Effective gluon operators and the dipole moment of the neutron

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The role of the pure gluon part of the *CP*-odd effective operator of dimension six, which appears in various models of *CP* violation, is discussed. This operator is of greatest interest in models with a nonminimal Higgs sector, models with a "right-handed" *W*, and in supersymmetric theories, where it can induce a dipole moment for the neutron even at the experimental bound. Methods for estimating the magnitude of d_n are proposed and arguments are presented to the effect that the original Weinberg estimate, based on "naive dimensional analysis," is a significant overestimate. The effect of the Peccei-Quinn mechanism on the magnitude of d_n, which can in general be quite significant, is discussed.

1. INTRODUCTION

The violation of *CP* invariance in elementary particle interactions is, without a doubt, a fundamental phenomenon which must also play a significant role in the determination of the surrounding world at the present stage of evolution of the Universe. However at the moment positive experimental information about its nature is confined to the two-pion decay of K^0 mesons, which leaves a variety of different possibilities in principle for its theoretical explanation. Indeed, at this time it is not even possible to say what the energy scale is at which the *CP*-odd processes giving the main contribution to ε_K act—it could be the *W*- or *Z*-boson mass scale, or the mass of the corresponding, much heavier, particles, in the range of tens of TeV.

Significant information on the nature of *CP* nonconservation is provided by experiments measuring the electric dipole moment (EDM) of the neutron d_n. The quantity ε_K is determined by processes violating fermion flavor conservation, and in that respect is different in principle from d_n, which is involved in the physics of fermion transitions diagonal in flavor. This difference leads to a broad spectrum of predictions for d_n in a number of models of *CP* violation, even though they are all normalized to give the same experimental value of ε_K. The corresponding estimates encompass the range from the "milliweak" scale $\sim 10^{-25}$ $e\cdot$cm, close to the experimental bound, to "superweak" values of order 10^{-31} $e\cdot$cm. This depends mainly on whether or not the appearance of *CP* violation requires the existence of several generations.

Regardless of the details of the model in a renormalized theory, the effects of highly virtual particles can be taken into account by integration over the corresponding degrees of freedom, which gives rise to local operators of various dimensions. In the simplest cases the *CP*-odd effects are determined by hadronic matrix elements of these operators. Purely dimensional considerations make it clear that each additional dimension of the operator leads to an additional power of the large mass in the denominator of the coefficient in front of the operator in the effective Lagrangian; then the matrix elements acquire a factor characteristic of the strong interaction scale $\mu_{\rm str}$. Consequently, in situations when the reduced degrees of freedom are indeed considerably heavier than ordinary hadrons the main effect is due to the operators of lowest dimension.

The requirement that the effective Lagrangian be hermitian substantially limits the possible form of the *CP*-odd operators. In QCD the lowest operators have dimension 4 and consist of the γ_5-containing quark-mass terms and the operator $(\alpha_S/4\pi)G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$ (the θ term). The former can be eliminated by chiral rotations which change the value of θ. The possible presence of the θ term constitutes the so-called θ puzzle, which has two aspects. In the first place, being an operator of dimension 4 it could already be present in the bare Lagrangian with an arbitrary coefficient. However even if for some symmetry reasons it is absent in pure QCD, the θ term would appear due to radiative corrections in the weak interaction. Depending on the specific model its coefficient could be logarithmically divergent in the ultraviolet region or could be finite. In the case of spontaneous symmetry breaking we certainly have the second possibility, although for explicit breaking of *CP* invariance the radiative corrections to θ could also be finite [for example, in the Kobayashi-Maskawa (KM) model for $g' = 0$]. In either case renormalization of the θ term could give the contribution to d_n, which if not dominant is close to it,[2)] and in many cases it turns out to be unacceptably large. Consequently the first question that needs answering in a broad class of models of *CP* nonconservation is the problem of suppression of the θ term. Since the zero-mass hypothesis for the *u* and *d* quarks is not consistent with the experimental data we assume the existence of the Peccei-Quinn (PQ) mechanism in some form or other.

The one gauge-invariant *CP*-odd operator of dimension 5 containing only light fields is $o_5 = \bar{q}g_S\sigma G\gamma_5 q$; for chiral reasons its coefficient contains the fermion mass so that in reality it can be viewed as an operator of dimension 6.

Even the number of *CP*-odd operators of dimension 6 is rather large. The most popular among them are the four-fermion operators, appearing in the simplest models with Higgs exchange. Estimates of the moment they induce d_n were accurately analyzed in Ref. 3. However, the only pure gluon operator

$$o_6 = (g_S{}^3/16\pi^2) f^{abc} G_{\mu\nu}{}^a G_{\nu\rho}{}^b \tilde{G}_{\rho\mu}{}^c \qquad (1)$$

was not considered in Ref. 3. The present paper is devoted mainly to discussing the effects of this operator—an estimate of its size in certain general models of *CP* nonconservation

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and the related size of d_n. We also discuss the effect of the PQ mechanism on d_n specially. We shall see that indeed the operator o_6 can sometimes make a significant contribution. It is relevant that such effective operators could be the sole trace of the interaction of heavy objects in the energy region of the order of tens of TeV, fully decoupled from the ordinary decays of known particles. In that sense the presence of "new physics" even for energies that are certainly unachievable experimentally in the foreseeable future could quite naturally, without fine tuning, ensure a value of d_n at the precision level of the current bound. Correspondingly further experiments on the search for the EDM of elementary particles and *CP* nonconservation in nuclei could be viewed as real attempts to "peek" into this energy region.

It should be noted that naive estimates of the powers of mass entering the coefficients of these operators are only valid in situations, when only one mass scale of heavy particles exists. In the general case a more detailed analysis is required. Thus, in Ref. 4 for the Weinberg model of *CP* nonconservation in the case of relatively light (~ 10 GeV) H bosons as an example of the effects of nontrivial hadron dynamics at low energies, a contribution was found to d_n, which is nonvanishing in the chiral limit and exceeds many times the standard contributions. In fact the operator discussed there was the operator $G^2 G\tilde{G}$ of dimension 8 with coefficient $c_8 \sim G_F/m_H^2$, although that operator also took into account loops of t quarks. Further, in the case under discussion the coefficient c_6 of the operator o_6 of dimension 6 was additionally suppressed by the factor $\sim m_H^2/m_t^2$ (or, in high loops, by Higgs field self-interaction constants) in comparison with the naive dimensional estimate. For $m_H \gtrsim M_W$ the operator o_6 certainly dominates, but then the Higgs exchanges by themselves cannot ensure the experimental size of ε_K. This case was recently discussed by Weinberg.[5]

The plan of the present paper is as follows. In Sec. 2 we discuss the size of the operator o_6 in models with right-handed currents, Higgs exchanges, and supersymmetric particles to lowest order in the number of loops. In the calculations we use the technique of an external gluon field. The corresponding effect is also analyzed in the KM model. In Sec. 3 we consider QCD renormalization. Section 4 is devoted to an estimate of the matrix element of o_6 determining d_n. We propose an alternate way of estimating it and provide arguments for the assertion that its initial estimate in Ref. 5 is too high by, at least, 1–2 orders of magnitude. In Sec. 5 we discuss the effect of PQ mechanism on the size of d_n. For the operator under discussion it turns out to be suppression, but in general a nonvanishing θ_{eff} can lead to d_n of the same size as the direct contribution. In Sec. 6 we discuss numerical values of d_n, and in the Conclusion we summarize the results of this work.

2. THE OPERATOR $GG\tilde{G}$ IN MODELS OF *CP* NONCONSERVATION

In this Section we obtain expressions for the coefficients c_6 of the *CP*-odd pure gluon operator of dimension six in the effective Lagrangian with QCD corrections ignored. We set

$$L_{odd}=c_6 o_6, \quad o_6=(g_s^3/16\pi^2) f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b \tilde{G}_{\rho\mu}^c, \quad \tilde{G}_{\rho\mu}^c={}^1/_2\varepsilon_{\rho\mu\lambda\tau} G_{\lambda\tau}^c. \tag{2}$$

It is not hard to see that all possible gauge-invariant structures reduce with the use of equations of motion to this operator or to the usual four-quark operators. In principle, having in mind the later application to the electric dipole moment, we should also consider the similar operators containing the electromagnetic fields. However, it can be shown that the only possible combination $F_{\mu\nu}(G^a_{\nu\rho}\tilde{G}^a_{\rho\mu} - G^a_{\mu\rho}\tilde{G}^a_{\rho\nu})$, which happens to have negative C parity, vanishes identically. To see that we write, for example, $F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\tilde{F}_{\alpha\beta}$ and express the product of the two ε tensors in terms of Kronecker symbols. Then this operator should be expressible in terms of $\tilde{F}$ and two G^a, which vanishes due to antisymmetry.

Technically we calculate the coefficient c_6 by considering the fermion loop with the corresponding *CP*-odd interactions in an external gluon field. In the models discussed below the operator o_6 appears in the two-loop approximation, except in the KM model. We briefly discuss the method of calculation, using as an example the simplest model with exchange of charged Higgs particles.

a) Exchange of charged scalar particles

We assume that the spinless fields $H^{\pm}$, which are states of definite mass, interact with both left-handed and right-handed components of quark fields of either charge. In analogy with the simplest scheme we make use of the notation

$$L_H=h_t\bar{t}(1-\gamma_5)bH^+ + h_b\bar{t}(1+\gamma_5)bH^+ + \text{h.c.} \tag{3}$$

The general diagram of Fig. 1 corresponds to the expression

$$L_{eff}=-\mathrm{Tr}\, i^{-2}\int d^4x_1 d^4x_2 \Gamma_1 G_{12}{}^f \Gamma_2 G_{21}{}^f G_{12}{}^H, \tag{4}$$

where $G^{f,H}$ is the Green's function of the quark and scalar particle, respectively, in the external gluon field, the indices 1 and 2 denote the coordinates x_1 and x_2, $\Gamma_{1,2}$ are the vertices arising from the interaction (3), and Tr means summation over Lorentz and color (and flavor) indices. Expanding the quark propagators in powers of the interaction with the gluon field we obtain for L_{eff} a series in powers of local gauge-invariant gluon operators, containing derivatives with respect to the coordinates. These operators are completed to dimension 4 by inverse powers of masses of the internal particles.

It is easily seen that for the interaction of the form (3) *CP* nonconservation appears only when both terms are taken into account; for $h_t = 0$ or $h_b = 0$ an additional symmetry appears, making possible the elimination of the phase of the coupling constant. Therefore the operator o_6 is proportional to $h_t h_b^*$ and corresponds to the case when helicity flip occurs

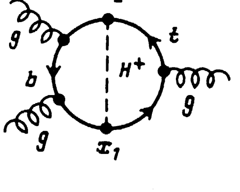

FIG. 1.

in both quark propagators. For this reason the integrand in (4) contains explicitly the product $m_t m_b$. The remaining factor in c_6 is dimensionally a fourth power of the mass in the denominator. In the following we make no distinction in dimensional analysis between the mass scale of the t quark and the H boson and, therefore, have just two scales: m_b and m_t. The naive expectation that the result is determined by the larger mass m_t is, obviously, false for two and more loops. Indeed, the integral over the right-hand loop could be dimensionless and, consequently, the fourth power of the mass could be gotten from the propagator of the b quark, which would yield $1/m_b^4$. However this most singular contribution vanishes, as can be demonstrated from general considerations.

Indeed, let us consider the right-hand loop, containing the t quark, as an operator of the $\bar{b}b$-type in general form. Then only the coefficients of the operators $\bar{b}i\hat{\nabla}(\gamma_5)b$ and $\bar{b}(i\gamma_5)b$ can not contain m_t in the denominator, and requiring it to be hermitian and CP-odd leaves just the operator $\bar{b}i\gamma_5 b$. Operators of higher dimension, $\bar{b}g_S\sigma G\gamma_5 b$ and others that are reduced to it by the equations of motion, already contain $1/m_t^2$. Integration over the b quark in the external gluon field with the "insertion" $\bar{b}i\gamma_5 b$ gives only the operator $G\tilde{G}$, but no higher operators—this is one of the properties of the axial anomaly.[3] Thus, the leading contribution in m_t to the effective action only renormalizes the θ term and does not produce the operator $GG\tilde{G}$.

Direct calculation leads to the answer

$$c_6=-\frac{2}{3}\frac{1}{16\pi^2}\operatorname{Im}(h_b h_t^*)\frac{m_t}{m_b}\frac{1}{m_H^2}\left\{\frac{1}{(1-m_t^2/m_H^2)^3}\ln\frac{m_H^2}{m_t^2}-\frac{3-m_t^2/m_H^2}{2(1-m_t^2/m_H^2)^2}\right\}, \tag{5}$$

where the expression in the braces equals 1/3 for $m_H = m_t$. In the derivation we made use of the following identities:

$$4O_6=\frac{g_S^3}{16\pi^2}\operatorname{Tr}(\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\sigma_{\rho\lambda}[G_{\rho\alpha},G_{\alpha\lambda}])=\frac{1}{2}\frac{g_S^3}{16\pi^2}\operatorname{Tr}\gamma_5(\sigma_{\mu\nu}G_{\mu\nu})^3,\quad G_{\alpha\beta}={}^1/_2\lambda^a G_{\alpha\beta}{}^a, \tag{6}$$

$$\{\nabla^2,G_{\mu\nu}\}=2\nabla_\alpha G_{\mu\nu}\nabla_\alpha+2ig[G_{\mu\alpha},G_{\alpha\nu}]-[\nabla_\mu,J_\nu]+[\nabla_\nu,J_\mu],$$
$$J_\nu=[\nabla_\alpha,G_{\nu\alpha}],$$

$$\operatorname{Tr}\tilde{G}_{\mu\nu}[\nabla_\mu,X_\nu]=\operatorname{Tr}(-[\nabla_\mu,\tilde{G}_{\mu\nu}]X_\nu)+\partial_\mu\operatorname{Tr}(\tilde{G}_{\mu\nu}X_\nu)=\partial_\mu\operatorname{Tr}(\tilde{G}_{\mu\nu}X_\nu),$$

$$\operatorname{Tr}[\nabla_\alpha,Y]=\partial_\alpha\operatorname{Tr}Y,$$

where X and Y are arbitrary functions. Note that the equations of motion imply that the commutator J_μ is equal to the quark current: $[\nabla_\nu,G_{\mu\nu}]_\alpha^\beta=g\bar{q}_\alpha\gamma_\mu q^\beta$. The second of the relations given above is easily obtained by twice applying the Jacobi identity

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$$

(A, B, C are arbitrary operators) to the commutator $[\nabla_\alpha,(\nabla_\alpha,G_{\mu\nu})]$ with the relation $gG_{\alpha\beta}=i[\nabla_\alpha,\nabla_\beta]$ taken into account.

b) Interaction of neutral scalars, off diagonal in generations

In analogy with the previous case we set

$$L_H=h_t\bar{t}(1-\gamma_5)cH^0+h_c\bar{t}(1+\gamma_5)cH^0+\text{h.c.} \tag{7}$$

where again the presence of two chiral structures is needed for CP violation. The answer for c_6 is given by expression (5) with the obvious replacement of b by c.

c) Diagonal interactions of the scalar particles

Here CP violation appears as a result of mixing of scalar and pseudoscalar bosons. Having in mind as an example the Higgs particles of the Standard Model with a nonminimal scalar sector, we consider the Yukawa coupling constants to be biggest for the heavy quarks. We consider therefore the contribution of the t-quark loop and write the corresponding Lagrangian in the form

$$L_H=h_S\bar{t}tH+h_P\bar{t}i\gamma_5 tH. \tag{8}$$

Here the masses of all particles are quantities of the same order and the coefficient c_6 is given by the two-loop integral

$$c_6=-\frac{1}{3}\frac{1}{16\pi^2}h_S h_P\frac{1}{m_H^2}F\left(\frac{m_H^2}{m_t^2}\right), \tag{9}$$

$$F(z)=z\int_0^1 dx\int_0^1 dy\frac{x^3y^3(1-x)}{[x(1-xy)+z(1-x)(1-y)]^2}. \tag{10}$$

An expression similar to (10) was given in Ref. 5. For $z\gg 1$ we have $F(z)\approx\ln z$; for $z\ll 1$, i.e. for $m_H^2\ll m_t^2$, we have $F(z)\approx z/4$. However, in all realistic models this term cancels out in the sum of the contributions from the exchanges of all neutral H bosons, so that in the latter case c_6 is additionally suppressed by the factor $\sim m_H^2/m_t^2$ relative to the naive estimate $\sim 1/m_t^2$. Numerically, we find $F\approx 0.2$ for $m_H^2=m_t^2$.

d) Models with right-handed W bosons

The simplest and most attractive are the so-called "manifestly L–R symmetric" models (see, for example, Ref. 6), where the discrete symmetry between the interactions of left-handed and right-handed fermions is broken spontaneously. These models are also somewhat less arbitrary in the choice of parameters in comparison with the general case. Besides the elements of the KM matrix and a few additional CP-odd phases, determining the form of the right-handed charged current, this scheme contains yet one more CP-odd phase which determines in the general case the complex mixing of W_L and W_R. If this phase is not small then the main effect is due precisely to the CP violation in the W_L–W_R mixing. Starting from this assumption we consider only this last source and set

$$L_{LR}=g_2\bar{t}\gamma_\mu(1-\gamma_5)bW_L{}^\mu+g_2\bar{t}\gamma_\mu(1+\gamma_5)bW_R{}^\mu+\text{h.c.} \tag{11}$$

with the transition propagator of W bosons in the R_ξ gauge given by

$$\langle 0|W_L{}^\mu W_R{}^\nu|0\rangle=\rho e^{i\eta}\frac{g_{\mu\nu}+k_\mu k_\nu(1-\xi)/(\xi k^2-M_W^2)}{k^2-M_W^2},$$
$$\rho=(M_{W_L}^2/M_{W_R}^2)\sin 2\theta, \tag{12}$$

where θ is the mixing angle.

Technically we performed the calculations for arbitrary ξ and then let ξ tend to zero. Indeed, in the R_ξ gauge the mass of the unphysical "Goldstone" state is equal to M^2/ξ, and in accordance with expression (5) the contribution from the exchange by scalar particles dies out when their masses diverge. It is interesting to note that despite the formal finiteness of the integral in the unitary gauge ($\xi = 0$), its literal use gives an incorrect result.

In summary, assuming $m_t^2 \ll M_{W_R}^2$, we obtain

$$c_6=-\frac{1}{24}\frac{1}{16\pi^2}g_2^2\rho(\sin\eta)\frac{m_t}{m_b}\frac{1}{M_W^2}K\left(\frac{m_t^2}{M_W^2}\right), \qquad (13)$$

$$K=1+3\frac{(1+m_t^2/M_W^2)}{(1-m_t^2/M_W^2)^2}-6\frac{m_t^2/M_W^2}{(m_t^2/M_W^2-1)^3}\ln\frac{m_t^2}{M_W^2}$$
$$=\begin{cases} 2, & m_t=M_W \\ 1+3M_W^2/m_t^2, & m_t\gg M_W \end{cases}. \qquad (14)$$

e) Minimal supersymmetric model (super-KM)

Here we discuss the minimal supersymmetric generalization of the Standard Model (SM) in the low-energy form that is obtained when supersymmetry is violated due to effects of supergravity. In that model all the interactions of the superparticles that are off-diagonal in generations are determined in the end by the standard KM matrix and are subject to the same chiral selection rules as in the SM. As a result, the general considerations demonstrating the inescapable smallness of the effects being considered in the SM (see below, and also Refs. 1, 7 and 8) remain largely in force, which ensures an extremely small value for c_6.

On the other hand, a new independent source of *CP* violation exists in a realistically broken supersymmetric model—the relative phase of the mass of the gauginos and the parameter A, determine the size of the trilinear terms in the superpotential in the Lagrangian of the broken theory (see, for example, the review in Ref. 9). The effect of these phases was discussed in detail in Ref. 10. The main contribution comes here from diagrams involving the gluino, and its form is simplest for $m_q, \Delta M_{\tilde Q} \ll \tilde M$ (where $\tilde M$ are the masses of the superparticles):

$$c_6=\frac{\alpha_S}{8\pi m_{\tilde g}^2}\,\mathrm{Im}\left(\frac{A}{\gamma_3}\right)f'(x),\quad x=\frac{M_{\tilde Q}^2}{m_{\tilde g}^2},$$
$$f'(x)=\frac{8}{9}-\left[\frac{1}{2(1-x)^2}+\frac{3}{(1-x)^3}-\frac{2\ln x}{(1-x)^3}+\frac{3\ln x}{(1-x)^4}\right]$$
$$+5\left[\frac{2}{(1-x)^2}-\frac{\ln x}{(1-x)^2}+\frac{2\ln x}{(1-x)^3}\right]. \qquad (15)$$

Here γ_3 is a dimensionless factor determining the mass of the gluino $m_{\tilde g}: m_{\tilde g} = \gamma_3 m_{3/2}$. In (15) values of α_S enter for momenta $\sim \tilde M^2$.

f) Standard KM model

The operator o_6 appears in the SM, obviously, only in the three-loop approximation, when two W-exchanges can take place. Then the value of c_6 and, consequently, of d_n will be exceptionally small independently of the details of the calculation and estimates of the matrix elements. We nevertheless discuss this case and show the presence of additional suppression in comparison with the naive expectations.

First of all it should be noted that *CP* nonconservation appears in the SM only in situations involving all six quarks, in particular the u and d quarks. Therefore, strictly speaking, it is not possible to integrate over the quark degrees of freedom and obtain the local o_6 operator, in any event in taking the strong interaction into account; indeed one can only determine the contribution to o_6 from distances small in comparison with the radius of the strong interactions. To overcome this difficulty one can formally consider the same six-quark system in which, however, the quarks of the first generation are also relatively heavy. In fact this is precisely how the short-range contribution can be distinguished in practice. The corresponding consequences for the real world will be discussed only later.

We reconstruct the o_6 operator from the three-gluon vertex. Let us consider the three-loop diagram (Fig. 2a) in the local limit, i.e., for $M_W \to \infty$ and for fixed G_F. After a Fierz transformation we arrive at three independent quark loops (Fig. 2b), where at the vertices stand either vector or axial currents. If the extreme (right or left) loop contains altogether one external gluon then the only nonvanishing

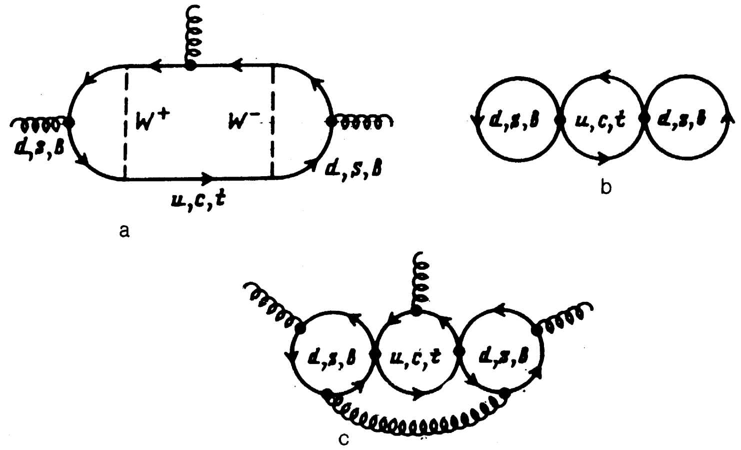

FIG. 2.

contribution comes from the vector vertex, reducing to the usual polarization operator of the vector current. Since the latter is proportional to q^2 and the three-gluon vertex induced by the o_6 operator is linear in momenta, such diagrams do not contribute to c_6. Consequently, at least one of the extreme loops should be a vacuum loop with one Lorentz index, which vanishes in the local limit.[4)] Similar considerations show that c_6 vanishes also in the case when only one of the weak vertices is considered to be local. Indeed, either two or all three gluons should come from the "local" extreme loop. In the latter case the two remaining loops already give a vacuum object with a Lorentz index. If on the other hand one gluon comes from them, then they give a certain transition correlator of the vector gluon current with a certain vector or axial current. This correlator is unavoidably quadratic in the momenta due to the condition of transversality on the gluon.[5)]

In view of the strong suppression of the three-loop contribution the main contribution to c_6 evidently comes from four-loop diagrams of the form shown in Fig. 2c with additional gluon exchange, just as in the case of the θ term.[1] An estimate of the corresponding contribution gives

$$c_6 \sim \sin\theta_c |V_{ub}|\ |V_{cb}| \sin\delta \frac{\alpha_S}{3\pi} \frac{G_F^2}{(16\pi^2)^2} m_c^2 \ln^3 \frac{m_t^2}{m_j^2}. \tag{16}$$

It is easily seen that the main contribution to (16) comes from momenta of the order of the mass of the heavy quarks. The sole exception is ln (m_s^2/m_d^2), which in fact is determined by the physics at large distances. In the real world with a relatively light s quark the contribution of the corresponding processes, for example to d_n, should be determined by operators that explicitly contain the fields of the d and s quarks. On the other hand a reasonable estimate of the size of c_6, determined by short distances, can be obtained by setting the $\ln(m_s^2/m_d^2)$ equal to unity or even to $m_s^2/\mu_{\rm str}^2$ in (16), where $\mu_{\rm str}$ is a characteristic hadronic mass scale.

3. RADIATIVE CORRECTIONS TO THE o_6 OPERATOR

The next step after the calculation of the coefficient c_6 in a specific model is the estimate of its matrix elements between hadronic states entering the process of interest. Since the exact calculation of the matrix elements is with very rare exceptions impossible, one must unavoidably make use of some model of the strong interactions or some other approximation methods. In all these cases the field objects are presumed normalized at the characteristic scale of the strong interactions $\mu^2 \sim 1$ GeV^2. Thus, the usual factorization is certainly violated by virtual gluons and refers only to operators renormalized at that scale. Consequently, it is necessary for practical purposes to take into consideration the modification of the o_6 operator by gluons and quarks at short distances.

For virtuality in excess of m_t^2 and m_H^2 the corresponding processes renormalize only the coupling constants determine the CP-odd interactions producing o_6. Taking them into account reduces simply to the use of the renormalized values.

To discuss the region of smaller momenta we consider first the simplest case when o_6 is determined by the t-quark loop (neutral Higgs exchange). In that case the relations given in the preceding Section fix the value of c_6 at $q^2 \sim m_t^2$. In the evolution to the low-energy region the o_6 operator could, in the first place, produce operators of lower dimension, in particular $m_q \bar{q} g_S \sigma G \gamma_5 q$. For light quarks, however, this effect is small. For heavy quarks, on the momentum scale of order m_q, when the fields of the q quarks are eliminated these operators generate the o_6 operator, as was discussed above. But the corresponding correction is proportional to $\alpha_S(m_q^2)$ and does not contain the logarithm, i.e., it vanishes in the approximation under discussion. Secondly, *a priori* o_6 could mix with the usual four-quark operators of dimension 6, but it can be shown that such mixing is absent. Indeed, the corresponding quark operator should be P-odd but C-even; the only such operators are $\sigma_{\mu\nu} \times \sigma_{\mu\nu}\gamma_5$ (with or without color λ matrices). However, as a consequence of the chiral-invariant form of the gluon-fermion interaction vertices such operators cannot appear without explicit factors in the form of quark masses. Thus in this case the evolution of c_6 is determined only by the anomalous dimension of the o_6 operator itself, which is calculated in Ref. 11 and turns out to be equal to -18:

$$c_6(\mu^2) = c_6(q^2) \left[\frac{\alpha_S(\mu^2)}{\alpha_S(q^2)}\right]^{-18/b},$$

$$b = \frac{11}{3} N_c - \frac{2}{3} n_f. \tag{17}$$

Numerically the corresponding renormalization factor $\varkappa_t$ turns out to be ≈ 0.1 for $\mu^2 \sim 1$ GeV^2.

A more detailed discussion is needed if the quarks that propagate in the fermion loop are t and b quarks. As was seen in Sec. 2 the effective CP-odd interaction is determined in the region $m_b^2 \ll q^2 \ll m_t^2$ by the operator $m_b \bar{b} g_S \sigma G \gamma_5 b$. Therefore the renormalization is governed here by the anomalous dimension of this operator, equal to $-14/3$:[12]

$$c_6(\mu^2) = c_6(q^2) \left[\frac{\alpha_S(\mu^2)}{\alpha_S(m_b^2)}\right]^{-18/b} \left[\frac{\alpha_S(m_b^2)}{\alpha_S(q^2)}\right]^{-14/3b} \tag{18}$$

or, numerically, $\varkappa_b \sim 0.3$.

4. ESTIMATE OF THE SIZE OF THE ELECTRIC DIPOLE MOMENT OF THE NEUTRON INDUCED BY THE o_6 OPERATOR

Keeping in mind the definition of the electric dipole moment we have to calculate the nucleon matrix element

$$\langle N | i \int d^4x T\{o_-(0) J_\mu^{em}(x)\} | N\rangle = d_n \bar{\psi}_N i \sigma_{\mu\nu} q_\nu \gamma_5 \psi_N, \tag{19}$$

where o_- is the CP-odd operator under consideration. In view of the obvious impossibility of calculating it exactly we formulate a prescription which uses simple dimensional considerations and permits estimating the order of magnitude of d_n.

In accordance with (19) in the general case d_n is given by the sum of the three diagrams of Fig. 3, where the one-nucleon-reducible diagrams a and b corresponding to a nucleon in the intermediate state are explicitly separated. Although for the P-odd operator o_- such a separation of the "pole" diagrams is conditional, it is convenient for the analysis.[6)] The sum of the diagrams a and b is easily calculated. Indeed, if we define

$$\langle N | o_-(0) | N\rangle = m_- \bar{\psi}_N i \gamma_5 \psi_N, \tag{20}$$

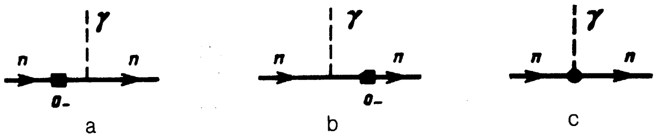

FIG. 3.

then their contribution is equal to

$$d_N|_{1Nred}=\mu_N m_-/M_N, \tag{21}$$

where μ_N is the anomalous magnetic moment of the nucleon.

In the general case the "one-nucleon-irreducible" contribution of the diagram c consists of two parts. First, it contains contributions of all low-lying levels of hadronic states except the one-nucleon state. This part is characterized as before by the typical scale of the strong interactions and should be close in size to the contribution (21). In addition we must take into account the contribution from short distances to the T-product of the operators in (19). This contribution, however, should be expressible in terms of operators of dimension no higher than six. It was shown in Sec. 2 that no such operators exist not containing quark fields. The only operators are the dipole moments of the light quarks, but their contribution to d_n, being proportional to current masses, is small and can be omitted.

Thus we can expect that the contribution of the CP-odd vertex of Fig. 3c has no particular enhancement in comparison with the size of (21). Since our aim is an order-of-magnitude estimate of d_n, it is natural to simply ignore this contribution and set

$$|d_N|\sim\mu_N m_-/M_N. \tag{22}$$

Actually it makes sense to view (22) as an upper bound. Experiment shows that the "irreducible" part turns out to be, as a rule, of opposite sign and has a tendency to partially cancel the one-particle-reducible contributions. More precisely it is easy to show that this cancellation indeed takes place to a large degree in the case when the CP-odd operator o_- is $G\tilde{G}$. In that specific case the irreducible vertex exactly cancels the one-nucleon contribution to leading order in the mass of the light quarks, so that the total moment d_n vanishes in the chiral limit. However, for the operator $G^2\tilde{G}$ of interest to us there is no reason to expect either the vanishing of d_n in the chiral limit or strong cancellation between the two contributions.

Thus we have to determine $\langle N|o_6|N\rangle$. Here we shall again rely on very simple dimensional considerations and write

$$\langle N|(g_s^3/16\pi^2)f^{abc}G_{\mu\nu}^aG_{\nu\rho}^b\tilde{G}_{\rho\mu}^c|N\rangle$$
$$=\mu_{(N)}\langle N|(\alpha_s/4\pi)G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a|N\rangle, \tag{23a}$$

$$\langle 0|(g_s^3/16\pi^2)f^{abc}G_{\mu\nu}^aG_{\nu\rho}^bG_{\rho\mu}^c|0\rangle=\mu_{(0)}^2\langle 0|(\alpha_s/4\pi)G_{\mu\nu}^aG_{\mu\nu}^a|0\rangle, \tag{23b}$$

and then set the two mass scales introduced in this fashion equal to each other:

$$\mu_{(N)}=\mu_{(0)}=\mu_h^2.$$

Consequently we are in fact setting

$$\langle N|(g_s^3/16\pi^2)f^{abc}G_{\mu\nu}^aG_{\nu\rho}^b\tilde{G}_{\rho\mu}^c|N\rangle=\langle N|(\alpha_s/4\pi)G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a|N\rangle\mu_h^2$$
$$=\langle N|(\alpha_s/4\pi)G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a|N\rangle\frac{\langle 0|(g_s^3/16\pi^2)f^{abc}G_{\mu\nu}^aG_{\nu\rho}^bG_{\rho\mu}^c|0\rangle}{\langle 0|(\alpha_s/4\pi)G_{\mu\nu}^aG_{\mu\nu}^a|0\rangle}. \tag{24}$$

Relying on the estimates of the vacuum expectation values obtained in the framework of QCD sum rules in Refs. 13 and 14

$$\langle 0|(\alpha_s/4\pi)G_{\mu\nu}^aG_{\mu\nu}^a|0\rangle\approx 3\cdot 10^{-3}\,\text{GeV}^4,$$
$$\langle 0|(g_s^3/16\pi^2)f^{abc}G_{\mu\nu}^aG_{\nu\rho}^bG_{\rho\mu}^c|0\rangle\approx 4\cdot 10^{-4}\,\text{GeV}^6, \tag{25}$$

we arrive at the result

$$\mu_h\approx 0{,}36\ \text{GeV}. \tag{26}$$

We note that for a particular choice of the operators entering (24), the relation (24) does not depend on the normalization point (in any case, in one loop). In the general case such an approach should be applied to operators normalized at $q^2\sim 1$ GeV^2, since it is precisely in that case, in accordance with the experience with QCD sum rules, that the standard assumptions such as factorization are applicable. We emphasize that the second numerical value in (25) corresponds to this normalization point.

Lastly, the nucleon matrix element of the operator $G\tilde{G}$ is easily calculated in the chiral limit. Indeed, for vanishing quark masses,

$$\langle N|(\alpha_s/4\pi)G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a|N\rangle=\langle N|\partial_\mu(J_{\mu5}^0$$
$$+{}^1/_3J_{\mu5}^8)|N\rangle=2m_N[g_A^0+{}^1/_2(F-D/3)]\bar{\psi}_N i\gamma_5\psi_N, \tag{27}$$
$$J_{\mu5}^0+{}^1/_3J_{\mu5}^8={}^1/_2(\bar{u}\gamma_\mu\gamma_5u+\bar{d}\gamma_\mu\gamma_5d),$$

where gA^0 is the $SU(3)$ analog of the usual nucleon axial coupling constant. Collecting all the numerical factors we obtain

$$|d_N|\sim 5{,}1\cdot 10^{-15}[g_A^0+(F-D/3)/2][c_6|_{1\,\text{GeV}^2}$$
$$(1\ \text{GeV})^2]e\cdot\text{cm}=1{,}8\cdot 10^{-22}[g_A^0+(F-D/3)/2]d_6e\cdot\text{cm}, \tag{28}$$

where for convenience we have introduced the dimensionless coefficient d_6:

$$\frac{2^{1/2}G_F}{48\pi^2}d_6=c_6|_{1\,\text{GeV}^2}. \tag{29}$$

What value should be used for gA^0? A few years ago in QCD sum rules for baryons the estimate $gA^0\approx 0.5$ was obtained.[15] However, recent data for deep inelastic scattering on a polarized target suggest that $gA^0=0.1\pm 0.2$. This result can be interpreted in two ways. It is possible that such a small value is the result of accidental cancellation of different contributions to gA^0. In that case, keeping in mind that we are interested in the operator $G^2\tilde{G}$ and not $G\tilde{G}$, it is natural to use for $gA^0+(F-D/3)/2$ a numerical value ~ 0.5. On the other hand, the result $gA^0\ll 1$ can be interpreted as suppression for some reason of the gluon-nucleon interaction in the pseudoscalar channel. In that case similar suppression is to be expected for the operator $G^2\tilde{G}$ as well and for the factor under consideration we should use the literal value 0.25.

For definiteness we use in the following the value 0.5, keeping in mind that this might again result in an overestimate for d_n. Thus we arrive finally at the following result:

$$|d_N| \sim 9 \cdot 10^{-23} d_6 \; e \cdot \text{cm}. \quad (30)$$

In Ref. 5 d_n was estimated using the method of Ref. 16 called by the authors "naive dimensional analysis" (NDA). It gave an estimate approximately 300 times larger than (30).[7)] We consider it necessary to briefly discuss the reasons for such a large discrepancy.

1. In the definition of the operator in Ref. 16 a factor of 1/6 is explicitly introduced, so that aside from the obvious factor $g_S^3/16\pi^2$ the two definitions of o_6 differ also by the factor 1/3. In the NDA method the literal estimate of the matrix element of an arbitrary operator does not depend on the explicit form of the coefficient, included *ad hoc* in its definition; at the same time a redefinition of this factor obviously modifies the coefficient with which the operator enters the effective Lagrangian automatically. This leads to an increase of the estimate of Ref. 5 by precisely a factor of three. The factor 1/3!, introduced by Weinberg, corresponds to the standard combinatoric factor, but within the framework of this logic the suppression of the matrix element should be due to the absence of factored contributions. In our approach the ambiguity described above is fully absent.

2. The characteristic hadronic mass scale μ_h is used in both approaches and in the final analysis the dependence on it in the NDA approach turns out to be $\propto \mu_h$, while in our method it is $\propto \mu_h^2/M_N$. The size of μ_h in NDA is determined by the expansion parameter $4\pi F_\pi$ of the chiral Lagrangian and amounts to about 1.2 GeV, while in our approach the corresponding mass turns out to be 0.36 GeV. This leads to a difference of approximately a factor of ten. It should be emphasized that the quantity $4\pi F_\pi$ can hardly be used as a universal strong interaction scale, least of all for the description of gluon operators.

3. In the NDA approach the result is proportional to the cube of the QCD coupling constant g_S, which enters on the momentum scale of typical hadronic processes. In Ref. 5 its value was determined from the condition for equality of the one-loop and two-loop contributions to the QCD β-function, and the corresponding value of α_S turned out to be ≈ 2. In our approach this problem does not arise and, as was made clear above, the renormalization should be achieved on a scale ~ 1 GeV ($\alpha_S \approx 0.25$). This difference gives, apparently, a factor ~ 7.

On the whole we believe that it is difficult to expect correct estimates the NDA method in nontrivial cases, if one takes into account the experience accumulated in QCD analysis, both by sum rules and by other methods, based on the inclusion of nonperturbative phenomena in the instanton vacuum. Although the concept of reduced coefficients introduced in NDA is self-consistent in perturbation theory, it is hard to believe that the NDA prescriptions can describe correctly nonperturbative phenomena.

Without a doubt, the principal property of QCD is gauge invariance. At the same time it is easily seen that for operators containing a covariant derivative NDA gives for the two terms in the ∇ operator reduced coefficients differing by 4π. Consequently, from the point of view of NDA, for the effects of the two terms in the covariant derivative to be approximately equal it is necessary that the quantity α_S be of order 4π! (Strictly speaking, both that number and the final answer in NDA depend, e.g., on the representation used for the gauge field.) At the same time in all realistic models of QCD the "running" constant α_S is in the region of nonperturbative physics "frozen" by the nonperturbative fluctuations at the relatively small value $\alpha_S \lesssim 0.5$–1. This stabilization occurs precisely because of the nonperturbative fields and not the equality of contributions of higher loops in perturbation theory, as is supposed in NDA; these nonperturbative fields have the characteristic scale $\sim g_S^{-1}$, i.e., it is precisely the quantity $g_S A$ that is determined by the typical hadronic mass scale. Further it turns out, as a rule, that the fermions have no strong influence on the structure of the nonperturbative fields, and the pion loops, described by the chiral Lagrangian, give altogether a small contribution (we recall that the interaction terms in the chiral Lagrangian are made dimensionless by powers of the factor F_π^2, proportional to N_c in the limit of a large number of colors). For this reason the identification of the nonperturbative QCD mass scale with the parameter $4\pi F_\pi$ of chiral perturbation theory, which grows with the number of colors, is unjustified.

The method for estimating d_n described in this work was developed to overcome the shortcomings of the NDA approach indicated above and is more or less free of most of the ambiguities. Nevertheless it is of interest to compare its predictions with the results of independent calculations by other methods. To this end one can carry out similar estimates for, say, operators of the form

$$o_5 = \bar{q} g_S \sigma G \gamma_5 q, \quad L_- = c_5 o_5,$$

which were studied in Ref. 3 by sum-rule methods. Here it is not possible to literally repeat the discussion outlined above for the o_6 operators. Indeed, in the general case the matrix element $\langle N | \bar{q} g_S \sigma G \gamma_5 q | N \rangle$ is singular in the chiral limit, having a contribution $\propto m_q^{-1}$ due to the pion pole. However, this singularity disappears from d_n due to the cancellation with a similar contribution in the irreducible diagram (Fig. 3c). This fact can be verified with the help of current algebra (more precisely, the Ward identities for axial currents). This difficulty can be eliminated by, for example, considering the combination of operators $\bar{q} g_S \sigma G \gamma_5 q$ that is isosinglet in the light quarks or by simply applying the dimensional arguments to the matrix elements from which the pole terms were extracted. Afterwards the dimensional estimate of the nucleon matrix element can be carried out similarly to the prescription (24), using as the "normalizing" operator literally that same operator $G\tilde{G}$, or, alternately, $\bar{q} i \gamma_5 q$. In the second approach, however, the unknown nucleon matrix element of the isoscalar pseudoscalar density appears, so that for numerical answers we use only the first version. Using the value

$$\langle 0 | \bar{q} i g_S \sigma G q | 0 \rangle \approx -10^{-2} \text{ GeV}^5 \quad (31)$$

(see, for example, Ref. 15) we arrive literally at the result

$$d_n \approx 3{,}3 (c_5^u + c_5^d) \quad (d_n \approx 0{,}11 (c_5^u - 2c_5^d) - [3]). \quad (32)$$

which approximately overestimates that of Ref. 3 by an order of magnitude.

We are inclined to interpret this disagreement as an indication that the diagram considered in Ref. 3 does not give the main contribution to d_n.

Indeed, to estimate d_n produced by the CP-odd o_5 operator we can calculate the most singular contribution in the chiral limit, $\propto \ln m_\pi^2$, as was done in Ref. 17 for the θ term. It is determined for $m_u = m_d$ by the diagram in Fig. 4, where one of the pion vertices is CP-even and the other is produced by the CP-odd Hamiltonian. Its evaluation gives the expression

$$d_n \approx -\frac{1}{M_N} h_S h_P \frac{1}{16\pi^2} \ln\left(\frac{M^2}{m_\pi^2}\right), \tag{33}$$

where h_S and h_P are the corresponding vertices for zero pion momentum. The CP-even constant $h_P = g_{\pi NN}$ can be written with the help of the Goldberger-Treiman relation in the form $h_P = 2M_N g_A / f_\pi$. The CP-odd vertex h_S can also be expressed with the help of the Ward identities in terms of the nucleon matrix element $\langle p|\bar{u} i g_S \sigma G d|n\rangle$ (see Sec. 5). Unfortunately, the corresponding operator is isotriplet and little is known about its expectation value. If we set as our goal only order-of-magnitude estimates then we can use the similar matrix element of the isosinglet operator and set

$$\langle p|\bar{u} i g_S \sigma G d|n\rangle = \langle p|\bar{u} i g_S \sigma G u - \bar{d} i g_S \sigma G d|p\rangle \\ \equiv \tau \langle p|\bar{q} i g_S \sigma G q|p\rangle, \tag{34}$$

where τ is some factor of order unity. In Ref. 18 the latter matrix element was related under certain definite assumptions to the nucleon isoscalar density:

$$\langle p|\bar{q} i g_S \sigma G q|p\rangle \approx {}^5/_3 m_0^2 \langle N|\bar{q}q|N\rangle, \\ \langle N|\bar{q}q|N\rangle \equiv s\bar{\psi}_N \bar{q} q \psi_N, \\ m_0^2 \approx 0{,}8\,\mathrm{GeV}^2,\ s \approx 7. \tag{35}$$

Using (34) and (35) we obtain for the dipole moment

$$d_n \approx \frac{10}{3} \frac{1}{16\pi^2} \frac{g_A}{f_\pi^2} m_0^2 \tau s \ln\left(\frac{M^2}{m_\pi^2}\right)(c_5^u + c_5^d) \approx 4{,}2\tau s(c_5^u + c_5^d), \tag{36}$$

where we have assumed $M = m_\rho = 770$ MeV. Setting[8] $\tau s \sim 1$, we arrive at practically the same result (32). Of course, such a close agreement between the estimates must be viewed as only accidental.

In Ref. 4 in the analysis of the Weinberg model of CP nonconservation for relatively light Higgs particles the d_n, induced by the operator $G^2 G\tilde{G}$, was in fact estimated. The method used there, also quite approximate, in effect reduced to considering as intermediate states in the T-product (19) of nucleon-antinucleon pairs (entering only into the one-nucleon-irreducible diagram of Fig. 3c) and utilizing the simplest factored contribution for the CP-odd vertex $(\bar{N}N)(\bar{N}N)$. In that case, were one to follow the prescription

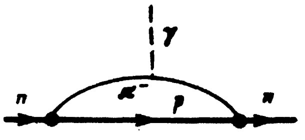

FIG. 4.

for estimating d_n described at the beginning of the Section and use for the determination of $\langle N|G^2 G\tilde{G}|N\rangle$ the naive factorization

$$\langle N|G^2 G\tilde{G}|N\rangle \approx \langle N|G\tilde{G}|N\rangle\langle 0|G^2|0\rangle,$$

then the result would turn out to be approximately five times larger than that obtained with the help of the mechanism of Ref. 4. Consequently, this example confirms the correctness of the order-of-magnitude estimates performed by the proposed method, and illustrates the present hypothesis that the irreducible vertices of Fig. 3c hardly dominate in d_n significantly.

5. EFFECT OF THE PECCEI-QUINN MECHANISM ON THE NEUTRON DIPOLE MOMENT

As was noted in the Introduction, the θ term is unavoidably induced in all natural models which have the operator o_6 appear at a noticeable level. The θ term, being an operator of dimension four, should give rise to a much bigger d_n. Therefore the phenomenological discussion of the o_6 operator makes practical sense only in the presence of the Peccei-Quinn (PQ) mechanism, which automatically eliminates any θ term introduced. In the presence of the corresponding symmetry the true vacuum, which spontaneously breaks it, is determined by minimizing the total energy with respect to the pseudo-Goldstonon degree of freedom, which in fact coincides here with the effective value of θ

$$\theta_{eff} = \theta - \arg\det M_{RL}, \tag{37}$$

where θ denotes the coefficient of the operator $(\alpha_S/4\pi) G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$ in the effective Lagrangian and M_{RL} is the quark mass matrix. Since θ is a CP-odd quantity, the point $\theta = 0$ is certainly an extremum. It is natural to assume that it is the true minimum of energy (arguments are presented in Ref. 20 in favor of this being the case in QCD). However, the situation changes if a CP-odd interaction is present in the theory. In that case the dependence on θ_{eff} of the vacuum energy is "distorted" and the vacuum value θ_{eff} is different from zero, being proportional to the introduced CP nonconservation. As a result we have in specific models two contributions to d_n that differ in nature: the direct contribution d_-, independent of the presence or absence of the PQ symmetry, and the contribution d_θ, induced by the nonzero value of θ_{eff}.

Generally speaking, the second contribution could turn out to be quite substantial. Thus, if the CP-odd weak interaction is determined by operators of the form $\bar{q} i \gamma_5 q$ or $G\tilde{G}$, d_θ identically cancels the direct contribution d_-. Below we shall formulate an approach to the determination of the relative role of d_θ for an arbitrary CP-odd operator o_- and discuss in more detail the cases of the o_6 and o_5 operators.

Thus, let the CP-odd weak Lagrangian have the form $L_{\text{odd}} = c_- o_-$. Identifying the axion field with the parameter θ_{eff} (37), and leaving off the subscript eff to simplify the notation, we write out the vacuum energy to lowest nontrivial order in c_- and θ as

$$V = {}^1/_2 (g_{11}\theta^2 + g_{22} c_-^2 + 2 g_{12} c_- \theta) + M^2 v^2 (\theta - \theta_0)^2, \\ v^2 = (2^{1/2} G_F)^{-1}, \tag{38}$$

where for purposes of illustration we have added the last term, violating the PQ symmetry and corresponding to a possible explicit violation of this symmetry in the Higgs potential; θ_0 stands for the bare value of $\theta_{\rm eff}$ (37). The quantity M can be of the order of the electroweak mass scale, but the axion model corresponds to $M = 0$. The coefficients g_{ij} in (38) represent vacuum correlators at zero momentum:

$$g_1 = \langle 0 | i \int d^4x\, T\{Q(x)Q(0)\} | 0 \rangle,$$

$$g_{12} = \langle 0 | i \int d^4x T\{Q(x) o_-(0)\} | 0 \rangle, \qquad (39)$$

$$g_{22} = \langle 0 | i \int d^4x T\{o_-(x) o_-(0)\} | 0 \rangle,$$

where

$$Q \equiv (\alpha_s/4\pi) G\tilde{G}. \qquad (40)$$

The vacuum value of θ, θ_v is determined for $M = 0$ by the minimum of the quadratic form (38):

$$\theta_v = -(g_{12}/g_{11}) c_-, \qquad (41)$$

and, consequently, the neutron dipole moment is now equal to

$$d_n = d_- + d_\theta = c_- k_- (1 - r), \qquad (42)$$

where

$$r = \frac{g_{12}}{g_{11}} \frac{k_\theta}{k_-}. \qquad (43)$$

We have introduced the factors k_- and k_θ, connecting the neutron dipole moment to the coefficients of the o_- operator and the θ term respectively:

$$d_n = k_- c_- + k_\theta \theta.$$

For a Higgs potential with a nonzero value of M the quantity θ_v is determined exclusively by the last term in (38) and $\theta_r = \theta_0$; for $\theta_0 = 0$ the additional contribution to d_n is absent and we have $r = 0$. Thus the quantity r represents the relative contribution to d_n of the PQ mechanism.

In fact formulas (42) and (43) represent the contribution to d_n of the additional diagram (Fig. 5) that takes into account exchange by neutral pseudoscalar Higgs bosons. In the general case the scale of the mass of the intermediate scalars is M_W, and the corresponding corrections turn out to be negligible, $\sim G_F m_\pi^2 f_\pi^2 / M_H^2$. However in the presence of

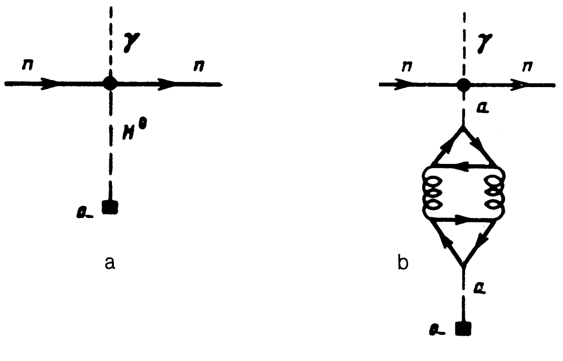

FIG. 5.

the PQ symmetry one of the states, the axion, is massless, and, literally, this diagram is equal to infinity. Summation of the "anomalous" pole diagrams of the form in Fig. 5b gives rise to a nonvanishing axion mass of order $G_F m_\pi^2 f_\pi^2$, determined by the correlator g_{11}, which in the end gives the additional contribution (43).

In the expression (43) the coefficients g_{12} and k_- depend on the explicit form of the operator o_-, and their calculation presents a nontrivial dynamical problem. One can, however, attempt to establish the behavior of the quantity r in the chiral limit, when the masses of the light quarks tend to zero. For simplicity we consider here the case $m_u = m_d \to 0$; the general case in which we have $m_q \to 0$ and the ratio m_u/m_d is fixed but different from unity requires a more accurate analysis and leads to the same expressions. (We work in the basis in which the quark masses are positive and free of γ_5. The corresponding redefinition of θ is included in θ_0.)

In the chiral limit the quantity g_{11} is easily calculated with the help of the anomalous Ward identities (see, for example, Ref. 21) and is linear in the quark masses:

$$g_{11} \approx [-2 m_u m_d / (m_u + m_d)] \langle \bar\psi \psi \rangle. \qquad (44)$$

The chiral behavior of the coefficient k_θ was calculated in Ref. 17 and has the form

$$k_\theta \propto m_\pi^2 \ln m_\pi^2 \propto m_q \ln m_q. \qquad (45)$$

The necessary relation for g_{12} is also easily obtained with the help of the anomalous Ward identities. Indeed, making use of the equality

$$\partial_\mu J_{\mu 5}^0 = m_q \bar q i \gamma_5 q + Q, \quad J_{\mu 5}^0 = \frac{1}{2} \sum_q \bar q \gamma_\mu \gamma_5 q, \qquad (46)$$

we can rewrite g_{12} in the form

$$g_{12} = \langle 0 | \int d^4x iT\{Q(x) o_-(0)\} | 0 \rangle = \langle 0 | \int d^4x iT\{(\partial_\mu J_{\mu 5}^0(x)$$
$$- m_q \bar q i \gamma_5 q(x)) o_-(0)\} | 0 \rangle = \int d^4x \partial_\mu \langle 0 | iT\{J_{\mu 5}^0(x) o_-(0)\} | 0 \rangle$$
$$- \int d^4x \delta(x_0) \langle 0 | [iQ_5^0(x), o_-(0)] | 0 \rangle$$
$$- m_q \int d^4x \langle 0 | iT\{\bar q i \gamma_5 q(x) o_-(0)\} | 0 \rangle, \qquad (47)$$

where $Q_5^0 = \frac{1}{2} \bar q \gamma_0 \gamma_5 q$ is the axial charge density operator.

The first term on the right-hand side of (47) vanishes, being the integral of a total derivative; the last term is linear in the quark mass and vanishes in the chiral limit, since for the isosinglet pseudoscalar the pion pole does not contribute to the correlator. Consequently, to within terms vanishing for $m_q = 0$, g_{12} is determined by the vacuum expectation value of the equal-time commutator in (47), which is easily calculated making use of the canonical commutation relations and represents by definition the axial charge of the operator o_-.

For the o_6 operator, which contains no quark fields, the commutator with Q_5^0 is equal to zero and we arrive at the conclusion that the correlator g_{12} is linear in the mass of the light quarks. As regards k_-, in accordance with the estimates of the preceding Section it does not vanish in the chiral limit; moreover we can see no reasons for it to vanish for $m_q = 0$. Consequently the quantity r is equal to zero in the chiral limit:

$$r \propto m_\pi^2 \ln m_\pi^2 \propto m_q \ln m_q. \qquad (48)$$

Therefore it is to be expected that in the real world r should be small, i.e., the presence of the PQ symmetry in the theory should have no effect on the estimate of the neutron dipole moment induced by the o_6 operator. We emphasize that this conclusion is based on two facts: the direct contribution to d_n from the o_6 operator is finite in the chiral limit and the axial charge of the o_6 operator is equal to zero.

The situation is different for the o_5 operators. In this case the equal-time commutator in (47) is equal to

$$\delta(x_0)[iQ_5^0(x), o_5(0)] = \delta^4(x)\bar{q}ig_s\sigma Gq, \qquad (49)$$

and in the chiral limit g_{12} takes the form

$$g_{12} = \langle 0|\bar{q}ig_s\sigma Gq(x)|0\rangle. \qquad (50)$$

As before k_- does not vanish in the chiral limit. Moreover, the most singular contribution to d_n, proportional to $\ln m_\pi^2$, can be calculated with the help of chiral perturbation theory just as was done in Ref. 17 for the ordinary pseudoscalar. For $m_u = m_d$ it is given by the diagram of Fig. 4 and determined by the vertex $\langle p\pi^-|o_-|n\rangle$ at zero pion momentum. That vertex can be easily expressed with the help of Ward identities in terms of the corresponding nucleon matrix element:

$$\langle N\pi^a|o_-|N\rangle_{q=0} \approx (1/F_\pi)\langle N|[Q_5^a, o_-]|N\rangle. \qquad (51)$$

As a result in the chiral limit the quantity r for the o_5 operator is given by the expression

$$r = \frac{\langle 0|\bar{q}ig_s\sigma Gq|0\rangle}{\langle 0|\bar{q}q|0\rangle} \frac{\langle p|\bar{u}d|n\rangle}{\langle p|\bar{u}ig_s\sigma Gd|n\rangle} = \frac{\langle 0|\bar{q}ig_s\sigma Gq|0\rangle}{\langle 0|\bar{q}q|0\rangle} \frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}ig_s\sigma Gu - \bar{d}ig_s\sigma Gd|p\rangle}. \qquad (52)$$

We again encounter here the difficulty of estimating isovector nucleon matrix elements. Lacking a better way we replace the last fraction in (52) by the ratio of the corresponding isoscalar quantities, i.e., we set

$$\frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}ig_s\sigma Gu - \bar{d}ig_s\sigma Gd|p\rangle} \approx \frac{\langle p|\bar{q}q|p\rangle}{\langle p|\bar{q}ig_s\sigma Gq|p\rangle}. \qquad (53)$$

If we then make use of the estimates of Ref 18 for the above expectation values, the quantity r is found to be about 3/5, where this last number is simply the ratio of the canonical dimensions of the $\bar{q}q$ and $\bar{q}ig_S\sigma Gq$ operators. Thus, in the case of the o_5 operator the effect of the PQ mechanism is quite substantial and the "axion" can "eat" a significant part of d_n.

6. NUMERICAL ESTIMATES

It is clear from general considerations that the phenomenologically most interesting consequences are to be expected for models where CP nonconservation is due to the exchange of Higgses (both in the standard form of interaction of H bosons with quarks like those of the Weinberg model, and with "horizontal" H bosons), for models with "right-handed" W bosons and for SUSY phases of realistic supersymmetric theories. We therefore start the discussion for just such cases. In the estimates we take the t-quark mass to be $m_t = (1–3)M_W$.

a) Charged and neutral "standard" Higgses

Making use of the value $M_H = m_t$ as a starting point, we obtain for the exchange of neutral bosons with (10), (17) and (30) included

$$d_n \approx 2\cdot 10^{-24} \sin\chi \; e\cdot\text{cm}, \qquad (54)$$

where we have introduced the notation

$$h_s h_p = 2^{1/2} G_F m_t^2 \sin\chi. \qquad (55)$$

based on the analogy with the Yukawa coupling constants of the SM. For the contribution of the charged bosons interacting with t and b quarks according to

$$\text{Im}(h_b h_t^*) = (1/2^{1/2}) G_F m_b m_t \sin\varphi, \qquad (56)$$

the numerical coefficient is found to be larger.

$$d_n \approx 10^{-23} \sin\varphi \; e\cdot\text{cm}. \qquad (57)$$

This difference is due, in the first place, to a weaker suppression of the QCD corrections, and also to a different explicit form of the coefficient functions (5) and (10).

Strictly speaking, d_n should be determined by the sum of the contributions (54) and (57) over all the Higgs states. In the most general case in the summation over all particles the CP-odd propagators for the neutral, as well as for the charged exchanges, either totally cancel in the limit $q^2 \to \infty$ (for spontaneous breaking of CP), or appear only in higher order loops. It is therefore reasonable to expect some cancellations in d_n when taking into account all contributions, although for $m_H > m_t$ this cancellation can hardly be significant except accidentally. We note that in the simplest models the CP-odd propagators for $H^\pm$ and H^0 are connected to each other at large momenta by simple relations.[4]

The notation (55) and (56) is introduced so that in the general case for moderate violation of CP in the Higgs sector the values of $\sin\varphi$ and $\sin\chi$ should be of order unity. Therefore comparison with the experimental bound

$$d_n = -(3\pm 4)\cdot 10^{-26} \; e\cdot\text{cm},$$

indicates that in the case of an extended Higgs sector the CP violation in it cannot be maximal. However, for bosons heavy in comparison with the t quark and if we take into account the uncertainty in the estimates realistically, only the bound on the charged bosons remains, and it is not very stringent.

b) $SU(2)_L \times SU(2)_R \times U(1)$ model

Here we again take for simplicity the case $m_t = M_W$; for $m_t = 250$ GeV the estimate of d_n goes up by a factor of 1.9. Putting together all the numerical factors we obtain

$$d_n \approx 1{,}5\cdot 10^{-21}\rho \sin\eta \; e\cdot\text{cm} \quad \rho = (M_{W_L}^2/M_{W_R}^2)\sin 2\theta. \qquad (58)$$

What is the order of magnitude of the CP-odd parameter $\rho\sin\eta$? In the simplest, "manifestly $L–R$ symmetric" models the interaction of W_R with the quarks is determined by the usual KM matrix and some additional phases. For simplicity we ignore them, as well as the ordinary CP nonconservation in KM. In that case the CP-odd $L–R$ mixing (12) completely determines the quantity ε_K, and at first glance the parameter $\rho\sin\eta$ should be of order ε_K. However such a naive estimate of the contribution of the $W_L–W_R$ mixing to the K^0–$\overline{K}^0$ transition turns out for several reasons to be much too low, and a more accurate relation is[6]

$$\rho \sin \eta \sim \varepsilon_K/500. \qquad (59)$$

Consequently, in these very simple models the contribution of the o_6 operator to d_n can be at the level

$$d_n \sim 10^{-26}\ e\cdot\text{cm}, \qquad (60)$$

while the bound on *CP*-odd W_L-W_R mixing turns out to be of the form

$$\rho \sin \eta \lesssim O(10^{-5}). \qquad (61)$$

In accordance with the results of Ref. 3 the direct four-fermion interaction of the constituent quarks, produced by the *CP*-odd W_L-W_R mixing, gives a somewhat larger dipole moment $d_n \sim 8\cdot 10^{-21}\rho \sin\eta\ e\cdot$cm. Nonetheless the estimate (60) is of independent interest, since that contribution is connected to the interaction with heavy quarks, which does not manifest itself directly in the physics of ordinary hadrons.

c) SUSY phases

In accordance with (15) the contributions of all heavy quarks to c_6 should all be of the same order; however in view of the significant negative anomalous dimension of o_6 the contribution of the lighter quarks is somewhat enhanced. We therefore consider here the case of the *b* quark. The parameters of the supersymmetric models are fairly undefined, and for purposes of illustration we take "typical" values, obtained as a result of solution of the renormalization group equations for $m_t \approx 100$ GeV. Setting $\alpha_S(\widetilde{M}^2) = 0.1$ from (15) for $m_{\tilde{g}} \approx 120$ GeV, $M_{\tilde{Q}} \approx 100$ GeV, and $\gamma_5 \approx A_b \approx 2$ we obtain numerically[10]

$$d_n \approx 2\cdot 10^{-22} \sin \tilde{\theta}\ e\cdot\text{cm}. \qquad (62)$$

This value is substantially larger than the direct contribution to d_n from the electric dipole moments of the constituent quarks, analyzed in Ref. 22, since the latter are proportional to the small current masses. Thus the potential effect of the o_6 operator strongly restricts the SUSY phases for the case of "near" supersymmetry:

$$\sin \tilde{\theta} \lesssim 10^{-3}-10^{-4}. \qquad (63)$$

d) Models with "horizontal" bosons and the standard KM model

Here in both cases the *CP*-odd amplitude for the transition $K^0 \to \overline{K}^0$ with $\Delta S = 2$ appears, generally speaking, in the same order in the weak interaction as the *CP*-odd processes without flavor change. For the "horizontal" bosons they appear in the tree approximation. Therefore, at first glance, the quantity c_6 should be extraordinarily small, at the "superweak" level. However, for scalar Higgs-type particles interacting predominantly with heavy quarks, considerable enhancement of d_n is possible. It occurs because the coupling of *H* bosons to light quarks, present in the *K* mesons, is very weak, while for heavy quarks such suppression is absent. To get a rough idea of the possible magnitude of the effect we consider neutral spinless bosons, having all possible diagonal and off-diagonal interactions of the form

$$L_H = (1/V)\,(m_i m_j)^{1/2}\bar{q}_i(f_{ij} + i g_{ij}\gamma_5)\,q_j, \qquad (64)$$

and assume that the complex coefficients f and g are of the order of unity and the masses of all bosons have the common scale M_H. The exchange of these particles also gives rise to the transitions $D^0 \to \overline{D}^0$ and $B^0 \to \overline{B}^0$, but the most stringent condition come from the parameter ε_K:

$$1/V^2 M_H^2 \sim 5\cdot 10^{-10}\varepsilon_K\ \text{GeV}^{-4} \sim 1\ \text{TeV}^{-4}. \qquad (65)$$

Making use of this estimate we arrive at the contribution of the *t* quark

$$d_n \sim 10^{-26}\ e\cdot\text{cm}. \qquad (66)$$

Thus we see that such "superweak" models can give rise to a neutron dipole moment even at the level of the present-day bound!

For vector gauge bosons the combined bound on the mass and coupling constant of the type (65) gives a much more stringent limit on the mass for a coupling constant on the order of g_2: $M_V \gtrsim 3\cdot 10^2$ TeV. At the same time, assuming that their coupling to quarks of different generations is of the same order, we obtain no enhancement for the loops with *t* quarks, and the resultant estimate for d_n turns out to be smaller by four orders of magnitude:

$$d_n \sim 10^{-30}\ e\cdot\text{cm}. \qquad (67)$$

On the whole it should be somewhat suppressed in comparison with the contribution of the four-fermion interaction of the light quarks at the tree level. We note that for the vector bosons the two-loop contribution to c_6 appears only for the off-diagonal interactions and only in the presence of interaction with both chiral components of the quarks.

For the KM model, as was noted in Sec. 2, the contribution to the o_6 operator from short distances is further suppressed due to the light quarks. Even if this suppression is ignored, the quantity d_n turns out to be extraordinarily small: in accordance with (16) the four-loop contribution is estimated as

$$d_n \sim 10^{-34}-10^{-35}\ e\cdot\text{cm}.$$

which is substantially smaller than the standard contribution from large distances.[2]

7. CONCLUSION

In this work we have analyzed the possible role of the pure gluon *CP*-odd operator of dimension six

$$o_6 = (g_S^3/16\pi^2) f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b \widetilde{G}_{\rho\mu}^c,$$

which could appear in the low-energy region of the theory due to *CP*-odd processes connected with heavy objects. As was to be expected, this operator plays a role that is most significant in models with the exchange of Higgs particles like those of the Weinberg model. The reason for this is quite transparent—the interaction of *H* bosons with fermions is proportional to the quark mass, so the *CP*-odd interaction generated by them is suppressed to a tremendous degree for light valence quarks. At the same time, for heavy quarks it is of "normal" size, reflecting directly the degree of *CP* violation in the Higgs sector. The generation by this interaction of a *CP*-odd gluon operator permits its "transfer" to the low-energy region without particular losses—the price is only the presence of an additional loop and suppression due to hard gluons. On the other hand, operators of the type of quark dipole moments (electric or color) are small, since for chiral reasons they are strongly suppressed by the mass of the light quark. Restrictions imposed by the experimental upper bound on d_n then permit one to assert that for an extended Higgs sector the *CP* violation in it cannot be maximal, $\sin \varphi \lesssim 0.01$–$0.1$, depending on the mass scale of the H bosons. Restrictions on the exchange by neutral particles are somewhat weaker.

Important information results also for *CP*-odd SUSY phases in broken supersymmetric models. Although the interactions with the superparticles are universal with respect to generations, minimal *CP*-odd operators containing only light quarks either again contain explicitly the quark mass or appear in higher orders and are therefore suppressed by additional powers of mass of the superparticles in the denominator. For heavy quarks the quark mass does not act as a suppression factor and the operators with heavy quarks directly reproduce new *CP*-odd interactions. The o_6 operator again permits their transfer to the region of low energies. The corresponding upper bounds on the phases for typical parameters of schemes with relatively light superparticles turn out to be at the level of 10^{-3}–10^{-4}, i.e., also stronger than the previously discussed effects of EDM of valence quarks.

For models with right-handed bosons, where the source of *CP* violation is the W_L–W_R mixing, the effective four-fermion interaction has the same constant for heavy and for light quarks and, obviously, no chiral selection rules for light quarks appear. Therefore here, in complete agreement with expectations, the induced o_6 operator gives a somewhat smaller contribution to d_n than the interaction of the valence quarks. Nonetheless, since o_6 is determined principally by the independent heavy-particle physics, its contribution is of separate interest as it provides information about new objects with mass of tens of TeV. In view of the estimates of d_n given here it is of the greatest importance to improve on the experimental accuracy of its measurement to a level of the order of 10^{-27} $e\cdot$cm.

Of great interest is the observation that even in "superweak" models with "horizontal" Higgs particles the o_6 operator can give a neutron dipole moment even at the 10^{-26} $e\cdot$cm level. Besides obvious consequences for experiment this fact imposes additional restrictions on the building of models required to answer the question of the origin of generations and quark mixing with the help of "horizontal" interactions.

In the Standard Model and for "horizontal" gauge bosons the effects of the o_6 operator are very small. These models require the existence of more than one generation of quarks and more or less successfully imitate the "superweak" model of Wolfenstein. Correspondingly the predictions for the contribution to d_n also begin with the "superweak" scale $\sim 10^{-30}$ $e\cdot$cm with some additional suppression, reflecting the specific details of the specific model.

The estimate of the size of the neutron dipole moment generated by the o_6 operator is fairly unambiguous and based in fact on dimensional considerations. We have proposed a more consistent method of applying these considerations and as a result obtained an answer much smaller than in the initial paper of Ref. 5. It is not hard to trace through the origin of this discrepancy, and we bring arguments in favor of our method of action, and also verify it in the case of other operators investigated by other methods.

When discussing possible models in which the o_6 operator arises phenomenologically at the level of interest, one must always assume the presence of the axion solution of the θ problem to suppress the unacceptably large contribution of the θ term to d_n. We have shown that in the concrete case of the o_6 operator the effect of the presence of the Peccei-Quinn symmetry is parametrically small in the chiral limit and in the real world can hardly affect the estimate of d_n substantially. At the same time this is not so for axial-charge-carrying operators, say, of the form $\bar{q}g_S\sigma G\gamma_5 q$, and we have presented arguments in favor of the idea that for such an operator the axion mechanism could substantially diminish the value of d_n.

In the process of formulating this work we have become aware of the preprint of Ref. 23, where in the framework of the Weinberg model of *CP* violation the role of the operator $\bar{q}g_S\sigma G\gamma_5 q$ is discussed. In accordance with the dimensional expectations of Sec. 4 its contribution to d_n has turned out to be smaller than that of the o_6 operator by about two orders of magnitude. As was emphasized above the additional suppression could be due to the PQ symmetry, which was not discussed in that work.

With the feeling of a pleasant duty one of us (N. G. U.) expresses acknowledgment to A. I. Vainshtein and I. B. Khriplovich for exceptionally fruitful discussions, thanks P. Kravchik and M. Eides, and also the coauthor of Ref. 10, A. A. Ioganse n, for useful discussions. For discussion of the QCD corrections I. Bigi thanks Prof. E. Braaten.

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[2)] In the KM model this contribution is smaller than the full d_n by about an order of magnitude. This is connected with the fact that it is dominated by nonlocal contributions to d_n, appearing in the *T*-product of two effective weak-interaction Hamiltonians, see Refs. 1 and 2.

[3)] Indeed, it can be shown that the one-loop expression for the pseudoscalar density in the external gluon field is $(-\alpha_S/4\pi)\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$ minus the total divergence of the sum of a series of well-defined local gauge-invariant operators. This series represents an expansion in inverse powers of the fermion mass of the one-loop expression for the axial current regularized according to Pauli-Villars.

[4)] One of us (N. G. U.) is grateful to I. B. Khriplovich for a discussion of this set of questions. He used similar considerations in Ref. 1.

[5)] It should be noted, however, that above considerations are based on the assumption that all the diagrams under discussion are finite in the local limit, including in that number loop diagrams. Even though the generalized GIM mechanism lowers the degree of divergence, *a priori* corresponding cancellations in the general case turn out to be insufficient. These remarks are also applicable to the analysis of Ref. 1.

[6)] In a number of cases what turns out to be distinguished is the intermediate state with a nucleon and a pair of soft pions, described by the diagram of Fig. 4 and leading to a unique infrared logarithm in d_n. For the o_6 operator this logarithm is absent due to the vanishing axial charge o_6.

[7)] It is necessary to note, however, that in Ref. 5 Weinberg has used an incorrect anomalous dimension of the o_6 operator, differing in sign from (17).

[8)] We use this value, keeping in mind that $\langle p|\bar{u}u - \bar{d}d\,|p\rangle \approx 0.5$. The latter equality follows for $m_s = 150$ MeV and $SU(3)$ relations for the mass splittings in the baryon octet; such an estimate is obtained approximately by the QCD sum-rules method.[19]

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