ dependence to zero temperature. Since the transverse component of the conductivity $\sigma_{\perp}$ at temperatures $1.4 < T < 4.2$ K in the subcritical region of magnetic fields $H < H_c$ is virtually independent of T, the value of $\sigma_{\perp}$ at $T = 1.4$ K can be taken as the value of $\sigma_{\perp}$ extrapolated to $T = 0$ K. In contrast to the transverse component of the conductivity, $\sigma_{\parallel}(T)$ does not saturate right down to the lowest temperatures. The dashed lines in Figs. 2–4 show $\sigma_{zz}(H)$ obtained by linear extrapolation.

The field dependence $\sigma_{zz}(H)$ at $T = 0$ K obtained in this manner for all samples disagrees completely with the dependence predicted by the proposed theoretical model both for the CS region, where the dependence H^{-6} was predicted, and for the quantum region, where the conductivity should increase as H^2. Of course, the entire region of the UQL can be essentially regarded as transitional, since neither the condition $\lambda \ll r_s$ nor the condition $\lambda \gg r_s$ is satisfied sufficiently well. The case of a transitional region was studied above. It follows from Eq. (29) that here the dependences $\sigma_{\perp} \propto H^{4/3}$ and $\sigma_{\parallel} \propto H^{-2}$, which are quite close to the experimental dependences for sample No. 4, which has the highest carrier density among all samples studied, can be observed. However if the field dependence of the components of the conductivity tensor is traced as a function of the electron density n, then a boundary between the regions of classical and quantum screening can indeed be seen well in $\sigma_{xx}(H)$ (starting with quite large values of n). Conversely, such a boundary is not seen in $\sigma_{zz}(H)$ and, moreover, there is no tendency for σ_{zz} to increase. For this reason, it cannot be stated unequivocally that the observed field dependence is explained only by the transitional character of the region under study.

It should also be kept in mind that starting at $T \approx 0.4$ K as T approaches zero the character of $\sigma_{\parallel}(T)$ and $\sigma_{\perp}(T)$ changes sharply to a square-root dependence (Refs. 8 and 9) or even a logarithmic dependence (Ref. 23). In Ref. 23 the appearance of such a logarithmic singularity was explained by invoking quantum corrections to interelectronic scattering.[24] These corrections, however, should vanish in a strong magnetic field, so that the singularities in $\sigma_{\perp}(T)$ and $\sigma_{\parallel}(T)$ at low temperatures more likely indicate the existence of a gap $\Delta\varepsilon \approx 0.005$–$0.01$ K near the Fermi surface. The existence of such a gap can be explained by collective electron effects in the form of charge-density waves.[3,25] Indeed, the forgoing analysis was performed using the Thomas-Fermi model, on the basis of which the many-electron problem reduces to a single-electron problem, and the role of the other electron reduces merely to screening. Quasi-one-dimensionality was assured here by the smallness of the parameter γ, i.e., by the condition that scattering along the z axis in the time interval of displacement transverse to z by an amount l_H is of a multiple-scattering nature.

The overlapping of the local states which arise here, however, will be quite strong, since the localization radius satisfies $l_{b0} \sim N\alpha_0/n \gg r_0 = (3/4\pi n)^{1/2}$, where r_0 is the average distance between the conduction electrons. Under these conditions, as shown in Ref. 25, the electronic system at zero temperature consists of a series of electron chains which are oriented parallel to **H** and which break down at $T > 0$. As T decreases and **H** increases the length of such chains as well as the number of electrons in them should increase. The Hall drift of the chains in the transverse direction is in no way limited as long as $l_H > r_0$ holds, i.e., ordering of chains transverse to **H** does not yet arise. The longitudinal conductivity, however, should decrease owing to the increase in the number of electrons localized in a chain. In the process, the transverse conductivity also decreases because the length of the chains increases and, therefore, the mean random field giving rise to transverse diffusion should decrease in the process.

The region near the critical localization field requires a special investigation. We determined the magnitudes of the critical fields H_{c1}, presented in Table I, based on the point at which the dependence $\sigma_{xy}(H) \propto H^{-1}$ first breaks down and σ_{xy} starts to depend on the temperature. They can all be fit by $H_c(n) \propto n^{0.87}$. It should be noted that the temperature dependence $\sigma_{xy}(T)$ becomes appreciable only for $H > H_{c1}$, and in addition extrapolation of $\sigma_{xy}(T)$ to $T = 0$ gives $\sigma_{xy}(0) = 0$ only starting with the field H_{c2}. This is because in the large-scale fluctuations electron localization should start, as indicated above, in fields H_{c1} less than the field H_{c2} of electron freeze-out on separate impurity centers. Indeed the dependence $\ln(n) \propto H^{-1/3}$, which is characteristic for magnetic freeze-out, is observed only in fields above H_{c2}.

The fields H_{c2} could be observed only for the two samples with the lowest carrier densities, for which, as one can see from the table, the values of H_{c2} are approximately two times higher than H_{c1}. If it is assumed that $H_{c2}(n)$ and $H_{c1}(n)$ are the same, then it can be asserted that they are close to the theoretical estimate $n^{0.86}$, made in Ref. 26 for magnetic freeze-out on isolated impurities. This estimate follows from the condition $n\alpha(H_c) l^2 H_c \sim 1$, where the dependence $\alpha(H) \propto H^{-1/6}$ is taken into account. The field dependence of the longitudinal and transverse mobilities is also interesting. If we assume that the Hall component of the

conductivity in fields $H > H_{c1}$ still reflects the current carrier density $n(H)$, then, dividing the remaining components of the conductivity by $n(H)$, we can obtain the H dependence of the longitudinal and transverse mobilities in this region. The transverse mobility remains increasing right up to H_{c2} and the derivative of the longitudinal mobility changes sign and starts to increase with H also up to H_{c2}, after which these dependences interchange with one another.

Wigner crystallization can also be studied as a possible reason for localization. From the condition of uncoupling of the chains, which leads to Wigner crystallization,[3]

$$l_H \sim r_0 = (3/4\pi n)^{1/3},$$

it follows that in this case we have $H_c(n) \propto n^{2/3}$, and from the condition of transition to the UQL

$$l_{H_2} \sim \lambda_F \sim (2^{3/2}\pi^2 n)^{-1/3}$$

it is found that $H_c/H_2 \approx 4$. This relation is numerically approximately the same as the relation obtained experimentally, but the carrier-density dependence differs somewhat from the experimental dependence. This difference can be explained by the broadening (which is neglected here) of the electronic states at the Fermi level. It can thus be asserted that near H_c interelectronic interaction should play a definite role. This fact also confirms the proposition stated above that the difficulties, which we have discussed, of achieving agreement between the single-electron theory and the experimental data can be explained by noting that collective effects, which were studied qualitatively above, must be taken into account.

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