Optical hysteresis, switching, and self-pulsation during resonant excitation of excitons in semiconductors

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(Submitted 27 July 1990, resubmitted 12 November 1990)
Zh. Eksp. Teor. Fiz. **99**, 899–910 (March 1991)

A theory is derived for optical bistability in the exciton spectral region in ring-cavity geometry. The theory is based on generalized Keldysh equations describing coherent excitons and photons which are slightly nonuniform in space and time. Optical switching in an optical-bistability regime of excitons is studied for the first time. It is shown in the mean-field approximation that triangular pulses incident on the cavity acquire a temporal deformation and that there is the possibility in principle that these pulses will convert into stochastic "photon flares." A spatial turbulence might be observed in a system of coherent excitons and photons in a crystal.

1. INTRODUCTION

Optical bistability has been the subject of numerous theoretical and experimental studies and has essentially become an independent field of nonlinear physics. This topic has attracted interest because it is one of the clearest examples of optical self-organization in systems far from thermodynamic equilibrium and because it opens up vast opportunities for practical applications, primarily in optical information processing and the development of a new generation of computers with optical logic. Optical bistability is described most comprehensively in a monograph by Gibbs.[1] In that book, the theoretical foundations of optical bistability are presented, bistable materials and devices are described, various types of optical switching are discussed, and instabilities and other phenomena are analyzed. There are also some fairly comprehensive reviews of optical bistability in semiconductors in Refs. 2 and 3.

Optical nonlinearities are known to be particularly pronounced in the exciton spectral region in semiconductors. As a result, the nonlinear interaction of light with matter is seen most vividly in specifically this frequency region. The circumstance that the typical exciton relaxation times in semiconductors are very short ($t \sim 10^{-10}$–10^{-11} s) may play a decisive role in the development of optoelectronic devices in which ultrafast switching is required.

The exciton bistability was first studied by Elesin and Kopaev.[4] Analyzing a Bose–Einstein condensation of excitons in semiconductors caused by a strong coherent electromagnetic field, they derived a cubic equation for the exciton density. Analysis of that equation revealed that the exciton density is a nonmonotonic function of both the intensity and the frequency of the field; i.e., there are both an amplitude hysteresis and a frequency hysteresis. A theory of optical bistability in the exciton spectral region was later developed in our own studies[5–11] and in studies by other investigators (Refs. 12–15, among others). A common shortcoming of these studies is that they dealt with only a static optical bistability. From the practical standpoint, the most important topics for study are those related to the optical dynamics, a dynamic optical bistability, various types of optical switching, auto-oscillations, and so forth. There has been no previous study of these topics in the exciton spectral region.

Dneprovskiĭ *et al.*[3,16] first observed an optical bistability during the resonant excitation of excitons through an exciton–exciton interaction at relatively low values of the exciton density ($n_{ex} \sim 10^{15}$ cm^{-3}) and the intensity (1 kW/cm^2). The observation of an optical bistability at such low levels of the crystal excitation opens up some fundamentally new possibilities for the development of optical elements which could operate over a broad temperature range and which would draw only a small amount of power.

The coherent nonlinear phenomena which occur in the exciton spectral region have some important features which distinguish them from the nonlinear effects seen in the model of two-level atoms. For example, at relatively low densities, at which the excitons can be regarded as bosons, the Hamiltonian of the exciton–photon interaction is quadratic, and the relationship between the amplitude of the electric vector of the electromagnetic field, E, and the amplitude of the exciton wave, a, is linear. The nonlinearity in the case of excitons stems from a dynamic and kinematic exciton–exciton interaction.

Keldysh has derived[17] equations describing coherent excitons and photons which are slightly nonuniform in space and time. He took the exciton–exciton interaction into account. These equations have served as a foundation for the study of many aspects of the coherent nonlinear propagation of light through dense condensed media in the exciton part of the spectrum. In Refs. 5 and 18–20, for example, a theory of a self-induced transparency and a theory of static optical bistability in the exciton spectral region were derived. Finally, we have recently demonstrated[21–23] that there is the possibility in principle of a new cooperative effect: a self-pulsation on the long-wavelength fundamental absorption edge of a crystal during the resonant excitation of excitons present in a high density. The Keldysh equations, generalized to the case in which a coherent pump is applied and in which there is damping, were used to derive the conditions for the appearance of nonlinear periodic and random auto-oscillations in a system of coherent excitons and photons in the spatially uniform case.

At the present stage of research on optical bistability, on various types of optical switching, and on self-pulsations, these effects are being considered in connection with specific optical instruments and a corresponding experimental geometry. For the most part, the theoretical and experimental

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research on these cooperative effects in the model of two-level atoms[24-31] has been carried out in the geometry of a ring cavity and a Fabry–Perot cavity.

In contrast with Ref. 21, where a study was made of the static optical bistability of the (exciton density)–light type and of self-pulsations in an unbounded crystal, without the appropriate consideration of the boundary conditions and the specific experimental geometry, the present paper is concerned with the light–light optical bistability in a ring–cavity geometry. Working from generalized Keldysh equations in the mean field approximation, we show, for the first time, that two types of light–light hysteresis are possible, depending on the pulse shift of the field transmitted through the cavity. The first type is a clockwise hysteresis, and the second is a hysteresis with a self-intersection. Various types of optical switching between the branches of the hysteresis loop and a dynamic optical bistability in the exciton spectral region are studied for the first time. It is predicted that "photon flares" might occur under conditions corresponding to a dynamic optical bistability. We also examine the nonlinear dynamics of the light emerging from the cavity, and we find the power spectrum of the auto-oscillations which arise.

There is an important distinction between this problem and the theory of optical bistability in the model of two-level atoms, in which the nonlinearity stems from a trilinear term in the Hamiltonian of the light–light interaction, and the evolution of the atom-plus-field system is described by the Maxwell–Bloch equations. The nonlinearity in the exciton part of the spectrum stems from an exciton–exciton interaction, and the space–time evolution of the coherent excitons and photons is described by Keldysh equations, which are equations of the Ginzburg–Landau type. As a result, there are two types of hysteresis which must be taken into consideration in the exciton part of the spectrum: an internal "density–light" hysteresis and an external (i.e., observable) "light–light" hysteresis. Whether the latter is manifested depends on whether the former exists; whether the former exists is determined in turn by the detuning from a resonance between the frequency of the electromagnetic pump field and the frequency of the mechanical exciton.

As we will see below, a variation of this detuning makes is possible to qualitatively alter the transmission regime of the cavity. This circumstance is a unique manifestation of an exciton nonlinearity and is substantially different from the optical bistability in the model of two-level atoms, where an increase in the detuning from resonance leads to a degradation of the optical bistability.

2. DYNAMIC EQUATIONS FOR COHERENT EXCITONS AND PHOTONS IN THE MEAN-FIELD APPROXIMATION

The starting point for this theoretical look at the nonlinear transmission of light through a ring cavity during resonant excitation of excitons is the system of Keldysh equations.[17] We have generalized these equations to the case in which excitons and photons leave the corresponding coherent modes, through the introduction of damping rates $\gamma_{\rm ex}$ and $\gamma_{\rm ph}$. For waves which are propagating along the x axis, this system of equations is

$$i\frac{\partial a}{\partial t}=\left(\Omega_{ex}-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\right)a+\frac{g}{\hbar V}|a|^2a-\frac{d}{\hbar}E^+-i\gamma_{ex}a, \quad (1)$$

$$c^2\frac{\partial^2 E^+}{\partial x^2}-\frac{\partial^2 E^+}{\partial t^2}-2\gamma_{ph}\frac{\partial E^+}{\partial t}=\frac{4\pi d}{v_0}\frac{\partial^2 a}{\partial t^2}, \quad (2)$$

where $a(x,t)$ is the macroscopic amplitude of the coherent excitons, $E^+(x,t)$ is the positive-frequency part of the alternating electromagnetic field, g is the exciton–exciton interaction constant, d is the dipole moment of the transition from the ground state of the crystal to the exciton state, $\Omega_{\rm ex}$ is the limiting frequency of mechanical excitons, m is the translational mass of an exciton, v_0 is the volume of the unit cell, and V is the volume of the crystal.

For the discussion below it is convenient to transform to the dimensionless quantities $\tilde{A}$ and $\mathscr{E}^+$, through the substitutions

$$a=\tilde{A}\left(\frac{\hbar\gamma_{ex}V}{g}\right)^{1/2}, \quad E^+=\tilde{\mathscr{E}}^+\left(\frac{\hbar^3\gamma_{ex}^3}{gd^2}\right)^{1/2}. \quad (3)$$

We write the amplitudes of the excitons and the field as modulated plane waves with a carrier frequency Ω and a wave vector k:

$$\tilde{A}=A\exp(ikx-i\Omega t), \quad \tilde{\mathscr{E}}^+=e\exp(ikx-i\Omega t), \quad (4)$$

where A and e are slowly varying functions.

Substituting (3) and (4) into (1) and (2), and using the approximation of smooth envelopes, we find

$$c^2\left(-k^2e+2ik\frac{\partial e}{\partial x}\right)+\left(\Omega^2e+2i\Omega\frac{\partial e}{\partial t}\right)-2\gamma_{ph}\left(\frac{\partial e}{\partial t}-i\Omega e\right)=4\pi\frac{d^2/v_0}{\hbar\gamma_{ex}}\left(\Omega^2A+2i\Omega\frac{\partial A}{\partial t}\right), \quad (5)$$

$$i\left(\frac{\partial A}{\partial t}-i\Omega A\right)=\Omega_{ex}A-\frac{\hbar}{2m}\left(-k^2A+2ik\frac{\partial A}{\partial x}\right)+\gamma_{ex}|A|^2A-i\gamma_{ex}A-\gamma_{ex}e. \quad (6)$$

In general, the amplitudes A and e are complex quantities. We introduce $A_r={\rm Re}\,A$, $A_i={\rm Im}\,A$, $e_r={\rm Re}\,e$, $e_i={\rm Im}\,e$. We also introduce $\sigma=\gamma_{\rm ph}/\gamma_{\rm ex}$, $\tau=\gamma_{\rm ex}t$ (a dimensionless time), and $\delta=(\Omega-\Omega_{\rm ex})/\gamma_{\rm ex}$ (a dimensionless detuning from resonance). Working from Eqs. (5) and (6), and ignoring spatial–dispersion effects, which are unimportant in the pertinent part of the spectrum, we find

$$\frac{\partial A_r}{\partial\tau}=-\delta A_i+(A_r^2+A_i^2)A_i-A_r-e_i, \quad (7)$$

$$\frac{\partial A_i}{\partial\tau}=\delta A_r-(A_r^2+A_i^2)A_r-A_i+e_r, \quad (8)$$

$$\frac{\partial e_r}{\partial\tau}=-\sigma\frac{\Omega^2+c^2k^2}{2\Omega^2}e_r-\frac{\Omega^2-c^2k^2}{2\Omega\gamma_{ex}}e_i-\frac{c^2k}{\Omega}\frac{\partial e_r}{\partial x}-2\pi\frac{d^2/v_0}{\hbar\gamma_{ex}}\frac{\Omega}{\gamma_{ex}}A_i, \quad (9)$$

$$\frac{\partial e_i}{\partial\tau}=-\sigma\frac{\Omega^2+c^2k^2}{2\Omega^2}e_i+\frac{\Omega^2-c^2k^2}{2\Omega\gamma_{ex}}e_r-\frac{c^2k}{\Omega}\frac{\partial e_i}{\partial x}+2\pi\frac{d^2/v_0}{\hbar\gamma_{ex}}\frac{\Omega}{\gamma_{ex}}A_r. \quad (10)$$

Equations (7)–(10), a system of nonlinear differential equations, describe the space-time evolution of coherent ex-

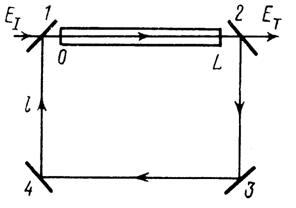

FIG. 1. Ring cavity. Mirrors 1 and 2 have a reflectance R; mirrors 3 and 4 have a reflectance of 100%; the distance between mirrors 1 and 2 is L; that between mirrors 2 and 3 is l.

citons and photons in condensed media in the approximation of smooth envelopes. These equations are the starting point for the analysis below.

We assume that a sample of length L lies between the entrance and exit mirrors of a ring cavity. These mirrors have a transmittance T. The two other mirrors are assumed to be ideally reflecting (Fig. 1). The corresponding boundary conditions (E_I is the pump, and E_T is the field at the exit from the cavity) are

$$(1-R)^{1/2}E_I+RE^+(L,\, t-\Delta t) \\ =E^+(0,\, t), \tag{11a}$$

$$E_T(t)=(1-R)^{1/2}E^+(L,\, t), \tag{11b}$$

where $R = 1 - T$ is the reflectance of cavity mirrors 1 and 2, and Δt is the delay time introduced by the feedback. Introducing the dimensionless entrance amplitude (y) and the dimensionless exit amplitude (x) of the fields, and using (4), we find the following boundary conditions on the normalized amplitudes:

$$Ty+R[e_r(L,\, t-\Delta t)\cos F-e_i(L,\, t-\Delta t)\sin F]=e_r(0,\, t), \tag{12a}$$

$$R[e_r(L,\, t-\Delta t)\sin F+e_i(L,\, t-\Delta t)\cos F]=e_i(0,\, t). \tag{12b}$$

where

$$E_I=y\left(\frac{\hbar^3\gamma_{ex}{}^3v_0}{gd^2}\right)^{1/2}T^{1/2},\qquad E_T=x\left(\frac{\hbar^3\gamma_{ex}{}^3v_0}{gd^2}\right)^{1/2}T^{1/2},$$

$F = k_0(L + 2l) + kL$ is the phase shift of the field in the ring cavity, and $k_0 = k/\varepsilon_\infty^{1/2}$ is the wave vector of the field in vacuum.

It is not possible to find exact analytic solutions of this system of nonlinear partial differential equations. However, the basic features of the nonlinear transmission of the light can be determined in the mean field model, which is widely used in the theory of optical bistability.[1] Mathematically, this model corresponds to the replacement of $\int_0^L E(x)dx$ by $E(L)L$. In this approximation, Eqs. (9) and (10) can be integrated along the coordinate:

$$\dot{e}_r=-\sigma e_r-\Delta e_i-\frac{c}{\gamma_{ex}L}[e_r-e_r(0)]-\frac{\alpha^2}{2}A_i, \tag{13}$$

$$\dot{e}_i=-\sigma e_i+\Delta e_r-\frac{c}{\gamma_{ex}L}[e_i-e_i(0)]+\frac{\alpha^2}{2}A_r, \tag{14}$$

where

$$\alpha^2=\frac{\Delta_0\,\Omega}{\gamma_{ex}\,\gamma_{ex}},\qquad \Delta_0=4\pi\frac{d^2/v_0}{\hbar},\qquad \Omega\approx ck,\qquad c=\frac{c_0}{\varepsilon_\infty{}^{1/2}}.$$

These results, along with Eqs. (7) and (8) and boundary conditions (12), describe the dynamics of coherent excitons and photons in the mean field approximation.

3. OPTICAL SWITCHING

Equations (7), (8) and (13), (14) fall in the category of nonlinear ordinary differential equations which describe open dynamic systems. However, not all the stationary states are stable; whether they are depends on the relations among the parameters. These states can be determined from the condition $\dot{A} = \dot{e} = 0$. From Eqs. (7) and (8) we immediately find an equation for the density–light bistability, which was first studied by Elesin and Kopaev[4] and ourselves:[5,9]

$$z=[(z-\delta)^2+1]=x^2, \tag{15}$$

where $z = A_r^2 + A_i^2$ is the dimensionless exciton density. A bistability of this type is a "thing in itself." To reveal the internal hysteresis, we need to study the intensity of the output light as a function of the intensity of the incident light. In other words, we need to study a light–light bistability. Using (7), (8), and (12), we find the following from (13), (14) for the steady state:

$$y^2=\left[\left(\frac{1-R\cos F}{1-R}+\frac{\sigma\tau_1\varepsilon_\infty{}^{1/2}}{1-R}\right)x+C\frac{z}{x}\right]^2 \\ +\left[\left(\frac{R\sin F}{1-R}+\frac{\Delta\tau_1\varepsilon_\infty{}^{1/2}}{1-R}\right)x+C\frac{z(z-\delta)}{x}\right]^2, \tag{16}$$

where $\tau_1 = \gamma_{ex}L/c_0$, $\Delta = (\Omega^2 - c^2k^2)/2\Omega\gamma_{ex}$, and $C = (1/2)(\Delta_0/\gamma_{ex})(kL/T)$ is the optical–bistability parameter. Equation (16) is the equation of state in the theory of the optical bistability in the exciton spectral region in a semiconductor ring cavity.

Figures 2 and 3 show the nonlinear $x(y)$ dependence, i.e., the amplitude of the output field as a function of the amplitude of the incident field for various values of the parameters. With increasing detuning (δ) from the resonance between the frequency of the electromagnetic pump field and the frequency of the mechanical exciton, this functional dependence changes from single-valued to multivalued. The change occurs when the transmission becomes bistable. With increasing δ, the optical bistability becomes progressively more obvious.

When the phase shift in the cavity satisfies $F = 2\pi n$, where n is an integer, the optical hysteresis in the system has a clockwise traversal direction. At certain values of δ and C,

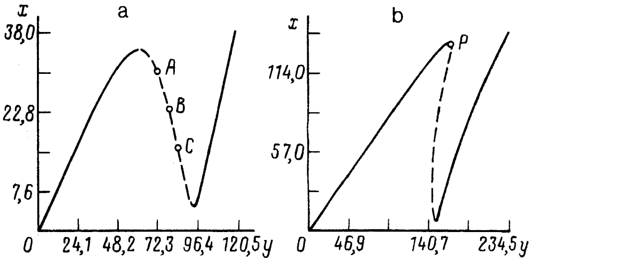

FIG. 2. Various types of "light–light" hysteresis with $F = 2\pi n$ and (a) $\delta = 20$ and (b) $d = 50$ ($C = 20$, $\varepsilon_\infty = 3.117$, $l/L = 2.94$, $\gamma_{ph}/\gamma_{ex} = 5$). The dashed part of the curve corresponds to $dz/dx < 0$.

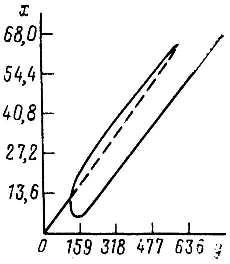

FIG. 3. The same as in Fig. 2, for $F = 2\pi n + \pi/2$ and $\delta = 20$ (the part of the curve shown by the dashed line corresponds to $dz/dx < 0$).

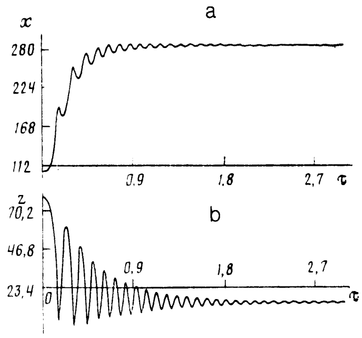

FIG. 5. Dynamics of the switching from the lower branch of the optical-bistability curve to the upper branch for the same parameter values. a—Photons; b—excitons.

the hysteresis loop converts into a close approximation of a switching step. This effect may find applications in optical logic cells for optical computers. In addition, when the phase shift has a value $F = \pi/2 + 2\pi n$, a self-intersection occurs on the hysteresis curve. These features are characteristic of optical bistability in the exciton spectral region when the exciton–exciton interaction is taken into account. They differ substantially from the corresponding effect in a system of two-level atoms, in which the hysteresis has a counterclockwise circuiting direction, and the bistability worsens with increasing detuning from resonance.[1] These features have been detected experimentally by Dneprovskiĭ *et al.*,[16] who observed an optical bistability as the result of an exciton–exciton interaction in semiconducting CdSe.

Since the $x(y)$ plot has a narrow, high hysteresis loop at certain values of δ, a switching of the system does not require a substantial change in the pump. The large height of the hysteresis means that a reliable switching can occur and that there will be no random operations of the logic cell. In contrast with bistable systems based on two-level atoms,[32,33] in which it is frequently possible to adiabatically exclude certain variables or others and to thus simplify the system of equations substantially, only a numerical analysis of the switching processes is possible in the case at hand.

We have carried out a numerical simulation in which Eqs. (7), (8) and (13), (14) were solved exactly with the boundary conditions for a ring cavity. The initial values of e_r, e_i, A_r, and A_i were chosen to correspond to the value of y near the threshold for an upward (or downward) switching. An abrupt change Δy in the pump is specified at the time $t = 0$; this change is of such a magnitude that $y \pm \Delta y$ lies on the other side of the corresponding switching threshold.

Figures 4 and 5 show downward and upward switching, respectively. We see that after a time $(1–1.5)\,\gamma_{ex}^{-1}$ the system goes from the upper branch of the optical bistability curve to the lower branch (Fig. 4) or from the lower branch to the upper one (Fig. 5). In contrast with two-level systems, where the switching times are substantially different for switching downward and upward, they are comparable in magnitude for the case of an optical bistability due to an exciton–exciton interaction. Since the typical relaxation times of the excitons in semiconductors are very short ($\tau_{ex} \sim 10^{-10}–10^{-12}$ s), optical switching in the exciton part of the spectrum is in the picosecond range.

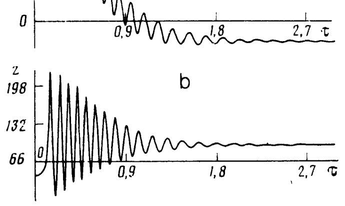

FIG. 4. Dynamics of the switching from the upper branch of the optical-bistability curve to the lower branch for $C = 40$ and $\delta = 90$. a—Photons; b—excitons.

4. NONLINEAR DYNAMICS OF COHERENT EXCITONS AND PHOTONS; DYNAMIC BISTABILITY

Up to this point we have been discussing the steady-state transmission of light through a semiconductor during resonant excitation of excitons. We turn now to the dynamic properties of the system of coherent excitons and photons as described by Eqs. (7), (8) and (13), (14).

At detunings $\delta < \delta_c = 3^{1/2}(C + 1)$ (Fig. 2a), at which there is no optical bistability, the $x(y)$ curve has a region with a negative slope. This region corresponds to an intermediate branch of the $z(x)$ curve in a triple-valued region, which is unstable.[9] We will refer to this region below as the "instability window."

A numerical analysis for values of y and δ corresponding to this window yields the following results. At the edges of the window one observes a sharp transition from stable solutions to an undamped nonlinear periodic oscillation. Toward the center of the window, we find period-doubling bifurcations; the oscillation becomes more complex; new harmonics appear in their spectrum; and, finally, in the central part of the window the oscillation converts into a stochastic self-pulsation (Fig. 6) with a continuous power spectrum.

Finally, Fig. 7 shows the time evolution of the light leaving the ring cavity in the case $\delta > \delta_c$, in which an optical bistability occurs in the system. We see from Fig. 7 that in this case there is an infinite chain of damped pulses.

The nonlinear periodic stochastic oscillation which arises can be utilized to convert the steady-state electromagnetic radiation incident on the cavity into pulsating radiation.

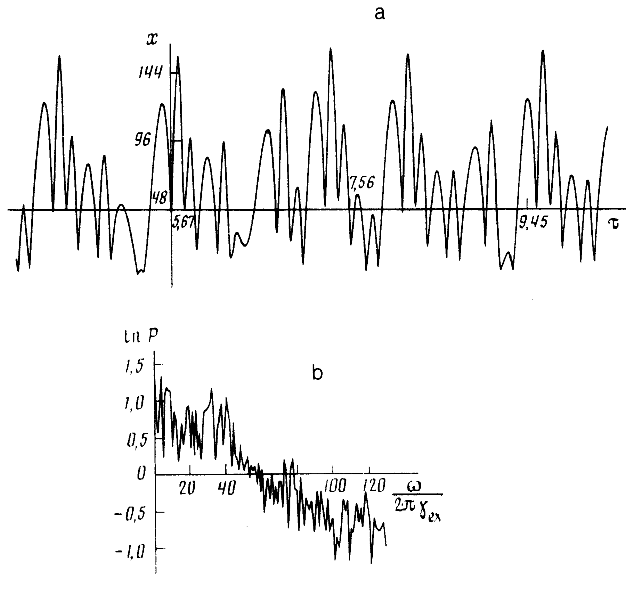

FIG. 6. Time evolution of this system for $\delta < \delta_c$ and $F = 2\pi n$ (point C in Fig. 2a). a—Time evolution of x; b—spectrum of the power P.

In experiments on optical bistability one observes not a static optical bistability but a dynamic one, which is found by comparing the time-dependent external pump–a Gaussian or triangular pulse— with the corresponding temporal response of the system. An optical bistability of this type was first studied by Bishofberger and Shen.[34] A theoretical and experimental study was made of the behavior of a nonlinear Fabry–Perot interferometer filled with a Kerr medium and exposed to pulses of various shapes. Lugiato[33] obtained excellent agreement of theory with experiment.

As to optical bistability in the exciton region of the spectrum, this problem has not been solved as yet. We have carried out a numerical simulation in which nonlinear differential equations (7), (8), (12)–(14) were solved numerically. These equations describe the dynamics of coherent excitons and photons, with boundary conditions corresponding to a ring cavity. The external pump $y(\tau)$ was a function of the time, specifically, an isolated pulse, either triangular or Gaussian. It was thus possible to find the correspondence between the static hysteresis and the dynamic hysteresis and to determine how the properties of the two types of hysteresis depend on the pulse length.

The experiment was carried out for the basic situations possible in the system, as given above: (1) $F = 2\pi n$, $\delta > \delta_c$. There is a static optical bistability in the system, without a window. (2) $F = 2\pi n$, $\delta < \delta_c$. There is no static optical bistability, while there is an "instability window." (3) $F = \pi/2 + 2\pi n$. The curve of the static optical bistability has a self-intersection point. The results of this simulation are shown in Figs. 8–10. Each figure has three curves: a plot of $y(\tau)$, a plot of $x(\tau)$, coordinated in time, and a plot of the output light as a function of the incident light, $x(y)$, i.e., the dynamic optical bistability. The first of these cases leads to the simplest result: With a pulse length $\tau = 100$ we find a "parallelogram" hysteresis with a clockwise direction, as observed experimentally in Ref. 16.

The second case also leads to a clockwise hysteresis, but both the upper and lower branches have regions with large-amplitude self-pulsations. Optical surges of this type were also found in a regime of dynamic optical bistability in Ref. 34, but they were of a regular nature there. It can be seen from Fig. 9 that they are random in our case. The reason for this randomness is that the system of coherent excitons and photons goes into a regime of stochastic self-pulsation at certain parameter values. The third case (Fig. 10) leads to a hysteresis which has a self-intersection point. The unstable part of the static hysteresis, corresponding to $dz/dx < 0$, becomes stable in the dynamic hysteresis, and vice versa.

Finally, we note an important aspect, which is of a fundamental nature. The self-pulsation in the system of coher-

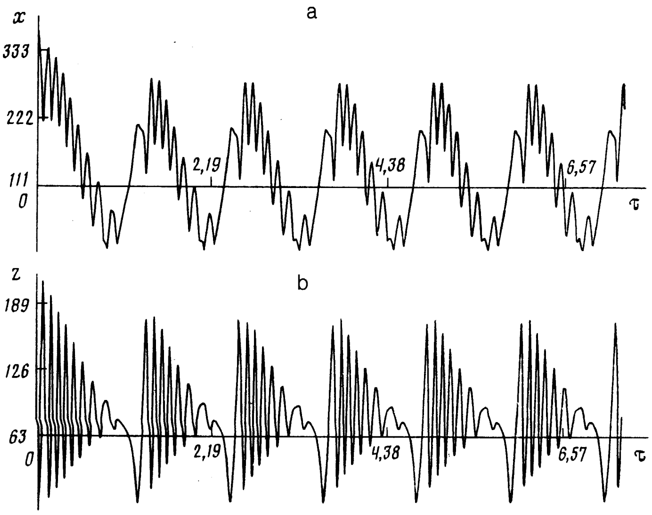

FIG. 7. Time evolution of x and z for $\delta > \delta_c$ for point P in Fig. 2b. a—Photons; b—excitons.

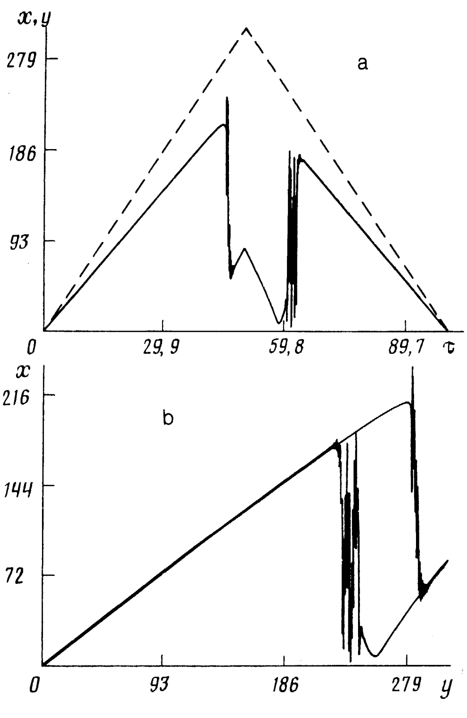

FIG. 8. Dynamic hysteresis ($\tau = 100$, $\delta = 67$). a—Time evolution $Y(\tau)$ (dashed line) and $x(\tau)$ (solid line); b—corresponding $x(y)$ dependence (optical–bistability regime).

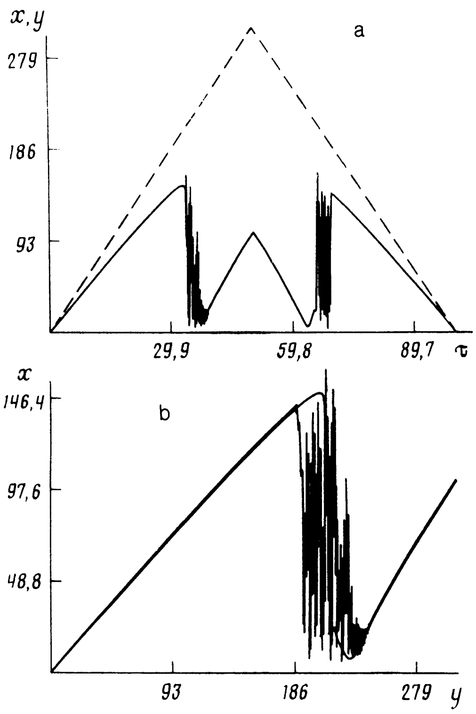

FIG. 9. The same as in Fig. 8, for $\delta = 53$. Flares of random self-pulsations can be seen.

ent excitons and photons which we have studied in the approximation of the mean field theory is a clear example of the onset of temporal structures in nonlinear dynamic systems. The Keldysh equations are equations of the Ginzburg–Landau type. A theory of a spatial turbulence has been derived[35] for equations of this type. A new class of order–chaos transitions has been observed, in the form of moving transition fronts. Corresponding phenomena may occur in a system of coherent excitons and photons. Along with the dynamic turbulence, spatial turbulence may develop, and order–chaos and chaos–order structures may appear. These transitions would arise because of a switching wave.[35]

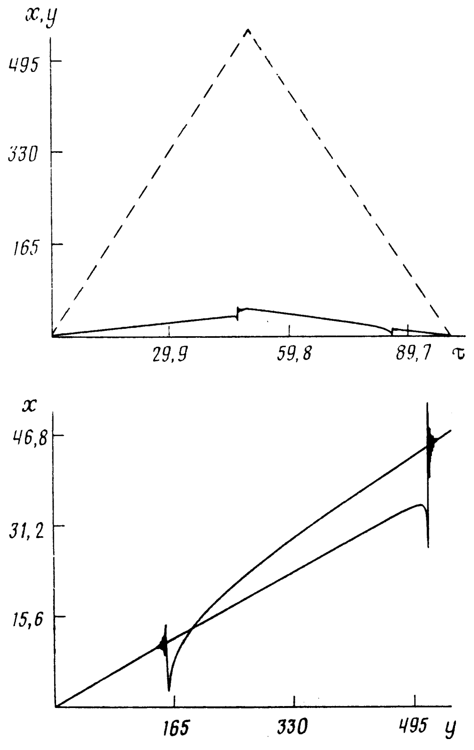

FIG. 10. The same, for $\delta = 20$ and $F = \pi/2 + 2\pi n$. Hysteresis with a self-intersection point.

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Translated by D. Parsons