Types of instability in ordered domain structures

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Using a phenomenological approach, we have carried out a theoretical analysis of the types of instability that are possible for a completely ordered stripe domain structure in a magnetic film with the easy-axis type of anisotropy. We show that changes can occur in the period of a metastable domain structure caused by kink-like or translational instabilities in the positions of its domain boundaries, and that transitions to hexagonal lattices of cylindrical magnetic domains are possible as well. Experiments carried out on films made of quasi-uniaxial magnetic garnets confirm the basic assumptions of our theory.

Within the class of completely ordered domain structures it is possible to identify certain structures that are in thermodynamic equilibrium, i.e., with parameters [for example, the period d_s of a stripe domain structure or the cell parameter a_s and radius r_s of a cylindrical magnetic domain (CMD) for a hexagonal lattice] that correspond to an absolute energy minimum for specified values of the external parameters, i.e., temperature T, magnetic field $\mathbf{H}$, etc. The behavior of a domain structure in thermodynamic equilibrium near a second-order magnetic phase transition curve (or near a first-order curve that is close to second-order) was analyzed in Refs. 1 and 2; the authors of these references treated ferromagnetic films with strong "perpendicular" uniaxial anisotropy (i.e., $\beta_u \gg \max\{4\pi,\beta_p,\beta_c\}$ and $\vartheta_u \ll 1$, where β_u, β_p, and β_c are the uniaxial, rhombic, and cubic anisotropy constants, and ϑ_u is the angle between the easy magnetization axis and the normal $\mathbf{n}$ to the film surface). For a spontaneous phase transition (near the Curie temperature T_C) the driving external parameter is the temperature T, while for an orientational phase transition it is a magnetic field $\mathbf{H}$ applied roughly parallel to the film surface (i.e., $H_\perp \gg H_\parallel$, where $H_\parallel$ and $H_\perp$ are projections of the vector $\mathbf{H}$ on the surface of the film and along its normal $\mathbf{n}$, respectively).

The analysis carried out in Refs. 1 and 2 showed that, depending on the values of the external parameters T and $\mathbf{H}$, such films can exhibit the following types of inhomogeneous magnetic states: a "crystalline" phase (i.e., a fully ordered domain structure), a Berezinskii-Kosterlitz-Thouless (BKT) phase,[3-5] (i.e., a domain structure with bound magnetic dislocations), a "liquid-crystal" phase (with free dislocations), and a "liquid" phase (with free magnetic dislocations). In what follows we will investigate films with completely ordered domain structures, i.e., films whose inhomogeneous magnetic state corresponds to the "crystal" phase; such films are commonly found, e.g., at $T = 0$.

The parameters of an equilibrium domain structure (in the region where it exists) are continuous functions of the temperature T and magnetic field $\mathbf{H}$; however, this does not imply that a quasistatic variation of the external forces will result in synchronous "retuning" of the domain structure parameters so as to follow their equilibrium values. In films with unbounded transverse dimensions (in the plane of the structure) smooth changes in the period d of the domain structure are forbidden by symmetry considerations alone. This is because the symmetry groups of two domain structures that differ in their periods (i.e., with respect to translations) by an infinitesimally small quantity δd cannot be subgroups of one another; therefore, a phase transformation that takes one domain structure into the other can only be first-order. If the transverse size of the film is finite, continuous changes in the period are hindered by energy barriers between states with differing numbers of domains.[1)] Therefore, if an equilibrium domain structure (e.g., stripe-like) with period $d_0 = d_s(T_0,\mathbf{H}_0)$ can exist in the film for certain values of $T = T_0$ and $\mathbf{H} = \mathbf{H}_0$, it must become metastable if the field or temperature changes in any way. For $d_0 > d_s(T,\mathbf{H})$, the domain structure is under tension. When the difference $\Delta d(T,\mathbf{H}) = |d_0 - d_s(T,\mathbf{H})|$ exceeds a certain value, the domain wall system undergoes either a kink-like instability, i.e., a tendency for the domain wall profiles to distort sinusoidally, or a translational (modulation) instability of the domain wall positions; a further possibility is a first-order phase transition to a hexagonal CMD lattice. For $d_0 < d_s(T,\mathbf{H})$ this type of domain structure is under compression; here, too, there is a certain critical value of $\Delta d(T,\mathbf{H})$ for which either translational (modulation) instability of the domain walls or a transition to a CMD lattice is possible.[7-13]

If we take into account the possibility of forming magnetic dislocations, then for $T \neq 0$ relaxation of a metastable stripe domain structure to its equilibrium state from a state under tension can take place by nucleation, translation, and annihilation of dislocations. These processes will not be discussed in this paper.

1. THEORY

1.1. Derivation of basic equations

The behavior of the magnetization vector $\mathbf{M}$ in a ferromagnetic film is described by the Landau-Lifshits equation:[14-15]

$$\dot{\mathbf{M}} = \omega_0\{-[\mathbf{m}\mathbf{H}^{eff}] + \lambda_{i1}[\mathbf{H}^{eff} - \mathbf{n}(\mathbf{n}\mathbf{H}^{eff})] + \lambda_r\mathbf{H}^{eff} - \lambda_e\nabla^2\mathbf{H}^{eff}\} \tag{1}$$

and the equations of magnetostatics [curl $\mathbf{H}_D = 0$, div$(\mathbf{H}_D + 4\pi\mathbf{M}) = 0$], together with the following boundary conditions at the surface of the film:

$$(\mathbf{n}\nabla\mathbf{M})|_s = 0, \quad (\mathbf{n}(\mathbf{H}_{Di} + 4\pi\mathbf{M} - \mathbf{H}_{De}))|_s = 0,$$
$$[\mathbf{n}(\mathbf{H}_{Di} - \mathbf{H}_{De})]|_s = 0. \tag{2}$$

Here λ_{i1} and λ_r are relaxation constants of relativistic origin that take into account contributions from induced uniaxial and cubic anisotropy, respectively, λ_e is the exchange relaxation constant, $\mathbf{m} = \mathbf{M}\,M^{-1}$, $\mathbf{H}^{eff} = -\delta\mathcal{F}/\delta\mathbf{M}$ is the ef-

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fective magnetic field, $\mathscr{F}$ is the free energy of the system, $\mathbf{H}_{Di} = M\mathbf{h}_{Di}$ and $\mathbf{H}_{De} = M\mathbf{h}_{De}$ are the demagnetization and scattering fields, respectively, $\omega = gM$, where g is the gyromagnetic ratio, and $\mathbf{n}$ is a unit vector parallel to the easy magnetization axis along the normal to the film surface. In spherical coordinates $[\theta = \arcsin([SB:m:z]/m)$, $\psi = \arctan(m_x/m)]$ the expression for the free energy of the film in a field $\mathbf{H} = M(h_\perp \mathbf{e}_y + h_\parallel \mathbf{e}_z)$, where $\mathbf{n}\|\mathbf{e}_z$, has the form

$$\mathscr{F} = \frac{1}{2} M^2 \int d\mathbf{r} \{\alpha[(\nabla\theta)^2 + \cos^2\theta(\nabla\psi)^2] - \beta_u \cos^2\theta - 2h_\perp \cos\theta\cos\psi - 2h_\parallel \sin\theta - \mathbf{m}\mathbf{h}_D\}. \quad (3)$$

Let us assume that the relaxation constants in Eq. (1) and the longitudinal magnetic susceptibility are small, and limit our discussion to the case of a thick uniaxial ferromagnetic film (i.e., $l_z \gg \alpha^{1/2}$, where l_z is the film thickness and α is the interaction constant for inhomogeneous exchange) with its easy magnetization axis parallel to $\mathbf{n}$. We also assume that the film is close to an orientational phase transition, i.e., $h_\parallel \ll h_\perp$, and $|\xi| \gg \max\{4\pi, \beta_u\}$, where $\xi = \beta_u - h_\perp$. Since we have $\max\{\theta,\psi\} \ll 1$ in this case, we find from (1)–(3) and the equations of magnetostatics that

$$(\mu\nabla_x^2 + \nabla_y^2)\left[-\alpha\nabla^2\theta + (h_\perp - \tilde{\beta})\theta + {}^1\!/_2\beta_u\theta^3 + \frac{(\lambda_r^* - \lambda_e\nabla^2)\dot{\theta}}{\omega_0} + \frac{\ddot{\theta}}{(h_\perp\mu\omega_0^2)}\right] + 4\pi\nabla_z^2\theta = 0, \quad (4a)$$

$$\nabla_x[(\omega_0 h_\perp \mu)\psi + (\lambda_r - \lambda_e\nabla^2)\dot{\psi}] - \nabla_z\dot{\psi} + 4\pi\omega_0\nabla_z\theta + [\nabla_x + (\lambda_r^* - \lambda_e\nabla^2)\nabla_z]\dot{\theta} = 0, \quad (4b)$$

where

$$\tilde{\beta} = \beta_u - \frac{3}{32\pi^2}\beta_u h_\parallel^2, \quad \mu = 1 + \frac{4\pi}{h_\perp}, \quad \lambda_r^* = \lambda_r + \lambda_{11}.$$

In calculating the distribution of magnetization and the spectrum of spin waves in the film, we can write the angle θ as the sum of a static component $\theta_0(\mathbf{r})$ and a dynamic component $\tilde{\theta}(\mathbf{r},t)$, where $|\theta_0| \gg |\tilde{\theta}|$. Assuming that the period of the stripe domain structure in the film is fixed (in a film with unbounded transverse dimensions the period d can be arbitrary; for a film with transverse dimensions l_x the period equals $d = l_x N_x^{-1}$, where N_x is a whole number), a solution to the equations of state (4) can be sought in the form[16,17]

$$\theta_0(\mathbf{r}) = \sum_{n=0}^{\infty} \lambda^n A_n(z)\cos(nkx);$$

$$\psi_0(\mathbf{r}) = \sum_{n=0}^{\infty} \lambda^n B_n(z)\sin(nkx),$$

where $\lambda \ll 1$ is the order parameter, and

$$k = 2\pi d^{-1}, \quad q = \pi/l_z, \quad A_0 = (4\pi)^{-1}h_\parallel,$$
$$A_1 = a_1\cos(qz) + {}^1\!/_2\beta_u\lambda^2 a_1^3 a_3\cos(3qz);$$
$$A_2 = {}^1\!/_2\beta_u A_0 a_1^2[b_0 + b_2\cos(2qz)],$$
$$A_3 = {}^1\!/_2\beta_u a_1^3[c_1\cos(qz) + c_3\cos(3qz)].$$

Explicit expressions for the coefficients a_n, b_n, and c_n are obtained from expression (13) of Ref. 17 if we make the replacement $\delta \to \beta_u/2$.

Substituting the series (5) into (3) and (4) and including the boundary conditions (2), we obtain an expansion of the free-energy density in terms of the parameter $\lambda a = \lambda a_1$ for fixed values of k and ξ in the form

$$\frac{\mathscr{F}}{V} = \frac{\mathscr{F}_0}{V} + \sum_{p=1}^{3}(\lambda a)^{2p}B_{2p}(\xi, k)\frac{M^2}{2p} \quad (6)$$

and an equation that determines the dependence of the parameter λa on k and ξ:

$$B_2 + (\lambda a)^2 B_4 + (\lambda a)^4 B_6 = 0, \quad (7)$$

where V is the volume of the film and $\mathscr{F}_0$ is the free energy of the film in the uniformly magnetized state, i.e.,

$$\mathscr{F}_0 = -(A_0 M)^2\left(2\pi - \frac{\xi}{2} + \frac{3\beta_u A_0^2}{8}\right),$$

the functions B_2, B_4, and B_6 are determined by the expressions

$$B_2 = {}^1\!/_4\varkappa^2(1 + \zeta_0^4 - \tilde{\xi}\varkappa^{-2}),$$
$$B_4 = \frac{9\beta_u}{128}\left[1 - \frac{\beta_u A_0^2}{4\varkappa^2 - \tilde{\xi}}\frac{3(4 - \tilde{\xi}\varkappa^{-2}) + 2\zeta_0^4}{4 - \tilde{\xi}\varkappa^{-2} + \zeta_0^4}\right], \quad (8)$$
$$B_6 = 6231\beta_u^2(512\varkappa_{c0})^{-2}.$$

Here, $\tilde{\xi} = \xi - \frac{3}{2}\beta_u A_0^2$, $\varkappa = k\alpha^{1/2}$, $\varkappa_{c0} = k_{c0}\alpha^{1/2} = (4\alpha\pi^3/\mu_\perp l_z^2)^{1/4}$, $\zeta_0 = \varkappa_{c0}/\varkappa = k_{c0}/k$; the expression for B_6 is valid for $\tilde{\xi}\varkappa^{-2} = 1 + \zeta_0^4$ and $\varkappa \simeq \varkappa_{c0}$.

1.2. Region of stability for a stripe domain structure with fixed period

In Ref. 2 it was shown that in the course of an orientational phase transition the uniformly magnetized state that exists in a strong transverse magnetic field, i.e., for $h_\perp > \beta_u$, becomes unstable against a transition to a stripe domain structure in thermodynamic equilibrium along the curve

$$h_{\perp c}(h_\parallel) = \beta_u - 2\varkappa_c^2 - \frac{3}{32\pi^2}\beta_u h_\parallel^2; \quad (9a)$$

this domain structure in turn becomes unstable with respect to a transition to the uniform state for

$$h_{\perp c}^*(h_\parallel) = h_{\perp c}(h_\parallel) + 0{,}388\frac{(\varkappa_{c0}\varepsilon_{kc})^2}{h_{\parallel kc}^4}w(\varepsilon_{kc}), \quad (9b)$$

while the critical values of normalized inverse period for such a domain structure on the curves given by Eqs. (9a) and (9b) equal

$$\varkappa_c(h_\parallel) = \frac{2\pi\alpha^{1/2}}{d_c} = \varkappa_{c0}\left(1 - 3{,}19\cdot 10^{-5}\frac{\beta_u h_\parallel^2}{\alpha^{1/2}}l_z\right), \quad (10a)$$

$$\varkappa_c^*(h_\parallel) = \varkappa_c(h_\parallel) - 0{,}17\varkappa_{c0}h_{\parallel kc}^{-2}\varepsilon_{kc}w(\varepsilon_{kc}). \quad (10b)$$

Here d_c is the critical period, $\varkappa_{c0} = \varkappa_c(0)$, $w(\varepsilon_{kc})$ is the Heaviside step function, and $\varepsilon_{kc} = h_\parallel^2 - h_{\parallel kc}^2$, $h_{\parallel kc} = 2\cdot 3^{1/2}\pi\varkappa_{c0}\beta_u^{-1/2}$ is the ordinate of a tricritical point which separates the first- and second-order phase transition curves across which a stripe domain structure with equilibrium critical period d_c changes to a uniformly magnetized state. Since $\varepsilon_{kc} < 0$ implies a second-order phase transition, the curves for $h_{\perp c}^*(h_\parallel)$ and $h_{\perp c}(h_\parallel)$, and also for $\varkappa_c^*(h_\parallel)$ and $\varkappa_c(h_\parallel)$, must coincide in this range of $h_\parallel$ (see expressions (9a), (9b), (10a), and (10b), along with Fig. 1); in addi-

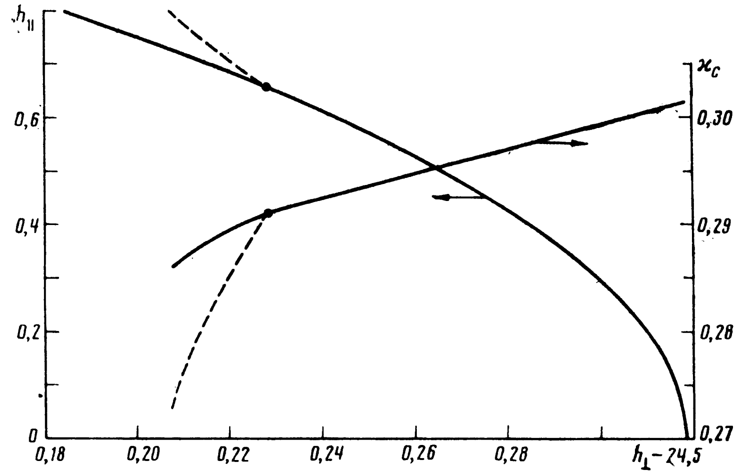

FIG. 1. Theoretical state diagram for a film corresponding to a first-order phase transition (dashed curves) and a second-order phase transition (solid curves) between uniformly magnetized states and an equilibrium domain structure, and the corresponding field dependence of the normalized wave vector $\varkappa_c$ of the critical domain structure. The value $\varkappa_c = \varkappa_{c0}$ at the tricritical point is 0.3014, $\alpha = 10^{-10}\,\text{cm}^2$, $l_z = 10^{-3}$ cm, $\beta_u = 25$; for simplicity we have plotted the difference between the normalized transverse field $h_\perp$ and a constant chosen to equal 24.5 along the abscissa.

tion, the order parameter λa (i.e., the amplitude of the z-component of the magnetization in the domain structure) must vanish on a second-order phase transition curve. In plotting the curves we have used values $\alpha = 10^{-10}\,\text{cm}^2$ and $l_z = 10^{-3}$ cm that are typical of the magnetic garnet films used in experiment. The values of the uniaxial anisotropy constant were intentionally taken to be rather small ($\beta_u = 25$), so as to separate the curve for loss of stability of the uniform state from the upper branch of the curve corresponding to a transition from the stripe domain structure to a CMD lattice (see below).

Equations (9) and (10) were obtained by minimizing Eq. (6) for the free energy with respect to the order parameter, i.e., from

$$\frac{\partial \mathcal{F}}{\partial(\lambda a)}=0, \qquad \frac{\partial^2 \mathcal{F}}{\partial(\lambda a)^2}\geqslant 0 \tag{11}$$

with subsequent minimization with respect to $\varkappa = 2\pi d^{-1}\alpha^{1/2}$, i.e., the normalized inverse period of the domain structure. If this last operation is not carried out, i.e., if we assume that the parameter $\varkappa$ is arbitrary but fixed, then the system (11) will determine the behavior of a nonequilibrium domain structure with $\varkappa \neq \varkappa_c$. The first of conditions (11) is identically Eq. (7); by choosing the equal sign in the second equation, we determine the location of the curve along which the stability of a domain structure with this $\varkappa$ is lost:

$$h_{\perp f}=h_{\perp c}-\varkappa^2(1-\zeta^2)^2, \tag{12a}$$

where $\zeta = \varkappa_c/\varkappa$, for a second-order phase transition and

$$h_{\perp f}^{*}=h_{\perp f}+0{,}388(\varkappa_c\varepsilon_k)^2 h_{\parallel k}^{-4} w(\varepsilon_k) \tag{12b}$$

for a first-order phase transition; the amplitudes of the z component of the magnetization in the domain structure on these curves are given by

$$(\lambda a)_f = 8\frac{\varkappa_{c0}}{\pi\varkappa}\left|\frac{\varepsilon_k w(\varepsilon_k)E_1}{2077E_2}\right|^{1/2}. \tag{13}$$

Here $\varepsilon_k = h_\parallel^2 - h_{\parallel k}^2$, $E_1 = 9-\zeta_0^4$, and $E_2 = 3-\zeta_0^4$; the ordinate $h_{\parallel k}$ of the tricritical point, which separates the first- and second-order phase transition curves across which the stripe domain structure with nonequilibrium period d changes to a uniformly magnetized state, is given by the expression

$$h_{\parallel k}=4\pi\varkappa\left(\frac{3E_2}{\beta_u E_1}\right)^{1/2}.$$

It follows from (12) that the curve for loss of stability of a domain structure with fixed period d on the plane $(h_\perp, h_\parallel)$ is displaced to the left of the corresponding curve for a domain structure with the equilibrium period; for $d > d_{c0}$, i.e., $\varkappa < \varkappa_{c0}$, this curve is tangent to the equilibrium curve for $d = d_c$. Thus, the curve which delineates loss of stability for the domain structure with equilibrium period consists of the envelope of the family of curves $h_{\perp f} = f(h_{\perp f}, \varkappa)$, for $\varkappa < \varkappa_{c0}$. We note, however, that quasistatic variation of the magnetic field cannot cause a transition from the equilibrium domain structure to a uniformly magnetized state, due to the development of prior instabilities of a different type, as we will show below. Since these instabilities occur at finite amplitudes of the order parameter, we determine the field dependence of λa by using Eq. (7). As a result we find that

$$(\lambda a)^2=\frac{(B_4^2-4B_2B_6)^{1/2}-B_4}{2B_6}. \tag{14}$$

In the limit $\Delta h_\perp = h_{\perp f} - h_\perp \ll \varkappa^2 h_{\parallel k}^{-2}|\varepsilon_k|$, we have

$$(\lambda a)^2=\frac{32}{9}\frac{\Delta h_\perp h_{\parallel k}^2}{\beta_u|\varepsilon_k|}, \tag{15a}$$

while for $\varkappa^2 h_{\parallel k}^{-2}|\varepsilon_k| \ll \Delta h_\perp \ll \varkappa^2$ we have

$$(\lambda a)^2=3{,}24\beta_u^{-1}\varkappa_{c0}(\Delta h_\perp)^{1/2}. \tag{15b}$$

In order to determine the boundaries of stability of a domain structure with fixed period, we first determine the spectrum of spin waves for a film with this domain structure. Linearizing Eq. (4a) with respect to θ, we obtain the equation

$$(\mu\nabla_x^2+\nabla_y^2)\left[\left(-\alpha\nabla^2+h_\perp-\beta_u+\frac{3\beta_u\theta_0^2}{2}\right)\tilde\theta + \frac{(\lambda_r^*-\lambda_e\nabla^2)\dot{\tilde\theta}}{\omega_0}+\frac{\ddot{\tilde\theta}}{\mu h_\perp\omega_0^2}\right]+4\pi\nabla_z^2\tilde\theta=0, \tag{16}$$

whose solution with the boundary conditions (2) has the form[2,18]

$$\tilde\theta=\sum_{n=-\infty}^{+\infty}\sum_{p=1}^{+\infty}\left(A_{np}\cos\frac{\pi pz}{l_z}+B_{np}\sin\frac{\pi pz}{l_z}\right)\exp[i(\mathbf{k}_n\mathbf{r}-\omega t)], \tag{17}$$

where $\mathbf{k}_n = \mathbf{Q} + nk\mathbf{e}_x$. The frequencies of the spin waves are smallest for $p=1$ and $k_n^2 = k^2$; therefore, modes with $n = \pm 1$ are strongly coupled at the center of the Brillouin zone ($|\mathbf{Q}| \ll k$), for which

$$\omega_{1,2}^2 + \frac{i\Gamma\omega_{1,2}}{\rho} = \frac{1}{\rho}\Big\{ C_0\frac{\Delta h_\perp}{\alpha} + (C_0E_3 - i\omega\eta_y)Q_x^2 + C_y^* Q_y^2 + \tilde{C}k^2 \pm C_0\Big[\frac{(\Delta h_\perp)^2}{\alpha^2} + 4k^2Q_x^2\Big(E_4 - \frac{i\omega\eta_y}{C_0}\Big)^2\Big]^{1/2}\Big\}; \quad (18)$$

here $\rho = (\lambda a)^2 k^2 (2\mu\beta_u g^2)^{-1} l_z$ is the effective density of the stripe domain structure, $C_i^*(\omega) = C_i - i\omega\eta_i$ is the effective elastic modulus ($i = x, y$), and

$$E_3 = 1 + 3\zeta_0^4, \quad E_4 = 1 - \zeta_0^4, \quad \tilde{C} = (C_1Q_x^4 + 2C_2(Q_xQ_y)^2 + C_3Q_y^4)k^{-4}, \quad C_1 = C_0\zeta_0^4, \quad C_2 = C_1\mu^{-1}, \quad C_3 = C_1\mu^{-2},$$
$$C_0 = {}^1/_2(\lambda a)^2\varkappa^2M^2l_z \quad \Gamma = \rho\mu\beta_u\omega_0(\lambda_r^4 + \lambda_e k^2),$$
$$\eta_y = \lambda_e C_0(\alpha\omega_0)^{-1};$$

the value of C_y is determined by the expression

$$C_y = C_0\Big\{1 - \frac{\zeta_0^4}{\mu} + \frac{6\Delta h_\perp h_\parallel^2}{|\varepsilon_k|\varkappa^2}\frac{E_2}{E_4}\Big[\frac{2}{E_2^2} + \frac{1}{9}\Big(1 - \frac{1}{4}\frac{\zeta_0^4}{\mu}\Big)\Big]\Big\}. \quad (19)$$

For $\Delta h_\perp \gg |\alpha k Q_x E_4|$ we have

$$\omega_1^2 + \frac{i\Gamma\omega_1}{\rho} = 2\mu\beta_u\omega_0^2\Delta h_\perp + C_0\Big[E_3 + \frac{2\varkappa^2E_4^2}{\Delta h_\perp} - \frac{i\omega\lambda_e}{\alpha\omega_0}\Big(1 + \frac{4\varkappa^2E_4}{\Delta h_\perp}\Big)\Big]\frac{Q_x^2}{\rho} + \frac{C_y^*Q_y^2}{\rho}, \quad (20)$$

$$\omega_2^2 + \frac{i\Gamma\omega_2}{\rho} = \frac{1}{\rho}\Big(C_x^*Q_x^2 + C_y^*Q_y^2 + \tilde{C}k^2 + \frac{1}{4}\frac{C_0}{k^2}\Big(\frac{E_3}{E_4}\Big)^2 Q_x^4\Big), \quad (21)$$

where

$$C_x = C_0E_3(1 - \Delta h_{\perp m}/\Delta h_\perp), \quad \Delta h_{\perp m} = 2\varkappa^2E_3^{-1}E_4^2, \quad \eta_x = \frac{\lambda_e C_0}{\alpha\omega_0}\Big(1 - \frac{4\varkappa^2}{\Delta h_\perp E_4}\Big). \quad (22)$$

The frequency of the optical mode ω_1 ($Q=0$), which is determined by expression (20), reduces to zero along the curve $h_\perp = h_{\perp f}(h_\parallel)$; this corresponds to loss of stability of the domain structure with fixed period with respect to a transition to a uniformly magnetized state. The mode with frequency ω_2 [see (18)] is acoustic, and reproduces the translational symmetry of the system. The moduli C_x and C_y determine the rigidity of the domain structure with respect to longitudinal displacements of the domain walls and kink-like deformations, respectively.

Making use of Eq. (18) and requiring that

$$\omega_2^2(h_\perp, Q_x^2) = 0, \quad \partial\omega_2^2(h_\perp, Q_x^2)/\partial(Q_x^2) = 0,$$

we can determine the stability boundary for a stripe domain structure with respect to longitudinal displacements of the domain walls:

$$h_\perp = h_{\perp m}(h_\parallel) = h_{\perp f} - \Delta h_{\perp m},$$

and also the critical value of wave vector Q_{xm}, which is found to equal zero; in this case, the elastic modulus also reduces to zero. The instability corresponding to this soft mode, which appears for a finite-amplitude z component of the magnetization in the domain structure, corresponds to a uniform shift of the domains.

Note that the function $h_{\perp m}(h_\parallel)$ determines the curve for loss of translational stability of the domain structure only for the case of a quasistatic (infinitely slow) increase in the magnetic field intensity. In reality, changes in the field always takes place at a finite rate, which may alter the situation considerably. Thus, if the field intensity $h_\perp$ starts out smaller than $h_{\perp m}$ in the original state, and then is rapidly (compared to the relaxation time of the domain structure) increased to a value $h_\perp + \delta h_\perp > h_{\perp m}$, the mode with maximum growth rate will have a nonzero value of wave vector

$$Q_{xm}^{cr} = 2^{1/2}\Big(\frac{kE_4}{E_3}\Big)\Big(1 - \frac{\Delta h_\perp}{\Delta h_{\perp m}}\Big)^{1/2},$$

while in this case

$$\omega_2^{cr} = i\Big(\frac{C_0\Delta h_{\perp m}}{2\alpha\rho}\Big)^{1/2}\Big(1 - \frac{\Delta h_\perp}{\Delta h_{\perp m}}\Big).$$

In a film with finite transverse dimensions l_x, for which the allowed values of wave vector are $Q_{xn} = (2n-1)l_x^{-1}$, where $n = 1,2,...$, a stripe domain structure becomes unstable against harmonic modulation of the positions of the domain walls with period $D_n = 2\pi Q_{xn}^{-1}$ at a field intensity $h_\perp = h_{\perp f} - \Delta h_{\perp n}$, where

$$\Delta h_{\perp n} = \Delta h_{\perp m}\Big[1 - \frac{\pi^2(2n-1)^2\zeta_0^4}{(kl_x)^2E_3}\Big].$$

Naturally other instabilities may develop earlier for $n = 1$. In addition, the assumption that a distribution of coercive forces exists in the film (even an unbounded film) leads to a finite value for the wave vector of the soft mode.

In order to analyze the stability of stripe domain structures with fixed period against a transition to a hexagonal CMD lattice, we follow Ref. 2 and define a minimum value of the spin-wave frequency for

$$Q_x = k/2, \quad Q_y = Q_x[1 + E_5(2\beta_uE_6)^{-1}]E_6^{1/2},$$

where $E_5 = 2\zeta_0^2 - 1$, $E_{16} = 4\zeta_0^2 - 1$.

The calculation shows that

$$\omega^2 + i\rho^{-1}\Gamma\omega = \mu\beta\omega_0^2|h_{\perp p} - h_\perp|, \quad (23)$$

where the equation for the curve of loss of stability of the domain structure has the form

$$h_\perp = h_{\perp p}(h_\parallel) = h_{\perp f} - \varkappa_{c0}^2\{\tilde{h}_{\parallel p}^2 - \tilde{h}_p^2 \pm [\tilde{h}_{\parallel p}^2(\tilde{h}_{\parallel p}^2 - 2\tilde{h}_p^2)]^{1/2}\}, \quad (24)$$

while

$$\tilde{h}_{\parallel p} = \frac{4h_\parallel}{3\pi^2}\Big(\frac{\beta_u}{\varkappa_{c0}^2}\Big)^{1/2}, \quad \tilde{h}_p = \frac{1}{\zeta_0^2}\Big(\frac{\pi}{\beta_u}E_6 - E_7^2\Big),$$
$$E_7 = \zeta_0^2 - 1.$$

In a stripe domain structure under tension ($d > d_s(T,H)$), relaxation of "elastic" stresses can be mediated by the appearance of sinusoidal distortions of the domain wall profiles.[7-11] If we limit ourselves to a discussion of films for which $l_x \gg (\alpha l_z)^{1/4}$, we find $Q_x \approx \pi l_x^{-1}$; then the critical value of the magnetic field intensity $h_{\perp s}$ and the period of the sinusoidal distortion Λ_s, which are determined by the conditions

$$\omega^2(h_{\perp s}, Q_y^2)=0, \quad \frac{\partial\omega^2(h_{\perp s}, Q_y^2)}{\partial(Q_y^2)}=0,$$

are given by the expressions

$$h_{\perp s}=h_{\perp f}-\frac{8}{7}\frac{\varkappa^2}{h_\parallel^2}\varepsilon_k\left[1-\frac{\zeta_0^4}{\mu}+\frac{2\pi(C_3C_x)^{1/2}}{l_xC_0k}\right], \tag{25a}$$

$$\Lambda_s\approx(l_xd)^{1/2}; \tag{25b}$$

while

$$C_y=-2\pi(C_3C_x)^{1/2}, \quad Q_{ys}=\frac{2\pi}{\Lambda_s}=\frac{(\pi^2k^2C_x)^{1/2}}{(C_3l_x^2)^{1/4}}. \tag{26}$$

Thus, in a film with finite transverse dimensions the period of sinusoidal distortions is finite and proportional to $l_x^{1/2}$. Since the equilibrium period d_s increases as $h_\perp$ decreases and $h_\parallel$ increases, sinusoidal distortions of the shape of the domain walls occur in that region of the $(h_\perp, h_\parallel)$ plane of the phase diagram in which $h_\perp(h_\parallel) > h_{\perp s}$. If the transverse dimensions of the film are unbounded, then the wave vector Q_{ys} of the soft mode corresponding to the appearance of kink-like instability reduces to zero, and the period of the sinusoidal distortions Λ_s goes to infinity. The expression for the critical field intensity $h_{\perp s}$ is determined by Eq. (25a) as $l_x\to\infty$. The presence of a distribution of coercive forces in an unbounded film causes the wave vector of the soft mode Q_{ys} to become finite.[7,11] In real films (with finite transverse dimensions and coercive fields) the value of Q_{ys} will be determined by the combined action of these two factors.

When the magnetic field intensity $h_\perp$ is increased rapidly (from a value smaller than $h_{\perp s}$ in the "above-threshold" region $h_\perp > h_s$), the mode with maximum growth rate (for an ideal unbounded film) has a wave vector

$$Q_{ys}^{cr}=\frac{k}{2^{1/2}}\left[\left(\frac{\zeta_k^4}{\mu}-1\right)\left(1-\frac{\Delta h_\perp}{\Delta h_{\perp s}}\right)\right]^{1/2},$$

for which

$$\omega_2=\frac{ikC_0^{1/2}}{2\rho}\left(\frac{\zeta_k^4}{\mu^{-1}}-1\right)\left(1-\frac{\Delta h_\perp}{\Delta h_{\perp s}}\right).$$

In order to investigate the nonlinear dynamics of shape distortions in the domain structure it is necessary to introduce a field $u\equiv u_x(x,y)$ that specifies the displacement of a point with a given value of magnetization from its position in the regular domain structure, i.e., to represent the distribution of magnetization in the form[1,8,9]

$$\theta(\mathbf{r},t)=\sum_{n=0}^{\infty}\lambda^nA_n(z)\cos\{nk[x-u(x,y,t)]\},$$
$$\psi(\mathbf{r},t)=\sum_{n=0}^{\infty}\lambda^nB_n(z)\sin\{nk[x-u(x,y,t)]\}. \tag{27}$$

If we limit ourselves to a discussion of long-wavelength shape distortions in the domain structure, we can write the "elastic" part of the free energy associated with displacements of the domain walls in the form

$$\mathscr{U}=\mathscr{U}_L+\mathscr{U}_{NL}, \tag{28}$$

where

$$\mathscr{U}_L=\frac{1}{2}\int dx\,dy[C_x(\nabla_xu)^2+C_y(\nabla_yu)^2+k^{-2}C_3(\nabla_y^2u)^2], \tag{28a}$$

$$\mathscr{U}_{NL}=\frac{1}{2}\int dx\,dy[C_{1,2}\nabla_xu(\nabla_yu)^2+C_{0,4}(\nabla_yu)^4]. \tag{28b}$$

Here $C_{1,2}=-4\zeta_0^4C_0$ and $C_{0,4}=\zeta_0^4C_0$. In expression (28b) we have included only those terms in the free energy expansion that are necessary for further calculations, i.e., those terms that are determined by the symmetries of the stripe domain structure (the point group D_{2h}).

In order to obtain the equations of motion for the domain wall displacement, we will also introduce a kinetic energy and dissipation function:

$$\mathscr{T}_{kin}=\frac{1}{2}\int dx\,dy\,\rho\dot{u}^2, \tag{29}$$

$$\mathscr{W}_d=-\frac{1}{2}\int dx\,dy[\Gamma\dot{u}^2+\eta_x(\nabla_x\dot{u})^2+\eta_y(\nabla_y\dot{u})^2]. \tag{30}$$

Then the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial\mathscr{L}}{\partial\dot{u}}+\frac{\partial}{\partial x_i}\frac{\partial\mathscr{L}}{\partial(\nabla_iu)}=\frac{\partial\mathscr{W}_d}{\partial\dot{u}}-\frac{\partial}{\partial x_i}\frac{\partial\mathscr{W}_d}{\partial(\nabla_i\dot{u})} \tag{31}$$

($\mathscr{L}=\mathscr{T}_{\rm kin}-\mathscr{U}_{\rm NL}$ is the Lagrangian), which completely determines the nonlinear behavior of the domain walls, takes the form

$$\rho\ddot{u}+\Gamma\dot{u}=C_i^*\nabla_i^2u-C_3\frac{\nabla_y^4u}{k^2}+\frac{\partial}{\partial x_i}\frac{\partial\mathscr{U}_{NL}}{\partial(\nabla_i\dot{u})}. \tag{32}$$

In the linear approximation, the following function is a solution to Eq. (32):

$$u=u_0\exp[i(\mathbf{Q}_\perp\mathbf{r}_\perp-\omega t)].$$

When this function is substituted into (32), it yields the dispersion relation for acoustic modes in the form (21).

In order to calculate the amplitude of sinusoidal distortions of the domain wall profiles for $h_\parallel > h_\perp$ we use the method developed in Ref. 6 for analyzing the distribution of magnetization in a domain wall formed during a second-order phase transition. We seek the static ($u=0$) solution to Eq. (32) in the form

$$u=u_1\cos(Q_xx)\cos(Q_yy)+{}^1/_{16}C_{1,2}Q_xQ_y^2u_1^2\sin(2Q_xx)$$
$$\times[-C_x^{-1}Q_x^{-2}+3\cos(2Q_yy)C_Q^{-1}], \tag{33}$$

where $C_Q=C_xQ_x^2+C_yQ_y^2+4C_3k^{-2}Q_y^4$; here u_1 is a small parameter. Using the boundary condition

$$u=0|_{x=\pm l_x/2},$$

we find that $Q_x=\pi(2n+1)l_x^{-1}$ ($n=0, 1,\ldots$). The minimum in the free energy (28) is reached for $n=0$. Using Eq. (32), we obtain an equation for the amplitude of the sinusoidal distortion:

$$\pi^2C_xl_x^{-2}+C_yQ_y^2+C_3k^{-2}Q_y^4+{}^9/_8u_1^2C_{0,4}^*Q_y^4=0, \tag{34}$$

where $C_{0,4}^*=C_{0,4}-\frac{1}{36}C_{1,2}^2(2C_x^{-1}+9C_Q^{-1}Q_x^2)$. Substituting (33) into (28), we find that the free energy of the system is $\mathscr{U}=-(9/128)C_{0,4}^*(u_1Q_y)^4$. The minimum value of $\mathscr{U}$ is reached for the maximum value of u_1^2, i.e., for $Q_y=Q_{ys}$. In this case it follows from Eq. (34) that

$$Q_y^2u_1^2=\frac{7}{9}\frac{C_0(h_\perp-h_{\perp s})h_\parallel^2}{C_{0,4}^*\varepsilon_k\varkappa^2}, \tag{35}$$

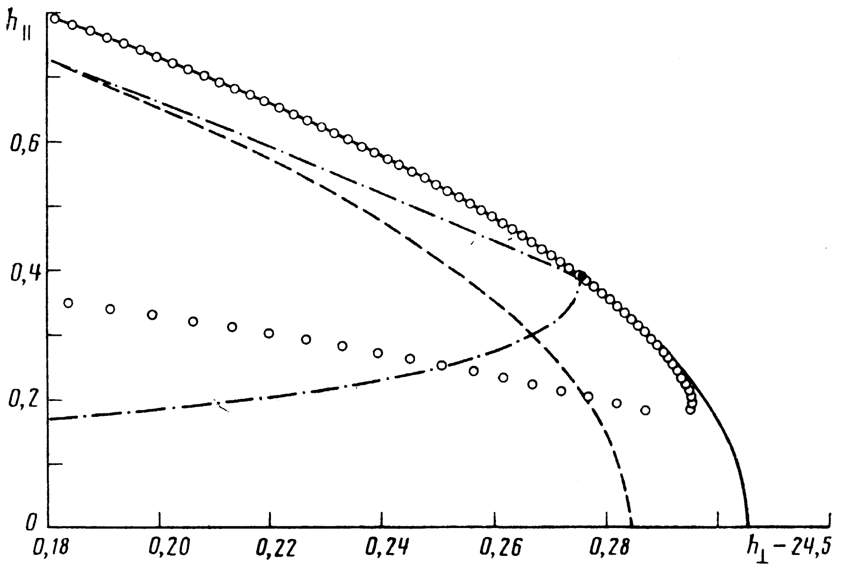

FIG. 2. Theoretical functions $h_{\|}(h_{\perp})$ corresponding to the onset of instabilities of various types for a nonequilibrium domain structure with normalized wave vector $\varkappa = 0.25$ (the values of the remaining parameters are the same as in Fig. 1). The dotted-dashed curves are for sinusoidal distortion of the domain wall profiles, the dashed curves are for translational instability, the open circles are for a transition to a hexagonal CMD lattice, the solid curves are for a transition to a uniformly magnetized state (for a nonequilibrium domain structure this is not possible).

where

$$C_{0,4}^{*}=C_{0,4}-\frac{5}{36}\frac{C_{1,2}}{C_x}.$$

The arguments presented above indicate that the phase transition which results in sinusoidal distortions of the domain wall profiles is second-order for $C_{0,4}^{*}>0$. However, as $h_{\perp}$ approaches $h_{\|}$ the quantity C_x decreases, so that the coefficient $C_{0,4}^{*}$ can change sign. In this case the phase transition becomes first-order, so that the sinusoidal distortions that appear have finite amplitude.

Results of our theoretical investigation of the stability of stripe domain structures with fixed periods are shown in Figs. 2–5, where we have plotted curves on the $(h_{\perp}, h_{\|})$ plane along which such domain structures lose their stability with respect to formation of sinusoidal shape distortions (the dotted-dashed curves), transitions to a CMD lattice (the open circles), and displacement of the positions of the domain walls (dashed curves) for various values of the normalized period $\varkappa$. The values of α, l_z, and β_u were chosen in the same way as in Fig. 1. Calculations for large values of β_u show that no qualitative changes in the results occur; however, it becomes difficult to plot these results graphically, because a number of curves almost merge (see the remarks at the beginning of Sec. 1.2). The curve for loss of stability of the domain structure with respect to a transition to a uniformly magnetized state, which cannot be produced by a quasistatic increase in the magnetic field intensity, is shown as a solid trace.

It is clear that for $\varkappa \leqslant \varkappa_{c0}$ the nonequilibrium domain structure is unstable with respect to both a transition to a CMD lattice and the appearance of sinusoidal distortions of the domain wall profiles (Fig. 2); as $\varkappa$ approaches $\varkappa_{c0}$ it also becomes possible for translational instability to occur (Figs. 3 and 4). For $\varkappa > \varkappa_{c0}$, the only possibilities are either a transition to a CMD or translational instability (Fig. 5); the curve for the latter shifts towards the region of small values of $h_{\perp}$ as $\varkappa$ increases.

Including the finite transverse dimensions of the film affects the position of these curves only slightly. Thus, e.g., even for $l_x = 1$ mm the curve for nonuniform (modulated) translational instability in the positions of the domain walls

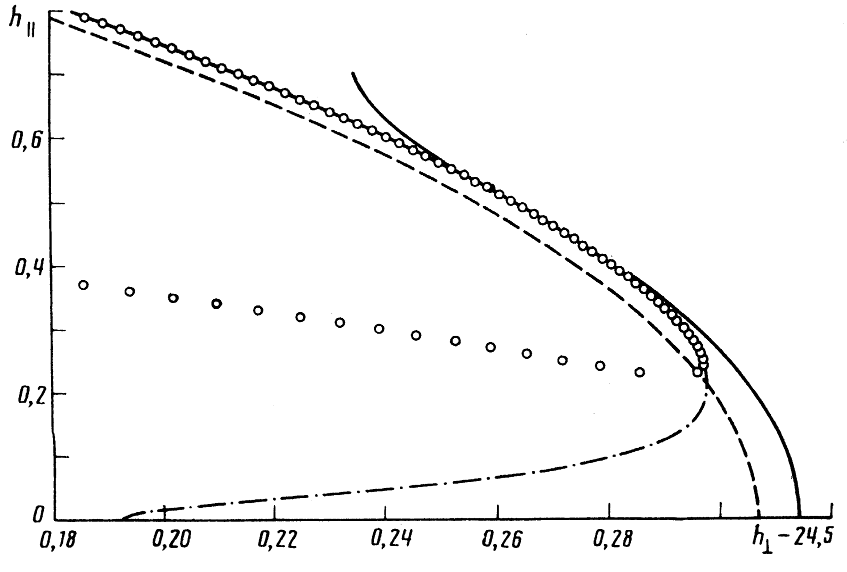

FIG. 3. Theoretical functions $h_{\|}(h_{\perp})$ corresponding to the onset of instabilities of various kinds for a nonequilibrium domain structure with normalized wave vector $\varkappa = 0.271$ (the values of the other parameters are the same as in Fig. 1; the notation of the curves is analogous to that used in Fig. 2).

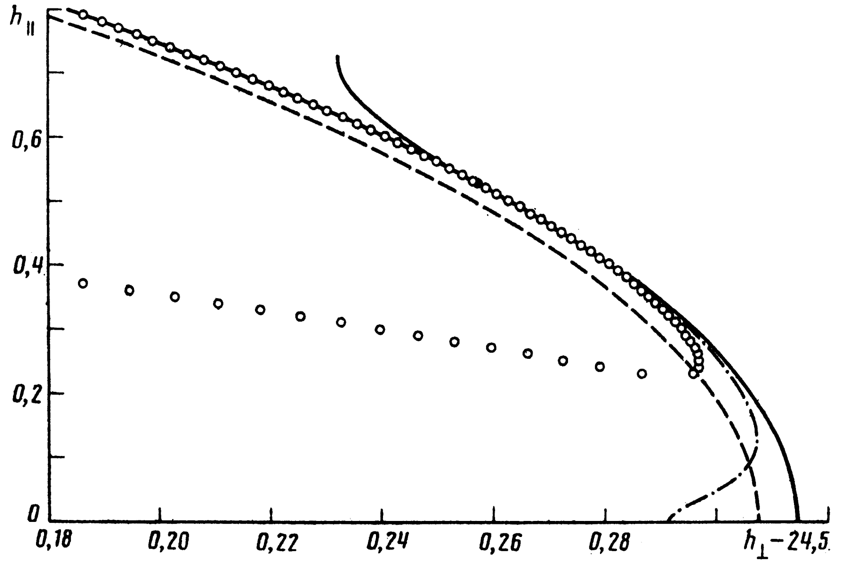

FIG. 4. Theoretical functions $h_{\parallel}(h_{\perp})$ corresponding to the onset of instabilities of various types for a nonequilibrium domain structure with normalized wave vector $\varkappa = 0.272$ (the values of the remaining parameters are the same as in Fig. 1, and the notation of the curves is analogous to that used in Fig. 2).

deviates from the curve for $l_x \to \infty$ by only 10^{-8}.

As we have already mentioned, for $T \neq 0$ the period of the domain structure can be changed not only by the appearance of instabilities of various types, but also by the formation of magnetic dislocations and subsequent translation of these dislocations under the action of "elastic" stresses.

It is noteworthy that for $h_{\parallel} \neq 0$ domains with $\mathbf{M}\cdot\mathbf{H} > 0$ and with $\mathbf{M}\cdot\mathbf{H} < 0$ have different widths (d_1 and d_2); for $|h_{\parallel}| \leqslant 8\lambda a$ the period of the domain structure is given by the expression

$$d = d_1 + d_2 = {}^1/_2 d(1+\varepsilon_d) + {}^1/_2 d(1-\varepsilon_d), \qquad (36)$$

where $\varepsilon_d \approx \arcsin(h_{\parallel}/2\pi^2\lambda a)$. The equilibrium period d_s for $|h_{\parallel}| \ll h_{\parallel kc}$ depends on the field $\mathbf{H}$ in the following way:

$$d_s = d_{1s} + d_{2s} = {}^1/_2 d_s(1+\tilde{\varepsilon}_d) + {}^1/_2 d_s(1-\tilde{\varepsilon}_d)$$
$$= d_{c0}\left\{1 + \frac{\Delta h_{\perp}}{4\varkappa_{c0}^2}\left[\frac{7}{8}\frac{h_{\parallel}^2}{|\varepsilon_{kc}|} + \frac{\alpha^{1/4}}{L^{1/2}(4\pi\mu)^{3/4}}\right]\right\}, \qquad (37)$$

where

$$\tilde{\varepsilon}_d = \varepsilon_d(\lambda a)_s, \qquad (\lambda a)_s = \frac{32}{9\beta_u}\frac{h_{\parallel kc}^2}{|\varepsilon_{kc}|}(h_{\perp c} - h_{\perp}).$$

Generally speaking, this type of asymmetry in the domain structure can modify the conditions for the appearance of instabilities of various types, especially far from the second-order phase transition curve, i.e., for $H_{\perp} \ll \beta_u M$.

In order to generalize the theory to domain structures in biaxial ferromagnetic films located in a magnetic field $|H_{\perp}| \ll \beta_u M$ and $|H_{\perp}| \sim 4\pi M$, we add to the free-energy density a term $\Delta\mathscr{F} = 1/2\beta_x M_x$, where β_x is the rhombic anisotropy constant ($0 < \beta_x \ll \beta_u$), and make use of the results of Refs. 13 and 18, in which the ground-state and spin-wave spectra were investigated for films of this kind with stripe domain structures and $\beta_x = 0$, $H_{\perp} = 0$. Following Ref. 18, we can show that the "elastic" part of the free energy $\mathscr{U}$ and the spectrum of the acoustic branch of the spin excitations are described by Eqs. (28) and (18), respectively, where

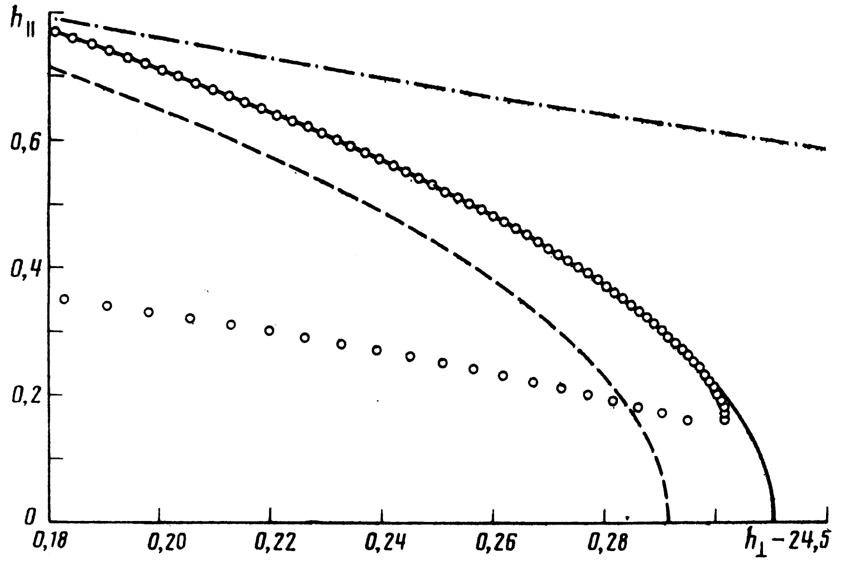

FIG. 5. Theoretical functions $h_{\parallel}(h_{\perp})$ corresponding to the onset of instabilities of various types for a nonequilibrium domain structure with normalized wave vector $\varkappa = 0.350$ (the values of the remaining parameters are the same as in Fig. 1; the notation of the curves is analogous to that used in Fig. 2).

$$C_x=4\pi k l_z^2 M^2\left(W_{\nu_1\nu_1}-\frac{W_{\nu_1\nu_2}^2}{W_{\nu_2\nu_2}}\right),$$

$$C_y=4\pi l_z M^2\left[W_{\nu_1}+\left(1+\frac{\beta_x}{\beta_u}+\frac{\pi H_\perp}{\beta_u M}\right)\frac{l_w}{l_z}\right],$$

$$C_3=12\frac{l_z M^2}{\nu_1}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[1-\left(1+\nu_1 n+\frac{\nu_1 n^2}{3}\right)\exp(-\nu_1 n)\right]\sin^2\frac{n\nu_2}{2},$$

$$\Gamma=\frac{2\lambda_r^* M(1+\zeta)}{g\Delta},\quad \rho=\bar{m},\quad \eta_i=0, \tag{38}$$

$$W_{\nu_i}=\frac{\partial W}{\partial \nu_i},\quad W_{\nu_i\nu_k}=\frac{\partial^2 W}{\partial\nu_i\,\partial\nu_k},\quad W=\nu_2\left[\frac{H_\parallel}{4\pi M}-1\right]$$

$$+\frac{2}{\pi\nu_1}\sum_{n=-\infty}^{\infty}\frac{1}{|n|^3}[1-\exp(-\nu_1|n|)]\sin^2\frac{\nu_2 n}{2}-\frac{1}{4}\frac{H_\parallel}{M}$$

$$+\frac{\pi}{2}+\nu_1\frac{l_w}{l_z}\left[1+\frac{\beta_x}{\beta_u}+\pi H_\perp\frac{\beta_u}{M}\right],\quad \zeta=\frac{\lambda_e}{3\lambda_r^*\Delta^2};$$

here $l_w=\sigma_0(4\pi M^2)^{-1}$ is a characteristic length in the material, and

$$\Delta=\Delta_0\left[1+\frac{\beta_x}{\beta_u}+\frac{\pi H_\perp}{2\beta_u M}\right];$$

m, σ_0, and Δ_0 are the effective mass, energy density, and width of a domain wall, respectively,[19] and $\nu_1=kl_z$, $\nu_2=kd_2$. The width of a domain d_2 at the center of which $\mathbf{M}\approx-M\mathbf{e}_z$ is calculated from the equation $W_{\nu_2}=0$, while the equilibrium period d_s is determined from the condition $W_{\nu_1}=0$. As in Ref. 18, in deriving Eq. (38) we have assumed that $\Delta\ll l_z$, and that the value of σ does not depend on the curvature of the domain wall.

Thus, with proper regard for the changes in notation implied by the relations given in (38), we can use results we have already obtained to find the boundaries of stability for the stripe domain structure. In particular, we can use Eq. (25) to determine the stability boundaries of a stripe domain structure with respect to the appearance of sinusoidal shape distortions of the domain walls, along with the period Λ_s of these distortions, by numerical calculations. In bounded samples, the distortion period Λ_s is finite (compare with Ref. 18), while for $l_x\to\infty$ it goes to infinity. A distribution of coercive forces can lead to a nonzero Λ_s in films with arbitrary transverse dimensions.

Because the period d_s increases as $|H_\parallel|$ increases, the stripe domain structure which forms for $|H_\parallel|=\text{const}\neq 0$ becomes unstable with respect to sinusoidal distortions in the domain wall profile as $|H_\parallel|$ decreases. As $|H_\parallel|$ increases, a reconstruction of the domain structure takes place due to a decrease in the thickness d_2 of energetically unfavorable domains, which can cause them to collapse or break up with the formation of dislocations.

Note that we cannot use Eq. (38) to obtain conditions for stability of a stripe domain structure with respect to a transition to a CMD lattice, or for the appearance of translational (acoustic) soft modes, since this equation was derived by using the approximation of geometric domain walls and distributions of the vector **M** within individual domains that are unaffected by the action of the field $H_\parallel$. If we build into the theoretical model the vortex character of the distribution of the vector **M** in a stripe domain structure (i.e., the fact that there exist twisted domain walls and a nonuniform distribution of **M** within the domains themselves that depends on magnetic field), these instabilities can indeed arise; however, they do so only in the case where all the domain walls in the domain structure are "single-polarity," i.e., where the direction of the vector **M** is the same at the center of every twisted domain wall. In this case, the vector **M** at the center of a domain at the film surface makes an angle with the surface normal which increases as the period of the domain structure d decreases, while the amplitude of the change in the component M_z in the domain structure decreases at the same time. Strictly speaking, the distribution of the vector **M** in a domain structure with single-polarity twisted domain walls is topologically equivalent to the distribution of **M** in a domain structure close to an orientational phase transition; therefore, all the conclusions of the theory developed at the beginning of this section apply to this case as well.

Another possible type of stripe domain structure is one with opposite-polarity twisted domain walls. For this structure, a translation along the x-axis by $d/2$ causes the vector **M** at the center of a domain wall to change direction by 180°. In this case, the vector **M** at the center of a domain at the film surface is oriented along the normal for any period d of the domain structure, and only sinusoidal instabilities of the domain wall profile are possible for $d>d_s$. For $d<d_s$, no instability occurs as d decreases, although in this case the energy of the domain structure goes to infinity.[2)] Since the domain structure model with geometric domain walls used in Ref. 13 is essentially equivalent to a domain structure with opposite-polarity domain walls, the conclusions of that paper regarding the possible existence of instabilities of such domain structures with respect to discontinuous changes in the period are incorrect.

Let us consider how a field $H_\perp$ affects these results. Since the magnitude of d_s decreases as $H_\perp$ increases (for $H_\parallel=\text{const}$), a stripe domain structure that forms for some value of $H_\parallel$ with $H_\perp=0$ becomes unstable with respect to the appearance of a sinusoidal modulation in the domain wall profiles as $H_\perp$ increases for any type of initial domain wall polarization. If the field $H=\mathbf{H}_\perp$ is directed along the domain walls, under certain conditions repolarization of the vectors **M** is possible at the centers of these domains, i.e., a transition from a structure with opposite-polarity domain walls to a single-polarity domain wall structure.[19] This repolarization can occur via the creation of pairs of vertical Bloch lines; their subsequent mutual repulsion then drives them beyond the edges of the film.

These results can be easily generalized to other phase transitions, both first- and second-order, that are accompanied by reconstruction of the period of a stripe domain structure as the external parameters change (temperature, magnetic field, pressure, etc.): for example, in phase-space regions where reorientation of the magnetic moments occurs in orthoferrites, in regions of existence of intermediate states in antiferromagnets, in the neighborhood of compensation points in ferromagnets, etc.

The theory we have developed can also be used to describe the behavior of metastable domain structures in a film of uniaxial ferroelectric placed in a plane capacitor with a gap of $d/2$ between the electrodes and the surface of the ferroelectric, which is in the neighborhood of its Curie point. For this it is sufficient to write the free-energy density in the form[20]

$$\mathcal{F} = \frac{1}{V}\left\{\int\left[\frac{1}{2}\tilde{\alpha}(\nabla P)^2 - \frac{1}{2}\xi P_z^2 + \frac{1}{2}\xi_\perp P_z^2 + \frac{1}{4}\tilde{\delta} P_z^4 + \frac{E^2}{8\pi}\right]dv + \sum_{i=1}^{2}\int_{S_i} dS_i \sigma_i U_i\right\},$$

where $\xi = (T_0 - T)\xi_0^{-1}$, $\mathbf{P}$ is the polarization vector, $\tilde{\alpha}$ is the gradient energy parameter, $\xi_\perp^{-1}$ is the polarizability in the plane of the film, $\mathbf{E}$ is the electric field intensity, eU_i and σ_i are, respectively, the chemical potential and surface charge density at the ith electrode, e is the electron charge, and V is the volume of the ferroelectric, while in the expressions of Secs. 1.1 and 1.2 we make the following replacements:

$$h_\perp \to T\xi_0', \quad \beta_u \to 2\tilde{\delta}, \quad \mu \to 1 + 4\pi\xi_\perp^{-1},$$
$$C_0 \to {}^1/_2[\varkappa^2(\lambda a)^2 l_z], \quad \mu g^2 \beta_u M_0^2 \to \tilde{\omega}_0^2,$$
$$\mu\beta_u\omega_0^2 \to \tilde{\omega}_0^2, \quad A_0 \to E_0/(4\pi d l_z^{-1} - \xi).$$

Here $E_0 = U/e$, $U = U_1 + U_2$, and $\tilde{\omega}_0$ is the characteristic frequency in the Landau-Khalatnikov equation

$$\tilde{\omega}_0^{-1}\lambda_r \dot{\mathbf{P}} = -\delta\mathcal{F}/\delta\mathbf{P}.$$

2. EXPERIMENT AND DISCUSSION OF RESULTS

In our experimental investigations of instabilities of a regular stripe domain structure we used epitaxial films made of uniaxial ferrite garnets with the composition $(YGdYbBi)_3(FeAl)_5O_{12}$, grown on a nonmagnetic substrate of $Gd_3Ga_5O_{12}$ with (111) orientation. The intrinsic domain structure of these films is usually labyrinthine; a regular stripe domain structure is created using the following method. A wide-gap magnetic head is used to record a harmonic oscillogram on magnetic tape. The spatial wavelength d_0 of this oscillogram is close to the equilibrium period of the corresponding labyrinthine domain structure of the film d_s at room temperature. This tape with its recording is pressed against the magnetic film in close contact with it, and is then removed by sliding it across the entire surface of the film in such a way that the direction of translation of the tape coincides with the direction of the "dashes" of the recording. This procedure allows us to create an almost ideally regular stripe-domain structure in the film (Fig. 6a), which is preserved even after the magnetic tape is removed. As the external parameters ($H_\parallel$, $H_\perp$, and T) are varied, instabilities of various types develop in this regular stripe domain structure for certain critical values of these parameters. These critical values of the external parameters are interrelated through certain functional relations, which are conveniently represented by the corresponding curves plotted on state diagrams for the film in the planes $H_\perp H_\parallel$ (for T = const), $TH_\parallel$ (for $H_\perp$ = const), etc. The procedure for determining the form of these state diagrams is described in detail in Ref. 2; therefore, we limit ourselves here to listing the terminology used in that paper: O denotes a uniformly magnetized state, Π a stripe (labyrinthine) domain structure, and P_1 and P_2 different-polarity hexagonal (or amorphous) CMD lattices.[3)]

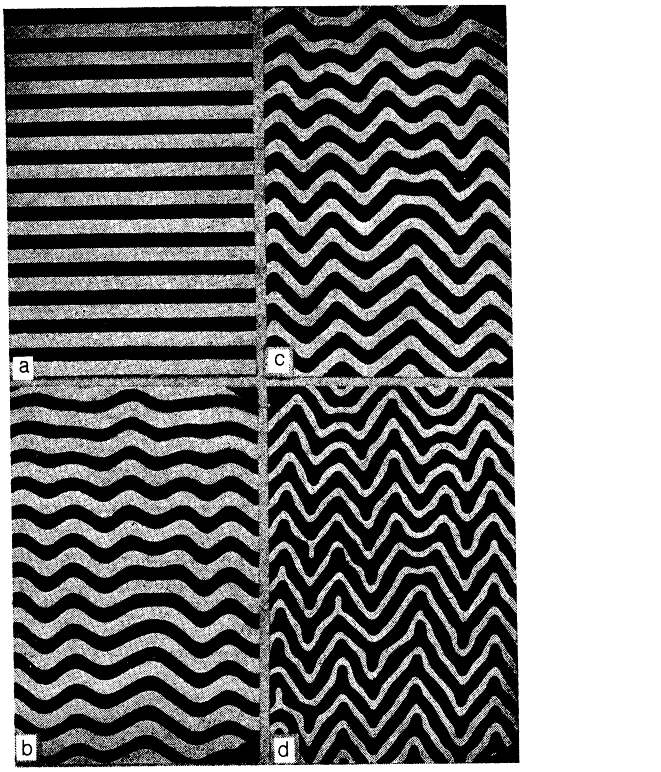

FIG. 6. Development of instabilities of a stripe domain structure as the curve for an orientational phase transition is approached in fields $H_\perp = 0$ (a), 1.17 kOe (b), 1.55 kOe (c), and 1.94 kOe (d).

Figure 7 shows a state diagram for film No. 1 and the location of curves for loss of stability of an original stripe domain structure with respect to a sinusoidal distortion of the domain wall profiles ($d_0 = 11$ μm; curve 1), a zigzag-

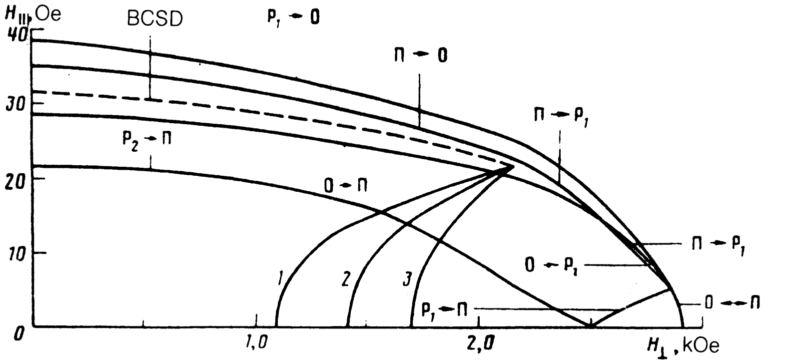

FIG. 7. Experimental state diagram of a domain structure with equilibrium period and the locations of curves for loss of stability of a domain structure with fixed original period with respect to the appearance of instabilities of various types on the $H_\perp H_\parallel$ plane; O is a uniformly magnetized state, Π is a stripe domain structure, and P_1 and P_2 are different-polarity CMD lattices; *1* is for sinusoidal instability for the domain wall profiles, *2* is for the appearance of a zigzag-shaped distortion of the domain wall profiles, and *3* is for the formation of fingers at the corners of the zigzags; BCSD denotes the beginning of collapse of the stripe domains (a manifestation of translational instability of the domain structure in real films).

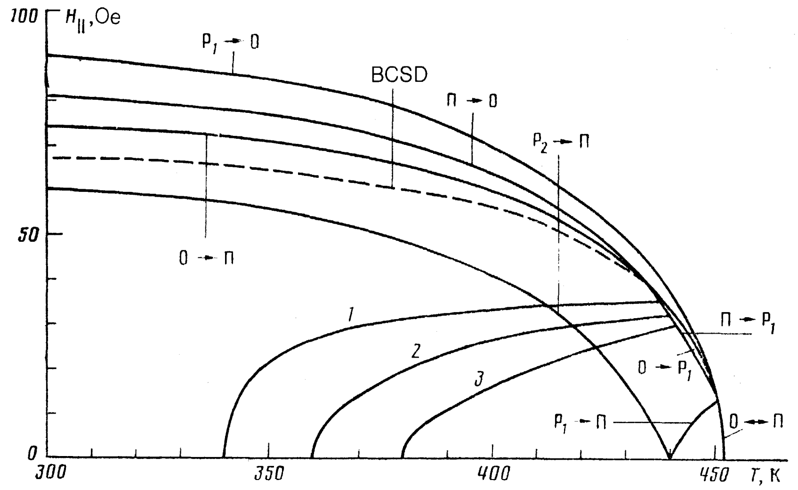

FIG. 8. Experimental state diagram of a domain structure with equilibrium period and the locations of the curves for loss of stability of a domain structure with fixed original period with respect to the appearance of instabilities of various types on the plane $H_{\parallel} T$. The notation is the same as in Fig. 7.

shaped distortion (2), and the appearance of fingers at the vertices of the zigzags (3); see also Fig. 3. The dashed curve marks the beginning of the collapse of individual stripe domains.[4)] The orientation of the projection $H_\perp$ in the plane of the film was chosen in such a way that the state diagram possessed mirror symmetry about the abscissa (compare with the diagram Fig. 2 in Ref. 2); in this case the period of the equilibrium domain structure is a monotonic function of $H_\perp$ for $H_{\parallel} = \text{const}$ (or $H_{\parallel}$ for $H_\perp = \text{const}$).

The locations of curves 1–3 and the curve which marks the beginning of the collapse of individual stripe domains are determined in the following way. We first create a regular stripe domain structure with period d_0 in the film by contact with magnetic tape, where d_0 coincides with the equilibrium period for $\mathbf{H} = 0$. Then the film is placed in the gap of an electromagnet, and for a preassigned value of $\vartheta_H = \arctan(H_{\parallel}/H_\perp)$ the reaction of the domain structure is observed visually as the field intensity increases. Once we have determined the position of a point on the curves 1–3 (or on the curve marking the beginning of the collapse of individual stripe domains), decreasing the intensity of the magnetic field to zero naturally does not lead to the reconstruction of the original stripe domain structure with period d_0; therefore, we once more carry out the operation of contacting the film with the magnetic tape, placing the film in the gap of the electromagnet, choosing another value of the angle ϑ_H, measuring the critical field, etc. The accuracy with which the critical fields were determined was ~ 1 Oe; in order to simplify the plots the experimental points are not shown in Figs. 7 and 8.

The location on the $H_\perp H_{\parallel}$ plane of the curve that marks the beginning of the collapse of individual stripe domains depends strongly on the density of magnetic dislocations in the domain structure. The curve shown in Fig. 7 was plotted for the minimum attainable dislocation density ($\leqslant 1\ \text{cm}^{-2}$); if there are many dislocations, then the process of dissociation of pairs and repulsion of the magnetic dislocations begins at much weaker fields $H_{\parallel}$.

For small ϑ_H (in the interval $|\vartheta_H| \leqslant 0.1$ for film No. 1) the period of the equilibrium domain structure decreases as the field intensity increases; therefore, an original domain structure with $d_0 = d_s(0) > d_s(H)$ has a tendency to become labyrinthine by passing through all the stages of instability. For larger values of $|\vartheta_H|$ we observed only a gradual collapse of the stripe domains, because $d_s(H) > d_s(0) = d_0$ holds in the presence of the magnetic field.

In films for which the anisotropy is predominantly uniaxial, the presence of cubic and rhombic components leads only to an insignificant change in the critical fields for appearance of instabilities of various types as we change the original direction of ordering of the domain walls in the stripe domain structure; the monotonic character of the function $d_s(H_\perp)$ for $\vartheta_H = \text{const}$ is not disrupted in this case. For $\beta_u \sim \beta_c$ (or $\beta_p \sim \beta_u$) this dependence can become nonmonotonic; therefore, e.g., after the development of a kink-like instability, further increases in $H_\perp$ can reestablish a structure with plane-parallel domain walls, etc. Strong anisotropy in the plane of the film suppresses kink-like instabilities of the domain walls.

Using analogous methods, we investigated the stability of the stripe domain structure with respect to spontaneous phase transitions. As an example we show in Fig. 8 the state diagram for film No. 2 ($d_0 = 9\ \mu\text{m}$) on the $TH_{\parallel}$ plane. The location of curves 1–3 and the curve for the beginning of the collapse of individual stripe domains (with notation the same as in Fig. 7) were determined by smoothly increasing the temperature with $H_{\parallel} = \text{const}$. The sole feature in the behavior of this domain structure that differs from what was discussed earlier is the existence of a certain narrow interval of values of $|H_\perp|$ for which the original domain structure transforms into a hexagonal CMD lattice (P_1 or P_2) on the curves $\Pi \to P_1$ or $\Pi \to P_2$ near the Curie temperature $T_c \approx 440$ K. The accuracy of our temperature measurements in these experiments came to ~ 0.1 K.

For films 1 and 2, the periods of the sinusoidal distortions in the shape of the domain walls were 26 and 25 μm, respectively (see, e.g., Fig. 6), and were weak functions of the external parameters. The results of Sec. 1.2 of this paper and Refs. 7 and 18 suggest that this may be due to the presence of a distribution of coercive forces in the film, in which case the wave vector of the "softened" mode with maximum growth rate depends strongly on the difference between the field $h_\perp$ and the threshold field $h_{\perp s}$ for the appearance of sinusoidal instability; it may also be due to the influence of the finite transverse dimensions of the film. Apparently, this latter reason can be discarded because films 1 and 2 were irregular in shape and possessed different transverse dimensions. Furthermore, we were unable to take into account nonuniformity of the properties of real films (in the plane of the structure) which first of all is equivalent to a coercive field, and secondly leads to a local variation of the critical field $h_{\perp s}$.

If magnetic dislocations are present in the original stripe domain structure, then the average period d of the domain structure can change within certain limits due to the motion of these dislocations. In a defectless film with finite dimensions for fixed values of $\mathbf{H}$ and T the dislocations are spaced in such a way that the average period d equals the thermodynamic equilibrium value $d_s(\mathbf{H},T)$ (if possible); a change in any of these parameters will cause translation of the dislocations without hysteresis (if they do not interact) to new positions.

Comparison of the experimental data with the conclusions of the theory indicate good agreement between them. All types of instabilities of a stripe domain structure predicted by the calculations were observed experimentally, including the kink-like distortions of the domain walls profile as well as their collapse and conversion to a CMD lattice.

We note that the problem of kink-like instabilities of a stripe domain structure was discussed in Ref. 11. However, the theoretical analysis carried out in Ref. 11 cannot be applied to real samples, because the model of an isotropic crystal used in that paper does not admit the existence of a regular stripe domain structure (the stable domain structure, according to the classification of Ref. 2, corresponds to the "liquid-crystal" phase). The expressions for the effective elastic modulus of the domain structure and the functional dependences of the domain structure period on the film thickness and field given in this paper are erroneous.

[1)] For sufficiently rigid boundary conditions the height of the barrier is large, and the probability of a transition between states with differing numbers of domains is vanishingly small.[6]

[2)] For an orientational phase transition from a uniformly magnetized state this type of domain structure does not nucleate.

[3)] The curves $\Pi \to O$, $\Pi \to P_1$, and $\Pi \to P_2$ determine the boundaries of the regions of stability for the equilibrium stripe (labyrinthine) domain structure, whose period for any attainable values of the external parameters $\mathbf{H}$ and T corresponds to an absolute minimum of the energy. For the creation of such structures we use the method of "magnetic jarring" (see Ref. 2). The curves $P_{1,2} \to O$ and $P_{1,2} \to \Pi$ for equilibrium CMD hexagonal lattices have an analogous meaning. However, domain structures that appear during $O \to \Pi$, $O \to P_{1,2}$, $\Pi \to P_{1,2}$, and $P_{1,2} \to \Pi$ first-order phase transitions are not in equilibrium as a rule.

[4)] The collapse of stripe domains in a regular domain structure takes place over a rather wide interval of variation of magnetic field; these latter domains collapse for values of $H_\parallel$ and $H_\perp$ that roughly correspond to the curve $\Pi \to O$ for the equilibrium domain structure. The overwhelming majority of stripe domains collapse by "breakup" of the domains, i.e., the formation of dislocation pairs with subsequent rapid repulsion of the dislocations to the film boundaries.

[1] I. E. Dikshtein, F. V. Lisovskii, E. G. Mansvetova, and V. V. Tarasenko, Fiz. Tverd. Tela (Leningrad) **25**, 3591 (1983) [Sov. Phys. Solid State **25**, 2066 (1983)].

[2] I. E. Dikshtein, F. V. Lisovskii, E. G. Mansvetova, and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **86**, 1473 (1984) [Sov. Phys. JETP **59**, 863 (1984)].

[3] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **59**, 907 (1970) [Sov. Phys. JETP **32**, 493 (1970)].

[4] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **61**, 1144 (1971) [Sov. Phys. JETP **34**, 610 (1971)].

[5] J. M. Kosterlitz and D. J. Thouless, J. Phys. C: Solid State Phys. **6**, 1181 (1973).

[6] V. A. Ignatchenko, I. D. Kim, E. Yu. Mironov, and D. Ch. Khvan, Zh. Eksp. Teor. Fiz. **98**, 593 (1990) [Sov. Phys. JETP **71**, 331 (1990)].

[7] T. W. Collins, J. Iazdag, and Ju. Kochan, IEEE Trans. Magnetics **MAG-11**, 1088 (1975).

[8] I. E. Dikshtein, F. V. Lisovskii, E. G. Mansvetova, and V. V. Tarasenko, *New Magnetic Materials for Microelectronics* [in Russian], p. 53. Mordvin State Univ., Saransk, 1984.

[9] I. E. Dikshtein, F. V. Lisovskii, E. G. Mansvetova, and V. V. Tarasenko, Preprint No. 25(256), Inst. Radiotekh. Elektron., USSR Acad. Sci. [in Russian]. IRE, USSR Acad. Sci., Moscow, 1989.

[10] P. Mohlo, J. Gouzerh, J. C. S. Levy, and J. L. Portesil, J. Magn. Magn. Mater. **54–57**, 857 (1986).

[11] D. Sornette, J. de Phys. **48**, 151 (1987).

[12] V. G. Baryakhtar, V. V. Gann, and Yu. I. Gorodets, Fiz. Tverd. Tela (Leningrad) **18**, 1990 (1976) [Sov. Phys. Solid State **18**, 1158 (1976)].

[13] L. E. Gurevich and E. V. Liverts, Zh. Eksp. Teor. Fiz. **82**, 220 (1982) [Sov. Phys. JETP **55**, 132 (1982)].

[14] V. G. Baryakhtar, Zh. Eksp. Teor. Fiz. **87**, 1501 (1984) [Sov. Phys. JETP **60**, 863 (1984)].

[15] V. G. Baryakhtar, Fiz. Tverd. Tela (Leningrad) **29**, 1317 (1987) [Sov. Phys. Solid State **29**, 754 (1987)].

[16] V. V. Tarasenko, E. V. Chenskii, and I. E. Dikshtein, Zh. Eksp. Teor. Fiz. **70**, 2178 (1976) [Sov. Phys. JETP **43**, 1136 (1976)].

[17] V. V. Tarasenko, Fiz. Tverd. Tela (Leningrad) **22**, 503 (1980) [Sov. Phys. Solid State **22**, 294 (1980)].

[18] V. G. Baryakhtar, Yu. I. Gorodets, and S. I. Denisov, Ukr. Fiz. Zh. **28**, 436 (1983).

[19] A. P. Malozemoff and J. C. Slonczewski, *Magnetic Domain Walls in Bubble Materials,* Academic Press, New York (1979) [Russ. transl. Mir, Moscow, 1982].

[20] E. V. Chenskii and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **83**, 203 (1982) [Sov. Phys. JETP **55**, 121 (1982)].

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