Interaction between a massive neutrino and a plane-wave field

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Invariant solutions of the generalized Dirac equation are obtained for a neutral particle endowed with an electric dipole moment and an anomalous magnetic moment in the field of a plane wave of the form $A = af[(kx)]$ and in a circularly polarized wave. These solutions are used to study the photoneutrino reactions $\gamma \to \nu\bar{\nu}$ and $\nu \to \nu\gamma$ in the field of a linearly polarized wave and in a constant crossed field. The feasibility of detecting the neutrino electric dipole and anomalous magnetic moments is discussed.

1. INTRODUCTION

At the present time there is no convincing experimental proof for the existence of nonvanishing mass for the electron neutrino, to say nothing for the neutrinos of other generations. However the existence of such a mass would solve a number of dead-end problems of modern astrophysics, the most important being the solar neutrino deficit and the question of dark matter in the Universe. With a minimal modification of the standard model the Higgs mechanism ensures the appearance of mass for the neutrino, although the theoretical situation is far from definitive due to the large number of unknown parameters, including the number of Higgs bosons and their properties. Thus arguments in favor of a neutrino mass, in general different for different types, are sufficiently nontrivial.

For $m_\nu \neq 0$ there arises the principal question of the origin of the mass contribution, which could be either Dirac or Majorana. The vanishing of the corresponding electromagnetic form factors of the Majorana neutrino, i.e., of the anomalous magnetic moment (AMM) μ_ν and electric dipole moment (EDM) ε_ν in particular, follows from the Lorentz invariance, and therefore *CPT* invariance, of the theory.[1] In the case of Dirac neutrinos *CPT* invariance is not in contradiction with nonzero values of AMM and EDM, while it follows from *CP* invariance that $\varepsilon_\nu = 0$ and the value of μ_ν proportional to the neutrino mass can be obtained theoretically from the minimally modified standard model and turns out to be equal to[2]

$$\mu_\nu \approx \frac{3eGm_\nu}{8\pi^2 2^{1/2}}.$$

If it is assumed that $m_\nu \sim 10$ eV, which is not in contradiction with the experimentally established upper bound, then $\mu_\nu \sim 10^{-17}\mu_B$. Estimates based on astrophysical data[1] and on experiments with reactor neutrinos[3] are significantly weaker: $\mu_\nu < 1.5\cdot 10^{-10}\,\mu_B$, and $\mu_\nu < 2\cdot 10^{-11}\,\mu_B$, respectively.

The presence of an EDM for a massive Dirac neutrino violates *CP* invariance, but modern experimental data do not exclude this possibility in the case of, e.g., the more definitive situation with the neutron EDM ε_n.[4,5] Thus, the K^0-decay data implies the possibility of existence of an EDM at the level $\varepsilon_n \sim 10^{-31}\,e\cdot$cm, while from the baryon asymmetry of the Universe follows $\varepsilon_n > 3\cdot 10^{-27}\,e\cdot$cm. On the whole, *CP*-invariance violation has not been unambiguously confirmed experimentally (see, for example, the review in Ref. 6), and theoretically in extended versions of the standard model there are two mechanisms for *CP* violation: the θ term and the *CP*-odd phase in the Kobayashi-Maskawa matrix, responsible for the appearance of an EDM.[7,8]

Note that the estimates for the neutrino AMM given above are based on integrated characteristics of the corresponding contributions into, for example, the photodecay of a massive neutrino into a less massive one $\nu_i \to \nu_j\gamma$, when the *CP*-violation effects are not seen and the contributions of AMM and EDM cannot be separated.[9] Thus these estimates should also apply to the presumed size of the neutrino EDM.

It is clear from what has been said that the study of electroweak processes, caused by the existence of neutrino AMM or EDM, is one of the principal means in the search for the final version of the standard model or its alternatives. Along that line it is of interest to investigate the channels that are opened by the stimulated influence of an external electromagnetic field, since its magnitude achievable in the focus of a laser or in the vicinity of collapsed objects makes it feasible to hope to detect neutrino AMM and EDM. In this work we study, with the help of the solutions found in Sec. 2 of the generalized Dirac equation in the field of a plane wave including the AMM and EDM of the neutral fermion, the process of photoproduction of massive neutrino pairs $\gamma \to \nu\bar{\nu}$ (Sec. 3) and of bremsstrahlung $\nu \to \nu\gamma$ (Sec. 4). The obtained results are applicable to the field of a linearly polarized plane wave and to the crossed field and generalize in part of our earlier work,[10–13] in which only the AMM of the particle was taken into account.

2. WAVE FUNCTIONS IN THE FIELD OF A PLANE WAVE

The generalized Dirac equation for a neutral particle endowed with an AMM μ and an EDM ε has the form

$$[i\hat{\partial} - m - {}^1/_2 i(\mu - i\varepsilon\gamma^5)\sigma_{\alpha\beta}F^{\alpha\beta}]\Psi = 0, \qquad \sigma_{\alpha\beta} = {}^1/_2(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha), \tag{1}$$

where F is the external field tensor. In the usual manner one verifies that

$$\partial_\alpha(\bar{\Psi}\gamma^\alpha\Psi) = 0 \tag{2}$$

and therefore the wave function can be normalized in the standard fashion to one particle per unit volume. We search for a solution of the squared equation in the field of a plane wave with the potential

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$$A=A(\varphi),\quad \varphi=(kx),\quad k^2=(kA)=(kA')=0$$

in the form

$$\Psi=\Phi(\varphi)e^{-i(px)}\frac{u(p)}{(2p_0)^{1/2}},\quad p^2=m^2, \tag{3}$$

where the matrix $\Phi(\varphi)$ satisfies as a function of the phase φ the equation

$$\Phi'=B'\Phi, \tag{4a}$$

$$B=\frac{1}{2(kp)}[\mu(\hat{k}\hat{A}\hat{p}+\hat{p}\hat{k}\hat{A})-i\varepsilon\gamma^5(\hat{k}\hat{A}\hat{p}-\hat{p}\hat{k}\hat{A})], \tag{4b}$$

and $u(p)$ is a Dirac spinor with normalization $\bar{u}u=2m$. In fields $A=af(\varphi)$, where the phase dependence factorizes in the form of a scalar function (for linear polarization $f=\sin\varphi$, for constant crossed field $f=\varphi$), the operators B and B' commute and the solution has the form

$$\Phi=\exp(B)\Phi_0. \tag{5}$$

Taking into account the fact that

$$B^2=A^2(\mu^2+\varepsilon^2),$$

it is easy to obtain the following final expression for the matrix function Φ:

$$\Phi=\cos z+\sin z(\tilde{\mu}M-i\tilde{\varepsilon}\gamma^5N), \tag{6}$$

$$z=[-A^2(\mu^2+\varepsilon^2)]^{1/2}, \tag{6a}$$

$$\tilde{\mu}=\frac{\mu}{(\mu^2+\varepsilon^2)^{1/2}},\quad \tilde{\varepsilon}=\frac{\varepsilon}{(\mu^2+\varepsilon^2)^{1/2}}, \tag{6b}$$

$$M=\frac{\hat{k}\hat{a}\hat{p}+\hat{p}\hat{k}\hat{a}}{2(kp)(-a^2)^{1/2}}, \tag{6c}$$

$$N=\frac{\hat{k}\hat{a}\hat{p}-\hat{p}\hat{k}\hat{a}}{2(kp)(-a^2)^{1/2}}=\frac{(ap)\hat{k}-(kp)\hat{a}}{(kp)(-a^2)^{1/2}}, \tag{6d}$$

with

$$\bar{M}=-M,\quad \bar{N}=N,\quad M^2=N^2=-1,\quad MN=NM, \tag{7}$$

and $\bar{\Phi}\Phi=1$, which corresponds to normalizing the wave function (3) to one particle per unit volume. In the absence of the field ($z=0$) we have $\Phi=1$, as we should; for $\varepsilon=0$ we obtain the solution we found previously for a neutral particle with an AMM.[10]

In a field with circular polarization

$$A=a_1\cos\varphi+a_2\sin\varphi, \tag{8}$$

$$a_1^2=a_2^2=a^2,\quad (a_1a_2)=0$$

the operators B and B' do not commute and to find the solution (4a) one should make use of the method outlined in Ref. 12. Taking into account the relations

$$\begin{gathered}B^2=B'^2=-z^2,\quad B''=-B,\quad z=[-a^2(\mu^2+\varepsilon^2)]^{1/2},\\ (BB')^2=-z^4,\quad BB'=z^2N,\\ N=\frac{1}{-a^2}\left(\hat{a}_1\hat{a}_2-\frac{a_1p}{kp}\hat{k}\hat{a}_2+\frac{a_2p}{kp}\hat{k}\hat{a}_1\right)\end{gathered} \tag{9}$$

and including the appropriate formulas from Ref. 12 we conclude that the solution scheme is unchanged and in the present case leads to the result

$$\begin{aligned}\Phi=\frac{1}{\sigma^{1/2}}\Big\{&\cos\varphi_-+\frac{\delta_-}{2(kp)z^2}[\mu(\hat{k}\hat{A}_+\hat{p}+\hat{p}\hat{k}\hat{A}_+)\\ &-i\varepsilon\gamma^5(\hat{k}\hat{A}_+\hat{p}-\hat{p}\hat{k}\hat{A}_+)]-N\sin\varphi_-\Big\},\end{aligned} \tag{10}$$

where

$$\begin{gathered}A_+=a_1\cos\varphi_++a_2\sin\varphi_+,\quad \varphi_\pm=\delta_\pm\varphi,\\ \delta_\pm={}^1/_2[(1+4z^2)^{1/2}\pm1],\quad \sigma=1+\frac{\delta_-^2}{z^2}.\end{gathered} \tag{10a}$$

It can be verified that in that case too the standard normalization $\bar{\Phi}\Phi=1$ applies.

3. THE PROCESS $\gamma\to\nu\bar{\nu}$ IN THE FIELD OF A PLANE WAVE

The effective $\gamma\nu\bar{\nu}$-interaction vertex, induced by the μ, ε-coupling to the radiation field $A(\gamma)$, has the form

$$\mathscr{L}=-i[\bar{\Psi}(\mu-i\varepsilon\gamma^5)\sigma^{\alpha\beta}\Psi]\frac{\partial A_\alpha^{(\gamma)}}{\partial x^\beta}. \tag{11}$$

Making use of expressions (3) and (6) for the neutrino wave function in the field of a linearly polarized plane wave, and also the expansion of trigonometric expressions in terms of Bessel functions,[10] we find for the matrix element of the process $\gamma\to\nu\bar{\nu}$:

$$\begin{aligned}\langle f|S|i\rangle=&\frac{i\pi^{1/2}(2\pi)^4}{(2p_02p_0'2\varkappa_0)^{1/2}V^{3/2}}\sum_s J_s\delta(\varkappa+sk-p-p')\\ &\times\bar{u}(p)\Big\{\frac{1+(-1)^s}{2}[(\mu-i\varepsilon\gamma^5)\sigma^{\alpha\beta}+(\tilde{\mu}M-i\tilde{\varepsilon}\gamma^5N)\\ &\times(\mu-i\varepsilon\gamma^5)(\tilde{\mu}M'-i\tilde{\varepsilon}\gamma^5N')]\\ &+i\frac{1-(-1)^s}{2}[(\mu-i\varepsilon\gamma^5)\sigma^{\alpha\beta}(\tilde{\mu}M'-i\tilde{\varepsilon}\gamma^5N')\\ &-(\tilde{\mu}M-i\tilde{\varepsilon}\gamma^5N)(\mu-i\varepsilon\gamma^5)\sigma^{\alpha\beta}]\Big\}\\ &\times u(-p')e_\alpha\varkappa_\beta,\end{aligned}$$

where s is the number of photons captured from the wave, $\varkappa$, p and p' are the momenta of the photon and the neutrinos, e_α is the photon polarization vector, the form of the matrices M and N is given by formulas (6c, d), and we have introduced the notation

$$M'=M(p\to p'),\quad N'=N(p\to p'),\quad \arg J_s=2[-a^2(\mu^2+\varepsilon^2)]^{1/2}.$$

By means of standard methods one can obtain the following expression for the probability per unit time

$$W=\frac{2}{\varkappa_0}\sum_s J_s^2\mathscr{F}[T], \tag{12}$$

where

$$\mathscr{F}[T]=\frac{1}{\pi}\int\frac{d^3p}{2p_0}\int\frac{d^3p'}{2p_0'}\delta(q-p-p')T, \tag{13}$$

$$q=\varkappa+sk, \tag{13a}$$

$$\begin{aligned}T=&(\mu^2+\varepsilon^2)(p\varkappa)(p'\varkappa)\\ &+\mu^2m^2(k\varkappa)\left[\frac{(k\varkappa)(pp')}{(kp)(kp')}-\left(\frac{\varkappa p}{kp}+\frac{\varkappa p'}{kp'}\right)\right]\\ &+(\mu^2-\varepsilon^2)\frac{m^2(kp)(kp')}{a^2}\left(\frac{ap}{kp}-\frac{ap'}{kp'}\right)^2,\end{aligned} \tag{13b}$$

where in (13b) the pseudoscalar term, which does not contribute to the total probability, was omitted.

For further simplification we need the following invariant integrals:

$$\mathscr{F}[p_\alpha p_\beta'] = \frac{b}{12}\left[\left(1+\frac{2m^2}{q^2}\right)q_\alpha q_\beta + \frac{1}{2}q^2 b^2 g_{\alpha\beta}\right], \tag{14a}$$

$$\mathscr{F}\left[\frac{p_\alpha}{kp}\right] = \frac{1}{2(kq)}\left[bq_\alpha - \frac{q^2}{kq}\left(b-\frac{1}{2}\ln Q\right)k_\alpha\right], \tag{14b}$$

$$\mathscr{F}\left[\frac{1}{(kp)(kp')}\right] = \frac{1}{(kq)^2}\ln Q, \quad b=\left(1-\frac{4m^2}{q^2}\right)^{1/2}, \tag{14c}$$

$$\mathscr{F}\left[\frac{p_\mu p_\nu p_\alpha'}{kp}\right] = \frac{1}{4(kq)}\left\{\frac{b}{3}\left(1+\frac{2m^2}{q^2}\right)q_\alpha q_\mu q_\nu \right.$$

$$\left. +\left[m^2\ln Q - \frac{b}{3}(q^2+2m^2)\right]g_{\mu\nu}q_\alpha + \frac{q^2 b^3}{6}(g_{\alpha\nu}q_\mu + g_{\alpha\mu}q_\nu)\right\},$$

$$Q=\frac{1+b}{1-b}. \tag{14d}$$

We have omitted in (14d) tensorial combination that do not contribute to (13) as a result of the properties

$$k^2 = (ka) = 0.$$

Making use of Eqs. (14) we obtain

$$\mathscr{F}[T] = (\mu^2+\varepsilon^2)\frac{b}{12}\left(1+\frac{2m^2}{q^2}\right)(q\varkappa)^2$$

$$+\mu^2 m^2[-b(q\varkappa)+bq^2-m^2\ln Q]$$

$$+(\mu^2-\varepsilon^2)\frac{m^2}{2}\left(m^2\ln Q - \frac{1}{2}bq^2\right). \tag{15}$$

With (13a) taken into account we introduce the notation

$$u_s = \frac{2s(k\varkappa)}{m^2}, \tag{16}$$

so that

$$\frac{q^2}{m^2} = u_s, \quad \frac{q\varkappa}{m^2} = \frac{u_s}{2}, \quad b = \left(1-\frac{4}{u_s}\right)^{1/2},$$

and the final result has the form

$$W = \sum_{s=s_m}^{\infty} W_s, \tag{17}$$

$$W_s = \frac{m^4(\mu^2+\varepsilon^2)}{\varkappa_0}J_s^2\left[\frac{bu_s}{12}\left(\frac{u_s}{2}+7\right) - \ln Q\right], \tag{17a}$$

$$s_m = E\left(\frac{2m^2}{k\varkappa}\right)+1. \tag{17b}$$

Near the partial threshold ($u_s = 4$) the expression in the square brackets in (17a) is approximately equal to $(u_s - 4)^{1/2}/2$, so that the pair $(\nu\bar{\nu})$ is produced in an s state. We call attention to the fact that μ and ε enter into the differential probability asymmetrically [see (13b)], while at the same time (17a) has $(\mu^2+\varepsilon^2)J_s^2$ as a factor.

In a constant crossed field, which approximates in the ultrarelativistic case constant fields of other configurations, the corresponding probability is obtained from (17) by leaving off the sum over s and making the replacements (see also Ref. 11)

$$J_s^2 \to {}^1/_2, \quad u_s \to 4\chi, \tag{18a}$$

where the field invariant is

$$\chi = \frac{[(\mu^2+\varepsilon^2)(\varkappa F^2\varkappa)]^{1/2}}{m^2}. \tag{18b}$$

Note that in a constant field there is the fixed reaction threshold $\chi = 1$, and near threshold we have

$$W_\perp \approx \frac{m^4(\mu^2+\varepsilon^2)}{2\varkappa_0}(\chi-1)^{1/2}. \tag{19}$$

4. THE PROCESS $\nu\to\nu\gamma$ IN THE FIELD OF A PLANE WAVE

The expression for the probability per unit time in the field of a linearly polarized plane wave has the form (12) with the replacement $2/\varkappa_0 \to 2/p_0$ and with the definition of the integral operator

$$\mathscr{F}[T] = \frac{1}{\pi}\int\frac{d^3p'}{2p_0'}\int\frac{d^3\varkappa}{2\varkappa_0}\delta(q-\varkappa-p')T, \tag{20}$$

where p and p' are the momenta of the initial and final neutrino, respectively, $\varkappa$ is the momentum of the photon, and the expression for T differs from that in (13b) by a change of sign in front of the second term. The necessary integral relations are given in Ref. 11, and with their help one readily obtains

$$W = \sum_{s=1}^{\infty} W_s, \tag{21}$$

$$W_s = \frac{m^4(\mu^2+\varepsilon^2)}{4p_0}J_s^2 f(u_s), \tag{21a}$$

$$f(u_s) = \frac{u_s(2+u_s)}{2(1+u_s)^2}(2+2u_s+u_s^2) - 2\ln(1+u_s), \tag{21b}$$

$$u_s = \frac{2s(kp)}{m^2}.$$

The argument of the Bessel functions is given in Sec. 3 and the properties of the function $f(u_s)$ are discussed in Ref. 11, where the process $n\to n\gamma$ was considered with the AMM of the neutron taken into account. The results of Ref. 11 follow from formula (21) for $\varepsilon = 0$. Transition to the case of a crossed field proceeds as in the preceding section with the replacement of $\varkappa$ by p.

5. DISCUSSION

As is obvious from the content of Secs. 3 and 4 the total probabilities contain μ and ε only in the symmetric combination $\mu^2+\varepsilon^2$, so that in the corresponding experiments it is not possible to determine the relative size of the AMM and EDM and in that sense inclusion of the external field in the indicated configurations does not change the situation (see Ref. 1). It should be noted, however, that the astrophysical estimates mentioned in Sec. 1 were based on the reaction $\nu_e \to \nu_x\gamma(m_{\nu_e} > m_{\nu_x})$, which is forbidden in the minimal version of the standard model by lepton-number conservation for individual generations, while the channels we have considered have no such restrictions. But even for the maximal value $\mu_\nu(\sim\varepsilon_\nu)\sim 2\cdot 10^{-11}\ \mu_B$ from the reactor neutrino data and for optimal characteristics of modern lasers $k_0 \sim 1$ eV, $E\sim 10^8$ G observation of the reactions $\nu\to\nu\gamma$ and $\gamma\to\nu\bar{\nu}$

under laboratory conditions is as yet impossible. As far as astrophysical applications are concerned we note that for $\varkappa_0 \sim 1$ GeV, $m_\nu \sim 10$ eV, $\mu_\nu (\sim \varepsilon_\nu) \sim 2 \cdot 10^{-11} \mu_B$, $F \sim 10^{16-17}$ G (this is an upper bound on the value of fields in magnetic neutron stars), the magnitude of the parameter χ in (18b) is found to be of order 10^3, which is adequate for using the crossed-field approximation. For these values one obtains from (17a), with (18) included, a mean free path against the decay $\gamma \to \nu\bar{\nu}$ larger than the dimensions of a neutron star by several orders of magnitude.

Thus under the most optimistic estimates of mass and of neutrino AMM and EDM, the characteristics of existing lasers and known astrophysical fields seem to use entirely insufficient for the identification of the electromagnetic form factors and, even more so, of the *CP*-symmetry violation effect in neutrino experiments in the discussed channels.

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