Classical mesoscopics

Yu. M. Gal'perin and V. I. Kozub

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted 11 February 1991)
Zh. Eksp. Teor. Fiz. **100**, 323–333 (July 1991)

The contribution of the classical correlation between consecutive lengths of electron scattering by impurity centers to the electrical conductance of samples with the metallic type of conduction is considered. This contribution changes as a result of jumps of impurities between their possible positions which gives rise to lf fluctuations of the conductance. These fluctuations are sensitive to a weak magnetic field that bends the electron paths. An important feature is that even a classically weak magnetic field influences the conductance fluctuations, so that the correlation can be destroyed by a very small deviation of the path from rectilinearity (the deviation can be of the order of one impurity potential radius in the distance equal to the mean free path of electrons). Estimates are obtained of fluctuations of the conductance due to the classical correlation. A comparison is made with quantum fluctuations due to the interference of the wave functions under multiple scattering conditions. It is shown that these classical fluctuations predominate at sufficiently high temperatures and also in the case of all the nondegenerate conductors.

INTRODUCTION

There is much interest in the characteristics of small samples which have what are called mesoscopic properties. Mesoscopic fluctuations is the name given to a fairly wide range of phenomena, of which we will be interested in two: the change in the resistance of a sample due to a spatial displacement of one of the scatterers and the stochastic dependence of the resistance on classically weak magnetic fields (known as "grass").

According to current views (see, for example, Refs. 1–3), these phenomena are due to quantum interference between electron states which are formed by scattering on the same impurities. Two types of such interference can be distinguished: global and local. The global interference occurs in the case of long paths (on the order of the smaller of two quantities, the characteristic size of a sample L and what is known as the dephasing length

$$L_T=(\hbar D/T)^{1/2}, \qquad (1)$$

where D is the diffusion coefficient of carriers and T is the absolute temperature). The global interference results in universal fluctuations of the conductance G, which are on the order of[1–3]

$$\delta G=(e^2/\hbar)(\lambda^{d-1}/L^{d-2}l)^{1/2}, \qquad (2)$$

where λ is the de Broglie wavelength of an electron; l is the mean free path of electrons, and d is the dimensionality of the sample. The estimate given by Eq. (2) applies for $L<L_T$; in the opposite limiting case, we have to replace L with L_T. The local interference (see, for example, Ref. 4) occurs in close pairs of defects and does not result in an anomalous dependence of the resistance on the dimensions. We shall consider only the local interference.

Our aim is to draw attention to the contribution made to fluctuations of the resistance by the classical correlation of the consecutive scattering by different impurities. We shall show that external manifestations of this contribution are similar to those of quantum mesoscopics. It is important to stress that the classical contributions appear also in some cases when the quantum contribution is small, particularly at sufficiently high temperatures.

We shall consider briefly the qualitative aspects of the phenomena under discussion. We assume that electrons are scattered by point defects without internal degrees of freedom. For simplicity, we regard the scattering as a collision with a sphere whose cross section is σ, i.e., whose radius is $(\sigma/\pi)^{1/2}$. To lowest order in the reciprocal of the number of impurities in a sample N_i^{-1}, the electron scattering events are totally uncorrelated. In this approximation the electrical conductance depends only on the impurity concentration n_i and is independent of the distribution of impurities. In fact, in traversing a distance equal to the mean free path to the next collision an electron retains its memory of the coordinate of the center on which it had been scattered. Therefore, the probability of the next collision depends on the specific distribution of the scattering centers in a sample.

By way of illustration, consider the scattering of electron by three centers (Fig. 1). Assume that the electron is first scattered by the center labeled 1. The probability that if subsequently scatters from the center 3 depends on the position of the center 2, which, in principle, can "cast a shadow" on the center 3. We consider this shadow effect. A measure of the shadow effect is the solid angle σ/R^2, where R is the distance between the impurities 1 and 2. The aim of our calculations will be to analyze quantitatively the correlation contribution to the conductance and to fluctuations of the latter when the impurity centers jump between different positions.

PRINCIPAL EQUATIONS

We use the semiclassical kinetic equation for the electron distribution function F in the presence of an electric field $\mathbf{E}$, which is not averaged over the positions of short-range impurities:

$$\mathbf{v}\frac{\partial F}{\partial \mathbf{r}}+e\mathbf{E}\frac{\partial F}{\partial \mathbf{p}}+I\{F\}=0, \qquad (3)$$

with the collision operator I given by

0038-5646/91/070179-06$03.00

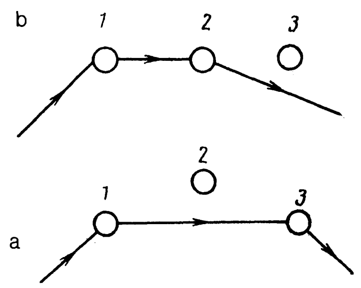

FIG. 1. "Shadow effect" in the scattering of an electron by a triplet of impurities.

$$\hat{I}\{F\}=\sum_{i,\mathbf{p}'}\delta(\mathbf{r}-\mathbf{R}_i)[F_{\mathbf{p}}(\mathbf{r})-F_{\mathbf{p}'}(\mathbf{r})]W_{\mathbf{pp}'},\qquad(4)$$

where $W_{\mathbf{pp}'}$ is the probability of a transition in the scattering center. Equation (3) was obtained in Refs. 5–8 from the equation of motion for the Wigner density matrix and is applicable in a situation when the de Broglie wavelength λ is much less than the average distance between the impurities $n_i^{-1/3}$. From now on we assume that the semiclassical condition $\lambda \ll n_i^{-1/3}$ is satisfied.

It is convenient to distinguish the fluctuating part of the impurity concentration:

$$\Phi(\mathbf{r})=\frac{1}{N}\sum_i[V\delta(\mathbf{r}-\mathbf{R}_i)-1],\qquad(5)$$

where V is the volume of the system, and also to distinguish the part of the collision operator averaged over the impurity positions. The kinetic equation then becomes

$$\mathbf{v}\frac{\partial F}{\partial \mathbf{r}}+e\mathbf{E}\frac{\partial F}{\partial \mathbf{p}}+\hat{\nu}\{F\}[1+\Phi(\mathbf{r})]=0,\qquad(6)$$

where

$$\hat{\nu}\{F\}=n_i\sum_{\mathbf{p}'}[F_{\mathbf{p}}(\mathbf{r})-F_{\mathbf{p}'}(\mathbf{r})]W_{\mathbf{pp}'}.\qquad(7)$$

It is now convenient to adopt the Fourier representation in terms of the coordinate $\mathbf{r}$. The Fourier component of fluctuations of the impurity concentration is

$$\Phi(\mathbf{q})=\frac{1}{N}\sum_i\left[e^{-i\mathbf{q}\mathbf{R}_i}-\frac{1}{V}\Delta(\mathbf{q})\right],\qquad(8)$$

where

$$\Delta(\mathbf{q})=\frac{2^3\sin(q_xL_x/2)\sin(q_yL_y/2)\sin(q_zL_z/2)}{q_xq_yq_z}.\qquad(9)$$

We require the quadratic fluctuation of the impurity concentration

$$|\Phi(\mathbf{q})|^2=\frac{1}{N^2}\sum_{i\neq k}(e^{i\mathbf{q}\mathbf{R}_i}-\langle e^{i\mathbf{q}\mathbf{R}_i}\rangle)(e^{i\mathbf{q}\mathbf{R}_k}-\langle e^{i\mathbf{q}\mathbf{R}_k}\rangle).\qquad(10)$$

The angular brackets mean averaging over the distribution of the impurities.

Analysis of the expression for the conductivity

Substituting into the kinetic equation (6) the nonequilibrium distribution function

$$F=F_0+f(-\partial F_0/\partial\varepsilon_{\mathbf{p}}),\qquad(11)$$

where $\varepsilon_{\mathbf{p}}$ is the electron energy, we find—in the approximation linear in terms of the electric field—that the following equation describes f:

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\hat{\nu}\{f\}[1+\Phi(\mathbf{r})]=e\mathbf{E}^{eff}\mathbf{v},\qquad(12)$$

where $\mathbf{E}^{\text{eff}}$ allows for a redistribution of the field in the sample and should be determined self-consistently from the continuity equation.

The contribution to the average current which is of interest to us appears in the second order of an expansion in terms of fluctuations of the impurity concentration Φ, since

$$\Phi(\mathbf{q}=0)=0.$$

We iterate Eq. (12) to second order in terms of the fluctuating function Φ:

$$f=\nu^{-1}e\mathbf{E}_0\mathbf{v}+f_1+f_2,\qquad(13)$$

where $\mathbf{E}_0$ is the electric field averaged over the sample, ν is the collision frequency, and f_1 is the result of the first iteration in powers of Φ, which satisfies the equation

$$(\nu+i\mathbf{q}\mathbf{v})f_1=-e\Phi(\mathbf{q})(\mathbf{E}_0\mathbf{v})+e(\mathbf{E}_{1\mathbf{q}}\mathbf{v}).\qquad(14)$$

The field $\mathbf{E}_{iq}$ is found from the equation of continuity $\operatorname{div}\mathbf{j}=0$ and is given by

$$\mathbf{E}_{1\mathbf{q}}=\Phi(\mathbf{q})\frac{\mathbf{q}(\mathbf{q}\mathbf{E}_0)}{q^2}.\qquad(15)$$

Consequently, in the first approximation we obtain

$$f_{1\mathbf{q}}=-\Phi(\mathbf{q})\frac{e(\mathbf{E}_t\mathbf{v})}{\nu+i\mathbf{q}\mathbf{v}},\qquad(16)$$

$$\mathbf{E}_t=\mathbf{E}_0-\mathbf{q}(\mathbf{q}\mathbf{E}_0)/q^2.\qquad(17)$$

We are interested only in the spatially homogeneous ($\mathbf{q}=0$) current (odd in $\mathbf{v}$) part of the function f_2. Solving the kinetic equation, we find

$$\tilde{\mathbf{j}}_2=-e^2V\int d\varepsilon\,\rho(\varepsilon)\left(-\frac{\partial F_0}{\partial\varepsilon}\right)$$

$$\times\int\frac{dn}{4\pi}\mathbf{v}\int(d\mathbf{q})[|\Phi(\mathbf{q})|^2-\langle|\Phi(\mathbf{q})|^2\rangle]$$

$$\times\frac{\nu}{\nu^2+(\mathbf{q}\mathbf{v})^2}\left[(\mathbf{E}_0\mathbf{v})-\frac{(\mathbf{q}\mathbf{E}_0)(\mathbf{q}\mathbf{v})}{q^2}\right],\qquad(18)$$

where $n=\mathbf{v}/v$ and $(d\mathbf{q})\equiv d^3q(2\pi)^3$. The angular brackets denote averaging over the positions of the impurities.

We now turn to Eq. (10) for $|\Phi(\mathbf{q})|^2$, which can rewritten in the form

$$|\Phi(\mathbf{q})|^2=\frac{1}{N^2}\sum_{i\neq k}\left(\cos[\mathbf{q}(\mathbf{R}_i-\mathbf{R}_k)]-\frac{1}{V}\Delta(\mathbf{q})\right)+\frac{N^2}{V^2}[\Delta(\mathbf{q})]^2. \tag{19}$$

The last term in Eq. (19) is easily shown to balance the contribution to the current density of the nonequilibrium function of the distribution f_1, which is due to the presence of the Δ function in Eq. (8) for $\Phi(q)$. Consequently, the correlation contribution to the current density is

$$\mathbf{j}_2=e^2\mathbf{E}_0\int d\varepsilon\,\rho(\varepsilon)\left(-\frac{\partial F_0}{\partial\varepsilon}\right)\frac{v^2}{\nu}\sum_{i\neq k}[A_{ik}-\langle A_{ik}\rangle], \tag{20}$$

where

$$A_{ik}=-\frac{V}{N^2}\int(d\mathbf{q})\,\frac{\pi}{4ql}\left(\cos[\mathbf{q}(\mathbf{R}_i-\mathbf{R}_k)]-\frac{\Delta(q)}{V}\right)$$
$$\times\left[1-\frac{(\mathbf{q}\mathbf{E}_0)^2}{q^2E_0^2}\right]. \tag{21}$$

In the derivation of the above expression we assumed that the condition $ql\gg 1$ is satisfied. The validity of this assumption will be discussed below.

It is worth noting the following important factor. Since we are dealing only with the pair correlation, it follows that in the summation over k (when i is given), we have to simplify Eq. (20) by excluding configurations in which an electron traveling ballistically from the ith to the kth impurity encounters an additional impurity. This means that the expression (16) for f_1 is valid only at distances from an impurity center that do not exceed the mean free path l. It is then found that in Eq. (20) we have to retain the summation only over the neighboring centers in a sphere of radius of order l centered on the ith impurity. Therefore, only the fluctuations with $q\geqslant l^{-1}$ are important; this justifies the above assumption. We can retain all the previous expressions without modification, provided we alter the normalization of the Δ function by assuming that it is normalized to the volume $V_0\approx l^3$. Note that V_0 is the minimum volume in which self-averaging is possible and for which the concept of electrical conduction still retains its meaning.

We now estimate the correlation contribution to the conductance created by one pair of centers, for which we shall select a fixed distance $\mathbf{R}\equiv\mathbf{R}_i-\mathbf{R}_k$, and transform the integral with respect to the vector $\mathbf{q}$. It is convenient to do this after introducing dimensionless variables

$$\xi\equiv qR,\quad c\equiv\cos(\widehat{\mathbf{R},\mathbf{E}_0}),\quad s\equiv\sin(\widehat{\mathbf{R},\mathbf{E}_0}). \tag{22}$$

Consequently, the integral

$$\int(d\mathbf{q})\frac{\cos(\mathbf{q}\mathbf{R})}{ql}\left[1-\frac{(\mathbf{q}\mathbf{E}_0)^2}{q^2E_0^2}\right]$$

is transformed to

$$\frac{2}{R^2l}\int_{\xi_{min}}^{\xi_{max}}d\xi\left[\left(1+\frac{2}{\xi^2}\frac{\partial^2}{\partial c^2}+\frac{1}{\xi^4}\frac{\partial^4}{\partial\xi^4}\right)\right.$$
$$\left.\times\frac{\sin(\xi(c^2+s^2))^{1/2}}{\xi(c^2+s^2)^{1/2}}\right]\Bigg|_{c^2+s^2=1}. \tag{23}$$

The lower limit of integration in the above expression is governed by the condition $ql\gg 1$: $\xi_{\min}\approx R/l$. The upper limit of integration is governed by the form factor of the scatterer $\Psi(qr_0)$, where $r_0\approx(\sigma/\pi)^{1/2}$ is the radius of the potential of this scatterer. In particular, in the case of a spherical potential well with a rectangular vertical profile, we have

$$\Psi(x)=i\frac{x\cos x-\sin x}{x^3}.$$

The integral in Eq. (23) should also include a factor $|\Psi(\xi r_0/R)|^2$, the influence of which can be allowed for by introducing the upper integration limit $\xi_{\max}\approx R/r_0$. This truncation of the integration domain is significant, because at high values of ξ the integrand in Eq. (23) behaves as $s^4\sin\xi$, whereas at low values of ξ its behavior is analytic.

We then find that the characteristic value of the correlation contribution of one pair of defects to the conductance of a sample is of order

$$\frac{\Delta G}{G}\approx\frac{V}{N_i^2R^2l}\approx\frac{1}{N_i}\frac{\sigma}{R^2}. \tag{24}$$

[Using the lower integration limit $\xi_{\min}$ in an expression of the form (23) for the contribution of the term in (21) proportional to $\Delta(\mathbf{q})$, we can easily show that this contribution is of order σ/N_il^2, i.e., it is $(R/l)^2\ll 1$ times less.]

The estimate given by Eq. (24) determines the order of the absolute value of the correlation contribution of the investigated pair. Therefore, this contribution depends on the direction of the vector $\mathbf{R}$ and can have different signs. Therefore, in the subsequent estimates we have to consider in somewhat greater detail the correlation contribution.

PHYSICAL RESULTS

Classical correlation contribution

We now consider the change in the conductance due to a displacement of one of the impurities (i) by a distance $\delta\mathbf{R}$. The contribution to the change in the quantity $|\Phi(\mathbf{q})|^2$, which is even in $\mathbf{q}$, is then

$$\delta|\Phi(\mathbf{q})|^2=\frac{4}{N^2}\left\{\sin^2(\mathbf{q}\,\delta\mathbf{R})\right.$$
$$\left.+{\sum_{k\neq i}}'\sin(\mathbf{q}\,\delta\mathbf{R}/2)\sin[\mathbf{q}(\mathbf{R}_i-\mathbf{R}_k-\delta\mathbf{R}/2)]\right\}. \tag{25}$$

We consider only the approximation linear in $\delta\mathbf{R}$. We then have

$$\delta|\Phi(\mathbf{q})|^2=\frac{2}{N^2}(\mathbf{q}\,\delta\mathbf{R})\sum_k{}'\sin(\mathbf{q}\mathbf{R}_k), \tag{26}$$

where the origin is shifted to the point $\mathbf{R}_i$.

We consider one term of the above sum. We select the z axis along $\mathbf{E}_0$ and we then have

$$(q_z\delta R_z)\sin(q_zR_z)\cos(\mathbf{q}_\perp\mathbf{R}_\perp)+(\mathbf{q}_\perp\delta\mathbf{R}_\perp)\cos(q_zR_z)\sin(\mathbf{q}_\perp\mathbf{R}_\perp).$$

Averaging this term over the directions of $\mathbf{q}$ with the weight

$$\left[1-\frac{(\mathbf{q}\mathbf{E}_0)^2}{q^2E_0^2}\right],$$

we obtain

$$q\int_0^1 d\mu\,(1-\mu^2)^2\{\mu\,\delta R_z\sin(qR_z\mu)J_0[qR_\perp(1-\mu^2)^{1/2}]$$

$$+(1-\mu^2)^{1/2}\delta R_\perp\cos(qR_z\mu)J_1[qR_\perp(1-\mu^2)^{1/2}]\},$$

where μ is the cosine of the angle between $\mathbf{q}$ and the z axis. The final result is

$$\delta A=\frac{\pi V s^3}{2N^2 l}[(4c^2-s^2)\delta R_\perp-5cs\delta R_z]$$

$$\times\left[\sum_k\frac{\cos(q_{min}R_k)-\cos(q_{max}R_k)}{R_k^3}-\frac{N_i}{V}\ln\left(\frac{q_{max}}{q_{min}}\right)\right].\qquad(27)$$

Here, c and s are, respectively, the cosine and sine of the angles between $\delta\mathbf{R}$ and $\mathbf{E}_0$. We have $q_{\min}\approx 1/l$ (we recall that the inequality $ql\gg 1$ was assumed). The value of $q_{\max}$ is limited by the ranges of validity of the adopted approach: $q_{\max}\leqslant\lambda^{-1}$, $(\delta R)^{-1}$, or by the characteristic form factor of the scattering center, which truncates the integration domain at values of q of order the reciprocal length of the scattering center $1/r_0$. The result is a characteristic estimate of the relative change in the conductance due to displacement of a single impurity:

$$\left|\frac{\Delta G}{G}\right|\approx\frac{\delta R}{R_{min}}\frac{1}{N_i}\frac{\sigma}{R_{min}^2}\approx\frac{1}{N_i}n_i\sigma\delta R\approx\frac{1}{N_i}\frac{\delta R}{l}.\qquad(28)$$

Here, $R_{\min}$ is the distance from the displaced impurity to its nearest neighbor. In obtaining the second equation it was assumed that for a typical impurity the value of $R_{\min}$ is on the order of $(N_i/V)^{-1/3}$, i.e., it is on the order of the average distance between the impurities.

It is important to stress that in a sphere of radius of the order of l the drop in the correlation contribution to the conductance has no characteristic scale and we are dealing in fact with the nearest neighbors.

It should be pointed out however that in obtaining the estimate (28) we ignored an important factor: the contribution of the neighbors in the shadow must be excluded in calculating the correlation term. The displacement of a single impurity center may change the nature of the neighboring centers governing the correlation contribution, because a displaced center may throw a shadow on some neighbors and lift the shadow from others. In estimating this effect we consider the centers i and k separated by a distance R. The probability of destruction or formation of an impurity triplet which includes these centers is equal to the probability that the third impurity falls within a cone with the base σ and the generator R, i.e., within $n_i\sigma R$; the existing triplet can be destroyed by moving an impurity a distance $\sigma^{1/2}$. We can estimate the dispersion of the correlation contribution to the conductance by selecting a sphere with the radius $R_{\max}\approx l$ around a displaced impurity. The correlation contribution of this region to the conductance dispersion is of order

$$\overline{\left(\frac{\Delta G}{G}\right)^2}\approx\frac{n_i}{N_i^2}\int_{R_{min}}^{R_{max}}d^3R\,(n_i\sigma R)\left[\frac{\phi^2}{R^2}-\frac{(\bar\phi)^2}{R_{max}^2}\right].\qquad(29)$$

Following the estimate given in Eq. (24) we can represent the contribution of one pair to the conductance in the form

$$\left.\frac{\Delta G}{G}\right|_{\mathbf{R}}\equiv\frac{1}{N_i}\frac{\sigma}{R^2}\phi\left(\frac{\mathbf{R}}{R}\right),\qquad(30)$$

where ϕ is a certain alternating-sign function of its argument and $\bar\phi$ is the average value of the function in a sphere of radius $R_{\max}$.

The integral in Eq. (29) depends logarithmically on the limits and $R_{\min}$ is governed by the actual distribution of impurities in a given sample. Therefore, an estimate of the shadow effect gives

$$\left|\frac{\Delta G}{G}\right|\approx\frac{1}{N_i}\frac{\sigma^{1/2}}{l}\ln\left(\frac{l}{R_{min}}\right),$$

which for $\delta R\approx\sigma^{1/2}$ is identical with the estimate given by Eq. (28). Note that the characteristic magnitude of the correlation contribution determined in one experiment depends on the experiment itself. In studies of the statistics of changes in the conductance due to jumps of different impurities in an ensemble of samples it is natural to assume $R_{\min}\geqslant\sigma^{1/2}$ for a totally uncorrelated distribution of impurities (otherwise there is no sense in discussing the scattering by individual defects). We then obtain

$$\left|\frac{\Delta G}{G}\right|\approx\frac{1}{N_i}\frac{\sigma^{1/2}}{l}\ln\left(\frac{l}{\sigma^{1/2}}\right).\qquad(31)$$

If we are interested in the correction to the conductance for jumps of the same impurity between, for example, two states, then $R_{\min}$ is the distance from an impurity to its nearest neighbor.

Note that this shadow-effect contribution is not determined by the adjacent impurities, but in general by all the impurities in a sphere of radius $\approx l$ around a displaced center.

Comparison with universal fluctuations of the conductance

Let us compare the classical fluctuations of the conductance with the quantum universal fluctuations of this quantity (see, for example, Refs. 1–3). In the three-dimensional case it follows from Eq. (2) that these fluctuations can be described by

$$\left|\frac{\Delta G}{G}\right|_{UCF}\approx\frac{\lambda^3}{(lL)^{3/2}}\min\left[1,\left(\frac{L_T}{L}\right)^{3/2}\right].\qquad(32)$$

Using the estimate given by Eq. (26) and also assuming that $L_T>L$, we obtain

$$\left|\frac{\Delta G_{CL}}{\Delta G_{UCF}}\right|\approx\frac{\delta R}{\lambda}\left(\frac{l}{L}\right)^{3/2}.\qquad(33)$$

It is natural to assume that the characteristic displacement of an impurity δR is on the order of the interatomic distance a and (in the case of a metal sample) also on the order of the de Broglie wavelength λ. Therefore, if $L_T>L>l$, the classical contribution is smaller than the quantum one.

It is important to stress, however, that the classical contribution "survives" at higher temperatures when the length L_T is small and the quantum fluctuations are suppressed. In fact, the quantum interference may appear only if the condition $L_T\gg l$ or $\hbar/T_\tau\gg 1$ is satisfied (here, τ is the relaxation time of the electron momentum). It therefore follows that the quantum effects can occur only if

$$T > T^{*} \equiv \hbar/\tau. \tag{34}$$

It should be pointed out that this condition is not obeyed by most nondegenerate systems. On the other hand, classical fluctuations, which are insensitive to this inequality, may appear also in the case of nondegenerate systems. If the electron velocity is $v \approx 3\times 10^{7}$ cm/s and we also have $l \approx 3\times 10^{6}$ cm, then $T^{*} \approx 77$ K.

Influence of an external magnetic field

Let us now consider the influence of an external magnetic field on the classical correlation effects. Bending of the electron path in a magnetic field alters the correlation contribution. Obviously, the influence of the field is greatest for distant pairs. On the other hand, as shown above, the classical correlation contribution is due to impurity clusters of dimensions $\leqslant l$. The deviation of the electron path from rectilinearity by an amount $\approx \sigma^{1/2}$ in a distance of length $\approx l$ is equivalent to a displacement of one impurity. We can therefore expect the correlation contribution of each pair to the conductance to change by an amount on the order of the conductance itself [for an estimate see Eq. (28)] in a magnetic field which satisfies the condition

$$\Omega_c \tau \approx \frac{\sigma^{1/2}}{l} \ll 1, \tag{35}$$

i.e., when the magnetic field is classically weak (Ω_c is the cyclotron frequency). Since the contributions of different impurities can be regarded as statistically independent, the total change in the conductance is of order

$$\left|\frac{\Delta G}{G}\right| \approx \frac{1}{N_i^{1/2}} \frac{\sigma^{1/2}}{l} \ln\left(\frac{l}{R_{min}}\right) \approx \frac{1}{N_i^{1/2}} \frac{\sigma^{1/2}}{l} \ln\left(\frac{l}{\sigma^{1/2}}\right). \tag{36}$$

In the case of a degenerate conductor, we obtain

$$|\Delta G| \approx \frac{e^2}{\hbar} \frac{\sigma}{\lambda^2} \left(\frac{l}{L}\right)^{1/2}, \tag{37}$$

which is $\approx (l/L)^{1/2} \ll 1$ times less than the scale of the "grass" due to universal fluctuations of the conductance. However, as in the absence of a magnetic field, the classical effect survives also at fairly high temperatures. For example, if $L_T < L$, the ratio of the classical and quantum scales of the "grass" is of order

$$\left|\frac{\Delta G_{CL}}{\Delta G_{UCF}}\right| \approx \frac{L}{L_T} \left(\frac{l}{L_T}\right)^{1/2}, \tag{38}$$

so that under certain conditions the classical contribution may predominate. The characteristic scale of the magnetic fields that govern the period of the "grass" is the condition (35), which for a degenerate conductor can be rewritten in the form

$$H^{*}_{CL} \approx \frac{\Phi_0}{l^2} \frac{\sigma^{1/2}}{\lambda}, \tag{39}$$

so that for $L_T > l$ this scale is greater than the scale of the fields governing the dependence of the universal fluctuations of the conductance of the magnetic field:[1-3] $H^{*}_{\mathrm{UCF}} \approx \Phi_0^2/L_T^2$. However, at high temperatures these scales may be of the same order of magnitude.

Comparison with the contribution of the local quantum interference

In addition to the universal fluctuations of the conductance, let us consider also the local quantum interference (see, for example, Ref. 4), which is the interference between the wave functions formed on scattering from adjacent impurities separated by a distance much less than the mean free path l. (It should be pointed out that the universal fluctuations of the conductance are due to the interference of the electron wave functions in the regions with dimensions of the order of $L_T \gg l$. Therefore, they are frequently referred to as manifestations of the global interference.)

We can make a comparison with the classical correlation contribution by analyzing the local interference in somewhat greater detail. We include in the scattering probability W some interference terms of the type $A^2 \cos(\mathbf{p} - \mathbf{p}', \mathbf{R}_{ik})$. (For simplicity, we assume that the scattering amplitudes A are real and the same for all the centers.) Using the same iteration equation as before, we obtain the following interference contribution to the collision integral:

$$\sum_{i\neq k,\mathbf{p}'} A^2 \mathbf{R}_{ik} \cos[(\mathbf{p}-\mathbf{p}'), \mathbf{R}_{ik}] ((\mathbf{p}-\mathbf{p}')\mathbf{E}_0) \left(\frac{\partial F_0}{\partial \varepsilon_{\mathbf{p}}}\right).$$

The corresponding contribution to the current density is proportional to

$$\sum_{\mathbf{p},\mathbf{p}',i\neq k} A^2 ((\mathbf{p}-\mathbf{p}')\mathbf{E}_0) \left(\frac{\partial F_0}{\partial \varepsilon_{\mathbf{p}}}\right) \frac{(\mathbf{p}\mathbf{E}_0)}{mE_0} [\cos(\mathbf{p}, \mathbf{R}_{ik}) \cos(\mathbf{p}', \mathbf{R}_{ik}) + \cos(\mathbf{p}, \mathbf{R}_{ik}) \cos(\mathbf{p}', \mathbf{R}_{ik})]. \tag{40}$$

This yields the following estimate for the interference contribution to the current due to one pair:

$$\left.\frac{\Delta G}{G}\right|_{\mathbf{R}} \approx \frac{4\pi}{N(pR)^2} \Big[-\cos(2pR)\cos^2(\mathbf{E}_0, \mathbf{R}) + \frac{2\pi}{(pR)} \sin(2pR) [4\cos^2(\mathbf{E}_0, \mathbf{R}) - \sin^2(\mathbf{E}_0, \mathbf{R})] + \frac{4\pi}{(pR)^2} [-3\cos^2(\mathbf{E}_0, \mathbf{R}) \sin^2(pR) + \sin^2(\mathbf{E}_0, \mathbf{R}) \sin^2(pR)] \Big]. \tag{41}$$

Obviously, in the summation over the neighboring impurities k it is sufficient to consider those which are located at distances $R \leqslant l$. In view of the size of the parameter $p_F l$, the main contribution comes from the first term in Eq. (38): this contribution is of order

$$\frac{1}{N_i} \left(\frac{\lambda}{R}\right)^2 \cos(2pR).$$

Because of the fast oscillations, the average over the realizations of the distribution of impurities vanishes and the dispersion is governed by the minimum distance between the impurities. Displacement of one impurity gives

$$\left|\frac{\Delta G}{G}\right|_{\mathbf{R}} \approx \frac{(p\delta\mathbf{R})}{N_i} \left(\frac{\lambda}{R}\right)^2 \approx \frac{1}{N_i} \left(\frac{\lambda}{R}\right)^2. \tag{42}$$

This contribution is also governed by the nearest neighbors. For a typical impurity we have $R \approx n_i^{-1/3}$ and for a close pair characterized by $R \approx \sigma^{1/2}$, the estimate given by Eq. (39)

agrees with the corresponding estimates for fluctuations of the resistance due to transitions of defects on the atomic scale (fluctuators) between locally stable states.[9,10] This is natural because such a close pair should behave as a single quantum system in the form of a particle in a two-well potential.

The contribution of the local interference described by Eq. (39) and corresponding to the displacement of one impurity is generally greater than the classical contribution:

$$\left|\frac{\Delta G_{CL}}{\Delta G_{LI}}\right| \approx \frac{R^2}{\lambda l}.$$

For a typical impurity we have $R \approx n_i^{-1/3}$ and therefore,

$$\left|\frac{\Delta G_{CL}}{\Delta G_{LI}}\right| \approx \frac{\sigma^{7/6}}{\lambda^2 l^{1/3}} \approx \left(\frac{\lambda}{l}\right)^{1/3}. \tag{43}$$

It is interesting to note that this contribution may also exceed the contribution of the universal fluctuations of the conductance:

$$\left|\frac{\Delta G_{LI}}{\Delta G_{UCF}}\right| \approx \left(\frac{l}{L}\right)^{3/2} \frac{l^{1/3}}{\sigma^{1/6}} \approx \left(\frac{l}{L}\right)^{3/2} \left(\frac{l}{\lambda}\right)^{1/3}. \tag{44}$$

However, like the contribution of the universal conductance fluctuations (UCF), the local interference should be manifested at sufficiently high temperatures when the thermal spreading of the electron momenta $\Delta p \approx T/v$ exceeds $\hbar/R \approx \hbar n_i^{1/3}$. The corresponding estimate for the temperature at which the local interference is suppressed gives

$$T_{LI}^* \approx \frac{\hbar}{\tau} \frac{l^{2/3}}{\sigma^{1/3}}. \tag{45}$$

The influence of the magnetic field has a characteristic scale in which the change in the contribution of the local interference is relatively of order unity, but is considerably greater than the classical contribution:

$$H_{LI}^* \approx \Phi_0 n_i^{2/3}.$$

It is therefore, meaningful to compare the local and classical interference contributions in fields of order H_{CL}^*, which strongly influence the classical contribution. In fields of this kind the magnetic field "feels" only the pairs with $R \approx l$. The interference contribution of each pair is, as pointed out already, of order $(\lambda/l)^2$ and the number of such pairs is of order $n_i R^3 \approx l^2/\sigma$. Since the contributions of the pairs are statistically independent, the relative contribution of the local interference is of order

$$(\lambda/l)^2 (l^2/N_i\sigma)^{1/2} \approx \lambda^2/l(N_i\sigma)^{1/2}.$$

Therefore, the contribution of the local interference is on the order of the classical contribution of Eq. (28). It is important to stress that the corresponding contribution of the classical interference appears at the same temperature as the UCF contribution described by Eq. (34).

CONCLUSIONS

We have considered the classical correlation contribution to the resistance due to consecutive scattering of conduction electrons by defect pairs. When defects jump between adjacent possible positions, the correlation contribution changes, giving rise to *lf* fluctuations of the resistance. At low temperatures these fluctuations are weaker than the fluctuations due to what is known as the global quantum interference over distances on the order of the dephasing length L_T (universal fluctuations of the conductance) and the local interference over distances smaller than the average distance between the impurities. However, as the temperature increases, the quantum fluctuations are suppressed and the classical ones survive. Therefore, at sufficiently high temperatures the classical contributions should predominate. This is also true of all nondegenerate conductors. It is important to stress that the classical and quantum fluctuations are sensitive to a classically weak magnetic field, which may help in their identification. Therefore, in observing the classical correlation effects in the conduction process it is preferable to investigate the fluctuations of the same quantity in degenerate conductors at temperatures of a few tens kelvin, and also fluctuations in nondegenerate systems.

The authors are grateful to V. L. Gurevich for numerous valuable discussions.

[1] B. L. Al'tshuler and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 363 (1985) JETP Lett. **42**, 447 (1985)].
[2] S. Feng, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. **56**, 1960 (1986).
[3] S. Feng, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. **56**, 2772 (1986) (Erratum).
[4] P. J. Hirschfeld, P. Wölfle, and D. Einzel, Phys. Rev. B **37**, 83 (1988).
[5] V. I. Kozub, Zh. Eksp. Teor. Fiz. **86**, 2239 (1984) [Sov. Phys. JETP **59**, 1303 (1984)].
[6] V. I. Kozub, Fiz. Tverd. Tela (Leningrad) **26**, 1955 (1984) [Sov. Phys. Solid State **26**, 1186 (1984)].
[7] V. I. Kozub, Fiz. Tverd. Tela (Leningrad) **26**, 1851 (1984) [Sov. Phys. Solid State **26**, 1121 (1984)].
[8] V. I. Kozub, Zh. Eksp. Teor. Fiz. **87**, 1410 (1984) [Sov. Phys. JETP **60**, 810 (1984)].
[9] Yu. M. Gal'perin, V. G. Karpov, and V. I. Kozub, Zh. Eksp. Teor. Fiz. **95**, 1123 (1989) [Sov. Phys. JETP **68**, 648 (1989)].
[10] Yu. M. Galperin, V. L. Gurevich, and V. I. Kozub, Europhys. Lett. **10**, 753 (1989).

Translated by A. Tybulewicz