Influence of spatial fluctuations of the modulus of magnetization on the properties of amorphous ferromagnets

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A study is made of a model of an amorphous ferromagnet with long-wavelength spatial fluctuations of the modulus of the magnetization M. It is shown that an allowance for the magnetodipole interaction, which enhances the contributions of fluctuations of M to the ground state, to the dispersion law, and to the damping of spin waves is essential to the correct understanding of the properties of this model. The spectral and correlation properties of an inhomogeneous ground state are studied and the law of approach of the magnetization to saturation is considered in this model. An investigation is made of the dispersion law and of the damping of spin waves. A new mechanism is proposed for the origin of a correction of the $T^{5/2}$ type to the Bloch law governing the low-temperature behavior of the magnetization.

INTRODUCTION

One of the main tasks in the physics of disordered systems is a study of the influence of fluctuations of various parameters of a system on its properties. We shall report a theoretical investigation of the influence of long-wavelength ($k_c a \ll 1$, where a is the interatomic distance and k_c is the characteristic wave number) fluctuations of the modulus of the magnetization M on the ground state and spectrum of elementary excitations of a randomly inhomogeneous ferromagnet (this definition covers amorphous and fine-grained materials, disordered crystalline alloys, and other ferromagnetic systems with an inhomogeneous distribution of the parameters). We shall study a model of a ferromagnet in which all these other parameters, such as the exchange and anisotropy, are assumed to be constant. The fundamental difference from earlier studies of this model[1,2] lies in an allowance for the magnetodipole interaction, which plays here a more important role than in the model of exchange fluctuations.[3] This is because the magnetization is a source of the magnetodipole field $\mathbf{H}_m(\mathbf{r})$ and, therefore, fluctuations of the magnetization create fluctuations of this field. Isotropic fluctuations of the modulus M give rise to an inhomogeneous distribution of the field $\mathbf{H}_m(\mathbf{r})$ between the various directions, which under certain conditions can have a drastic influence on the properties of the investigated system.

We shall base our investigation on a phenomenological theory of disordered magnetics developed in Ref. 4. According to this theory the fluctuations of M can be modeled by a homogeneous random function of the coordinates $\rho(\mathbf{r})$:

$$M(\mathbf{r})=M_0[1+\gamma\rho(\mathbf{r})]. \tag{1}$$

The statistical properties of this function are assumed to be known:

$$\langle\rho(\mathbf{r})\rangle=0, \quad \langle\rho^2(\mathbf{r})\rangle=1, \\ \langle\rho(\mathbf{r})\rho(\mathbf{r}')\rangle=K_\rho(|\mathbf{r}-\mathbf{r}'|), \tag{2}$$

where $K_\rho(\mathbf{r})$ is the normalized isotropic correlation function of fluctuations of M, characterized by a correlation radius r_c, which determines the characteristic spatial scale of the magnetization inhomogeneities. The parameter γ shown separately in Eq. (1) is the rms deviation of the modulus $M(\mathbf{r})$ from its average value M_0.

The physical reason for fluctuations of the magnetization modulus of ferromagnets is a random distribution of spins in space, which in turn may be due to (for example) disorder in the distribution of magnetic and nonmagnetic ions in multicomponent systems or due to lattice deformation, which gives rise to fluctuations of the density of the investigated material. In adopting a phenomenological description of such situations we have to distinguish two cases: $r_c \sim a$ and $r_c \gg a$ (the existence of such long-wavelength inhomogeneities in amorphous ferromagnetic systems is now reliably established by experimental investigations). In the former case the correlation radius r_c lies within a volume ΔV in which the averaging is carried out in the continuum approximation. This results in the loss of information in the short-wavelength part of the spectrum of inhomogeneities ($\lambda < r_c$) and therefore a phenomenological approach cannot be used to consider the processes of these values of λ, i.e., we have to assume that the inhomogeneities are delta-correlated.

In the second case the volume ΔV can be selected so that $\Delta V \ll r_c^3$. Consequently, the macroscopic quantities obtained by averaging over this volume "inherit" fully the inhomogeneities present in the system. The correlation radius r_c can then be regarded as finite and we can consider processes with characteristic scales greater than or smaller than r_c. We must stress once again that this is the case which will be considered here.

Following Ref. 4, we shall assume that the modulus of the magnetization is at each point an integral of motion and, consequently, the system in question can be described by the usual Landau–Lifshitz equation

$$\frac{\partial \mathbf{m}}{\partial t} = -g[\mathbf{m}\mathbf{H}^{eff}], \tag{3}$$

where $\mathbf{m}$ is the unit vector in the direction of the magnetization and $\mathbf{H}^{\text{eff}}$ is given by

$$\mathbf{H}^{eff} = -\frac{\partial \mathscr{H}}{\partial M} + \frac{\partial}{\partial x_i}\frac{\partial \mathscr{H}}{\partial(\partial \mathbf{M}/\partial x_i)}, \tag{4}$$

where $\mathscr{H}$ is the Hamiltonian which can be written in the form

$$\mathscr{H} = \frac{1}{2}\alpha M^2 \nabla^2 \mathbf{m} - M\mathbf{H}_0\mathbf{m} + \frac{1}{8\pi}\mathbf{H}_m^2. \tag{5}$$

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Here, α is the exchange parameter, whereas $\mathbf{H}_0$ and $\mathbf{H}_m$ are the external and magnetodipole fields, respectively.

The first section will deal with the ground state. An allowance for the magnetodipole interaction leads to the appearance of what is known as a stochastic magnetic structure (SMS), which is inhomogeneous in respect of the distribution of the directions of the magnetization. In general, an SMS appears in ferromagnetic systems under the influence of a disordered anisotropy of any nature. In particular, investigations have been made of the SMS induced by a local magnetic anisotropy[4,5] or the magnetostrictive anisotropy,[5] including that allowing for the magnetodipole interaction.[3] We must stress that we shall discuss an SMS associated with isotropic fluctuations of the scalar parameter M induced entirely by the anisotropy of the magnetodipole interaction itself.

The first intimation of the possibility of existence of such an SMS (we shall call it the magnetodipole SMS or MSMS) was given in Ref. 5, where however it was concluded that its dispersion is negligible ($\sim 10^{-6}$). We shall discuss the investigation reported in Ref. 5 in greater detail and identify the conditions under which the conclusion reached there is invalid.

In Secs. 2 and 3 we shall discuss the properties of spin waves such as the dispersion law and the damping. The results obtained in these sections are very different from those obtained earlier without an allowance for the magnetodipole interaction.[1,2] New results on the damping of spin waves are of special interest, since the recent experimental investigations[6,7] are not described by the theoretical relationships obtained in Refs. 1–4.

In the last section we shall give an interesting, in our opinion, example of the influence of an SMS of arbitrary nature on the thermodynamic properties of disordered magnetics. We shall propose a mechanism of adding a correction of the $T^{5/2}$ type to the Bloch temperature dependence of the magnetization reported in several experimental papers.[8,9]

1. MAGNETODIPOLE STOCHASTIC MAGNETIC STRUCTURE

The ground state of the investigated system is described by a statistical variant of equations of the (3) type with an effective field

$$\mathbf{H}^{eff}=\alpha M(\mathbf{r})\,\nabla^2\mathbf{m}+2\alpha\nabla\mathbf{m}\nabla M+\mathbf{H}_0+\mathbf{H}_m. \tag{6}$$

Here, $\mathbf{H}_m$ is a magnetostatic field which satisfies the Maxwell equations. Its average value $\langle\mathbf{H}_m\rangle$ in a sample of ellipsoidal shape is given by the familiar expression

$$\langle\mathbf{H}_m\rangle=-4\pi\hat{N}\mathbf{M}_0. \tag{7}$$

We shall assume that $\langle\mathbf{H}_m\rangle$ is included in $\mathbf{H}_0$; the remaining fluctuation component can be calculated by applying the Fourier transformation and assuming that the medium (ferromagnetic) is infinite.

In the zeroth approximation with respect to fluctuations of M the ground state is homogeneous of the $m_x = m_y = 0$ and $m_z = 1$ type with the z axis directed along the magnetic field $\mathbf{H}_0$ (allowing for $\langle\mathbf{H}_m\rangle$). If the fluctuations of M and, consequently, deviations from the homogeneous state are small, then in the first order of perturbation theory (in respect of γ), we obtain

$$m_i(\mathbf{k})=\gamma k_M{}^2\frac{k_ik_z}{k^2(k^2+k_H{}^2+k_M{}^2\sin^2\theta_\mathbf{k})}. \tag{8}$$

Here, the index i assumes the values of x and y; $\theta_\mathbf{k}$ is the angle between the vector $\mathbf{k}$ and the field $\mathbf{H}_0$; $\mathbf{m}(\mathbf{k})$ and $\rho(\mathbf{k})$ are the Fourier transforms of the functions $\mathbf{m}(\mathbf{r})$ and $\rho(\mathbf{r})$. The parameters k_H and k_M are given by the expressions

$$k_H{}^2=\frac{H_0}{\alpha M_0},\quad k_M{}^2=\frac{4\pi}{\alpha}.$$

Equation (8) describes an inhomogeneous distribution of the magnetization directions, i.e., it describes an SMS.

The main characteristics of this SMS are the spectral density $S_M(\mathbf{k})$, the correlation function $K_M(\mathbf{r})$, and the variance $D_M = K_M(0)$. The spectral density is given by[10]

$$\langle m_i(\mathbf{k})\,m_i^*(\mathbf{k}')\rangle=S_M{}^{(i)}(\mathbf{k})\,\delta(\mathbf{k}-\mathbf{k}'). \tag{9}$$

Substituting here Eq. (8), we obtain

$$S_M^{(i)}(\mathbf{k})=\gamma^2k_M{}^4\frac{k_z{}^2k_i{}^2S_\rho(\mathbf{k})}{k^4(k^2+k_H{}^2+k_M{}^2\sin^2\theta_\mathbf{k})^2}, \tag{10}$$

where S_ρ is the spectral density of the random function of inhomogeneities (2). The correlation function $K_M(\mathbf{r})$ and, consequently, the variance D_M are found from Eq. (10) using the Wiener–Khinchin theorem:[10]

$$K_M^{(i)}(\mathbf{r})=\langle m^i(0)\,m^i(\mathbf{r})\rangle=\int S_M(\mathbf{k})\,e^{i\mathbf{kr}}\,d^3k. \tag{11}$$

The expression for the variance is the same for the components m_x and m_y. It can be written down conveniently by introducing dimensionless parameters as follows:

$$D_M=\gamma^2\int\frac{(q_xq_z)^2S_\rho(\mathbf{q})}{(q^2\xi^{-2}+\nu^2+\sin^2\theta_\mathbf{q})^2q^4}\,d^3q, \tag{12}$$

where

$$\mathbf{q}=\frac{\mathbf{k}}{k_c},\quad \xi^2=\frac{k_M{}^2}{k_c{}^2}=\frac{4\pi}{\alpha k_c{}^2},\quad \nu^2=\frac{k_H{}^2}{k_M{}^2}=\frac{H_0}{4\pi M_0}, \tag{13}$$

and k_c is the correlation wave number which occurs in $S_\rho(k)$ ($k_c \propto r_c^{-1}$).

Equation (12) was first obtained in Ref. 5 on the assumption that the fluctuations $M(\mathbf{r})$ are delta-correlated. We can readily see from this equation why the authors of Ref. 5 concluded that the value of D_M for real materials is negligible. The authors of Ref. 5 integrated Eq. (12) with the spectral density $S_\rho(k) =$ const and thus limited the integration domain to wave numbers $k\sim a^{-1}$. In fact this implies introduction of a correlation radius $r_c \sim a$. Then the inequality $\xi^2 \ll 1$ is obeyed and the quantity D_M given by Eq. (12) is of the order of $\gamma^2\xi^3$.

The situation changes drastically if we assume that an MSMS is due to long-wavelength inhomogeneities of M characterized by $r_c \gg a$. In this case we can realistically expect $\xi\sim 1$ and even $\xi \gg 1$; then Eq. (12) has no additional smallness. The quantity D_M is then of the order of γ^2 and may be comparable with the variances for other types of SMS.

A characteristic feature of an MSMS, compared with a SMS is an anisotropy (which we shall denote an anisotropic

SMS by ASMS) responsible for its stability in the limit $H_0 \to 0$: we can see from Eq. (12) that in this limiting case the variance D_M remains finite. The field dependence $D_M(H_0)$ is also characteristic: a linear fall in the range $H < H_c = \alpha M_0 k_c^2$ (H_c is the correlation field in accordance with the terminology of Ref. 4), which changes to logarithmic in the range $H > H_c$. For comparison, we recall that the variance of an ASMS is proportional to $H^{-1/2}$ when $H < H_c$ and to H^{-2} when $H > H_c$. A further increase in the field results in a change in the logarithmic law in the vicinity of $4\pi M_0$ to the law H^{-2} typical of all types of SMSs in such high fields. The variance of an SMS determines the field dependence of the projection of the average magnetization along the field direction, $\langle M_z(H)\rangle$, which is known as the law of approach of the magnetization to saturation. In Ref. 4 this law was first used to find the correlation radius of fluctuations of the anisotropy[1)] in amorphous Co–P. However, in connection with the results in the present section, it is essential to consider more carefully the interpretation of the reported experiments.

In the case of a magnetodipole SMS the law of approach to saturation differs fundamentally from all other laws of approach that have been used to interpret the experimental data, beginning from the very first investigations of these topics (see, for example, Ref. 12); instead of expansion in terms of the reciprocal powers of $H^{1/2}$, we have a linear (for $H < H_c$) or a logarithmic (for $H > H_c$) field dependence.

In the case of thin films it is possible to obtain direct electron-optic images of an SMS (Ref. 13). An analysis of these images yields the one-dimensional cross sections of the correlation function and the corresponding one-dimensional spectral densities $S_M(k)$. Such an analysis was first made in Ref. 14 on the basis of a theory proposed in Ref. 15 for ASMSs in thin microcrystalline films. Subsequently the function $S_M(k)$ had been determined for a whole range of microcrystalline and amorphous alloys (for a review of some of the work, see Ref. 16).

A comparison of the theoretical expressions for the spectral densities of an ASMS and an MSMS shows that the electron-optic images of these structures should be very different from one another. In fact, whereas in the case of the ASMS the maximum fluctuations of the contrast are observed only along the direction of the vector $\mathbf{M}_0$, in the MSMS case we can expect the maximum fluctuations at some angle to $\mathbf{M}_0$.

We also carried out a numerical investigation of the correlation function of an MSMS corresponding to the spectral density of Eq. (10); the correlation properties of inhomogeneities of the modulus M were then modeled by the following expressions:

$$K_\rho(r) = \exp(-k_c r), \quad S_\rho(k) = \frac{k_c}{\pi^2 (k^2 + k_c^2)^2}. \tag{14}$$

The results of a numerical integration are presented in Fig. 1. This figure gives the dependence of the correlation function $K_M(r,\theta)$ on the distance between the points r for different values of the angle θ between the vector $\mathbf{r}$ and the magnetic field $\mathbf{H}_0$. As expected, the correlation properties of the MSMS are strongly anisotropic. However, the most important is the appearance of negative correlations for large values of r when $\theta < \theta_c$, where θ_c is a certain critical value of

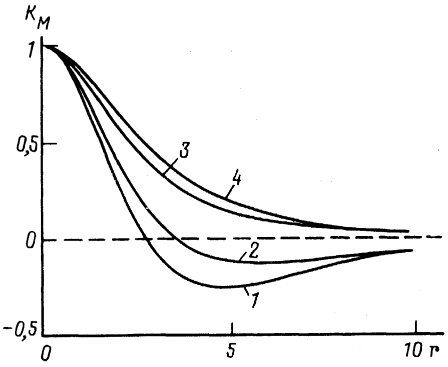

FIG. 1. Sections of the correlation function of a magnetodipole stochastic magnetic structure formed by cones passing at different angles θ relative to the field H_0. Curve 1 corresponds to $\theta = 0°$, curve 2 to $\theta = 30°$, curve 3 to $\theta = 60°$, and curve 4 to $\theta = 90°$.

the angle. This implies a qualitatively different distribution of the magnetization in the MSMS for the directions with $\theta < \theta_c$ and $\theta > \theta_c$. Such sections of the correlation function mean that the corresponding one-dimensional spectral densities have a maximum at a certain value $k^* \neq 0$ (i.e., in accordance with the terminology of Ref. 17, these functions include a contribution of correlation functions of the second type). Along these directions the SMS acquires some of the features of an ordered wave structure with a characteristic wave number k^*.

Callen[18] used a microscopic approach to consider the ground state of the spin system in which spins are distributed at random between the lattice sites, subject to an allowance for the dipole–dipole interaction, i.e., he solved a problem similar to ours. On the other hand, at first sight the results reported in Ref. 18 seem to be in conflict with the results obtained in the present section: the ground state is unstable in the limit $H_0 \to 0$ and the dependence $D_M(H_0)$ is similar to the field dependence of the ASMS variance. Consequently, we shall conclude this section with a discussion of the reason for this contradiction.

The microscopic energy of the dipole–dipole interaction is

$$\mathscr{H}_{dip} = \frac{1}{2} \sum_{l,m} \frac{g^2 \mu_B^2}{R_{lm}^5} [R_{lm}^2 \mathbf{s}_l \mathbf{s}_m - 3(\mathbf{R}_{lm}\mathbf{s}_l)(\mathbf{R}_{lm}\mathbf{s}_m)], \tag{15}$$

where $\mathbf{s}_l$ is a spin at a site l, and $\mathbf{R}_{lm}$ is the radius vector joining two sites l and m. The lattice sum in Eq. (15) can be separated in the usual way[19] into two: the immediate neighborhood of size of the order of a and for all the other sites. It is necessary to consider the following two cases: 1) when the characteristic size of the inhomogeneities in the distribution of the spins is of the order of a; 2) $r_c \gg a$. In the former case the sum over the region where $|\mathbf{R}_{lm}| \gg r_c$, responsible for the macroscopic field occurring in the Maxwell equations, becomes self-averaged and negligibly small [in full agreement with the results of Ref. 5 and the corresponding discussion of Eq. (12) in the present section]. The rest of Eq. (15), containing summation over the immediate environment, gives in this case the effective random magnetic field whose influence is similar to that of the random anisotropy, as demonstrated in Ref. 18.

However, in the case of long-wavelength inhomogeneities $r_c \gg a$, the summation in the immediate environment where $|\mathbf{R}_{lm}| \ll r_c$ in fact occurs for a homogeneous distribution of the spins. The corresponding sum either vanishes if we are speaking of the cubic lattice or makes a contribution to the anisotropy.[19] The statistics of this contribution is generally governed not by the distribution of the magnetization, but by the statistics of the lattice distortions and, consequently, will not be considered here. On the other hand, summation of the more distant region characterized by $|\mathbf{R}_{lm}| \sim r_c$ gives rise to the same fluctuation-induced magnetodipole field which was discussed by us earlier. Therefore, the difference between the results of Ref. 18 and ours is due to the fact that the statistical characteristics of the magnetodipole field H_m depend strongly on the correlation radius of fluctuations of the magnetization $M(\mathbf{r})$ ($r_c \sim a$ or $r_c \gg a$).

2. DISPERSION LAW OF SPIN WAVES

As pointed out already in Sec. 1, we shall deal with systems that can be described by the usual Landau–Lifshitz law of Eq. (3) with an effective field given by Eq. (7). A unit vector along the direction of the magnetization can be represented in the form.

$$\mathbf{m}(\mathbf{r},t)=\mathbf{m}(\mathbf{r})+\boldsymbol{\mu}(\mathbf{r},t),$$

where $\mathbf{m}(\mathbf{r})$ is the static distribution of the magnetization discussed in the preceding section, whereas $\boldsymbol{\mu}(\mathbf{r},t)$ is the spin-wave variable. Equation (3) should be linearized with respect to $\boldsymbol{\mu}$ [but not with respect to $\mathbf{m}(\mathbf{r})$: we have to retain also the nonlinear terms of order not exceeding γ^2]. The corresponding equations obtained after the Fourier transformation are

$$(\omega-\omega_0-\omega_M\sin^2\theta_{\mathbf{k}}-Jk^2)\mu_k-\omega_M\frac{k_+^2}{k^2}\mu_k^*=Q_1+Q_2+Q_3, \qquad (16)$$

where

$$Q_1=\gamma J\int \mathbf{q}(2\mathbf{k}-\mathbf{q})\rho_{\mathbf{k}-\mathbf{q}}\mu_{\mathbf{q}}d^3q$$
$$+\gamma\omega_M\int\rho_{\mathbf{k}-\mathbf{q}}\left(\mu_{\mathbf{q}}\sin^2\theta_{\mathbf{k}}+\frac{k_+^2}{k^2}\mu_{\mathbf{q}}^*\right)d^3q,$$
$$Q_2=\iint\Gamma(\mathbf{k},\mathbf{q}_1,\mathbf{q}_2)\left[m_x(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)m_x(\mathbf{q}_1)\right.$$
$$\left.+m_y(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)m_y(\mathbf{q}_1)\right]\mu_{\mathbf{q}_2}d^3q_1\,d^3q_2$$
$$-\frac{1}{2}\omega_M\iint\left[m_x(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)m_x(\mathbf{q}_1)+m_y(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)m_y(\mathbf{q}_1)\right]$$
$$\times\frac{q_{2+}^2}{q_2^2}\mu_{\mathbf{q}_2}^*d^3q_1\,d^3q_2+g\int h_z^m(\mathbf{q})\left[m_x(\mathbf{k}-\mathbf{q})+im_y(\mathbf{k}-\mathbf{q})\right]d^3q,$$
$$Q_3=g\int H_z^m(\mathbf{k}-\mathbf{q})\mu(\mathbf{q})d^3q.$$

Here, $\mu=\mu_x+i\mu_y$; $\mu^*=\mu_x-i\mu_y$; $\rho(\mathbf{k})$ is the Fourier transform of the function $\rho(\mathbf{r})$ modeling the inhomogeneities; $k_\pm=k_x\pm ik_y$; $\theta_{\mathbf{k}}$ is the angle between the vector $\mathbf{k}$ and the z axis, which is parallel to $\mathbf{H}_0$; $J=\alpha gM_0$; $\omega_0=gH_0$; $\omega_M=2\pi gM_0$. The function $\Gamma(\mathbf{k},\mathbf{q}_1,\mathbf{q}_2)$ and the field h_z^m occurring in Q_2 are described by the following expressions:

$$\Gamma(\mathbf{k},\mathbf{q}_1,\mathbf{q}_2)=J[\mathbf{q}_1(\mathbf{k}-\mathbf{q}_2)-{}^1/_2\mathbf{q}_2^2]$$
$$-{}^1/_2\omega_M\sin^2\theta_{\mathbf{q}_2},$$

$$h_z^m(\mathbf{k})$$
$$=-2\pi M_0\frac{k_z}{k^2}\left\{k_-\mu_k+k_+\mu_k^*+\gamma\int\rho_{\mathbf{k}-\mathbf{q}}[k_-\mu_{\mathbf{q}}+k_+\mu_{\mathbf{q}}^*]d^3q\right\}.$$

The structure of Eq. (16) reflects the various physical processes that occur in our system: the expression for Q_1 describes the scattering of a spin wave of frequency ω with a wave vector $\mathbf{k}$ directly by inhomogeneities of the modulus of the magnetization $M(\mathbf{r})$, whereas Q_2 allows for the interaction with the stochastic magnetic structure; finally, Q_3 describes the scattering of a spin wave by static inhomogeneities of the magnetostatic field H_z^m of the type

$$H_z^m=-4\pi M_0\frac{k_z}{k^2}\left\{k_xm_x+k_ym_y+\gamma k_z\rho(\mathbf{k})\right.$$
$$\left.+\gamma\int\rho_{\mathbf{k}-\mathbf{q}}[k_xm_x(\mathbf{q})+k_ym_y(\mathbf{q})]d^3q\right\}.$$

To avoid misunderstanding, we recall that $\mathbf{H}^m$ contains only the fluctuating part of the magnetostatic field: $\langle\mathbf{H}^m\rangle=0$; its average value is included in $\mathbf{H}_0$, exactly as in Sec. 1.

After averaging Eq. (16) and its complex conjugate in the same approximation as in Refs. 1–4, we obtain a system of equations for the average values $\langle\mu\rangle$ and $\langle\mu^*\rangle$. It is extremely cumbersome and we shall not give it here. For this reason we shall consider the simplest, but important in practice, case of a wave traveling along a magnetic field $\mathbf{H}_0$ in such a way that $k_x=k_y=0$. The dispersion law for this case is

$$\frac{\omega-\omega_0}{\omega_M}=\xi^{-2}p^2-\gamma^2\left[D_M(H_0)\xi^{-2}p^2-\int d^3q\frac{q_x^2q_z(p+q_z)S_\rho(\mathbf{q})}{q^2(\mathbf{p}+\mathbf{q})^2\Delta_1(\mathbf{q})}\right.$$
$$+\frac{1}{2}(p^2+\lambda^2)\int d^3q\frac{q_z(p+q_z)\sin^4\theta_{\mathbf{p}+\mathbf{q}}}{q^2\Delta(\mathbf{q},\mathbf{p})\Delta_1(\mathbf{q})}S_\rho(\mathbf{q})$$
$$+\int d^3q\frac{{}^1/_2\sin^2\theta_{\mathbf{p}+\mathbf{q}}+2\nu^2+\xi^{-2}(2p^2+2\mathbf{pq}+q^2)}{\Delta(\mathbf{q},\mathbf{p})}$$
$$\left.\times(\xi^{-2}L_1(\mathbf{q},\mathbf{p})+L_2(\mathbf{q},\mathbf{p}))S(\mathbf{q})\right], \qquad (17)$$

where

$$L_1=(p^2-q^2)(2pq+p^2)-\frac{(\lambda^2+q^2)\cos^2\theta_{\mathbf{q}}}{\Delta_1(\mathbf{q})}$$
$$\times\left[2p^2+2\mathbf{pq}+q^2-\frac{\lambda^2+q^2}{\Delta_1(\mathbf{q})}\cos^2\theta_{\mathbf{q}}\right],$$
$$L_2=\frac{1}{2}(p^2-q^2)\sin^2\theta_{\mathbf{p}+\mathbf{q}}+\frac{\sin^2\theta_{\mathbf{p}+\mathbf{q}}}{\Delta_1(\mathbf{q})}$$
$$\times\left[\frac{q_z(p+q_z)}{q^2}\left(p^2+2\mathbf{pq}-\frac{\lambda^2+q^2}{\Delta_1(\mathbf{q})}\cos^2\theta_{\mathbf{q}}\right)-\frac{1}{2}(\lambda^2+q^2)\cos^2\theta_{\mathbf{q}}\right],$$
$$\Delta(\mathbf{q},\mathbf{p})=\sin^2\theta_{\mathbf{q}+\mathbf{p}}[\lambda^2+(\mathbf{q}+\mathbf{p})^2]+$$
$$+\xi^{-2}(q^2+2\mathbf{pq})[2\lambda^2+2p^2+2\mathbf{pq}+q^2],$$
$$\Delta_1(\mathbf{q})=\xi^{-2}q^2+\nu^2+\sin^2\theta_{\mathbf{q}}.$$

The dimensionless parameters ξ and ν were determined in Sec. 1 [see Eq. (13)]; the other parameters are $\lambda=\nu\xi=k_H/k_c$ and $p=k/k_c$.

The first two terms in the square brackets in Eq. (17) are due to inhomogeneities of the ground state of the investigated system and are not related directly to the scattering effects in the sense that they do not contain a resonant denominator $\Delta(\mathbf{q},\mathbf{p})$ and do not contribute to the damping; they represent renormalization of the energy in its "pure form."

The third term describes the contribution made to modification of the dispersion law by the scattering processes involving a change in the polarization: waves with the right-hand polarization were scattered into left-hand-polarized waves. Finally, the last term corresponds to intrinsic scattering processes which do not involve a change in the polarization.

If the inequality $\xi^2 \gg 1$ is obeyed, we can distinguish a number of terms in Eq. (17) and they differ by factors governing their order of magnitude. Firstly, this is the term $\sim \gamma^2 \xi^{-2}$, which originates from the first term in L_1. This would be the term describing the modification in the dispersion law due to fluctuations of the modulus M in the absence of the magnetodipole interaction.[1] The second term in L_1 leads to, because of the logarithmic divergence of the corresponding integral in the limit $\xi \to \infty$, to the term $\sim \gamma^2 \xi^{-2} \ln \xi$. Finally, there are the terms $\sim \gamma^2$ and $\sim \gamma^2 \ln \xi$ related to the remaining terms in L_1 and L_2. The physical conclusion that follows from these estimates is that the magnetodipole interaction enhances considerably (by a factor of ξ^2) the contribution of fluctuations of the modulus of the magnetization M to the dispersion law of spin waves under the conditions of long correlations. An allowance for this effect is important in an experimental determination of the contributions made by various fluctuating parameters of a ferromagnet to the overall modification of the dispersion law.

We shall begin an analysis of Eq. (17) with the case of a homogeneous oscillation characterized by $k = 0$. An estimate of the relevant integrals in the limiting case $\xi \gg 1$ yields

$$\omega \approx \omega_0 + \gamma^2 \begin{cases} \frac{1}{2}\omega_M\left(\ln 2\xi - J_1 - \frac{17}{12}\right), & H_0 \ll H_c, \\ \frac{1}{2}\omega_M\left(\ln\frac{2}{\nu} - \frac{7}{2}\right), & H_c \ll H_0 \ll 4\pi M, \\ 2\omega_M[3 - 2\ln(2^{1/2}\xi) + 2J_1], & H_0 \gg 4\pi M, \end{cases} \tag{18}$$

where

$$J_1 = 4\pi \int q^2 \ln q S(q)\, dq.$$

The expression for the frequency ω_0, obtained allowing for the average demagnetizing field $\langle \mathbf{H}_m \rangle$, can be written in the form

$$\omega_0 = g(H_0 - 4\pi N_{zz} M_0) + 4\pi g \gamma^2 D_M(H_0) N_{zz} M_0, \tag{19}$$

where N_{zz} is the corresponding demagnetization factor (it is assumed that the field $\mathbf{H}_0$ is directed along the major axes of an ellipsoidal sample).

It is clear from Eqs. (18) and (19) that inclusion of the magnetodipole interaction gives rise to a new (for systems with an inhomogeneous magnetization) effect in the form of a shift of the spin-wave gap compared with the homogeneous ($\gamma = 0$) case. The existence of this effect has been simply mentioned in connection with an inhomogeneity of the anisotropy parameters.[3,4]

An analysis of the dispersion law of Eq. (17) in the $k \neq 0$ case is difficult because of the cumbersome nature of the relevant expressions. Therefore, in order to obtain at least qualitative information on this law, we shall limit it only to terms $\sim \gamma^2$ and $\sim \gamma^2 \ln \xi$. Retention of both these terms does not represent excessive precision (because of the inequality $\ln \xi \ll \xi$ when $\xi \gg 1$) and it also makes it possible to follow changes in the nature of the dispersion law when the parameters of the system are varied.

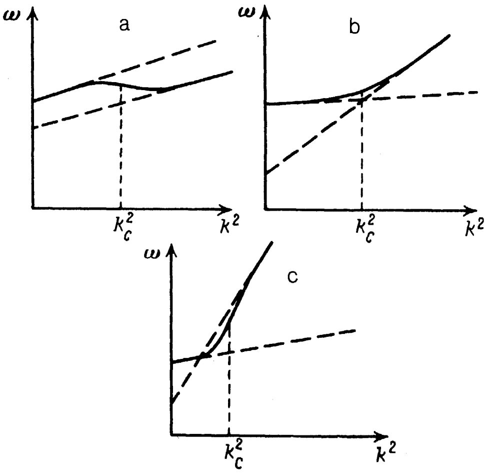

FIG. 2. Possible types of modification of the dispersion curves of spin waves: a) simulation of an "exchange kink"; b), c) "magnetization kink" (in accordance with the terminology adopted in Refs. 1, 3, 4, and 20).

Depending on the external field H_0 and on the parameters of the investigated material, we will observe one of the three types of dispersion curves shown in Fig. 2. These curves are plotted on the basis of an analysis of the asymptotic behavior of Eq. (17) when $p \ll 1$ and $p \gg 1$, and the difference between them is related to the difference between the relationships linking the angles of tilt of the immediate and distant asymptotes (the relative positions of the point of intersection of these asymptotes with the ordinate is always the same). A transition between these curves involves changes in ν and ξ and it occurs at those values of the two parameters that correspond to realistic experimental conditions. In fields $4\pi M_0 \ll H_0 \ll H_c$, apart from the region $k \sim k_c$, the modification of the dispersion law occurs also in the vicinity of the wave number k_H (not shown in Fig. 2), which is described by an expression of the type

$$\omega \approx \omega_0 - \frac{1}{2}\gamma^2 \omega_M \frac{p^2 + 2\lambda^2}{p^2 + \lambda^2} + \tilde{\alpha} p^2, \tag{20}$$

where $\tilde{\alpha}$ is an effective parameter but its exact value is unimportant.

Moreover, a change in the dispersion law occurs in the wave vector range $k \gg k_M$. Spin waves with $k \gg k_M$ play the dominant role in determination of the temperature dependence of the magnetization, so that we shall discuss the dispersion law for this range of k in Sec. 4.

3. INFLUENCE OF THE MAGNETODIPOLE INTERACTION ON THE DAMPING OF SPIN WAVES

One of the mechanisms of the influence of the magnetodipole interaction on the damping of spin waves, associated with the change in the nature of the unperturbed dispersion law, was investigated earlier[3] for models with exchange and anisotropy fluctuations. The magnetodipole interaction gives rise to a wave vector k_i in the vicinity of which there is a change in the behavior of the damping:

$$k_i = \{k_H[(k_H^2 + k_M^2)^{1/2} - k_H]\}^{1/2}.$$

In this case the inclusion of the magnetodipole interaction has more drastic consequences because it alters not only the unperturbed dispersion law, but also the nature of the interaction of a spin wave with inhomogeneities.

The expression for the damping factor can be written in the form

$$\omega''=-\pi^2[F_1(p)+F_2(p)], \tag{21}$$

where

$$F_1(p)=\frac{\xi^2}{4}(p^2+\lambda^2)\int dx \times\frac{q_p^2(q_px-p)x(1-x^2)S_\rho(\mathbf{p}-\mathbf{q})}{[(p^2+\lambda^2)^2+{}^1/_4\xi^4(1-x^2)^2]^{1/2}(\mathbf{p}-\mathbf{q})^2\Delta_1(\mathbf{p}-\mathbf{q})}\Bigg|_{|\mathbf{q}|=q_p},$$

$$F_2(p)=-\frac{1}{2}\int dx\left\{1+\frac{\lambda^2+p^2}{[(\lambda^2+p^2)^2+{}^1/_4\xi^4(1-x^2)^2]^{1/2}}\right\} \times q_p[\xi^{-2}L_1(\mathbf{q}-\mathbf{p},\mathbf{p})+L_2(\mathbf{q}-\mathbf{p},\mathbf{p})]S_\rho(\mathbf{p}-\mathbf{q})\,|_{|\mathbf{q}|=q_q},$$

q_p is found from the equation $\Delta(\mathbf{q},\mathbf{p})=0$:

$$q_p=\{[(\lambda^2+p^2)^2+{}^1/_4\xi^4\sin^4\theta_\mathbf{q}]^{1/2}-\lambda^2-{}^1/_2\xi^2\sin^2\theta_\mathbf{q}\}^{1/2},$$

where the limits of integration with respect to $x=\cos\theta_\mathbf{q}$ are selected in accordance with Ref. 3; L_1 and L_2 are defined in Eq. (17).

Since the integrals in the expressions for F_1 and F_2 cannot be calculated rigorously, we shall draw some general conclusions on the nature of the function $\omega''(k)$ which we shall support later by a detailed numerical analysis.

In the range of low values of $k<\min(k_i,\ k_c)$ the behavior of the damping is governed by a term of the k^{2n+3} type, where n represents the order of zero of the function $S_\rho(k)$ when $k=0$ [for details of the use of the functions $S(k)$ with different values of n, see Ref. 17]. On going through one of the characteristic points k_i or k_c $[k>\min(k_i,\ k_c)]$ the function $\omega''(k)$ should be described by the expression

$$\omega''(k)\sim c_1k+c_2k^3. \tag{22}$$

[It should be mentioned, for the sake of accuracy, that if $\min(k_i,\ k_c)=k_i$, then Eq. (22) must be multiplied by k_{2n}.] In the range of sufficiently large wave numbers $k\gg k_M$ the first term in L_1 begins to play the main role and this gives rise to the same k^3 law as in Ref. 1.

In our numerical analysis we selected for the fluctuations of M a correlation function of the type given by Eq. (14), where the parameter n was postulated to be zero. The results of integration of Eq. (21) are presented in Figs. 3a–c, where apart from the function $\omega''(k)$ we plotted also its derivative $d\omega''/dk$. We can see from Figs. 3a–c (when $k_i>k_c$ in all cases) that the linear term in Eq. (22) (corresponding to the wave-vector range $k_c<k<k_i$) predominates over the cubic term and this is true in a wide range of values of the parameters of the system. This is illustrated clearly by a graph of the derivative $d\omega''/dk$, which in this range of wave vectors has a wide maximum that sometimes even becomes a plateau (Fig. 3a).

This is of special interest in connection with recent experimental investigations[6,7] of Co–P and Co–Zr films which revealed a linear dependence of the width of a spin-wave resonance line on the mode number, in contrast to the k^5 and k^3 laws predicted in Refs. 1, 3, and 4 using models with exchange and magnetization fluctuations in the range $k>k_c$.

Figure 3d shows, for the sake of comparison, a plot of $\omega''(k)$ in the limiting case when $\xi\ll 1$, which corresponds to neglect of the magnetodipole interaction.

We shall conclude this section by pointing out another factor. All the relationships given above describe an experimental situation when the dependence of the damping factor on k is determined in a constant magnetic field H_0. However, similar measurements can be carried out also under different conditions when the frequency ω of the resonant field is constant as is true, for example, of spin-wave resonance experiments. In describing this case it is necessary to modify all the expressions in the present section by replacing

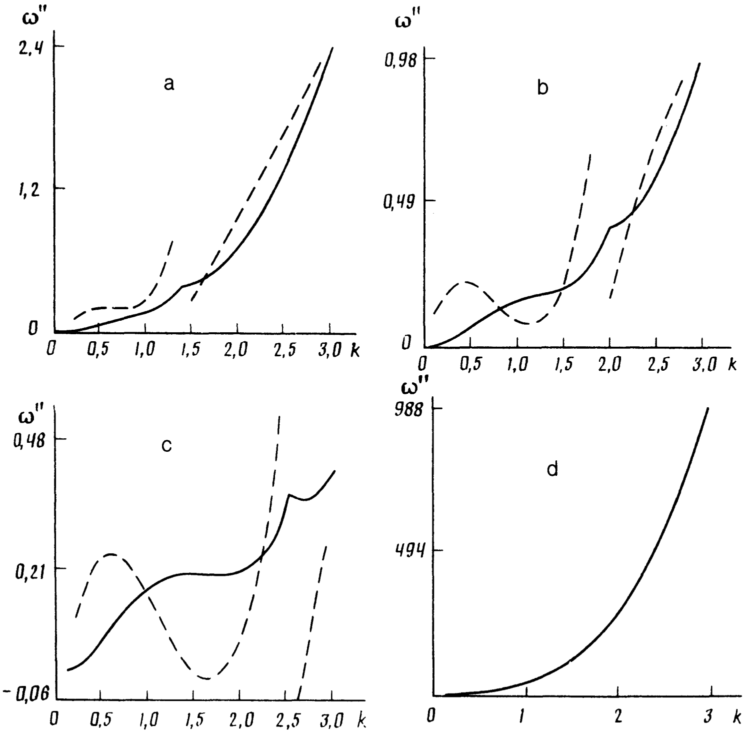

FIG. 3. Dependences of the damping of spin waves on the wave vector calculated for different values of the parameter ξ (continuous curves). The dashed curves give the derivatives $d\omega''/dk$; a) $\xi=2.0$, $k_i=1.3k_c$; b) $\xi=5.0$, $k_i=2.5k_c$; c) $\xi=3.2$, $k_i=1.9k_c$; d) $\xi=0.1$.

the field parameter λ^2 with the expression $f^2 - q^2$ corresponding to the unperturbed dispersion law where $f^2 = \omega/\alpha g M_0 k_c^2$. The graphs plotted in Fig. 3 apply precisely to this case, since our aim was to compare them with the experimental results of Refs. 6 and 7.

4. INFLUENCE OF AN INHOMOGENEOUS GROUND STATE ON THE LOW-TEMPERATURE BEHAVIOR OF THE MAGNETIZATION OF AN AMORPHOUS FERROMAGNET

One of the characteristic features of the thermodynamic properties of amorphous ferromagnets is the deviation of the low-temperature behavior of the magnetization from the temperature dependence given by the Bloch law $T^{3/2}$. Many experiments (see, for example, Refs. 8 and 9) have revealed a correction of the $T^{5/2}$ type to the usual $T^{3/2}$ Bloch law. Some of the mechanisms which can be responsible for this behavior had already been discussed in the cited papers.

The results of the preceding sections in this paper suggest one further mechanism which could account for the term $T^{5/2}$ in the temperature dependence of the magnetization.

At temperatures such that the effects in question are observed the magnetization is dominated by spin waves with $k^2 \gg 4\pi/\alpha$. In this range the dispersion law is

$$\omega = \omega_0 + \alpha g M_0 (1 - \gamma^2 D_M) k^2, \tag{23}$$

where D_M is the variance of the stochastic structure, discussed by us in Sec. 1, whereas ω_0 is given by Eq. (19). We ignored here the field-dependent corrections to ω_0.

As expected, in this range of wave vectors all the effects associated with the magnetodipole interaction (with the exception of a reduction in the value of M_z caused by the MSMS) disappear. However, this effect is not the prerogative of the MSMS: the expressions (19) and (23) should be similar for a system with any type of a stochastic magnetic structure. (A similar effect in the ASMS case was ignored in Ref. 4.) We shall therefore use the expressions (19) and (23) without specifying the actual nature of the quantity D_M. It will be important to us that D_M depends on the external field H_0.

The dependence of the magnetization on the temperature T of a sample is given by the expression[21]

$$M(T) - M_0 = \int \mu(\mathbf{k}) \left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1 \right]^{-1} \frac{d^3 k}{(2\pi)^3}, \tag{24}$$

where $\mu(\mathbf{k})$ is the magnetic moment of one magnon:

$$\mu = -\hbar \frac{\partial \omega}{\partial H_0} = -\hbar g \left(1 + 4\pi N_{zz} \gamma^2 M_0 \frac{\partial D_M}{\partial H_0} \right) + \alpha g \hbar M_0 \gamma^2 \frac{\partial D_M}{\partial H_0} k^2. \tag{25}$$

However, we are speaking here of a disordered system and Eq. (24) contains ensemble- or volume-average values of the magnetization, but the validity of using such expressions must be justified. This can be done in two stages. In the first stage we can change in Eq. (24) from integration with respect to **k** to integration with respect to ω and introduce the density of states $g(\omega)$, which is a self-averaging quantity[22] so that in averaging of (24) we can take it outside the symbols denoting the averages. On the other hand, we can readily see that the average density of states is equal to the density of states of an effective homogeneous system with the dispersion law given by (23). In the second stage we must show that the average value μ, which must be substituted in Eq. (24), is identical with that determined in Eq. (25), i.e., the average frequency of the natural fluctuations of the disordered system is in agreement with the dispersion law obtained using our approach. This is readily checked by direct calculation of the natural frequency employing the Rayleigh–Schrödinger perturbation theory: the necessary coincidence can be established in any order of perturbation theory. Such calculations (up to the second order) had been reported in, for example, Ref. 23.

It is clear from Eq. (25) that the existence of an SMS has two effects: the main one is the appearance of a dependence of μ on the wave number k. Moreover, the term independent of k also becomes renormalized.

The two terms in Eq. (25) correspond to two types of terms in

$$M(T) = M_0 - M_1(T) - M_2(T). \tag{26}$$

The first term gives the usual Bloch law $T^{3/2}$:

$$M_1(T) \sim \hbar \tilde{g} \left(\frac{T}{\tilde{A}\hbar} \right)^{3/2}, \tag{27}$$

where

$$\tilde{g} = g \left(1 + 4\pi N_{zz} \gamma^2 M_0 \frac{\partial D_M}{\partial H_0} \right),$$

$$\tilde{A} = \alpha g M_0 (1 - \gamma^2 D_M).$$

The second term gives

$$M_2(T) \sim -\gamma^2 \hbar \alpha g M_0 \frac{\partial D_M}{\partial H_0} \left(\frac{T}{\tilde{A}\hbar} \right)^{5/2}. \tag{28}$$

The temperature T in Eqs. (27) and (28) is measured in energy units.

Therefore, the term proportional to $T^{5/2}$ appears in the present case as a consequence of a stochastic magnetic structure because of the dependence of its variance on H_0. We shall distinguish this mechanism of formation of the correction to the Bloch law from others by comparing the magnetic-field dependences of the coefficients in front of the temperature factors in Eqs. (27) and (28).

Equation (23) was obtained earlier by a different method in Ref. 24 in order to account for the experimental results[25] of an investigation of the dependence of the spin-wave rigidity $\tilde{A}$ on H_0. However, it should be pointed out that in an analysis of these experiments no allowance was made for renormalization of the quantity g in Eq. (25), which—generally speaking—alters the field dependence of the corresponding coefficient in Eq. (27).

CONCLUSIONS

An investigation was made of the model of a disordered ferromagnet with fluctuations of the modulus of the magnetization M. It was found that an allowance for the magnetodipole interaction was fundamental for the correct understanding of the properties of this model. The results obtained demonstrated that the magnetodipole interaction can enhance significantly (by a factor $\xi^2 = 4\pi/\alpha k_c^2$) the influence of fluctuations of M on the ground state and on the spectrum of excitations of the investigated model (naturally, on condition that $\xi \gg 1$ which is satisfied in the case of systems with a long correlation radii).

An investigation was made of a stochastic magnetic structure due to the magnetodipole interaction (MSMS). It was found that the correlation characteristics of an MSMS differ significantly from the correlation characteristics of the stochastic magnetic structures investigated before. This makes it possible to distinguish an MSMS in electron-optic images of an SMS. A study was made of the law of approach of the magnetization to saturation and it was found that it differs considerably from the dependences $\langle M_z(H_0)\rangle$ typical of other types of inhomogeneities.

The main conclusion drawn from an analysis of the dispersion law of spin waves in this model was that, depending on the parameters of the material and the conditions during observation, there may be three different types of the special curves, which is in contrast to the universal nature of a single curve obtained in Ref. 1 without allowance for the magnetodipole interaction. This circumstance is important for the correct use of the method of spin-wave spectroscopy developed in a number of reports,[1,3,4,20] because the method relies greatly on the concept of a universal dispersion law typical of every fluctuating parameter of a system. In a detailed comparison of the results obtained with the relevant experiments it will be necessary to carry out a more thorough numerical analysis of Eq. (17) of the experimental data.

In addition to the nature of the dispersion curves, it would be of interest to study also the field-dependent shift of the spin-wave gap, which is also due to the magnetodipole interaction. An experimental detection and investigation of this effect would make it possible to determine independently both the correlation radius r_c and the *rms* deviation γ, which would increase the reliability of resonance magnetic structure methods for the investigation of amorphous materials (including the method of spin-wave spectroscopy).

The special interest in the damping of spin waves is due to the reports of experimental results[6,7] which cannot be described by the existing theories. One of these results is a linear law $\omega'' \propto k$ in the range $k > k_c$, whereas the existing theories[1,3] predict the laws k^5 and k^3 for models with fluctuations of the exchange and of the magnetization. This raises two problems: 1) what is the reason for the linear law? and 2) why the apparently stronger laws k^5 and k^3 are not manifested? An analysis presented in Sec. 3 makes it possible to suggest possible answers to these questions: 1) a randomly inhomogeneous magnetostatic field $\mathbf{H}_m(\mathbf{r})$ is responsible for the linear term; 2) predominance of the linear term over the terms with higher power exponents is due to the effect of a gain ξ.

The last section suggests a new mechanism of the $T^{5/2}$ correction to the Bloch law which is introduced to allow for the interaction of spin waves with an inhomogeneous ground state. A special feature of this mechanism is the existence of a quite definite relationship between the field dependences of the coefficients in front of $T^{3/2}$ and $T^{5/2}$, which follows from Eqs. (26)–(28). An experimental investigation of this relationship would make it possible to distinguish the proposed mechanism from those discussed earlier. One should mention also that this feature is specific to disordered materials, which is not true of the earlier explanations of the origin of the $T^{5/2}$ law.

[1)] The law of approach of the magnetization to saturation had been used earlier to obtain information on an amorphous material by Kronmüller and his colleagues (see, for example, Ref. 5), but the interpretation of this law was fundamentally different from that adopted in Ref. 11.

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