Electron-spinwave pairing in a one-dimensional Kondo lattice Integrable model

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We obtain an exact solution of the Kondo problem, taking the pairing of band electrons and spinwaves into account. We obtain a gap in the momentum distribution density near the Fermi energy and in the spectrum of the singlet excitations.

The Kondo lattice model is the simplest of the many-electron models which take the interaction between band electrons and localized electrons into account. The Falicov-Kimball model[1] is a more complicated one which describes the interaction between *s–p* band electron states and the localized *d* states. It is well known that its single-impurity analog—the resonance level model—is similar to the Kondo problem with an anisotropic exchange integral.[2] For $V^2/U \ll 1$ the Anderson model for a half-filled conduction band can also be reduced to the Kondo problem[3] (U is the magnitude of the Coulomb interaction of the d electrons on a single site and the parameter V determines the overlap of the wavefunctions of the *s–d* electrons). The Kondo lattice model can thus describe the features of considerably more complex models, which makes it even more attractive.

Exact solutions of one-dimensional models, obtained through the Bethe ansatz occur for Hamiltonians with particle number conservation. In the present paper we propose a Hamiltonian of an *s–d* model in which we introduce a term which does not conserve the particle number. We show that in that case the problem is also integrable; we evaluate the gap in the momentum distribution density, the ground state energy, and the spectrum of the triplet excitations.

The Hamiltonian of the model has in the approximation with a constant electron density near the Fermi surface ε_F the following form:

$$\mathcal{H}=-i\sum_{\sigma}\int dx c_\sigma^+(x)\frac{d}{dx}c_\sigma(x)+\frac{J}{2}\sum_n\sum_{\sigma,\sigma'}\int dx\,\delta(x-x_n)c_\sigma^+(x)\sigma_{\sigma\sigma'}c_{\sigma'}(x)\mathbf{S}_n+\frac{3}{4}J\sum_n\int dx\,\delta(x-x_n)\times[\Delta c_\uparrow^+(x)X_n^{\downarrow 0}+\Delta X_n^{\uparrow 0}c_\downarrow^+(x)+\text{h.c.}], \quad (1)$$

where we have taken into account only interactions between electrons with the same directions of their momentum. In the Hamiltonian (1) we consider a linear section of the electron spectrum near the Fermi momentum k_F so that the electron energy is measured from k_F. We use here the following notation: J is the exchange integral; $c_\sigma^+(x)$, $c_\sigma(x)$ are the creation and annihilation operators of the conduction electrons ($\sigma=\uparrow,\downarrow$); $\mathbf{S}_n$ is the spin operator localized on the nth site; and the $\sigma^\alpha_{\sigma\sigma'}$ are the Pauli matrices.

In the Hamiltonian (1) we have introduced anomalous averages which determine the singlet pairing between the band electrons and the spinwaves:

$$\Delta=\langle X_n^{0\uparrow}c_\downarrow(x_n)\rangle,$$

X_n^{pg} is a projection operator, $X^{pg}|r\rangle=|p\rangle\delta_{gr}$ and $|r\rangle$ is a state of the electron shell of an impurity atom.

The interaction in the Hamiltonian (1) is assumed to be a contact interaction; we consider the $S=\frac{1}{2}$ case. Assuming that the interaction in (1) is a two-particle one and that the T_j matrix is determined by the two-particle scattering matrices, in the Hamiltonian (1) we have split off the anomalous averages (the last terms which are proportional to Δ), which account for the nonconservation of the particle number.

We consider the solution of the Schrödinger equation for the wavefunction

$$|\hat{\psi}_k\rangle_\sigma^s=\int dx[f_{k\sigma}^s(x,x_n)c_\sigma^+(x)X_n^{s0}+\delta(x-x_n)\tilde{f}_{k\sigma}^s(x_n)]|0\rangle, \quad (2)$$

where $|0\rangle$ is the vacuum function: $c_\sigma(x)|0\rangle=0$, $S_n^\alpha|0\rangle=0$; k is the electron wavevector.

The last term in (2) takes account of the conduction-electron–spinwave pair creation. The presence of the function $\tilde{f}_{k\sigma}^s(x_n)$ in (2) means that the amplitude of the wavefunction $f_{k\sigma}^s(x,x_n)$ is determined from a Schrödinger equation with a potential equal to

$$[1+\delta J(k)/4]1+{}^1/_2\tilde{J}(k)\sigma\mathbf{S}. \quad (3)$$

The quantities $\delta J(k)$ and $\tilde{J}(k)$ depend here on the electron energy $E(k)$ which is measured relative to the Fermi energy:

$$\delta J(k)=J\Delta_0^2/E(k),\quad \tilde{J}(k)=J-\delta J(k),\quad \Delta_0^2={}^9/_8 J|\Delta|^2. \quad (4)$$

It follows from (3) and (4) that taking the singlet pairing into account renormalizes the value of the exchange integral. According to (3) the matrix for the scattering of an electron by a localized spin has the following form:

$$R_{ss'}^{\sigma\sigma'}(k,k_s)=\frac{g(k)-g(k_s)-icP_{ss'}^{\sigma\sigma'}}{g(k)-g(k_s)-ic}e^{i\vartheta}, \quad (5)$$

where

$$\vartheta=-2\,\mathrm{arctg}\frac{J}{8},\quad g(k)=\frac{\Delta_0^2}{E(k)-\Delta_0^2}\left(1+\frac{Jc}{8}\right),$$

$$c=\frac{J}{2(1-3J^2/64)},$$

$P_{ss'}^{\sigma\sigma'}$ is the spin exchange operator, and $P=\frac{1}{2}(1+2\sigma\mathbf{S})$. A localized magnetic moment corresponds to a wavevector k_s which is determined by the condition $g(k_s)=-1$.

The two-electron wavefunction is, according to (2), de-

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termined in each region of the x_1x_2 plane as a sum of the following form:

$$|\hat{\Phi}^{s}_{k_1\sigma_1k_2\sigma_2}(x_1,x_2)|0\rangle=\psi^{s}_{k_1\sigma_1k_2\sigma_2}(x_1,x_2)c_{\sigma_1}^{+}(x_1)c_{\sigma_2}^{+}(x_2)X_0^{s0}|0\rangle$$
$$+\exp(ik_1x_1)\tilde{f}^{s}_{k_2\sigma_2}(0)c_{\sigma_1}^{+}(x_1)|0\rangle+\exp(ik_2x_2)\tilde{f}^{s}_{k_1\sigma_1}(0)c_{\sigma_2}^{+}(x_2)|0\rangle, \quad (6)$$

where the last two terms describe a state with a single electron without an impurity spin. The amplitude $\tilde{f}^{s}_{k\sigma}(0)$ is independent of the electron coordinates, so that when we consider electron scattering processes we need take into account only the quantity

$$\psi^{s}_{k_1\sigma_1k_2\sigma_2}(x_1,x_2)=\{\exp[i(k_1x_1+k_2x_2)]-\exp[i(k_1x_2+k_2x_1)]\}$$
$$\times[\theta(x_1-x_2)\delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'}+S^{\sigma_1\sigma_1'}_{\sigma_2\sigma_2'}(k_1,k_2)\theta(x_2-x_1)]\psi^{s}_{k_1\sigma_1'k_2\sigma_2'}(x_1,x_2), \quad (7)$$

where

$$S^{\sigma_1\sigma_1'}_{\sigma_2\sigma_2'}(k_1,k_2)=\frac{g(k_1)-g(k_2)-icP^{\sigma_1\sigma_1'}_{\sigma_2\sigma_2'}}{g(k_1)-g(k_2)-ic}, \quad (8)$$

which is the two-particle electron scattering matrix. Note that the R- and S-matrices of (5) and (8) depend on the difference of the $g(k_i)$ arguments; they satisfy the Yang-Baxter equation defined for the Kondo problem.[3,4]

We consider a system consisting of N_e electrons and N_i impurity atoms. The many-particle wavefunction depends on the particle configuration. We define a region X_Q which is characterized by a definite configuration of electrons and localized magnetic moments:

$$X_Q=\{x_{q1}<x_{q2}<x_{q3}<\ldots<x_{qN_0}\},$$

and which corresponds to a set of quantities

$$Q=\{q1,\ q2,\ q3,\ldots,\ qN_0\},$$

which label these particles; let $q1$, $q2$, $q3$,..., qN_e be the coordinates of the electrons and $q(N_e+1)$, $q(N_e+2)$,..., qN_0 be the coordinates of the impurities ($N_0=N_e+N_i$). Accordingly, the region $X_{Q'}$,

$$X_{Q'}=\{x_{q'1}<x_{q'2}<x_{q'3}<\ldots<x_{q'N_0}\},$$

then corresponds to the set

$$Q'=\{q'1,\ q'2,\ q'3,\ldots,\ q'N_0\}.$$

For definiteness we assume that $N_e\geqslant N_i$.

We define the wavefunction of the N_e electrons for a given Q configuration according to the Bethe ansatz:

$$|\hat{\Psi}_{\{k\}}(Q)\rangle^{\{s\}}_{\{\sigma\}}$$
$$=\int\prod_{i=1}^{N_e}dx_{qi}\hat{\Psi}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(x_{q1},\ldots,x_{qN_0};k_1,\ldots,k_{N_e})|0\rangle.$$

The wavefunction $\hat{\Psi}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(x_{q1},\ldots,x_{qN_0};k_1,\ldots,k_{N_e})$ is determined in the second-quantization representation and can be written in the form

$$\hat{\Psi}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(x_{q1},\ldots,x_{qN_0};k_1,\ldots,k_{N_e})=\sum_{P}\hat{A}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(Q/P), \quad (9)$$

where the summation is over the permutations P. The operator $\hat{A}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(Q/P)$ is determined by the two-particle wavefunction (2):

$$\hat{A}^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(Q/P)$$
$$=\prod_{qi<qj}^{N_i}\hat{\Psi}^{s_j}_{kPi\sigma_i}(x_{qi},x_{qj})\prod_{l=N_i+1}^{N_e}\Psi_{kPl\sigma_l}(x_{ql})c^{+}_{\sigma l}(x_{ql})$$
$$\times A_{\sigma_1\ldots\sigma_{N_e}}(Q/P). \quad (10)$$

According to Eqs. (2)–(5) we can write the two-particle wavefunction $\hat{\Psi}^{s}_{k\sigma}(x,x_i)$ in the following form:

$$\hat{\Psi}^{s}_{k\sigma}(x,x_i)=\Psi_{k\sigma}(x)c_{\sigma}^{+}(x)X_i^{s0}$$
$$+\frac{1}{N_a^{1/2}}\sum_{n=1}^{N_i}\exp[ik(x-x_n)]\tilde{A}^{sn}_{k\sigma}(x-x_n),$$
$$\Psi_{k\sigma}(x)=\frac{1}{N_a^{1/2}}\sum_{n=1}^{N_i}\exp[ik(x-x_n)]A^{sn}_{k\sigma}(x-x_n), \quad (11)$$

while the coefficients $A^{s}_{k\sigma}(x)$ and $\tilde{A}^{s}_{k\sigma}(x)$ are defined as follows:

$$A_{k\sigma^s}(x)=1-\frac{ic}{g(k)-g(k_s)}(1-\delta_{s\sigma})\operatorname{sign}x,$$
$$\tilde{A}_{k\sigma^s}(x)=\frac{2}{3\Delta}\frac{g(k)}{1+cJ/8+g(k)}\delta(x)(1-\delta_{\sigma s}), \quad (12)$$

and N_a is the number of lattice sites.

Depending on the regions X_Q and $X_{Q'}$, which differ by the permutation of two electrons or of an electron and an impurity, the coefficients $A^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(Q/P)$ and $A^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_{N_e}}(Q'/P)$ for the single-electron operators in Eq. (10) are connected through the relations

$$A^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_i\ldots\sigma_j\ldots\sigma_{Ne}}(Q'/P)=S^{\sigma_i\sigma_i'}_{\sigma_j\sigma_j'}(k_i,k_j)A^{s_1\ldots s_{N_i}}_{\sigma_1\ldots\sigma_i'\ldots\sigma_j'\ldots\sigma_{N_e}}(Q/P),$$
$$A^{s_1\ldots s_i\ldots s_{N_i}}_{\sigma_1\ldots\sigma_j\ldots\sigma_{N_e}}(Q'/P)=R^{\sigma_j\sigma_j'}_{s_is_i'}(k_j,k_s)A^{s_1\ldots s_i'\ldots s_{N_i}}_{\sigma_1\ldots\sigma_j'\ldots\sigma_{N_e}}(Q/P). \quad (13)$$

The problem is thus equivalent to the Kondo problem with scattering matrices $S^{\sigma_i\sigma_i'}_{\sigma_j\sigma_j'}(k_i,k_j)$ and $R^{s_is_i'}_{\sigma_i\sigma_j'}(k_j,k_s)$ which depend on the electron momenta. The eigenvalues k_j are determined by taking into account the boundary conditions imposed on the wavefunction. The problem is reduced to finding the eigenvalues of the T_j matrix which, according to (13), has the following form:

$$T_j=S_{jj+1}(k_j,k_{j+1})\ldots S_{jN_e}(k_j,k_{N_e})R_{j1}(k_j,k_s)$$
$$\ldots R_{jN_i}(k_j,k_s)\ldots S_{jj-1}(k_j,k_{i-1}).$$

The eigenvalues of the T_j matrix are determined from a set of transcendental equations which in the present case can be written in the following form:

$$\exp(ik_jL)=\prod_{\alpha=1}^{M}\frac{g(k_j)-\lambda_\alpha-ic/2}{g(k_j)-\lambda_\alpha+ic/2}\exp(-iN_i\vartheta),$$

$$\prod_{j=1}^{N_e}\frac{\lambda_\alpha-g(k_j)-ic/2}{\lambda_\alpha-g(k_j)+ic/2}\left(\frac{\lambda_\alpha-g(k_s)-ic/2}{\lambda_\alpha-g(k_s)+ic/2}\right)^{N_i}=-\prod_{\beta=1}^{M}\frac{\lambda_\alpha-\lambda_\beta-ic}{\lambda_\alpha-\lambda_\beta+ic},\tag{14}$$

where L is the length of the chain and M the number of electron spins which have not been inverted.

The Bethe equations, written down for the distribution density of the real numbers k_j and λ_α, denoted respectively by $\rho(k)$ and $\sigma(\lambda)$, have the form

$$\rho(k)=\frac{1}{2\pi}-\frac{c}{2\pi}g'(k)\int d\lambda\sigma(\lambda)\frac{1}{[\lambda-g(k)]^2+c^2/4},$$

$$\sigma(\lambda)+\frac{c}{\pi}\int d\lambda'\sigma(\lambda')\frac{1}{(\lambda-\lambda')^2+c^2}$$

$$=\frac{c}{2\pi}\int dk\rho(k)\frac{1}{[\lambda-g(k)]^2+c^2/4}$$

$$+\frac{c}{4\pi}n_i\frac{1}{[\lambda-g(k_s)]^2+c^2/4}.\tag{15}$$

Since we neglect the interaction in the Hamiltonian (1) between electrons with oppositely directed momenta, Eqs. (15) are defined for positive and negative electron momenta and it is necessary to take into account that $g(k)=g(|k|)\operatorname{sign} k$. Since the function $g(k)$ is antisymmetric in k it follows from the form of Eqs. (15) that the momentum and the velocity distribution functions $\rho(k)$ and $\sigma(\lambda)$ are symmetric.

The integration limits in Eqs. (15) are determined from the following conditions:

$$\int_{-k_0}^{k_0}dk\rho(k)=\frac{N_e}{N_a}=n_e,\quad m=\frac{n_e+n_i}{2}-\int_{-B}^{B}d\lambda\sigma(\lambda).\tag{16}$$

The first equation in (16) is a consequence of the conservation of the number of electrons and the second determines the magnetization m per site of the system (n_e is the electron density and n_i the impurity density). We introduce the distribution density $p(z)$; it follows from (15) that it is given in the following form:

$$p(z)=\rho(k)/g'(k)|_{k=(k_F+\Delta_0^2)\operatorname{sign} k+\Delta_0^2(1+cJ/8)/z}.$$

If there is no magnetic field ($B=\infty$) the second equation can be integrated and the first one reduces to a linear integral equation with a difference kernel which for $z<0(k>0)$ has the following form:

$$p(z)-\int_{-z_2}^{-z_1}dz'\mathscr{L}(z-z')p(z')=-\frac{\Delta_0^2(1+cJ/8)}{2\pi z^2}-\frac{n_i}{2}\mathscr{L}(z+1),\tag{17}$$

where

$$\mathscr{L}(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\exp(i\omega z)\frac{1}{1+\exp(|\omega|c)},$$

$$z_1=\frac{\Delta_0^2(1+cJ/8)}{k_F+\Delta_0^2},\quad z_2=\frac{\Delta_0(1+cJ/8)}{k_F-k_0+\Delta_0^2}.$$

The solution for $p(z)$ can be obtained using a Fourier expansion of the functions in (17) on the section z_2-z_1. In the case of a weak interaction one can write the solution $p(z)$ as an expansion in Δ; the first terms of that series can be written as follows:

$$p(z)=-\frac{n_e}{2}\mathscr{L}(z)-\frac{\Delta_0^2(1+cJ/8)}{2\pi z^2}-\frac{n_i}{2}\mathscr{L}(z+1)+O(\Delta^3).\tag{18}$$

The equation for k_0 is obtained from the condition for the conservation of the number of electrons:

$$\pi n_e-k_0=\frac{\Delta_0^2(1+cJ/8)}{c(k_F-k_0)}[n_e\ln 2+\pi cn_i\mathscr{L}(1)].\tag{19}$$

It follows from (19) that the Fermi level is not changed when one takes into account the singlet electron pairing, $k_F=\pi n_e$. The quantity k_0 which corresponds to the highest filled electron state is less than k_F; there a gap appears in the momentum distribution function near the Fermi energy. The quantity $\mathscr{L}(1)$ is of the order of $\exp(-\pi/c)$ so that in (19) the main contribution comes from the conduction electrons:

$$k_0=k_F-|\Delta|(1+cJ/8)^{1/2}[n_e\ln 2+\pi cn_i\mathscr{L}(1)]^{1/2}.\tag{20}$$

As one should expect, the width of the region of forbidden states is determined by the quantity Δ. In the case of a weak interaction, with an accuracy of $n_i\Delta\mathscr{L}(1)$ the velocity distribution density $\sigma(\lambda)$ is unchanged for $\Delta\ll 1$ and the solution is the same as in the Kondo problem of Refs. 3 and 4.

We calculate the parameter Δ corresponding to a contact singlet pairing of the electrons and the spinwaves using the wavefunction determined above according to the Bethe ansatz (9) to (12). The equation for Δ has the form

$$\frac{1}{c'}=\frac{n_i}{N_e}\sum_{i=1}^{N_e}\frac{1}{E(k_i)}\left\{1+\left[\frac{c}{g(k_i)-g(k_s)}\right]^2+n_i\frac{J\Delta_0^2}{2E^2(k_i)}\right\}^{-1},\tag{21}$$

where $c'=\frac{3}{4}J$. The correlation function Δ determines the contact pairing of the electrons with the spinwaves so that in Eq. (21) which determines Δ there occurs the normalization of the Bethe function.

In high-density Kondo systems the solution for Δ obtained in the logarithmic approximation equals

$$\Delta=\tfrac{2}{3}[(1+cJ/8)n_e\ln 2]^{-1/2}k_F\exp(-\pi\nu/c'),\quad \nu=n_e/n_i.\tag{22}$$

The exact solution for the quantity Δ is obtained by substituting the solution for $p(z)$ into (21) and then integrating over z in the given ranges $[z_1,z_2]$ and $[-z_2,-z_1]$. In Eq. (22) given above the terms proportional to $\mathscr{L}(1)$ are neglected as being small corrections. The exponential dependence on the impurity density is characteristic for Δ so that the pairing effects are unimportant in the single-impurity Kondo model.

We determine the stability of the solutions, calculating the ground state energy E_0:

$$E_0=\sum_{j=1}^{N_e}k_j.$$

We use the first Eq. (14) and redefine the energy E_0 in the form

$$E_0=-2L\int_{-\infty}^{\infty}d\lambda\sigma(\lambda)\int_{-k_0}^{k_0}dk\rho(k)\left\{\operatorname{arctg}\left(\frac{2[\lambda-g(|k|)]}{c}\right)+\frac{\pi}{2}\right\}+\frac{2\pi}{L}\sum_j N_j-N_i\vartheta, \tag{23}$$

where the N_j are integers.

In the case of a weak interaction we can separate in E_0 the energy of the fermion gas, the energy of the disordered Kondo lattice E_K, and a term E_s connected with the pairing of the electrons and the spins. The quantity E_K was calculated in detail in Ref. 4 so that we give here the expression for E_s obtained from (23) in the weak-interaction approximation:

$$E_0=-2Lc\int_{-\infty}^{\infty}d\lambda\sigma(\lambda)\frac{1}{\lambda^2+c^2/4}\int_{z_1}^{z_2}dz\,zp(z).$$

Using the solutions for $p(z)$ and $\sigma(\lambda)$ we get the quantity E_s:

$$E_s=-4N_e\frac{\Delta_0^2(1+cJ/8)}{2\pi c}\ln 2\left[\ln\left(\frac{z_1}{z_2}\right)+\frac{1}{2}\right]+O(\Delta^3). \tag{24}$$

The term E_s is negative, so that the solutions obtained are stable when the quantity Δ is nonvanishing. The energy E_s is considerably larger than the difference in the energies of the superconducting and the normal phases which occur in the Bardeen-Cooper-Schrieffer theory. In the case of a weak interaction the first term dominates in Eq. (24) so that in the logarithmic approximation the quantity E_s for a Kondo lattice with $n_e=n_i$ has a simple form:

$$E_s=-\frac{L}{c'}\tilde{\Delta}^2,\quad \tilde{\Delta}=k_F\exp\left(-\frac{\pi}{c'}\right).$$

We consider the triplet excitations in the chain which have the lowest energy. The formalism for calculating the energies of excitations in one-dimensional models has been expounded in detail in Refs. 5 to 7, so we shall not give here the details of the calculation but only the results. The energy of the triplet excitations, measured from the ground state energy E_0 has the form

$$\varepsilon(k)=\varepsilon_e(k)+\varepsilon_i(k),$$

$$\varepsilon_e(k)=\int dzp(z)\left\{2\operatorname{arctg}\left(\exp\left[\frac{\pi(\lambda_k-z)}{c}\right]\right)-\frac{\pi}{2}\right\}, \tag{25}$$

$$\varepsilon_i(k)=2\pi n_i\int dz\delta p(z)W(|z|-1),$$

where

$$W(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\frac{\sin(\omega z)}{1+\exp(|\omega|c)},$$

while $\delta p(z)$ is the solution of the integral equation which for $z>0(k<0)$ is determined in the following form:

$$\delta p(z)-\int_{-z_2}^{-z_1}dz'\mathcal{L}(z-z')\delta p(z')=\frac{1}{2c\operatorname{ch}[\pi(\lambda_k-z)/c]}.$$

The quantity λ_k and the momentum k are connected through the relation

$$|k|=\frac{\pi n_e}{2}+2\pi\int_{-\infty}^{\lambda_k}d\lambda\,\sigma(\lambda).$$

In the case of a weak interaction the renormalization of the energy of the triplet excitations $\delta\varepsilon$ for $\Delta\ll 1$ is determined by the second term $\delta\varepsilon_i(k)$ in (25) since the quantity $\delta\varepsilon_e$ is proportional to Δ^3 and

$$\delta\varepsilon_i(k)=2\pi n_i W(1)\frac{\Delta_0^2(1+cJ/8)}{c(k_F-k_0)}\frac{1}{\operatorname{ch}(\pi\lambda_k/c)},$$

where λ_k is the weak coupling approximation is given in the form

$$|k|=\frac{\pi n_e}{2}+2n_e\operatorname{arctg}\left[\exp\left(\frac{\pi\lambda_k}{c}\right)\right].$$

The energy of the excitations, and hence also the wave-vector, is a continuous function of λ_k, but the presence of a gap in the momentum distribution density for $k_0<k<k_F$ introduces a gap in the excitation spectrum near the Fermi energy. The magnitude of the gap in the weak coupling case is determined by the ratio of the exchange interaction constant and the width of the conduction band. The solution for the momentum distribution density with a gap near the Fermi energy was obtained taking into account the anomalous averages which describe the creation of an electron-spin-wave pair. A nontrivial solution for Δ is obtained only when there is an effective attraction between the electrons and the spin moments; in the Kondo lattice model the antiferromagnetic *s–d* interaction is the analog of the attraction.

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