Resonant modification of quasistatic profiles of spectral lines of hydrogen in a plasma under the influence of noncollinear harmonic electric fields

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An analysis is made of resonant modification of the emission spectrum of an atom of hydrogen subjected to a static electric field **F** or a magnetic field **H**, as well as a superposition of hf noncollinear out-of-phase harmonic electric fields of frequency close to the Stark (in the field **F**) or the Zeeman (in the field **H**) splitting of an arbitrary energy level (upper or lower) of the hydrogen atom. A full set of quasi-energies and wave functions of the quasi-energy states of an atomic level is found and the results are used to calculate the splitting of an arbitrary spectral line of hydrogen. It is shown that in the case of inhomogeneous broadening of the hydrogen spectral lines in quasistatic internal plasma fields (electric or magnetic) the influence of such a superposition of hf noncollinear harmonic electric fields gives rise to singularities at certain points of the profiles of the spectral lines of hydrogen. Experimental detection of these singularities should make it possible to determine not only the characteristic intensity of the hf fields, but also their angular distribution, as well as the direction of rotation of the projection of the vector of the combined hf electric field on a plane perpendicular to the magnetic field.

1. INTRODUCTION

One of the most commonly used methods for diagnostics of hf oscillating electric fields in a plasma is spectroscopy based on modification of the emission spectra of atomic hydrogen under the influence of these fields (see Refs. 1–3 and the literature cited there). Such electric fields may exist in a plasma, for example, when it interacts with electromagnetic radiation, when large currents are passing, when a beam of charged particles is crossing a plasma, or when magnetic field lines become closed. At present, the most thoroughly investigated case is the modification of the emission spectrum of hydrogen in a plasma under the influence of a linearly polarized harmonic electric field of the form

$$\mathbf{E}_l(t)=\mathbf{E}_0\cos(\omega t+\vartheta). \tag{1}$$

Calculations reported in Refs. 4 and 5 deal with the intensities of the satellites of a hydrogen spectral line that appear in the emission spectrum under the influence of the field (1) at frequencies $\pm\omega$, $\pm 2\omega$,... measured relative to the unperturbed position of the spectral line of hydrogen. The concept of quasi-energy is introduced in Ref. 6 and the shifts and splittings of the quasi-energy levels in the field (1) are found for an atom of hydrogen. The transformation of the spectra of hydrogen under the influence of a superposition of the field (1) and a quasistatic electric field **F** is considered in Refs. 7–15, while superposition of the field (1) and a quasistatic magnetic field **H** is discussed in Refs. 13 and 16. In experimental situations a quasistatic field **F** may be in the form of ionic microfields of a plasma or it may be the field of an lf plasma turbulence, whereas a quasistatic field **H** may be used to confine a plasma.

A method for determining the intensity of oscillating electric fields in a plasma suggested in Ref. 17 is based on recording of the resonance "relief" exhibited by quasistatic Stark profiles of the spectral lines of hydrogen. Such relief is due to an appearance of a resonance between the frequency of the oscillating electric fields and the Stark splitting of the upper (or lower) atomic level under the influence of the quasistatic field **F**. A detailed investigation of the structure of such a resonance relief is made in Refs. 11, 12, 14, and 15 for some spectral lines of hydrogen subjected to a linearly polarized field of the kind described by Eq. (1). General expressions are obtained in Ref. 13 for the wave functions of the quasi-energy states of an atom of hydrogen which experiences a resonance between the frequency of the hf field (1) and the splitting (Stark or Zeeman) of an arbitrary level n in an electric field **F** or a magnetic field **H**; these expressions can be used to calculate the resonance relief for any spectral line of hydrogen.

However, in many experimental situations the oscillating electric fields acting on an atom of hydrogen in a plasma are not linearly polarized but represent a superposition of noncollinear out-of-phase oscillations of the type

$$\mathbf{E}(t)=\sum_k \mathbf{E}_k\cos(\omega t+\vartheta_k), \tag{2}$$

where the vectors $\mathbf{E}_k$ and $\mathbf{E}_{k'}$ are not parallel. The field (2) may represent an elliptically polarized wave (for example, many types of waves in a magnetically active plasma have elliptical polarization[18]) or it may be the field of hf plasma turbulence representing a set of a large number of noncollinear oscillations with random phases. We shall investigate resonance effects in the emission spectrum of the atomic hydrogen subjected simultaneously by an hf field of the type described by Eq. (2) and one of two types of quasistatic fields: electric **F** or magnetic **H**. Consider a situation defined by the inequality

$$(\hbar/m_e e)n^2|\mathbf{E}(t)|\,\omega^{-1}\ll 1, \tag{3}$$

which ensures the existence of a one-photon resonance between the frequency of the field (2) and the splitting of the level whose principal quantum number is n; we assume the presence of a quasistatic field **F** or **H**. It follows from Eq. (3) that in calculating the parameters of such resonant effects it is sufficient to include in Eq. (2) only the terms orthogonal to the direction of the quasistatic field. It is these compo-

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nents that can induce nonadiabatic resonant transitions between the Stark or Zeeman sublevels of the hydrogen atom which appear in the quasistatic field. Therefore, we concentrate our attention on calculations of the resonant modification of the hydrogen spectra under the influence of a superposition of an elliptically polarized electric field $\mathbf{E}_{el}(t)$ and a field **F** (Sec. 2) or **H** (Sec. 3) orthogonal to the elliptically polarized field. The most important results reported in the present paper are the quasi-energies and the wave functions of the quasi-energy states of the hydrogen atom for an arbitrary level n. The results obtained and their diagnostic applications are discussed in Sec. 4.

We assume that the frequency ω and the characteristic splitting of the level n of the hydrogen atom in fields **F**, **H**, and $\mathbf{E}_{el}(t)$ considerably exceed the fine-structure splitting, but are much smaller than the separation between the level n and the next level $n+1$.

2. RESONANT EFFECTS IN THE STARK SPECTRUM OF HYDROGEN

Consider a hydrogen atom subjected to a superposition of a quasistatic electric field $\mathbf{F} = F\mathbf{e}_z$ and an hf elliptically polarized electric field

$$\mathbf{E}_{el}(t) = E_x\cos(\omega t)\mathbf{e}_x + E_y\cos(\omega t+\delta)\mathbf{e}_y, \quad E_x \geqslant 0, \quad E_y \geqslant 0, \tag{4}$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors along the x, y, and z axes.

The wave function of the hydrogen atom satisfies the Schrödinger equation$^{1)}$

$$i\partial\Psi/\partial t = \mathscr{H}\Psi \tag{5}$$

with the Hamiltonian

$$\mathscr{H} = \mathscr{H}_a + xE_x\cos\omega t + yE_y\cos(\omega t+\delta) + zF, \tag{6}$$

where $\mathscr{H}_a$ is the unperturbed Hamiltonian.

We seek the solution of Eq. (5) for the level n in the basis of wave functions described using parabolic coordinates $\varphi_{n_1n_2m}(\mathbf{r})$ with the z quantization axis:$^{2)}$

$$\Psi(\mathbf{r},t) = \sum_k \exp(-i\mathscr{E}_n t - iz_{kk}Ft)C_k(t)\varphi_k(\mathbf{r}), \tag{7}$$

where $\mathscr{E}_n$ is the energy of the unperturbed level n; $k \equiv (n_1, n_2, m)$; $u_{kk'} \equiv \langle\varphi_k|u|\varphi_{k'}\rangle$; $u = x, y, z$. Substituting Eq. (7) into Eq. (5), we obtain

$$i\dot{C}_k = 2^{-1}\sum_p \{[x_{kp}E_x + y_{kp}E_y\exp(i\delta)]\exp(i\omega t)$$
$$+[x_{kp}E_x + y_{kp}E_y\exp(-i\delta)]\exp(-i\omega t)\}\exp(i\Delta z_{kp}Ft)C_p, \tag{8}$$

where $\Delta z_{kp} \equiv z_{kk} - z_{pp}$. The matrix elements x_{kp} and y_{kp} may differ from zero only between the adjacent Stark states separated by $|\Delta z_{kp}|F = 3nF/2$. Hence, we obtain the condition for a resonance in the interaction of the field $\mathbf{E}_{el}(t)$ with an atom of hydrogen which is in the quasistatic field **F**:

$$3nF/2 = \omega + \Delta, \tag{9}$$

where Δ is the frequency offset $|\Delta| \ll \omega$. If the condition (9) is obeyed, we can simplify the system (8) by ignoring in the resonance approximation the terms oscillating rapidly (at the frequency $\sim 2\omega$). The result is

$$i\dot{C}_k = 2^{-1}\sum_p [x_{kp}E_x + y_{kp}E_y\cos\delta$$
$$+ i(-1)^{(s_p-1)/2}y_{kp}E_y\sin\delta]C_p\exp(-is_p\Delta t), \tag{10}$$

where

$$s_p = \begin{cases} 1, & (n_1-n_2)_k - (n_1-n_2)_p < 0, \\ -1, & (n_1-n_2)_k - (n_1-n_2)_p > 0. \end{cases}$$

Using the relationships (given in the Appendix) between the matrix elements y_{kp} and $(l_x)_{kp}$, where l_u is the projection operator of the orbital momentum of an electron along the u axis, we can rewrite the system (10) in the form

$$i\dot{C}_k = 2^{-1}\sum_p [x_{kp}E_x + y_{kp}E_y\cos\delta$$
$$-{}^3/_2(l_x)_{kp}E_y\sin\delta]C_p\exp(-is_p\Delta t), \tag{11}$$

where

$$(l_x)_{kp} \equiv \langle\varphi_k|l_x|\varphi_p\rangle.$$

We can easily see that the system (11) describes evolution of a hydrogen atom in the state with the principal quantum number n under the influence of superposition of two effective static fields: an electric field

$$\mathbf{f} = 2^{-1}E_x\mathbf{e}_x + 2^{-1}E_y\cos\delta\mathbf{e}_y + \frac{2\mathbf{e}_z\Delta}{3n} \tag{12}$$

and a magnetic field

$$\mathbf{h} = -\frac{3nE_y\sin\delta\mathbf{e}_x}{4\mu_0}, \tag{13}$$

where μ_0 is the Bohr magneton. The solution of this problem is well known.[20] According to Ref. 20, the splitting of the level n in the crossed fields **f** and **h** is given by the expression

$$\lambda_{nn'n''} = |\boldsymbol{\omega}_1|n' + |\boldsymbol{\omega}_2|n'',$$
$$n', n'' = -j, -j+1, \ldots, j; \quad j = (n-1)/2, \tag{14}$$

where the vectors $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are of the form

$$\boldsymbol{\omega}_1 = \mu_0\mathbf{h} - {}^3/_2 n\mathbf{f}, \quad \boldsymbol{\omega}_2 = \mu_0\mathbf{h} + {}^3/_2 n\mathbf{f}.$$

In the case under consideration when the vectors **f** and **h** are given by Eqs. (12) and (13), the values of ω_1 and ω_2 are

$$\omega_s = [\Delta^2 + (3n/4)^2\sigma_s^2]^{1/2},$$
$$\sigma_s \equiv [E_x^2 + E_y^2 + (-1)^{s-1}2E_xE_y\sin\delta]^{1/2}, \quad s = 1, 2. \tag{15}$$

Under exact resonance conditions ($\Delta = 0$) the frequencies ω_s of Eq. (15) have a simple physical meaning. In fact, allowing for the fact that the major $E^{(\max)}$ and minor $E^{(\min)}$ semiaxes of the ellipse describing rotation of the vector $\mathbf{E}_{el}(t)$ of Eq. (4) are

$$E^{(max)}=2^{-1}[E_x^2+E_y^2+(E_x^4+E_y^4+2E_x^2E_y^2\cos 2\delta)^{1/2}]^{1/2},$$

$$E^{(min)}=2^{-1}[E_x^2+E_y^2-(E_x^4+E_y^4+2E_x^2E_y^2\cos 2\delta)^{1/2}]^{1/2}, \tag{16}$$

we find if $\Delta = 0$, then apart from the factor $3n/4$, one of the frequencies ω_s of Eq. (15) coincides with $E^{(\max)} + E^{(\min)}$ and the other with $E^{(\max)} - E^{(\min)}$. The wave functions corresponding to the frequencies (15) can be represented, as in Ref. 20, by the expression

$$\Phi_{nn'n''}(\mathbf{r})=\sum_{k_1k_2=-j}^{j}\exp[-i(k_1\alpha_1+k_2\alpha_2)]$$

$$\times\, d^{(j)}_{n'k_1}(\beta_1)\,d^{(j)}_{n''k_2}(-\beta_2)\,\varphi_{nk_1k_2}(\mathbf{r}), \tag{17}$$

where $d^{(j)}_{n'k_1}(\beta_1)$ and $d^{(j)}_{n''k_2}(-\beta_2)$ are the Wigner functions; α_i and β_i are the Euler angles[3)] which describe rotation from the coordinate system xyz to the system $x'y'z'$ with its z' axis parallel to ω_i $(i = 1, 2)$; and the quantum numbers k_1 and k_2 are in one-to-one relationship with the parabolic quantum numbers n_1 and n_2:

$$k_1=2^{-1}(m+n_2-n_1),\quad k_2=2^{-1}(m-n_2+n_1).$$

Note that the selection of the signs of the arguments of the functions $d^{(j)}_{n'k_1}$, $d^{(j)}_{n''k_2}$ is matched to the selection of the phases of the wave functions $\varphi_{nk_1k_2}(\mathbf{r})$. The Euler angles α_1, α_2, β_1, and β_2 are described by the following expressions:

$$\sin\alpha_1=-E_y\cos\delta/\sigma_1,\quad \cos\alpha_1=-(E_x+E_y\sin\delta)/\sigma_1,$$

$$\sin\alpha_2=E_y\cos\delta/\sigma_2,\quad \cos\alpha_2=(E_x-E_y\sin\delta)/\sigma_2,$$

$$\beta_1=\arccos(-\Delta/\omega_1),\quad \beta_2=\arccos(\Delta/\omega_2).$$

Using Eqs. (7), (9), (10), (14), and (17), we can represent the wave functions of the hydrogen atom with the Hamiltonian (6) undergoing a resonance described by Eq. (7) by the following wave functions of quasi-energy states:

$$\Psi_{nn'n''}(\mathbf{r},t)=\exp(-i\mathscr{E}_nt-i\lambda_{nn'n''}t)$$

$$\times\sum_{k_1,k_2=-j}^{j}\exp[-i(k_1\alpha_1+k_2\alpha_2)]$$

$$\times d^{(j)}_{n'k_1}(\beta_1)\,d^{(j)}_{n''k_2}(-\beta_2)\exp[i(k_1-k_2)\omega t]\,\varphi_{nk_1k_2}(\mathbf{r}), \tag{18}$$

where $\lambda_{nn'n''}$ is described by Eqs. (14) and (15). Using Eq. (18), we can readily calculate the emission spectrum for an arbitrary transition $n_a \to n_b$ in the hydrogen atom. In particular, the emission spectrum of the line L_α with the polarizations x, y, and z is described by

$$I^{(\nu)}(\Delta\omega)=\sum_{r=1}^{2}[A_{k,k}\delta(\Delta\omega-2^{-1}(-1)^r(\omega_1+\omega_2))$$

$$+A_{k+1,k}\delta(\Delta\omega-2^{-1}(-1)^r(\omega_1-\omega_2))], \tag{19}$$

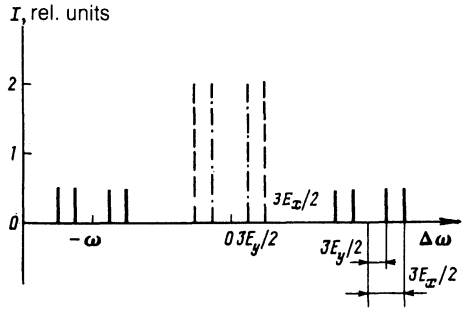

FIG. 1. Spectrum of the hydrogen line L_α (above the $\Delta\omega$ axis) under the conditions of an exact resonance of Eq. (9) (with an offset $\Delta = 0$) between the frequency of an elliptically polarized electric field of Eq. (4) and the Stark splitting of the $n = 2$ level in a static electric field $\mathbf{F} = F\mathbf{e}_z$, calculated on the assumption that E_x and E_y are, respectively, the major and minor semiaxes of the polarization ellipse. The dashed line is the spectrum with the x polarization; the chain lines represent the spectrum with the y polarization; the solid lines give the spectrum with the z polarization. The frequency $\Delta\omega$ is measured from the unshifted position of the L_α line. The intensities of the components in the spectrum of L_α are independent of the relationship between E_x and E_y.

$$A_{k,p}=2^{-1}\{1-(-1)^k[9(E_x^2-E_y^2)+4(-1)^p\Delta^2]/(4\omega_1\omega_2)\},$$

$$\nu=x,\ y,$$

$$I^{(z)}(\Delta\omega)=S(\Delta\omega)+S(-\Delta\omega),$$

$$S(\Delta\omega)=4^{-1}\sum_{r=1}^{2}\sum_{p=1}^{2}[1-(-1)^r\Delta/\omega_1]\times[1-(-1)^p\Delta/\omega_2]\delta(\Delta\omega$$

$$-\omega+2^{-1}(-1)^r\omega_1+2^{-1}(-1)^p\omega_2). \tag{20}$$

In Eq. (19), $k = 1$ corresponds to $\nu = x$, whereas $k = 2$ corresponds to $\nu = y$; the frequencies ω_1 and ω_2 are given by Eq. (15) with $n = 2$; the argument of the δ functions identifies the positions of the spectral components; the frequency $\Delta\omega$ is measured from the unperturbed position of the L_α line. The spectra $I^{(x)}(\Delta\omega)$ and $I^{(y)}(\Delta\omega)$ exhibit splitting of the central component of the L_α line, whereas the spectrum $I^{(z)}(\Delta\omega)$ describes a side line. Figure 1 shows, by way of example, the spectrum of L_α with the x, y, and z polarizations, calculated for the case of exact resonance [$\Delta = 0$ in Eq. (9)].

3. RESONANT EFFECTS IN THE ZEEMAN SPECTRUM OF HYDROGEN

We now consider an atom of hydrogen interacting with a superposition of a quasistatic magnetic field $\mathbf{H} = H\mathbf{e}_z$ and the field $\mathbf{E}_{el}(t)$ described by Eq. (4). In this case the wave functions of the hydrogen atom are found by solving the Schrödinger equation

$$i\partial\Psi/\partial t=[\mathscr{H}_a+xE_x\cos\omega t+yE_y\cos(\omega t+\delta)+\mu_0 l_zH]\,\Psi. \tag{21}$$

By analogy with Eq. (7) we seek the solution of Eq. (21) for the level n in the basis of wave functions described using parabolic coordinates with the quantization axis z:

$$\Psi(\mathbf{r},t)=\sum_{k}\exp(-i\mathscr{E}_n t-im\mu_0 Ht)C_k(t)\varphi_k(\mathbf{r}), \qquad (22)$$

where $k\equiv(n_1n_2m)$. Substituting Eq. (22) into Eq. (21), we obtain

$$i\dot{C}_k=2^{-1}\sum_{p}\{[x_{kp}E_x+y_{kp}E_y\exp(i\delta)]\exp(i\omega t)$$
$$+[x_{kp}E_x+y_{kp}E_y\exp(-i\delta)]$$
$$\times\exp(-i\omega t)\}\exp[i(m_k-m_p)\mu_0 Ht]C_p. \qquad (23)$$

In Eq. (23) the matrix elements x_{kp} and y_{kp} may differ from zero only for the states characterized by $|m_k-m_p|=1$. Hence, we obtain the condition for a resonance:

$$\mu_0 H=\omega+\Delta,\quad |\Delta|\ll\omega. \qquad (24)$$

Applying, subject to the condition (24), the resonance approximation to the system (23), we find that

$$i\dot{C}_k=2^{-1}\Big[\sum_{p'}(x_{kp'}E_x+y_{kp'}E_y$$
$$\times\cos\delta+iy_{kp'}E_y\sin\delta)C_{p'}\exp(-i\Delta t)$$
$$+\sum_{p''}(x_{kp''}E_x+y_{kp''}E_y\cos\delta-iy_{kp''}E_y\sin\delta)C_{p''}\exp(i\Delta t)\Big], \qquad (25)$$

where p' represents a set of parabolic quantum numbers $(n_1n_2m)_{p'}$ which is characterized by $m_{p'}>m_m$, whereas p'' is a set of parabolic quantum numbers $(n_1n_2m)_{p''}$, which is characterized by $m_{p''}<m_k$. Using Eqs. (A1) and (A2) in the Appendix, we replace the matrix elements iy_{kp} in the system (25) with the matrix elements x_{kp}. The result is

$$i\dot{C}_k=2^{-1}\sum_{p}[x_{kp}(E_x-E_y\sin\delta)+y_{kp}E_y\cos\delta]\exp(-is_p\Delta t)C_p, \qquad (26)$$

where

$$s_p=\begin{cases}1, & m_p=m_k+1,\\ -1, & m_p=m_k-1.\end{cases}$$

We can easily see that the system (26) describes the simultaneous Stark–Zeeman effect for the level n under the influence of an effective static electric field

$$\mathbf{f}_0=2^{-1}(E_x-E_y\sin\delta)\mathbf{e}_x+2^{-1}E_y\cos\delta\mathbf{e}_y \qquad (27)$$

and an effective static magnetic field

$$\mathbf{h}_0=\mu_0\Delta\mathbf{e}_z. \qquad (28)$$

Hence, the splitting of the level n is described by

$$\varepsilon_{nn'n''}=|\varkappa_1|n'+|\varkappa_2|n'',$$
$$n',n''=-j,-j+1,\ldots,j,\quad j=(n-1)/2, \qquad (29)$$

where

$$\boldsymbol{\varkappa}_1=\mu_0\mathbf{h}_0-{}^3/_2n\mathbf{f}_0,\quad \boldsymbol{\varkappa}_2=\mu_0\mathbf{h}_0+{}^3/_2n\mathbf{f}_0. \qquad (30)$$

Substituting Eqs. (27), (28), and (30) into Eq. (29), we obtain

$$\varepsilon_{nn'n''}=(n'+n'')[\Delta^2+(3n/4)^2\sigma_2^2]^{1/2}, \qquad (31)$$

where σ_2 is defined by Eq. (15). The solutions of Eq. (21) can be represented, by analogy with Eq. (18), in the form of the wave functions of quasi-energy states

$$\Psi_{nn'n''}(\mathbf{r},t)=\exp(-i\mathscr{E}_n t-i\varepsilon_{nn'n''}t)$$
$$\times\sum_{k_1,k_2=-j}^{j}\exp[-i(k_1+k_2)\alpha-ik_2\pi]$$
$$\times d^{(j)}_{n'k_1}(\beta)d^{(j)}_{n''k_2}(-\beta)\exp[-i(k_1+k_2)\omega t]\varphi_{nk_1k_2}(\mathbf{r}),$$
$$k_1=2^{-1}(m+n_2-n_1),\quad k_2=2^{-1}(m-n_2+n_1). \qquad (32)$$

In the system (32) the Euler angles α and β define rotation from the coordinate system xyz to the system $x'y'z'$ with the z' axis parallel to $\varkappa_1$. The angles α and β obey the following relationships:

$$\cos\alpha=\frac{-E_x+E_y\sin\delta}{\sigma_2},\qquad \sin\alpha=\frac{-E_y\cos\delta}{\sigma_2},$$
$$\beta=\arccos\{\Delta[\Delta^2+(3n/4)^2\sigma_2^2]^{-1/2}\}.$$

We can use the wave function of Eq. (32) to find readily the emission spectrum of the hydrogen atom for any transition $n_a\to n_b$. We give explicitly the spectrum of the line L_α with the polarizations x, y, and z:

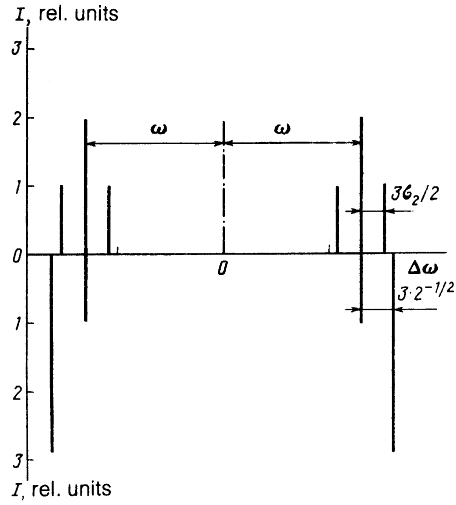

FIG. 2. Spectrum of the hydrogen line L_α with the x and y polarizations under the conditions of a resonance described by Eq. (24) between the frequency of an elliptically polarized electric field of Eq. (4) and the Zeeman splitting of the level $n=2$ in a static magnetic field $\mathbf{H}=H\mathbf{e}_z$. The solid vertical lines above the $\Delta\omega$ axis represent the spectrum of L_α in the exact resonance case when $\Delta=0$, whereas the spectrum below the $\Delta\omega$ axis represents L_α when $\Delta=3\sigma_2/2$; here, $\sigma_2=(E_x^2+E_y^2-2E_xE_y\sin\delta)^{1/2}$. The frequency $\Delta\omega$ is measured from the unshifted position of the L_α line.

$$I^{(x)}(\Delta\omega)=I^{(y)}(\Delta\omega)=\mathcal{S}(\Delta\omega)+\mathcal{S}(-\Delta\omega),$$
$$\mathcal{S}(\Delta\omega)=[9\sigma_2^2/8R_\Delta^2]\delta(\Delta\omega-\omega)+4^{-1}(1+\Delta/R_\Delta)^2\delta(\Delta\omega-\omega-R_\Delta)$$
$$+4^{-1}(1-\Delta/R_\Delta)^2\delta(\Delta\omega-\omega+R_\Delta),\quad R_\Delta\equiv(\Delta^2+9\sigma_2^2/4)^{1/2},$$
$$I^{(z)}(\Delta\omega)=2\delta(\Delta\omega),\tag{33}$$

where the frequency $\Delta\omega$ is measured from the unperturbed position of the line L_α. Figure 2 shows, by way of example, how the spectrum of L_α is modified as the offset Δ changes. Note that when the condition for a resonance given by Eq. (24) is obeyed, the emission spectrum of hydrogen depends strongly on the direction of rotation of the vector $\mathbf{E}_{\rm el}(t)$ in Eq. (4). This follows even from Eq. (27) where the effective static field $\mathbf{f}_0$, governing the additional splitting of the spectral components, depends on the sign of $\sin\delta$.

4. DISCUSSION: POSSIBILITY OF DIAGNOSTICS OF NONCOLLINEAR hf FIELDS IN A PLASMA

We found the quasi-energies and the wave functions of quasi-energy states for an arbitrary level n of the hydrogen atom under the conditions of resonance between the splitting of this level in a static field (electric or magnetic) and the frequency of an hf oscillating electric field representing a superposition of noncollinear quasimonochromatic oscillations. This was possible because we were able to reduce the problem in hand to the familiar problem of the hydrogen atom subjected to a superposition of static electric and magnetic fields. The conditions of validity of our results require that the frequencies of the resonant splitting [given by relationships (14), (15), and (31)] should be low compared with ω. If in the investigated plasma the dominant spectral line-broadening mechanism of hydrogen is an inhomogeneous broadening in quasistatic internal plasma fields (electric or magnetic), then the resonance splitting of the spectral components discussed in the present paper should be manifested as a relief in the spectral line profiles.

We first consider the resonant singularities in the profiles of the spectral lines of hydrogen in the case when the Stark mechanism of the broadening of these spectral lines predominates in inhomogeneous quasistatic electric fields **F**. We assume that the condition for a resonance (9) is satisfied for a certain group of the hydrogen atoms which are in the upper state n_a. Then, the emission spectrum for these atoms due to the $a\rightarrow b$ transition consists of a set of components located at frequencies differing by $\Delta\omega$ relative to the frequency of the unperturbed $a\rightarrow b$ transition:

$$\Delta\omega=\lambda_{(nn'n'')a}(\Delta)+(n_1-n_2)_a\omega-(n_b/n_a)(n_1-n_2)_b(\omega+\Delta)$$
$$=[(n_1-n_2)_a-(n_1-n_2)_b n_b/n_a]\omega+n_a'[\Delta^2+(3n_a/4)^2\sigma_1^2]^{1/2}$$
$$+n_a''[\Delta^2+(3n_a/4)^2\sigma_2^2]^{1/2}-(n_1-n_2)_b\Delta n_b/n_a,$$
$$n_a',\ n_a''=-j_a,\ -j_a+1,\ldots,j_a,\quad j_a=(n_a-1)/2,\tag{34}$$

where the relationships (9) and (18) are used and an allowance is made for the fact that the wave function of the lower level n_b outside the resonance is

$$\Psi_{(n_1n_2m)_b}(\mathbf{r},t)=\exp[-i\mathscr{E}_{n_b}t-{}^3/_2 n_b(n_1-n_2)_bFt]\varphi_{(n_1n_2m)_b}(\mathbf{r}).\tag{35}$$

A change in the offset Δ should generally shift the spectral components at the frequencies $\Delta\omega$ defined by Eq. (34), and this should be accompanied by changes in the intensities of these components. Consequently, the smooth quasistatic Stark profile of a hydrogen spectral line may have a resonance relief in the vicinity of the frequencies

$$\Delta\omega=[(n_1-n_2)_a-(n_1-n_2)_b n_b/n_a]\omega.\tag{36}$$

Consider first the case $(n_1-n_2)_b=0$, manifesting most clearly the difference between a resonance involving a linearly polarized field (investigated earlier in Refs. 7, 9, and 11–15) and a resonance involving an elliptically polarized field of the form described by Eq. (4). Since the relationship $E_xE_y\sin\delta=0$ is satisfied for a linearly polarized field, it follows from Eq. (34) that if $(n_1-n_2)_b=0$ holds, then

$$\Delta\omega=(n_1-n_2)_a\omega+(n_a'+n_a'')[\Delta^2+(3n_a/4)^2(E_x^2+E_y^2)]^{1/2}.\tag{37}$$

The relationship (37) demonstrates that the emission spectrum due to the $a\rightarrow b$ transition in the vicinity of the frequencies $\Delta\omega\approx(n_1-n_2)_a\omega$ consists of a set of components (specified by the quantum numbers n_a' and n_a'') and they include a component at the frequency $\Delta\omega=(n_1-n_2)_a\omega$ whose position is independent of the offset Δ. The intensity of this component is the sum of the intensities of the components at the frequency $\Delta\omega=(n_1-n_2)_a\omega$ in the emission spectra, which appear at all possible transitions $\Psi_{(nn'n'')a}(\mathbf{r},t)\rightarrow\Psi_{(n_1n_2m)}(\mathbf{r},\mathbf{t})$, where $\Psi_{(nn'n'')a}$ is given by Eq. (18) and $\Psi_{(n_1n_2m)b}$ is given by Eq. (35), when $(n_1-n_2)_b=0$ and $n_a'=-n_a''$. Therefore, when a linearly polarized field is applied, the resultant profile of the hydrogen spectral lines at frequencies $\Delta\omega=(n_1-n_2)_a\omega$ exhibit sharp peaks surrounded by dips on both sides. Such peaks are demonstrated, for example, in Fig. 4 of Ref. 12 for $\Delta\omega=\omega$ (in the case of the line L_α) and in Figs. 3 and 4 of Ref. 15 (3π components of the line H_α), as well as in Figs. 5 and 6 of the same paper (4σ and 8π components of the line H_β). When an elliptically polarized field ($E_xE_y\sin\delta\neq0$) is applied, such sharp peaks can appear for $(n_1-n_2)_b=0$ at frequencies $\Delta\omega=(n_1-n_2)_a\omega$ only when the upper state $\Psi_{(nn'n'')a}(\mathbf{r},t)$ is characterized by the quantum numbers $n_a'=n_a''=0$ [see Eq. (34)]. This is possible only in the case of the spectral lines which begin from the levels with odd n_a (for which j_a is an integer).

Let us assume that $(n_1-n_2)_b\neq0$. In this case the reliefs of the resultant Stark profiles of spectral lines of hydrogen do not have sharp peaks at the frequencies described by Eq. (36) in the case of linearly and elliptically polarized fields, because the presence of the term $(n_b/n_a)(n_1-n_2)_b\Delta$ in Eq. (34) results in "smearing out" into a spectral band when Δ is varied. We now consider the situation when the condition for a resonance (9) is satisfied by a group of hydrogen atoms which are in the lower state n_b ($n_b\neq1$). In this case the positions of the components in the emission spectrum representing the $a\rightarrow b$ transition can be represented by an analogy with Eq. (34) in the form

$$\Delta\omega=[(n_a/n_b)(n_1-n_2)_a-(n_1-n_2)_b]\omega$$
$$+(n_a/n_b)(n_1-n_2)_a\Delta-\lambda_{(n_1n_2m)b}(\Delta).\tag{38}$$

The results of an analysis of Eq. (38) are similar to those of an analysis of Eq. (34). In particular, if the resonance condition (9) is satisfied for the lower level n_b, the clearest differences in the profile of a spectral line of hydrogen between the cases of interaction with linearly elliptically polarized fields is observed when $(n_1 - n_2)_a = 0$ holds and the nature of the differences is identical with that considered in an analysis of the relationship (34). Therefore, recording of the resonant relief in the quasistatic Stark profile of the hydrogen spectral lines near two groups of frequencies, one of which is given by Eq. (36) and the other by

$$\Delta\omega = [(n_a/n_b)(n_1-n_2)_a - (n_1-n_2)_b]\,\omega, \qquad (39)$$

would have made it possible to estimate not only the intensity of the hf electric fields [using the characteristic splitting frequencies of the spectral components in the vicinity of the frequencies defined by Eqs. (36) and (39), and applying Eqs. (14) and (15)], but also to analyze the spatial angular distribution of the hf fields. In considering the angular distribution of these fields we have to analyze the nature of the resonant singularities near the frequencies $\Delta\omega_a = (n_1 - n_2)_a\omega$ and $\Delta\omega_b = -(n_1 - n_2)_b\omega$. Naturally, we must consider the singularities only close to those values $\Delta\omega_\nu$ ($\nu = a, b$), for which the corresponding spectral components $(n_1 n_2 m)_a \to (n_1 n_2 m)_b$ are of significant intensity [bearing in mind that $(n_1 - n_2)_b = 0$ for $\Delta\omega_a$ and $(n_1 - n_2)_a = 0$ for $\Delta\omega_b$].

For example, in the case of the line H_α these resonant singularities are strongest at the frequencies $\Delta\omega = \pm\omega$; for the line H_β this is true at frequencies $\Delta\omega = \pm\omega, \pm 2\omega$; for the line H_γ this applies at frequencies $\Delta\omega = \pm p\omega$ ($p = 1, 2, 3$); and for the line H_δ, at frequencies $\Delta\omega = \pm p\omega$ ($p = 1, 2, 3, 4$). Observation of sharp peaks at $\Delta\omega = \Delta\omega_\nu$ ($\nu = a, b$) would suggest that the hf oscillating field is predominantly linearly polarized, and its absence implies that this field is a superposition of noncollinear out-of-phase quasimonochromatic fields. The effect is observed most easily for spectral lines with the initial level characterized by an odd value of n (for example, the lines H_β and H_δ) when the resonance in the field of Eq. (4) results in complete splitting of the spectral components.

However, we must bear in mind that in the case of the "three-dimensional" angular distribution of the quasistatic field **F** (these may be, for example, the "Holtsmark" microfields of ions) and the "two-dimensional" angular distribution of hf fields (when the vectors of the oscillations comprising the combined hf field lie mainly in one plane) there is a relatively small group of hydrogen atoms for which the applied field **F** lies in the plane of the combined hf field. Since the resonant effects are governed by the hf field component orthogonal to the vector **F**, the emission spectrum of this group of hydrogen atoms is the same as under the influence of a linearly polarized hf field. If, however, the hf field has an angular distribution close to the three-dimensional isotropic case (for example, in the case of isotropic hf plasma turbulence), then for any direction of **F** they are noncollinear vectors of the hf field in a plane orthogonal to **F**.

We now consider singularities of the profiles of hydrogen spectral lines in the case of a resonance in a magnetic field of Eq. (24). In this case the Zeeman splittings of the upper n_a and lower n_b levels of the hydrogen atom are in resonance simultaneously. According to Eq. (32) the emission spectrum for the $a \to b$ transition then consists of a series of components at frequencies

$$\Delta\omega = (m_a - m_b)\,\omega + \varepsilon_{(nn'n'')_a} - \varepsilon_{(nn'n'')_b}. \qquad (40)$$

It follows from the selection rule for the magnetic quantum numbers that $m_a - m_b = 0, \pm 1$. Therefore, in the case of the resonance described by Eq. (24) the spectral line of hydrogen consists of three groups of closely spaced components: the central component in the vicinity of the frequency $\Delta\omega = 0$ and two side components in the vicinity of the frequencies $\Delta\omega = \pm\omega$. Using Eqs. (34) and (40), we can readily show that in each group there is a component whose position is independent of the offset Δ (this component corresponds to $n'_a = -n''_a$ and $n'_b = -n''_b$). Therefore, in the case of inhomogeneous broadening of a spectral line of hydrogen in magnetic fields there should be sharp peaks at the frequencies $\Delta\omega = 0, \pm\omega$. We analyze the intensity of a side peak at the frequency $\Delta\omega = \omega$ for the L_α line (a similar result is also obtained for a peak at a frequency $\Delta\omega = -\omega$). We assume that the magnetic field $\mathbf{H} = H\mathbf{e}_z$ has a constant direction but its intensity varies in accordance with the law $W(H)$. It then readily follows from Eq. (33) that for low values of R_0 the integral intensity J of a peak at a frequency $\Delta\omega = \omega$ (for the spectrum with the x or y polarization) is

$$J = {}^1\!/_2 \int \frac{R_0^2 W(H)\,dH}{(\mu_0 H - \omega)^2 + R_0^2} \approx \frac{3\pi}{4\mu_0}\,\sigma_2 W\left(\frac{\omega}{\mu_0}\right), \qquad (41)$$

where

$$\lim_{R_0 \to 0} R_0 (x^2 + R_0^2)^{-1} = \pi\delta(x).$$

At the same time for any value of H the emission of the shifted components in the spectrum L_α at frequencies $\Delta\omega = \omega \pm R_\Delta$ does not fall within the spectral interval $|\Delta\omega - \omega| < R_0$. In the case when the spectrum of the line L_α is not affected by the hf field, the interval $|\Delta\omega - \omega| < R_0$ can be characterized at low values of R_0 by the total intensity

$$\mathcal{J} = \int_{(\omega - R_0)\mu_0^{-1}}^{(\omega + R_0)\mu_0^{-1}} W(H)\,dH \approx 3\mu_0^{-1}\sigma_2 W\left(\frac{\omega}{\mu_0}\right). \qquad (42)$$

Using Eqs. (41) and (42), we conclude that the resonant influence of the hf field on the L_α line reduces the total intensity of the emitted radiation by about 20% at frequencies characterized by $|\Delta\omega - \omega| < R_0$ and $|\Delta\omega + \omega| < R_0$. Note that the splitting of the spectral components at the frequencies $\Delta\omega = \pm\omega$ can be used to determine the quantity

$$\sigma_2 = (E_x^2 + E_y^2 - 2E_x E_y \sin\delta)^{1/2}.$$

For $\sin\delta < 0$, it follows from Eq. (16) that $\sigma_2 = E^{(\max)} + E^{(\min)}$ whereas for $\sin\delta > 0$, we have $\sigma_2 = E^{(\max)} - E^{(\min)}$. Since the sign of $\sin\delta$ is in one-to-one correspondence with the direction of rotation of the vector $\mathbf{E}_{el}(t)$ of Eq. (4), it should be possible to determine spectroscopically the direction of rotation of the electric field vector of an elliptically polarized wave in a magnetically active plasma using the known value of $E^{(\max)}$. However, if we know the direction of rotation of the electric field vector and the degree of ellipticity of a wave, we can determine the parameters $E^{(\max)}$ and $E^{(\min)}$. The results of a calculation of the resonant modification of a spectral line of hydrogen under the simultaneous influence of a quasistatic field H and an hf field of the type described by Eq. (4) can be used to monitor the propagation and absorption of elliptically polarized microwaves used to heat a plasma in toroidal devices (see Refs. 21 and 22). In particular, it should be mentioned that heating of a plasma is often performed by a superposition of ordinary and extraordinary electromagnetic waves in the range of frequencies corresponding to the electron cyclotron resonance and the electric vectors of such waves have opposite directions of rotation.[21]

APPENDIX

We shall now give the relationships between the matrix elements of the operators x, y, and l_x calculated using the hydrogen wave functions in parabolic coordinates with the quantization axis z. For $m \geqslant 0$, we have

$$\begin{aligned} \langle n_1 n_2 m | iy | n_1+1,\ n_2,\ m-1\rangle &= \langle n_1 n_2 m | x | n_1+1,\ n_2,\ m-1\rangle \\ &= -(3n/2)\langle n_1 n_2 m | l_x | n_1+1,\ n_2,\ m-1\rangle, \\ \langle n_1 n_2 m | iy | n_1,\ n_2+1,\ m-1\rangle &= \langle n_1 n_2 m | x | n_1,\ n_2+1,\ m-1\rangle \\ &= (3n/2)\langle n_1 n_2 m | l_x | n_1,\ n_2+1,\ m-1\rangle. \end{aligned} \tag{A1}$$

For $m < 0$, the following relationships apply:

$$\begin{aligned} \langle n_1 n_2 m | iy | n_1+1,\ n_2,\ m+1\rangle &= -\langle n_1 n_2 m | x | n_1+1,\ n_2,\ m+1\rangle \\ &= -(3n/2)\langle n_1 n_2 m | l_x | n_1+1,\ n_2,\ m+1\rangle, \\ \langle n_1 n_2 m | iy | n_1,\ n_2+1,\ m+1\rangle &= -\langle n_1 n_2 m | x | n_1,\ n_2+1,\ m+1\rangle \\ &= (3n/2)\langle n_1 n_2 m | l_x | n_1,\ n_2+1,\ m+1\rangle. \end{aligned} \tag{A2}$$

[1)] Here and below we use atomic units such that $\hbar = m_e = e = 1$.

[2)] Here and below the wave functions $\varphi_{n_1 n_2 m}(\mathbf{r})$ expressed in parabolic coordinates are identical with the functions defined by Landau and Lifshitz.[19]

[3)] In the present treatment the Wigner functions $d^{(j)}_{nk}(\beta)$ and the Euler angles α_i and β_i are the same as those given by Landau and Lifshitz[19] in §58.

[1] H. R. Griem, *Spectral Line Broadening by Plasmas*, Academic Press, New York (1974).

[2] V. S. Lisitsa and V. I. Kogan, Itogi Nauki Tekh. Ser. Fiz. Plazmy **3**, 5 (1982).

[3] E. A. Oks, *Spectroscopy of Plasma with Quasimonochromatic Electric Fields* [in Russian], Énergoatomizdat, Moscow (1990).

[4] D. I. Blochinzew, Phys. Z. Sowjetunion B **4**, 501 (1932).

[5] E. A. Oks and Yu. M. Shagiev, Preprint No. 76 [in Russian], Institute of Applied Physics, Academy of Sciences of the USSR, Gorki (1983).

[6] V. I. Ritus, Zh. Eksp. Teor. Fiz. **51**, 1544 (1966) [Sov. Phys. JETP **24**, 1041 (1967)].

[7] A. Cohn, P. Bakshi, and G. Kalman, Phys. Rev. Lett. **29**, 324 (1972); Erratum: Phys. Rev. Lett. **31**, 620 (1973).

[8] W. R. Rutgers and H. W. Kalfsbeek, Z. Naturforsch. Teil A **30**, 739 (1975).

[9] V. P. Gavrilenko and E. A. Oks, Zh. Eksp. Teor. Fiz. **80**, 2150 (1981) [Sov. Phys. JETP **53**, 1122 (1981)].

[10] V. P. Gavrilenko, E. A. Oks, and V. A. Rantsev-Kartinov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 315 (1986) [JETP Lett. **44**, 404 (1986)].

[11] V. P. Gavrilenko and E. A. Oks, *Methods for Atomic Calculations* [in Russian], Council on Spectroscopy, Academy of Sciences of the USSR, Moscow (1986), p. 89.

[12] V. P. Gavrilenko and E. A. Oks, Fiz. Plazmy **13**, 39 (1987) [Sov. J. Plasma Phys. **13**, 22 (1987)].

[13] V. P. Gavrilenko, Zh. Eksp. Teor. Fiz. **94**(5), 88 (1988) [Sov. Phys. JETP **67**, 915 (1988)].

[14] B. V. Lyublin, D. G. Yakovlev, and V. Yu. Yasevich, Zh. Tekh. Fiz. **59**(2), 64 (1989) [Sov. Phys. Tech. Phys. **34**, 167 (1989)].

[15] D. G. Yakovlev, V. Yu. Yasevich, and B. V. Ljublin, J. Phys. B **22**, 3851 (1989).

[16] D. A. Volod'ko and V. P. Gavrilenko, Opt. Spektrosk. **64**, 263 (1988) [Opt. Spectrosc. (USSR) **64**, 155 (1988)].

[17] A. I. Zhuzhunashvili and E. A. Oks, Zh. Eksp. Teor. Fiz. **73**, 2142 (1977) [Sov. Phys. JETP **46**, 1122 (1977)].

[18] I. R. Gekker, *Interaction of Strong Electromagnetic Fields with Plasmas*, Oxford, New York (1982).

[19] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, 3rd ed., Pergamon Press, Oxford (1977).

[20] Yu. N. Demkov, B. S. Monozon, and V. N. Ostrovskiĭ, Zh. Eksp. Teor. Fiz. **57**, 1431 (1969) [Sov. Phys. JETP **30**, 775 (1970)].

[21] V. V. Alikaev, Itogi Nauki Tekh. Ser. Fiz. Plazmy **1**, Part 2, 80 (1981).

[22] V. E. Golant and V. I. Fedorov, *High-Frequency Methods for Heating Plasma in Toroidal Thermonuclear Facilities* [in Russian], Énergoatomizdat, Moscow (1986).

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