# Scaling one-parameter theory of phase transitions

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A scaling one-parameter variant of the theory of phase transitions is proposed in which the concept of the order parameter does not appear explicitly and which is conceptually close to the theory of the Anderson transition. The key parameter of the theory is the total free energy of a topological defect, which contains information about the character of the spontaneous symmetry-breaking in the ordered phase. When applied to ideal systems the proposed scheme gives the possibility of an asymptotically exact calculation of the phase-transition temperature and correlation-length index near the lower critical dimensionality. New results here are the following: expressions for the indices and transition temperature of the Ising model in the $1+\varepsilon$ approximation and of the roughness phase transition in the $2+\varepsilon$ approximation. In application to nonideal one- and two-component spin systems in the presence of defects of the random-field type the new results are the following: a prediction of a quasiferromagnetically ordered phase in a space of lower critical dimensionality with a weak degree of disorder; the construction of a disorder–temperature phase diagram on which, in dimensions greater than the lower or equal to critical dimensionality, there are two tricritical points; a proof of the absence of any phase transitions at nonzero temperature in dimensions less than the lower critical dimensionality; a proof of the impossibility in such systems of performing an $\varepsilon$ expansion about the lower critical dimensionality; a demonstration of the violation of the correspondence $d \to d-2$ between the indices of the ideal $XY$ system and the defect $XY$ system as $d \to 4$. In connection with defects of the random-temperature type, an accurate formula for estimating the indices of percolation theory is obtained, and it is also shown that at a nonzero temperature to first order in $\varepsilon = d-1$ the value of the correlation-length index coincides with the value for the ideal system.

## INTRODUCTION

In the modern theory of phase transitions the properties of matter near a critical point are described by a set of critical indices, only some of which are independent (see, e.g., the reviews in Refs. 1 and 2). One of these, certainly, is the correlation-length index $\nu$. By virtue of this, all the singular properties of matter can be divided into two groups: global properties (associated with the behavior of the correlation length), and the rest. This division appears reasonable in the light of the scaling hypothesis (scale invariance).[1,2] The only question that arises after this is the following: Is it possible to construct a theory of just the global properties of matter near a critical point? The present paper is devoted to answering this question.

Below we show that the global properties of a broad class of physical systems can be described by a simple scaling one-parameter theory that does not even require the explicit use of the concept of the order parameter. The new theory not only has methodological significance but also makes it possible to obtain a number of previously unknown results, pertaining both to conventional systems and to systems with defects. The proposed scheme is conceptually close to the scaling theory of Anderson localization (see, e.g., Ref. 1), and links together all areas of the physics of phase transitions, including the percolation problem.

The following is an outline of the article: First we introduce the concept of the rigidity of a spin system and perform an elementary analysis of the properties of the classical $d$-dimensional vector model of phase transitions. The subsequent analysis pertains to one-component (Ising) and two-component ($XY$) systems. For these systems the scale dependence of the rigidity is found under arbitrary assumptions about the causes of the phase transition (thermal fluctuations and (or) inhomogeneities). The general relations obtained are then used to analyze the following specific problems: the phase transitions and the forms of ordering in the presence of defects of the random-field type (for the classification of defect types, see, e.g., Refs. 1 and 2) and random-temperature type. As examples of possible applications of the one-parameter approach we also consider the phase transition in a spin glass and the roughness phase transition. Some of the results described, pertaining to the Ising model with defects of the random-field type, have been published in a brief communication,[3] and a complete summary of the new results is given in the concluding part of the article.

## 1. IDEAL SYSTEMS WITH DISCRETE AND CONTINUOUS SYMMETRIES

Traditionally, a state with spontaneously broken symmetry is described by an order parameter that vanishes at the transition to the disordered phase. There exists, however, another quantity, which characterizes not only the ordered state but also its stability against fluctuations. Consider, e.g., the possibility of the formation of a domain of the opposite sign in the ordered phase of the Ising model. If the surface-tension coefficient of the domain boundary is equal to $\alpha$ and the radius of the domain is $L$, the corresponding increase of the energy will be proportional to the area of the domain wall: $\alpha L^{d-1}$ where $d$ is the dimensionality of space. At not too high temperatures $T$ the probability of formation of a domain is small and proportional to $\exp(-\alpha L^{d-1}/T)$.

Considering the possibility of the reversal of the magnetization of the entire sample, we find it to be unrealizable in the thermodynamic limit $L \to \infty$ if $d > 1$ holds. This is the

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signal of a state with broken symmetry, and the well known inequality $d > 1$ is a necessary condition for the existence of an ordered phase at $T > 0$ (Ref. 1). The total domain-wall energy $G(L) = \alpha L^{d-1}$ is the key (and only) parameter of the theory.

However, a somewhat different definition is convenient. Consider a $d$-dimensional spin system of arbitrary nature and linear size $L$, and calculate the difference in the free energies when, in any of the directions, we impose antiperiodic boundary conditions (the spins at the edge of the sample are in opposite directions) and periodic boundary conditions (these spins are in the same direction), respectively. The energy difference thus obtained will be called the rigidity $G(L)$. If we have $G(L \to \infty) \to 0$, this is an indicator of the disordered phase. In any other case the system is in an ordered state. The rigidity introduced in this manner is a positive-definite quantity in systems with ferromagnetic ordering (since the periodic boundary condition corresponds to a lower energy) and negative-definite in systems with antiferromagnetic interactions. In spin glasses, $G(L)$ is of variable sign, since it is not known in advance which boundary condition will be preferable for the given configuration of random exchanges. In other words, the spin glass is characterized by a distribution of probabilities of different values of $G(L)$, and a measure of the order is given by the nonzero width of this distribution for $L \to \infty$ (Ref. 4).

Except for spin glasses, below, for definiteness, we shall be speaking of ferromagnetic ordering. If we have $G(L \to \infty) = \alpha L^{d-1}$, the latter is characterized by the Ising spontaneous magnetization. In the ordered phase of systems with continuous (Heisenberg) symmetry, it is obvious that $G(L \to \infty) = \rho L^{d-2}$ holds ($\rho$ is the so-called spin-wave stiffness), since the reversal of the magnetization here occurs continuously over the entire sample. In this case, a necessary condition for the existence of the ordered phase is that the well known inequality $d > 2$ be fulfilled.[1]

It is clear that the concept of rigidity embodies a substantially greater amount of information than does the traditional order parameter. In addition, it contains the *a priori* possibility of ordered states with zero spontaneous magnetization. For example, if in a system with discrete symmetry it turns out, for various reasons, that $G(L \to \infty) \propto L^{\theta}$ with $\theta < d - 1$, it may be asserted that the ordered state is characterized by zero spontaneous magnetization. Examples of this kind of phase will be given below. Apparently, the first to draw attention to the possibility of using the concept of rigidity to describe the ordered phase of an Ising spin glass were Fisher and Huse.[4] In the theory of Anderson localization the analog of the rigidity is the conductance.[1]

We turn now to the determination of the dependence $G(L)$ in the disordered phase. In the latter, change of the form of the boundary conditions affects only a narrow layer whose width is on the order of the correlation length $\xi$ near the edge of the sample, and therefore, as in the theory of localization, it is reasonable to assume that $G(L)$ decreases exponentially fast: $G(L) \propto \exp(-L/\xi)$. It should be noted that this dependence holds in any system, since all the disordered phases are the same. We now assume, in the spirit of the scaling hypothesis, that the rigidity $G$ is the only quantity that determines the behavior of the system as its size changes. We shall show that the information available to us is already sufficient to deduce the presence of a phase transition. From the examples given above it can be seen that the indicator of an ordered state was the power of $L$ ($d - 1$ or $d - 2$), and, therefore, instead of investigating the dependence $G(L)$, it is more convenient to consider the so-called Gell-Mann–Low function (GLF)

$\beta(G) = \partial \ln G / \partial \ln (L/a)$

($a$ is of the order of the interatomic spacing). The information given above permits us to write out the following asymptotic formulas for $\beta(G)$:

$$\beta = d - 1, \quad G \to \infty \quad (\text{Ising ordering}), \tag{1}$$

$$\beta = d - 2, \quad G \to \infty \quad (\text{Heisenberg ordering}), \tag{2}$$

$$\beta = \ln G + \text{const}, \quad G \to 0 \quad (\text{disordered phase}). \tag{3}$$

On the basis of these, it is possible to propose the dependence $\beta(G)$ shown in Fig. 1 for the Ising system. It can be seen that the curves have substantially different forms for different dimensionalities. For $d \leqslant 1$ the function $\beta(G)$ is always negative. It follows from this that as the size increases the function $G$ always decreases, i. e., the system is in the disordered phase. For $d > 1$ the situation is different. If we start from a small $G$, then $\beta$ is negative and $G$ decreases with increase of the size. But if we take $G$ to the right of the intersection point $G = G_c$, then $\beta > 0$ and $G$ increases as a function of $L$, approaching the asymptotic dependence $G \propto L^{d-1}$.

Thus, the unstable fixed point $G_c$ divides the ordered phase from the disordered phases, and the condition for the phase transition is that the initial value $G_0$ coincides with $G_c$. Taking the initial value of $L$ to be of the order of the interatomic spacing $a$, we arrive at the conclusion $G_0 \approx J$ ($J$ is the exchange-interaction constant). The analysis of the phase transition repeats exactly the investigation of the Anderson transition.[1] In the vicinity of the transition point the correlation length diverges as

$$\xi \approx a \left| \frac{G_0 - G_c}{G_c} \right|^{-\nu}, \quad \nu^{-1} = \left. \frac{\partial \beta}{\partial \ln G} \right|_{G = G_c}, \tag{4}$$

and, for $G_0 > G_c$, the surface-tension coefficient vanishes as

$$\alpha \approx G_c / \xi^{d-1} \approx \frac{G_c}{a^{d-1}} \left( \frac{G_0 - G_c}{G_c} \right)^{\mu}, \quad \mu = (d-1)\nu. \tag{5}$$

To calculate the index $\nu$ it is necessary to know the behavior of the function $\beta(G)$ near the fixed point $G_c$. We

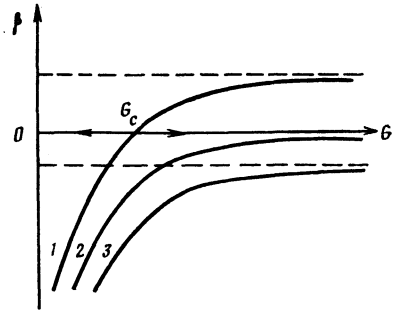


FIG. 1. Form of the Gell-Mann–Low function for the ideal $d$-dimensional Ising model: 1) $d > 1$, 2) $d = 1$, 3) $d < 1$; the arrows indicate the direction of change of the rigidity with increase of the scale, and the dashed lines correspond to the value $\beta = d - 1$.

show how the dependence (1) for $G \to \infty$ can be made more precise. Thermal fluctuations decrease the surface-tension coefficient of an Ising domain wall, and this, in turn, leads to a correction to the GLF (1). A regular procedure for obtaining this correction will be described below, but for the present case of an ideal system the form of this correction is easily estimated from scaling arguments. At a nonzero temperature it can be a function only of the ratio $T/G$, and, since we are speaking of an entropic correction, it should be linear in the temperature. Hence,

$$\beta(G)=\frac{\partial \ln G}{\partial \ln(L/a)}=d-1-c_dT/G, \tag{6}$$

where $c_d \sim 1$. The right-hand side of the latter equation vanishes at the value

$$G_c=c_dT/(d-1), \tag{7}$$

which tends to infinity as $d \to 1+0$. Therefore, Eq. (6) can be used to determine the transition temperature and indices only in the sense of the $1+\varepsilon$ expansion. Representing Eqs. (4) and (5) in a more familiar form, we finally find

$$\xi \approx a\left|\frac{T-T_c}{T_c}\right|^{-\nu}, \quad \nu^{-1}=d-1, \tag{8}$$

$$T_c \approx (d-1)J, \tag{9}$$

$$\alpha \approx \frac{T_c}{a^{d-1}}\left(\frac{T_c-T}{T_c}\right)^{\mu}, \quad \mu=1. \tag{10}$$

For $d=1$ the solution of Eq. (6) gives an expression for

$$G(L)=G_0-c_1T\ln(L/a),$$

that vanishes for $L=\xi \approx a\exp(G_0/c_1T)$.

If we choose $G_0/c_1=2J$, then $\xi$ will correspond to the well known rigorous result that in the one-dimensional Ising model the long-range order is destroyed as a result of fluctuational creation of kinks.[1] Thus, the results obtained above are correct for $d=1+\varepsilon$ ($\varepsilon \ll 1$). There are, however, grounds to suppose that they give a reasonably adequate description under less stringent restrictions too. Indeed, for $d=2$, it follows from (8) and (10) that $\nu=\mu=1$, which coincides with Onsager's exact solution of the two-dimensional Ising model.[5] Taken on its own, the fact that the result to first order in $\varepsilon$ coincides with the exact result for $\varepsilon=1$ appears to be accidental; however, it is by no means exotic. Below we shall give a further example of how, at one point, the result to first order in $\varepsilon$ coincides with the exact value.

All that has been said above about the Ising model pertains equally to the Heisenberg model as well if, in all the formulas, $d-1$ is replaced by $d-2$; correspondingly, the indices and transition temperature can be calculated in the $2+\varepsilon$ approximation. We shall write out certain results pertaining to this case:

$$\nu^{-1}=d-2, \tag{11}$$

$$T_c \approx (d-2)J, \tag{12}$$

$$\beta(G)=d-2-c_dT/G. \tag{13}$$

We draw attention to the fact that for $d=4$ (i.e., in the upper critical dimensionality) Eq. (11) reproduces the well known exact value $\nu=\frac{1}{2}$ that follows from the Landau theory. The relations (11) and (12) were obtained in a different way in Refs. (6) and (7), and this also argues in favor of a one-parameter theory.

The expressions (6) and (13) will be reproduced below as a particular case of a more general relation.

## 2. GENERAL ANALYSIS

We shall consider an Ising domain wall situated in a random medium. The corresponding Hamiltonian has the well known form[2]

$$H=\int d^{d-1}x\left[\alpha+\frac{1}{2}\Gamma(\nabla f)^2+V(f,x)\right]. \tag{14}$$

Here, $\Gamma$ is the effective stiffness of the domain wall, $f$ is its displacement relative to an arbitrary reference plane, and $V$ is the random potential, with zero average and with a correlation function

$$\langle V(f,x)V(f',x')\rangle=R(f-f')\delta(x-x')$$

that depends on the type of inhomogeneities.[2] The effect of fluctuations (of the defect or temperature type; in the latter case, $V \equiv 0$) is reflected in the roughness of the domain boundary—the dependence of the characteristic transverse displacement $w$ on the scale $L$ over which it is accumulated:

$$w \approx AL^{\zeta}, \tag{15}$$

where $A$ depends on the source of the fluctuations and $\zeta$ is the so-called roughness index. Since the relation (15) arises as a consequence of balancing the contributions from the increase of the elastic part of the energy of the deformed wall [the second term of (14)] and the decrease of the random term (the third term), the fluctuation contribution to the surface-tension coefficient can be estimated as the average value of the second term in the integrand of (14). Consequently, for the relative correction $\Delta\alpha/\alpha$ we have[2]

$$\frac{\Delta\alpha}{\alpha}=\text{const}\,\frac{\Gamma A^2}{\alpha}\,\frac{a^{2(\zeta-1)}-L^{2(\zeta-1)}}{2(\zeta-1)}. \tag{16}$$

For $\zeta<1$ the first term of Eq. (16) corresponds to a negative contribution to the surface tension, while the second term corresponds to a positive size correction that vanishes as $L \to \infty$. For $\zeta \geqslant 1$, however, the sign of (16) changes; it begins to grow, leading to the vanishing of the total surface-tension coefficient, and, consequently, to instability of the uniformly magnetized state.[2] We draw attention to the similarity of the expression (16) to the interference correction to the conductivity in the theory of localization.[1]

The form of the GLF is established in exactly the same way as in the theory of localization. In the zeroth approximation $\beta$ is expressed by Eq. (1). Using the zeroth-approximation result $G=\alpha L^{d-1}$ and the correction (16), we finally find

$$\beta_1=d-1-k\left(\frac{G}{\alpha a^{d-1}}\right)^{2(\zeta-1)/(d-1)}, \tag{17}$$

where $k=\Gamma A^2 a^{2(\zeta-1)}/\alpha$ is the dimensionless "intensity" of the source of fluctuations. Substituting for $A$ and $\zeta$ their values for the ideal, defect-free system at nonzero temperature[2]

$$A^2 \approx T/\Gamma, \quad \zeta=\frac{3-d}{2}, \tag{18}$$

we return to Eq. (6), obtained from elementary considerations.

We turn now to systems with continuous symmetry; we shall consider only two-component ($XY$) magnets. The ana-

log of the Hamiltonian (14) in this case is also well known (see, e.g., Refs. 8 and 9):

$$H=\int d^dx\left[\frac{\rho}{2}(\nabla\varphi)^2+V(\varphi,x)\right], \tag{19}$$

where $\varphi$ is the phase (angle of rotation of the spin) and $V(\varphi,x)$ is a random $2\pi$-periodic potential with zero average. The form of the $\beta$ function in this case is established in exactly the same way as was described above for the Ising model. Omitting the analogous calculation, we find

$$\beta_{XY}=d-2-k\left(\frac{G}{\rho a^{d-2}}\right)^{2\zeta/(d-2)}, \quad k\approx A^2a^{2\zeta}. \tag{20}$$

In the defect-free system, $A$ and $\zeta$ are given by formulas similar to (18):

$$A^2\approx T/\rho, \quad \zeta=\frac{2-d}{2}.$$

Substituting these values into (20), we again arrive at the already familiar formula (13).

The general relations (17) and (20) obtained above will be used below to analyze various specific cases.

### 3. ISING MODEL AND *XY* MODEL IN A RANDOM FIELD

We shall find it more convenient to analyze the Ising model and $XY$ model at the same time. We consider first the case of zero temperature. Substituting $\zeta=(5-d)/3$ (Ref. 2) into Eq. (17) and $\zeta=(4-d)/2$ (Ref. 10) into Eq. (20), we obtain

$$\beta_I=d-1-k\left(\frac{G}{J}\right)^{2(2-d)/3(d-1)}, \tag{21}$$

$$\beta_{XY}=d-2-k\left(\frac{G}{J}\right)^{(4-d)/(d-2)}. \tag{22}$$

In writing Eqs. (21) and (22) we have taken into account the fact that $\alpha a^{d-1}\approx\rho a^{d-2}\approx J$. It should also be noted that in (21) and (22), for simplicity, we have used the single symbol $k$ to denote the different dimensionless quantities that control the degree of disorder. On the basis of these formulas, and also the relation (3), we may postulate the dependence $\beta(G)$ depicted in Fig. 2 for different dimensionalities of space. Because of the qualitative similarity of the behavior of the systems under consideration, the values of the parameters pertaining to the $XY$ model are indicated in brackets. We note that, strictly speaking, Fig. 2b can be established only for $d<2$ ($d<4$, respectively), but there are no obvious reasons why, under less stringent restrictions, Fig. 2b could undergo qualitative changes, since the curves shown should be obtained from each other by continuous deformation upon decrease of the dimensionality of space.

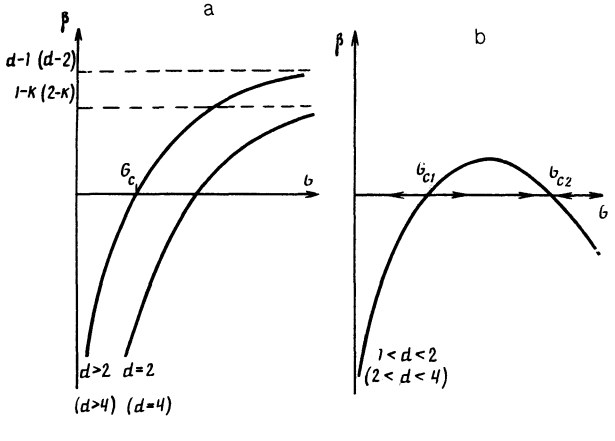


FIG. 2. a) Form of the Gell-Mann–Low function in dimensions greater than or equal to the lower critical dimensionality for the $d$-dimensional Ising model (the values corresponding to the $XY$ model are indicated in brackets) in the presence of defects of the random-field type; $k$ is the dimensionless degree of disorder; b) the same in dimensions less than the lower critical dimensionality, with $k\ll 1$; $G_{c1}$ is the unstable fixed point, $G_{c2}$ is the stable fixed point, and the arrows indicate the directions of the changes of the rigidity with increase of the scale.

For $d>2$ for the Ising model ($d>4$ for the $XY$ model) the behavior of the GLF qualitatively resembles that for the defect-free systems for $d>1$ and $d>2$, respectively. With increase of the degree of disorder (increase of $k$), $G_c$ intersects the initial value $G_0\approx J$ and a phase transition to the disordered phase occurs. In this case the indices are given, as before, by the general formulas (4) and (5), but a correct calculation of the index $\nu$ turns out to be impossible. In fact, the right-hand side of (21) vanishes at the value

$$G=J\left(\frac{k}{d-1}\right)^{3(d-1)/2(d-2)},$$

which, for any $d>2$, cannot be regarded as large (we recall that Eqs. (21) and (22) have been written out in the limit $G\to\infty$), and, therefore, the expression (21) can be used only for a rough estimate of $\nu$. Bearing this in mind, we find

$$\nu^{-1}\approx{}^2/_3(d-2), \tag{23}$$

$$\mu=(d-1)\nu\approx\frac{3}{2}\frac{d-1}{d-2}. \tag{24}$$

The estimates given turn out to be certainly inapplicable for $d\to 2$, since for $d=2$ the right-hand side of (21) does not depend on $G$ at all. For $d=6$ Eq. (23) gives $\nu=\frac{3}{8}$, which is fairly close to the exact value $\nu=\frac{1}{2}$ (we recall that the upper critical dimensionality is $d=6$, for which the value of the index $\nu$ should coincide with the result of the Landau theory[1,2]), so that far from $d=2$ the relations (23) and (24) are fully applicable.

All that has been said above about the indices of the Ising model with defects of the random-field type applies equally to the $XY$ model as well. The corresponding results have the form

$$\nu^{-1}\simeq d-4, \tag{25}$$

$$\mu=(d-2)\nu\approx\frac{d-2}{d-4}. \tag{26}$$

Formally, the relation (25) can be obtained from Eq. (11) by the replacement $d\to d-2$ (indeed, the literature contains statements about the correspondence $d\to d-2$ between the indices of the pure system and the defect system[1,2,9]). It should be remembered, however, that Eqs. (11) and (25) have entirely different meanings: The result (11) is asymptotically exact for $d\to 2$, whereas (25) is certainly inapplicable for $d\to 4$ and is a rough estimate. In other words, the correspondence $d\to d-2$ between the indices is not fulfilled for $d\to 4$ at least. It may be hoped that far from $d=4$ Eq. (25) is a reasonable approximation, since for $d=6$ it gives the exact value $\nu=\frac{1}{2}$.

Consequently, in the presence of defects of the random-field type the position of the fixed point of the renormalization-group equation is not controlled by the proximity to the lower critical dimensionality; therefore, in contrast to the assertions of a number of authors (see, e.g., the literature

cited in Refs. 1, 2, and 9), in such systems it is not possible to perform an $\varepsilon$ expansion about the lower critical dimensionality. The physical reason for this unusual behavior is that, at the critical dimensionality itself, complete destruction of the long-range order does not occur (see below).

As can be seen from Fig. 2, for $d = 2$ for the Ising model ($d = 4$ for the $XY$ model) the corresponding ordered phases are characterized by a rigidity with the asymptotic dependence $G(L) = YL^{\theta}$, with a nonuniversal, disorder-dependent index $\theta = 1 - k$ ($\theta = 2 - k$ for the $XY$ model) and zero spontaneous magnetization. At zero temperature and with increase of the degree of disorder in such systems a phase transition to the paramagnetic phase occurs. Since the initial value $G_0 \approx J$ is finite, it may be asserted that at the transition point we have $\theta_c > 0$ ($k_c < 1$ and $k_c < 2$ for the Ising and $XY$ models, respectively). In the critical region the correlation length increases in accordance with the law (4), and the specific rigidity $Y$ vanishes as

$$Y \sim \xi^{-\theta_c} \propto (G_0 - G_c)^{\nu\theta_c}.$$

The information at our disposal is not sufficient for an estimate of the index $\nu$. For $\theta > \theta_c$ and nonzero temperature, in the system under consideration a temperature phase transition occurs, which can be described as follows. The GLF for this case, for the same reason as for ordinary defect-free systems undergoing a phase transition as a result of thermal fluctuations, has, to lowest order in $T/G$, the form

$$\beta(G) = \theta - cT/G, \tag{27}$$

where $c \sim 1$, while the second term, just as in Eqs. (6) and (13), corresponds to the entropic correction. If $\theta \ll 1$, the use of the latter formula to find the critical behavior is fully justified. In this approximation, we find

$$\xi \approx a \left| \frac{T - T_c}{T_c} \right|^{-\nu}, \quad \nu \approx \theta^{-1}, \tag{28}$$

$$T_c \approx \theta J, \tag{29}$$

$$Y \approx T_c \xi^{-\theta} \approx T_c a^{-\theta} \left( \frac{T_c - T}{T_c} \right)^{\nu\theta} \approx a^{-\theta} (T_c - T). \tag{30}$$

We shall show that the results obtained lead to the conclusion that there exists a tricritical point in the $(k, T)$ plane. If we have in mind a second-order temperature phase transition from the paramagnetic phase to an ordered phase with rigidity $G \propto L^{\theta}$, the corresponding transition temperature vanishes at that degree of disorder for which $\theta = 0$. On the other hand, it was shown above that a zero-temperature phase transition in such a system occurs at a lower disorder $k_c$, i.e., when $\theta_c$ is still greater than zero. In our view, the only way to resolve this contradiction is to assume the existence of a tricritical point with approximate coordinates $(T_c(k_c), k_c)$, at which a line of second-order phase transitions, issuing from the transition point $T_c(0)$ of the pure system, terminates. The point with coordinates $(0, k_c)$ automatically turns out to be tricritical, and the line connecting the two tricritical points corresponds to first-order transitions.

Although, up to now, we have been speaking of the critical dimensionality, it is clear from continuity considerations that the picture outlined should be preserved qualitatively in higher dimensions as well. It is curious that the existence of a tricritical point in the Ising model is predicted in the mean-field approximation for a certain distribution of the random field.[11]

The effect of an external magnetic field on the systems under consideration, in a space of the critical dimensionality, can be investigated in exactly the same way as has been done in the phenomenological theory of a spin glass with a finite interaction range.[4] Let us consider the possibility of the formation, in the ordered phase of the two-dimensional Ising model (or four-dimensional $XY$ model), of a "droplet" of linear size $L$ with global reversal of the spin in an external field $H$. The change of the energy here is estimated as[4]

$$E \simeq YL^{\theta} - \text{const}\, mH(L/a)^{d/2}, \tag{31}$$

where $m$ is the spin magnetic moment, and the second term corresponds to the exchange-energy gain, which is found from considerations of the Imry-Ma type.[12] Since, in the systems under consideration, we have $\theta < 2$ (we recall that $\theta < 1$ hold for the two-dimensional Ising model and $\theta < 2$ for the four-dimensional $XY$ model), droplets of a size exceeding the "magnetic length"

$$L_H \simeq (Ya^{d/2}/mH)^{2/(d-2\theta)}$$

will be oriented along the field; consequently, the ground state is nondegenerate and the phase transition is smeared out.

For $2 < d < 4$ for the $XY$ model ($1 < d < 2$ for the Ising model) the Gell-Mann-Low equation has an unstable fixed point $G_{c1}$ and a stable fixed point $G_{c2}$. If the initial value satisfies $G_0 < G_{c1}$, the system is in the disordered phase. If we have $G_0 > G_{c1}$, as $L$ increases the measurable rigidity tends to $G_{c2}$ and does not depend on the size of the system. The ordered phase in this case resembles the one-dimensional Ising model at zero temperature (we recall that the latter is characterized by a size-independent rigidity, equal to $2J$). As $d \to 4$ from below ($d \to 2$, for the Ising model), $G_{c2}$ tends to infinity, and, therefore, the indices determining the spatial behavior of the spin-spin correlation functions should vanish as $d \to 4 - 0$. In addition, it can be seen from Fig. 2 that all the phases for $2 < d < 4$ ($1 < d < 2$, respectively) are similar. These properties of the phases with constant rigidity are in qualitative agreement with the results of Ref. 8, in which a renormalization-group analysis of the Hamiltonian (19) in real space was performed.

At an arbitrarily low temperature, just as in the one-dimensional Ising model, disordering occurs, since in the thermodynamic limit the probability of formation of a droplet of the opposite sign is nonzero and proportional to $\exp(-G_{c2}/T)$. The correlation length here is given by the formula

$$\xi \approx a \exp(G_{c2}/T). \tag{32}$$

For $d \to 4 - 0$, using the expression (22) we find

$$\xi \approx a \exp\left[ \text{const} \frac{J}{T} \left( \frac{2}{k} \right)^{2/(4-d)} \right].$$

An analogous formula can also be written out for the Ising model.

Thus, we arrive at the conclusion that, in the presence of defects of the random-field type, at a nonzero temperature no phase transitions are possible in the Ising system for $d < 2$ or in the $XY$ model for $d < 4$. The corresponding phases are a paramagnet with the correlation length (32).

## 4. ISING MODEL WITH DEFECTS OF THE RANDOM-TEMPERATURE TYPE

An interesting application of the method described above is the investigation of the phase transition in the Ising model, at zero temperature and in the presence of defects of the "random temperature" or "random coupling" type,[1,2] as the degree of disorder is increased, since it is known that in this case we are dealing with a percolation problem.[1,2] Substituting into (17) the value of $\zeta$ from Ref. 13, viz., $\zeta = (2/9)(5-d)$, we find the form of the GLF:

$$\beta = d-1-k\left(\frac{G}{J}\right)^{2(1-2d)/9(d-1)}. \tag{33}$$

Estimating in the same way as was done above for the $XY$ model and Ising model in a random field, for the correlation-length index we have

$$\nu^{-1} \approx {}^2/_9(2d-1). \tag{34}$$

This estimate is certainly inapplicable for $d \to 1$, but for not too small $d$ it gives reasonable results. For $d = 2$, from (34) we find $\nu \approx \frac{3}{2}$, which is fairly close to the exact result $\nu = \frac{4}{3}$ (Ref. 14); for $d = 3$ we obtain the value $\nu \approx 0.9$, which is in excellent agreement with the most accurate numerical data ($\nu = 0.88$).[14] The situation is analogous for larger $d$ as well: For $d = 6$, it follows from Eq. (34) that we have $\nu \approx \frac{9}{22}$, which is close to the exact value $\nu = \frac{1}{2}$ [we recall that the upper critical dimensionality for the percolation problem is $d = 6$ (Ref. 1)].

The index $\mu$, expressed here by the general formula (5), corresponds to a discrete lattice formulation of the percolation problem (see, e.g., Ref. 15 and the literature cited therein). If we are speaking of a random network of resistances, it describes the way in which the conductivity vanishes near the percolation limit.

We shall consider now the phase transition that occurs at nonzero temperature. To investigate it, we must take into account the renormalization of the random part of the Hamiltonian (14), and also the temperature dependence of the effective rigidity $\Gamma$. This can be taken into account phenomenologically by introducing an extra index into the general formula (17). Following the ideas described in Ref. 2, we shall demonstrate how it is possible to calculate this index. In Ref. 13 it was shown that a unified description of defects of different types is possible if the correlation function $R(f)$ of the random potential for $f \gg a$ is chosen in the form

$$R(f) = \Delta(f/a)^{-\gamma}.$$

For $\gamma = -1$ it corresponds to defects of the rando ı-field type, and for $\gamma = \gamma_c = \frac{1}{2}$ the index $\zeta$, as calculated using arguments of the Imry-Ma type, corresponds to defects of the random-temperature type, and also to correlated disorder with $\gamma \geqslant \gamma_c$, independently of $\gamma$.

The estimate for the energy of a distorted wall of linear size $L$ and with characteristic transverse displacement $w$ has the form[2]

$$E \approx \Gamma L^{d-1}\left(\frac{w}{L}\right)^2 - \left[\Delta L^{d-1}\left(\frac{w}{a}\right)^{-\gamma}\right]^{1/2}.$$

To take account of the above-mentioned renormalization, in the second term it is necessary to introduce an index $\sigma$ that allows for this effect:

$$E \simeq \Gamma L^{d-1}\left(\frac{w}{L}\right)^2 - \left[\Delta L^{d-1}\left(\frac{w}{a}\right)^{-\gamma-\sigma}\right]^{1/2}.$$

Minimizing the latter expression with respect to $w$ gives

$$w \approx \left(\frac{\Delta}{\Gamma^2 a^{\gamma+\sigma}}\right)^{1/(4+\gamma+\sigma)} L^{(5-d)/(4+\gamma+\sigma)}. \tag{35}$$

Substituting this dependence into the preceding formula and dividing by the area $L^{d-1}$ of the part of the wall under consideration, we find the contribution arising from the scale $L$ to the surface-tension coefficient:

$$\alpha(L) \approx -\Gamma\left(\frac{w}{L}\right)^2 \approx -\Gamma\left(\frac{\Delta}{\Gamma^2 a^{\gamma+\sigma}}\right)^{2/(4+\gamma+\sigma)} L^{2(1-d-\gamma-\sigma)/(4+\gamma+\sigma)}.$$

Summation over all scales from $a$ to $L$ leads to a formula of the form (16), from which the form of the GLF follows automatically:

$$\beta = d-1-k\left(\frac{G}{J}\right)^{2(1-d-\gamma-\sigma)/(4+\gamma+\sigma)(d-1)}. \tag{36}$$

We note that for $\sigma = 0$ and $\gamma = -1$ (defects of the random-field type; $T = 0$) we return to Eq. (21), while for $\sigma = 0$ and $\gamma = \frac{1}{2}$ (defects of the random-temperature type; $T = 0$) we return to (33). The unknown index can be established from considerations of scale invariance. In fact, near the critical point there is only one length scale in the system, and therefore, in the critical region, a distorted piece of a domain wall is transformed into an isotropic region of the high-temperature phase, of characteristic size $\xi$. Substituting the correlation length into (35) in place of $w$ and $L$, and also taking into account that $\Gamma \approx \alpha \propto \xi^{-(d-1)}$, we find the following relation between the indices:

$$\gamma+\sigma = d-1.$$

Substituting this into the expression (36), irrespective of the type of defects we find

$$\beta = d-1-k\left(\frac{J}{G}\right)^{4/(3+d)}. \tag{37}$$

In the limit $G \to \infty$ and $d > 1$, the correction term is much greater than the entropic correction $T/G$ to the GLF, and, therefore, from (37) and the general formula (4) there follows an expression for $\nu$:

$$\nu^{-1} \approx \frac{4(d-1)}{3+d}, \tag{38}$$

which is asymptotically exact for $d \to 1$ (i.e., coincides, to first order in $\varepsilon = d-1$, with the corresponding result (8) for the defect-free system). In the next orders in $\varepsilon$, however, as can be seen from the estimate (38), differences between the pure system and the defect system should appear, although numerically, for small $\varepsilon$, the indices should differ only slightly. This conclusion is in qualitative agreement with that of an exact investigation of the two-dimensional Ising model,[16] and also with the results of renormalizing-group analysis in the $4-\varepsilon$ approximation (see Ref. 1 and the literature cited therein). Although Eq. (38) is an asymptotically exact result for $d \to 1$, it can serve for estimates for large $d$ as well, since for $d = 4$ it gives a value $\nu \approx \frac{7}{12}$ close to $\frac{1}{2}$.

The formulas (37) and (38) are consistent from the standpoint of defects of the random-temperature type, and agree qualitatively with the conclusions of Ref. 17, in which it was shown that near the percolation limit and at $T > 0$ the critical behavior changes and is found to be asymptotically the same as in the case of weak disorder[1] (in particular, the upper critical dimensionality will be $d = 4$, instead of $d = 6$ for $T = 0$), with preservation of the usual relation (5) between the indices.

As regards defects of the random-field type [we recall that (37) and (38) were obtained, irrespective of the type of defect, under the assumption that the transition is continuous], the expression (38) is a further argument that, in the presence of such defects at zero temperature, at least in some part of the phase diagram, a first-order transition should occur, since it follows from (38) that the lower critical dimensionality for $T \neq 0$ is $d = 1$, which contradicts everything that has been said above about inhomogeneities of this type and also contradicts all the available data in the literature.

## 5. OTHER APPLICATIONS

The one-parameter approach turns out to be most effective in those cases when, in the system under consideration, there is no obvious microscopic order parameter. A classic example, here, evidently, is the spin glass. It has already been pointed out above that the ordered phase of the Ising spin glass is characterized by a distribution of probabilities of different values of the rigidity, where the width of this distribution, according to the results of numerical experiments,[18,19] behaves as $YL^{\theta}$. This distribution is symmetric about zero as a consequence of the evenness of the distribution of random exchanges, and therefore the sign of the rigidity is in no way distinguished here. This fact leads to the following important conclusion: The renormalization-group equation should be invariant under change of the sign of $G$. In other words, the GLF should be an even function of the ratio $T/G$.

Taking this circumstance into account, to lowest order in $T/G$ we have

$$\beta(G) = \theta - c(T/G)^2, \tag{39}$$

where $c \sim 1$. For $\theta \ll 1$ (according to the results of the numerical experiments of Refs. 18 and 19, this situation is realized in the three-dimensional case, where $\theta \simeq 0.2$), the latter relation and the general formula (4) can be used to determine the critical behavior. As a result, we find

$$\begin{gathered} \nu^{-1} \approx 2\theta, \\ Y \simeq T_g \xi^{-\theta} \simeq T_g a^{-\theta} \left( \frac{T_g - T}{T_g} \right)^{\nu\theta} \approx T_g a^{-\theta} \left( \frac{T_g - T}{T_g} \right)^{1/2}, \\ T_g \approx \theta^{1/2} J. \end{gathered} \tag{40}$$

Here, $T_g$ is the glass-transition temperature, and $J$ has the meaning of the absolute value of the characteristic exchange-interaction constant. The results (40) were obtained in a somewhat different way in Ref. 20. The above-mentioned symmetry property of the glass phase under change of the sign of $G$ leads to a linear dependence of the specific heat at low temperatures, since the entropic contribution to the free energy for $T \to 0$ should be quadratic in the temperature.

Up to now, the examples of the application of the one-parameter theory have been spin systems. From the analysis of just these, it is already clear that the rigidity of any system is equal to the total energy of the topological defect whose properties reflect the character of the spontaneous symmetry breaking. As soon as this aspect has been elucidated for any particular system, the subsequent analysis of the phase transition can be performed using the rules described above.

As a very simple example, consider the phase transition from the atomically smooth state to the rough state on the $(d-1)$-dimensional boundary of a crystal. By fixing (in opposite directions along the surface) the positions of the boundary to be at heights differing by a lattice constant, we obtain an elementary step. If its total energy $G(L \to \infty) \to 0$, the boundary does not sense the crystal relief and is rough. In any other case it will be smooth. In fact, if $G(L \to \infty) \to \infty$, the probability ($\propto \exp[-G(L)/T]$) of formation of a nucleus of size $L$ in the neighboring valley of the potential relief will vanish in the thermodynamic limit. Consequently, the crystal boundary will remain in the initial valley for an arbitrarily long time, and it is this which serves as a signal of a state with broken symmetry.

We consider first an ideal, defect-free surface in the smooth state. The step here is a $(d-2)$-dimensional object with total energy $G(L \to \infty) = \rho L^{d-2}$, while a kink on this step is a $(d-3)$-dimensional object with energy $E(L) = \delta L^{d-3}$ ($\rho$ and $\delta$ are the corresponding specific energies). From these definitions it rapidly follows that the inequalities $d > 2$ and $d > 3$ serve as necessary conditions for the existence of stable surfaces and steps, respectively. For the values $d \leqslant 3$ that have physical meaning, a step will always be rough; it is for this reason that we can disregard the periodicity along the surface, as is done in the usual investigation of the roughness transition for $d = 3$ by means of the sine-Gordon model.[21]

Consequently, we have the situation that we encountered in the analysis of the Heisenberg magnet, and all the results (11)–(13), together with the law of vanishing of the specific energy of a step (in a magnet, this is the spin-wave stiffness)

$$\rho \approx T_c \xi^{2-d}$$

are carried over in their entirety to the roughness phase transition. However, in contrast to the case of the magnetic system, the result (11) of the $2 + \varepsilon$ expansion gives a qualitatively incorrect result for $\varepsilon = 1 (d = 3)$. This fact is not surprising, since $d = 3$ is the upper critical dimensionality for the roughness phase transition (for $d > 3$ the crystal boundary is found to be smooth even without allowance for the crystal relief[2]) and in this case nonpower singularities near $T_c$ occur.[21] Nevertheless, in the $2 + \varepsilon$ approximation the results (11)–(13) remain asymptotically exact.

In defining the smooth state as a thermodynamic phase in which the step energy $G(L \to \infty) \to \infty$, we have included the *a priori* possibility of the existence of states with a dependence $G(L)$ that differs from the above-mentioned law $G(L) \propto L^{d-2}$. We now show that such an exotic smooth phase with $G(L) \propto L^{\theta} (\theta < 1)$ arises on the surface of an ordinary three-dimensional crystal in the presence of point defects. A step on the surface of the crystal is a one-dimensional object, similar to a domain wall in the two-dimensional Ising model. However, as follows from the above investigation, in the presence of defects of the random-field type in the two-dimensional Ising model a quasiferromagnetic phase with rigidity $G(L) \propto L^{\theta} (\theta < 1)$ is realized. The entire question is whether there exist defects of the random-field type (that "prefer" a higher to a lower step terrace, or vice versa) on the crystal surface. The inequivalence of a higher terrace and a lower terrace is connected with the fact that a step, being a surface defect, induces internal stresses in the crystal, from the standpoint of which, in the absence of inversion, the above-mentioned inequivalence arises.[22] Consequently, all point defects will play, in relation to the step, the role of defects of the random-field type. Thus, all that has been said

above about the properties of the two-dimensional Ising model with defects of the random-field type, including the phase diagram, tricritical points, indices, etc., carry over fully to the roughness phase transition on the defect surface of a crystal, with the role of the quasiferromagnetic phase being played by the smooth phase and that of the paramagnet being played by the rough phase.

## DISCUSSION OF THE RESULTS AND CONCLUSIONS

Before proceeding to a direct discussion of the results, we must discuss what place is occupied by the proposed variant of the theory of phase transitions among the existing approaches, and whether there are any fundamentally new aspects in this variant. The existing scheme of the renormalization-group description is a many-parameter scheme, its parameters being the coefficients of the Landau functional and also the characteristics of the random interactions (if such are present).[1,2] The results obtained from such an analysis are asymptotically exact near the upper critical dimensionality. At the same time, the investigation of such questions as the existence of long-range order in ideal and defect systems[1,2,5,12] has shown the fundamental importance of topological defects, the properties of which reflect the character of the spontaneous symmetry breaking and determine the lower critical dimensionality. It is clear, therefore, that they are responsible for the character of the phase transition (at least at the descriptive level) in dimensions higher than the lower critical dimensionality as well.

Since, in the final analysis, the ordered phase and disordered phase have, respectively, a nonzero and a zero topological-defect energy in the thermodynamic limit, it is natural to attempt to construct a one-parameter theory whose macroscopic parameter will be the rigidity that has been mentioned repeatedly above. Since, in such a theory, the concept of the order parameter is absent, one of its weak points is the incompleteness of the information obtainable about the phase transition (in the Introduction it was noted that the theory gives the possibility of calculating only the correlation-length index and the indices related to it by the scaling laws). At the same time, in a number of cases (the spin glass, spin systems with defects of the random-field type in a space of the lower critical dimensionality, and the roughness transition), when in the object investigated there is no obvious microscopic order parameter, this weak aspect turns out to be strong, since even in these cases the rigidity, as before, is well defined.

Thus, the fundamentally new point is the idea of using, instead of many parameters, just one—the rigidity, which subsumes all the important information about the properties of the ordered phase. Correspondingly, in the theory, instead of many renormalization-group equations[1] there is one, reflecting the character of the ordering and the cause of the phase transition. The results thus obtained "gravitate" toward the lower critical dimensionality. Thus, the one-parameter approach supplements the existing theory,[1] and, in a number of cases, leads to new results.

For ideal systems, we obtain asymptotically exact expressions for the transition temperature and indices near the lower critical dimensionality, and of these results the following are new: Expressions for the indices and transition temperature of the Ising model in the $1+\varepsilon$ approximation; expressions for the indices and transition temperature of the roughness phase transition in the $2+\varepsilon$ approximation.

In addition, the one-parameter theory in the $2+\varepsilon$ approximation reproduces the known results pertaining to the indices and transition temperature of the Heisenberg magnet.[6,7]

We turn now to a discussion of the results pertaining to systems with defects.

The central results in the presence of defects of the random-field type are as follows: The prediction of a new quasiferromagnetic phase in spin systems in space of the lower critical dimensionality; the construction of a disorder-temperature phase diagram on which, in dimensions not less than the lower critical dimensionality, there are two tricritical points; the prediction of new types of quasiferromagnetically ordered phases that are unstable against thermal fluctuations in dimensions less than the lower critical dimensionality, and, consequently, a proof of the absence of any phase transitions at a nonzero temperature; a proof that in such systems it is impossible to perform an $\varepsilon$ expansion about the lower critical dimensionality; a demonstration of the violation of the relation $d \to d-2$ between the indices of the ideal $XY$ system and the defect $XY$ system for $d \to 4$.

The results listed above are in many respects a consequence of the fact that the lower critical dimensionalities for the ideal system and the defect system do not coincide. They are also insensitive to the actual values of the roughness indices; it is important only that the values of the latter give the same critical dimensionality. For example, in deriving Eq. (22) we used the result of Ref. 10, obtained by means of perturbation theory. Although, at present, there are doubts as to its correctness,[9] the fact that the lower critical dimensionality is $d=4$ is not in dispute. Therefore, even if the criticism made by Fisher[9] is justified, the exponent in the second term of Eq. (22) should vanish for $d=4$ in any case, and this turns out to be sufficient for all the subsequent conclusions. Of course, the estimates (25) and (26) will be changed, but these are only estimates and all the qualitative conclusions remain unchanged.

It is also curious that the conclusion that a stable quasiferromagnetic phase and unstable quasiferromagnetic phases are present follows even without knowing the form of the extra terms in (21) and (22). Consider, e.g., the two-dimensional Ising model at zero temperature and with weak disorder, and assume that we know (e.g., from arguments of the Imry-Ma type[12]) that the system is in a space of the lower critical dimensionality. We try to guess the behavior of the GLF as $G \to \infty$. First, it is clear that $\beta(G)$ cannot tend to unity (otherwise, we have obtained ferromagnetism). Second, it is obvious that, for a small degree of disorder, defects begin to have an effect only for sufficiently large $G$, i.e., as soon as $\beta > 0$. For the largest value of the $G$ the GLF cannot intersect the abscissa, since when the dimensionality is increased infinitesimally the assumed intercept should move away discontinuously to infinity, and, because of continuity considerations, this is impossible. Thus, two variants of the behavior of $\beta(G)$ as $G \to \infty$ are conceivable: The GLF tends to a constant value, differing from unity in proportion to the small magnitude of the degree of disorder; the GLF tends to zero from above. In either of these cases, it is obvious that $G(L \to \infty) \to \infty$, which is the signature of a phase that is stable against thermal fluctuations and "softer" than a ferro-

magnet. The specific calculation performed in the paper shows that the first of the above variants is realized. For $d<2$ the second intercept $G_{c2}$ (Fig. 2) arises in a natural manner, with all the consequences that flow from this.

It is interesting that, in some numerical experiments on the two-dimensional Ising model with defects of the random-field type in the presence of weak disorder, a certain ordered phase, called by the authors a spin-glass phase, is found (see Ref. 23 and the references therein), with an ordered state of an obscure nature. In the light of the results obtained above, it is obvious that a spin-glass phase in such a system is impossible, since the rigidity is positive-definite. It is not ruled out that this phase and the quasiferromagnetic phase under discussion are identical. As shown in the preceding section, an experimental object on which the theoretical predictions could be checked is the surface of a defect crystal undergoing a roughness phase transition.

Note also that Fisher[9] has drawn attention to the fact that the position of the fixed point of the renormalization-group equation is not controlled by the proximity to the lower critical dimensionality, and also that the relation $d \to d-2$ between the indices in the $XY$ model may possibly not be fulfilled. However, a physical reason for this behavior (incomplete destruction of the longer-range order in a space of the critical dimensionality) and an explicit demonstration of the nonfulfillment of the above relation between the indices for $d \to 4$ are indicated only in the present paper.

As regards defects of the random-temperature type, here we have found the following new results: An accurate estimation formula for the indices of percolation theory has been obtained; it has been shown that, at a finite temperature to first order in $\varepsilon = d-1$, the value of the correlation-length index coincides with that for the ideal system, but in the next orders differences should appear that are greater the higher the dimensionality.

Thus, using this example in which the properties of a broad class of physical systems are studied, we have demonstrated the effectiveness of the one-parameter approach. It may be hoped that it will be found useful in the investigation of other systems too.

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