Is the fundamental Schrödinger soliton stabilized in its own nonlinear light guide?

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The classical answer to the question posed above would be positive: an optical soliton can "free itself" of the initial noise modulation in the course of nonlinear propagation in a fiber. However, the situation is different for quantum fluctuations. It is shown that the latter tend to accumulate and can destabilize the soliton, even if it moves through the light guide without losses.

1. INTRODUCTION

It is commonly assumed that an ideal fundamental soliton, satisfying the nonlinear Schrödinger equation, can propagate indefinitely in a nonlinear optical fiber without losses. Such a light guide should possess a cubic nonlinearity and be adequately described by the second-order approximation of dispersion theory, and the initial amplitude and shape of the pulse should correspond to the single-soliton propagation regime, i.e., they should be sufficiently similar to the corresponding amplitude and shape of a soliton. Indeed, under these conditions the initial noise phase modulation of the pulse over the course of time transforms into an amplitude modulation, after which it is "ejected" to the wings, and eventually all that remains is a "pure" well-formed soliton.[1,2]

However, this "self-freeing" property is not universal with regard to other types of fluctuations. The authors of Ref. 3 were apparently the first to call attention to this fact. In that article they considered propagation of a fundamental soliton periodically amplified to compensate for losses in the fiber. The spontaneous noise that arises together with the soliton is also amplified. It accumulates and increases, and the first symptom of its presence is random departures of the frequency of the carrier wave with resultant variations of the rate of propagation; this is extremely undesirable in information communication lines, for which optical solitons are the carriers of choice. Thus, there arises a limiting distance of travel of the soliton, bounding the range of action of the information channel.

In the present paper it is shown that an analogous tendency of accumulation and magnification is also present for quantum fluctuations (phase as well as amplitude) which are always present in real radiation, even in the absence of absorption and/or amplification, i.e., in an ideal light guide which does not introduce losses. The analysis, which takes account of all possible amplifications and losses, allows us to make this assertion confidently at least in the initial stage of propagation of the soliton, where the implemented approximations are guaranteed to be valid.

It should be noted that quite a large number of publications (see, e.g., Refs. 4–12 and the references cited therein) have already been dedicated to the study of the evolution of the quantum state of the fundamental soliton in its own nonlinear channel without amplification or losses. However, practically all of them have to do with the so-called squeezed quantum states,[13–15] i.e., with questions of the suppression of one of the quadrature components of the field, not with the spontaneous accumulation of noise. Since a statement of simultaneous diminution and growth of the fluctuations has a paradoxical ring to it, we will clarify this assertion at once.

We introduce the quadrature components of the electric component of the field in the form

$$X=E^{(+)}e^{-i\varphi}+E^{(-)}e^{i\varphi},\quad Y=(E^{(+)}e^{-i\varphi}-E^{(-)}e^{i\varphi})/i, \tag{1}$$

where $E^{(+)}$ and $E^{(-)}$ are the slowly varying (in time) operators of the positive and negative-frequency parts of the electric field, and the angle φ determines the orientation of the coordinate axes in the phase plane. This angle can be such that the variance of the fluctuations of one of the quadrature components of a soliton that has passed through an interval of the nonlinear waveguide is below the corresponding level of the vacuum or coherent state. This is a criterion for the formation of a squeezed state of the field.

The first prediction of the existence of squeezed states in optical solitons was made in Refs. 4 and 5. Without in any way calling these two papers into question, a certain error in the treatment both in the indicated papers and in a subsequent paper (Ref. 6) should be noted, namely that the authors of these papers in fact analyzed not self-action leading to a nonlinear phase advance in a medium with cubic nonlinearity and being the cause of the squeezing, but synchronous degenerate four-photon parametric amplification of fluctuations in the field of a regular soliton component, as in pumping. This question has been considered in greater detail in Refs. 7–9. We will touch on this point below. In this sense the analyses of the evolution of the quantum states of a soliton presented in Refs. 7 and 8, which make use of the Heisenberg representation, and in Refs. 10 and 11, which use the Schrödinger representation, are more on the mark. In the first case (Refs. 7–9) a comparatively greater ease of visualization is achieved, which reveals the paths of preparation[1)] of the sub-Poisson solitons,[8,9] but at the expense of a quite rigid approximation of the given channel. The use of the Schrödinger representation[10,11] leads to more general results but of substantially more complicated form.

Following the history of this question to its conclusion, it should be noted that the possibility of onset of squeezed quantum states of a single-mode monochromatic signal in a nonlinear phase advance in cubic-nonlinearity media was first pointed out in Ref. 16.

Thus, for a fixed orientation of the coordinate axes in phase space the fluctuations of one of the quadrature components, e.g., the X component, are suppressed. But does this imply a corresponding decrease of the amplitude or quantum phase noise of the signal in the given case, i.e., of a soliton? Not necessarily, since by virtue of the Heisenberg uncertainty principle a decrease of the fluctuations of one

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quadrature component (X) implies a corresponding increase of the fluctuations of the other (Y). Thus, for a single-mode field we can write to within a normalization factor

$$X_{(1)}=a+a^{+}, \quad Y_{(1)}=(a-a^{+})/i,$$

where a^{+} and a^{-} are the photon creation and annihilation operators, and the product of the variances of the fluctuations obeys the inequality $\langle \Delta X_{(1)}^{2} \rangle \langle \Delta Y_{(1)}^{2} \rangle \geqslant 1$.

What then happens to the signal noise? It all depends on the orientation in the phase plane of the vector of the complex amplitude of the regular component. It if is aligned in the X direction, i.e., the longer axis of the so-called squeezing ellipse, perpendicular to the vector of the regular component, then there takes place a decrease of the amplitude noise and a corresponding increase of the phase noise. The field in such a state is called sub-Poisson.[17] We now rotate the squeezing ellipse by $\pi/2$ with respect to the vector of the regular component. Then there appears a state with suppressed phase fluctuations and increased amplitude fluctuations. In an intermediate orientation, that noise as well as the other noise can grow. This is why there can be a simultaneous increase of the amplitude and phase fluctuations of the field in the squeezed quantum state.

2. BASIC EQUATION

The evolution of the electric field of radiation entering a transparent medium with cubic nonlinearity can be described in the second-order approximation of dispersion theory by the following equations:[4-12]

$$\frac{\partial E^{(+)}}{\partial z}=\left[-u^{-1}\frac{\partial}{\partial t}+\frac{i}{2}g\frac{\partial^{2}}{\partial t^{2}}+\frac{ik\varepsilon_{\mathrm{nl}}}{2\varepsilon_{0}}E^{(-)}E^{(+)}\right]E^{(+)}. \tag{2}$$

Here $E^{(+)}(t,z)$ and $E^{(-)}(t,z)$ are the slowly varying (in time) operators of the positive- and negative-frequency parts of the field in the Heisenberg representation, the z axis points in the direction of propagation, t is the time, $u=(\partial k/\partial\omega)^{-1}$ is the group velocity at the carrier frequency ω, k is the wave number of the carrier wave, the parameter $g=\partial^{2}k/\partial\omega^{2}$ characterizes the dispersion of the group velocity, and $\varepsilon_{n\,l}$ and ε_0 are the nonlinear and linear parts of the dielectric constant of the medium. We assume that the interaction is collinear, the spatial mode is a plane wave, and the nonlinearity is instantaneous. The derivation of Eq. (2) is given in sufficient detail, for example, in Ref. (8); therefore we will not pay it any special attention here. We note only that to go over to the well-known classical analog[1] it suffices to replace $E^{(+)}$ and $E^{(-)}$ in Eq. (2) by the conjugate complex amplitudes A and A^*.

Equation (2) describes the influence of dispersion and of the nonlinearity on the passing radiation in the Heisenberg representation. In general, however, it is desirable also to take account of losses and possible amplification. We assume that all these processes take place independently and successively, but that in the end we combine them, i.e., we will assume them to act uniformly over the length of the light guide.

Thus, we take a small section of the light guide of length Δz and arbitrarily divide it into three successive layers. In the first only dispersion and nonlinearity are manifested, in the second—only absorption, and in the third—amplification of the Stokes component thanks to stimulated scattering in the laser pump field. Since transmission by the first layer is described by Eq. (2), we go now at once to the second.

We assume that linear losses appear as a result of the interaction of the optical radiation with an infinite Markov system of phonons. The conversion of one mode of this radiation with detuning frequency Ω from the carrier frequency ω is described by the evolution of the Heisenberg photon annihilation operator a_Ω (Refs. 18 and 19):

$$a_{\Omega}(\Delta z)=\mu(\Delta z)a_{\Omega 0}+\sum_{j}\nu_{j}(\Delta z)b_{j0}. \tag{3}$$

Here μ and ν are C-number functions which depend on the length of the layer Δz. We do not need the explicit form of $\nu_j(\Delta z)$ in what follows. It can be found in Refs. 18 and 19, and $\mu(\Delta z)$ is written out below. b_j is the phonon annihilation operator, and the sum is carried out over the phonon modes j. The index "0" labels operators at the entrance to the considered layer.

By virtue of the commutation relations for bosons

$$[a_{\Omega}, a_{\Omega'}{}^{+}]=\delta_{\Omega\Omega'}, \quad [b_j, b_{j'}{}^{+}]=\delta_{jj'}, \tag{4}$$

$\delta_{\Omega\Omega'}$, and $\delta_{jj'}$ are Kronecker symbols that differ from zero and equal unity only for $\Omega=\Omega'$ and $j=j'$, respectively, we have

$$|\mu|^{2}+\sum_{j}|\nu_{j}|^{2}=1. \tag{5}$$

But since[18]

$$\mu(\Delta z)=e^{-\varkappa\Delta z/2}\approx 1-\varkappa\Delta z/2, \tag{6}$$

where $\varkappa$ is the intensity absorption coefficient, we obtain

$$\sum_{j}|\nu_{j}(\Delta z)|^{2}\approx\varkappa\Delta z. \tag{7}$$

We let Δz approach zero and replace the finite differences in Eq. (3) by differentials. Then

$$\partial a_{\Omega}(z)/\partial z=-\varkappa a_{\Omega}(z)/2+\Gamma_{\Omega\varkappa}(z). \tag{8}$$

Here the random Langevin force $\Gamma_{\Omega\varkappa}$ possesses the following statistical properties (see also Ref. 19):

$$\begin{gathered}\langle\Gamma_{\Omega\varkappa}(z)\Gamma_{\Omega\varkappa}{}^{+}(z')\rangle=\varkappa(\langle N_{\varkappa}\rangle+1)\delta(z-z'),\\ \langle\Gamma_{\Omega\varkappa}{}^{+}(z)\Gamma_{\Omega\varkappa}(z')\rangle=\varkappa\langle N_{\varkappa}\rangle\delta(z-z'),\\ \langle\Gamma_{\Omega\varkappa}(z)\Gamma_{\Omega\varkappa}(z')\rangle=\langle\Gamma_{\Omega\varkappa}{}^{+}(z)\Gamma_{\Omega\varkappa}{}^{+}(z')\rangle=0.\end{gathered} \tag{9}$$

In these expressions the averaging is carried out over the quantum state of the photon–phonon system $|\Phi\Phi\rangle$, and by virtue of the Markov nature of the phonons the state vectors of the photons and phonons separate: $|\Phi\Phi\rangle=|\Phi_{\mathrm{phot}}\rangle|\Phi_{\mathrm{phon}}\rangle$. The quantity $\langle N_\varkappa\rangle=\langle\Phi_{\mathrm{phon}}|b_{j0}{}^{+}b_{j0}|\Phi_{\mathrm{phon}}\rangle$ is the mean number of phonons giving rise to absorption in the mode, and which is as a rule significantly less than one. Here we assume that all the phonon modes take part identically in the absorption process and that $\langle N_\varkappa\rangle$ does not depend on j.

The next step of our treatment is to go now to a multimode optical quantum field. We assume that the spectral absorption line is uniformly broadened within the limits of spectrum of the signal. Then all the optical modes will find themselves under independent and equal (from the point of view of losses) conditions. Here relations (9) can be rewritten in more general form:

$$\langle \Gamma_{\Omega\varkappa}(z)\Gamma^{+}_{\Omega'\varkappa}(z')\rangle = \varkappa(\langle N_\varkappa\rangle + 1)\delta_{\Omega\Omega'}\delta(z-z') \qquad (10)$$

and analogously for $\langle \Gamma_{\Omega\varkappa}(z)\Gamma^{+}_{\Omega'\varkappa}(z')\rangle$.

We sum the modes of the field:

$$E^{(+)}(t,z) = C\sum_{\Omega} a_\Omega e^{-i\Omega t}, \qquad (11)$$

where the normalization factor $C = (2\pi\hbar\omega/V)^{1/2}$ can be assumed to be constant for a narrow-band signal, when $|\Omega| \gg \omega$. V is the quantization volume, which in our case is simply equal to the volume of the active core of the fiber. As a result we obtain

$$\partial E^{(+)}(t,z)/\partial z = -(\varkappa/2)E^{(+)}(t,z) + C\Gamma_\varkappa(t,z). \qquad (12)$$

Here the force $\Gamma_\varkappa$ has the following nonzero correlators:

$$\langle\Gamma_\varkappa(t,z)\Gamma_\varkappa^{+}(t',z')\rangle = \varkappa(\langle N_\varkappa\rangle+1)\delta(t-t',z-z'),$$
$$\langle\Gamma_\varkappa^{+}(t,z)\Gamma_\varkappa(t',z')\rangle = \varkappa\langle N_\varkappa\rangle\delta(t-t',z-z'), \qquad (13)$$

whence it is clear in particular that

$$\langle \mathrm{Re}\,\Gamma_\varkappa(t,z)\,\mathrm{Re}\,\Gamma_\varkappa(t',z')\rangle = \langle \mathrm{Im}\,\Gamma_\varkappa(t,z)\,\mathrm{Im}\,\Gamma_\varkappa(t',z')\rangle = {}^1/_4\varkappa(2\langle N_\varkappa\rangle+1)\delta(t-t',z-z'). \qquad (14)$$

Here and in Eq. (13) the δ-correlatedness in time is a result of taking the limit of an infinite number of modes Ω and a continuous signal spectrum. The sum over a finite number of modes can be taken, but the explicit form the correlators take on in this case is less compact.

Let us turn now to a discussion of the next step in the conversion of the signal—linear amplification in the process of stimulated scattering in the third layer of the short segment of fiber. Analogous considerations for the interaction of the photons and phonons excited by the laser pumping, under the condition that the amplification band is homogeneously broadened within the limits of the signal spectrum, lead to the following results for the evolution of the amplified Stokes component (see also Ref. 19):

$$\partial E^{(+)}(t,z)/\partial z = (\gamma/2)E^{(+)}(t,z) + C\Gamma_\gamma(t,z), \qquad (15)$$

where γ is the intensity gain coefficient, and the statistical properties of the Langevin force $\Gamma_\varkappa$ have the form

$$\langle\Gamma_\gamma(t,z)\Gamma_\gamma^{+}(t',z')\rangle = \gamma\langle N_\gamma\rangle\delta(t-t',z-z'),$$
$$\langle\Gamma_\gamma^{+}(t,z)\Gamma_\gamma(t',z')\rangle = \gamma(\langle N_\gamma\rangle+1)\delta(t-t',z-z'), \qquad (16)$$
$$\langle\Gamma_\gamma(t,z)\Gamma_\gamma(t',z')\rangle = \langle\Gamma_\gamma^{+}(t,z)\Gamma_\gamma^{+}(t',z')\rangle = 0.$$

Here $\langle N_\gamma\rangle$ is the mean number of pump-excited phonons in the mode, which can also be assumed to be significantly less than unity.

Combining all the considered processes in one equation, we finally obtain

$$\left[\frac{\partial}{\partial z} + u^{-1}\frac{\partial}{\partial t} - \frac{ig}{2}\frac{\partial^2}{\partial t^2} - \frac{\gamma-\varkappa}{2} - \frac{ik\varepsilon_{nl}}{2\varepsilon_0}E^{(-)}(t,z)E^{(+)}(t,z)\right]E^{(+)}(t,z) = C[\Gamma_\varkappa(t,z) + \Gamma_\gamma(t,z)]. \qquad (17)$$

Equation (17) can be reduced to a simpler and more obvious form by transforming to a moving coordinate system: $t \to t - z/u$, and to dimensionless variables: $\tau = t/\tau_c$, where τ_c is the duration of the soliton, $\zeta = z/L_p$, where $L_p = \tau_c^2/g$ is the dispersive spreading length, $\psi = E^{(+)}/|A_0|$, $A_0 = \langle E^{(+)}(t=0, z=0)\rangle$, $b = L_p L_{nl}$, and $L_{nl} = 2\varepsilon_0/k\varepsilon_{nl}|A_0|^2$ is the nonlinear length. For the stationary case of radiation propagation in the light guide, when the amplification completely compensates the losses ($\gamma = \varkappa$), we have then

$$\partial\psi/\partial\zeta - (i/2)\partial^2\psi/\partial\tau^2 - i\beta\psi^{+}\psi^2(\tau,\zeta) = C(F_\varkappa + F_\gamma), \qquad (18)$$

where

$$F_{\varkappa,\gamma} = L_p\Gamma_{\varkappa,\gamma}(\tau\tau_c, \zeta L_p)/|A_0|.$$

Unfortunately, the nonlinear operator equations (17) and (18) cannot be solved analytically, wherefore in what follows we will consider various types of approximations which allow us to draw definite conclusions about the evolution of the quantum fluctuations of the soliton during its nonlinear propagation.

3. QUASISTATIC PRESCRIBED-CHANNEL APPROXIMATION

This approximation allows us to describe the quantum state of the soliton in the initial stage of its propagation in the absence of amplification and losses, i.e., with the right side of Eq. (18) set to zero. It reduces to the following.[7-9] Equation (18) is represented in continuum-integral form. Then the regular and fluctuational components separate out in the operators ψ and ψ^{+}, e.g., $\psi(\tau,\zeta) = \bar{\psi}(\tau,\zeta) + \xi(\tau,\zeta)$, where $\langle\xi(\tau,\zeta)\rangle = 0$, and the regular part

$$\bar{\psi}(\tau,\zeta) = e^{i\zeta/2}\,\mathrm{sech}\,\tau \qquad (19)$$

is the classical fundamental soliton, formed at $\beta = 1$. Next the kernel operator of the continuum integral is linearized with respect to the fluctuations and this same integral is calculated under the assumption that the fluctuational components do not influence the trajectories over which the integration is carried out. Thus, the nonlinear channel, surmounted by the soliton, is completely determined in this approximation by the regular part $\bar{\psi}(\tau,\zeta)$. This is the meaning of the term "prescribed channel."

The criterion of validity of linearization over the fluctuations is fulfillment of the condition

$$|\bar{\psi}(\tau,\zeta)|^2 \gg \langle\Delta X^2(\tau,\zeta)\rangle, \qquad (20)$$

where the fluctuation quadrature is

$$\Delta X(\tau,\zeta) = \xi e^{-i\varphi} + \xi^{+}e^{i\varphi}.$$

Here inequality (20) must be satisfied for any value of the angle φ.

After carrying out the indicated procedures we obtain for the fluctuational component[7-9]

$$\xi(\tau,\zeta) = [1 + i\Psi(\tau,\zeta)]\xi_0(\tau)e^{i\zeta/2} + i\Psi(\tau,\zeta)\xi_0^{+}(\tau)e^{i(\zeta/2-2\theta_0)}. \qquad (21)$$

Here $\xi_0(\tau) = \xi(\tau,\zeta=0)$ corresponds to fluctuations at the entrance of the light guide, $\Psi(\tau,\zeta) = |\bar{\psi}(\tau)|^2\zeta$ is the nonlinear phase incursion in the absence of dispersion, and θ_0 is the initial phase of the regular component of the signal:

$\bar{\Psi}(\tau,\zeta=0)=|\bar{\Psi}(\tau,\zeta=0)|e^{-i\theta_0}$.

Note that the corresponding expression for nonlinear conversion of single-mode monochromatic radiation during self-action (30) in the absence of amplification and losses differs from expression (21) only in that τ enters in the latter as a parameter. It is for this reason that the title of this section begins with the word "quasistatic."

If a free soliton entering the fiber is initially in the coherent state (we will adhere to this assumption throughout the remainder of this discussion), then the operator $\xi_0(\tau)$ corresponds to the multimode vacuum since the regular part of the coherent signal separates out as self-sustaining.

The approximation under consideration allows one to elucidate the evolution of the phase fluctuations of the soliton as it propagates in an ideal transparent light guide. For this it is necessary to identify the direction in the phase plane perpendicular to the complex amplitude vector of the regular component, and to find the projection on it of the fluctuations. The ratio of this projection to the length $|\bar{\Psi}(\tau)|$, taking inequality (20) into account, is the signal phase operator:

$$\Phi(\tau,\zeta)=\frac{\operatorname{ch}\tau}{2i}\{[1+2i\Psi(\tau,\zeta)]\xi_0(\tau)e^{i\theta_0}-[1-2i\Psi(\tau,\zeta)]\xi_0^+(\tau)e^{-i\theta_0}\}. \tag{22}$$

Generally speaking, the phase operator is nonhermitian (Ref. 20).[2)] In our case its hermiticity is ensured, again thanks to the fulfillment of condition (20). Note also that $\langle\Phi(\tau,\zeta)\rangle=0$.

With the help of Eq. (22) the variance of the phase fluctuations, relative to the initial state of a soliton all whose modes were initially coherent, can be easily calculated:

$$\frac{\langle\Phi^2(\tau,\zeta)\rangle}{\langle\Phi^2(\tau,0)\rangle}=1+4\Psi^2(\tau,\zeta). \tag{23}$$

Here and below, the averaging is carried out over the initial vacuum states of the fluctuational components of the soliton modes since the evolution of the quantum field in our case is described by Heisenberg operators.

Thus, the variance of the phase fluctuations in the nonlinear propagation of the soliton grows. We estimate the absolute value of this variance on the basis of the following considerations. In the initial state ($\zeta=0$), when the soliton is a set of coherent modes, it can be treated as some one temporal mode.[21] Then the initial variance of the quantum phase fluctuations of the soliton is equal to $\langle\Phi^2(\zeta=0)\rangle\approx\langle 4N\rangle^{-1}$, where $\langle N\rangle$ is the mean number of photons in the soliton. Hence

$$\langle\Phi^2(\zeta)\rangle\approx(1+4\zeta^2)/4\langle N\rangle. \tag{24}$$

Nowadays it is hard to say how significant this effect is from a practical point of view; however, the very fact of fluctuational "buildup" of the soliton instead of its stabilization during nonlinear propagation even in an ideal light guide is of important significance.

It is not hard to understand that an increase in the phase indeterminacy is in no way the only effect of the growth of quantum noise. Indeed, owing to dispersive spreading, the phase instability is transformed into an amplitude instability, etc. But unfortunately, the above-implemented prescribed-channel approximation is incapable of describing this process, since it takes account of the variance of only the regular component of the soliton (recall that we have neglected the influence of the fluctuations on the variation of the trajectories in the calculation of the continuum integral[7–9]). Therefore neither the instantaneous number of photons nor their stochastic characteristics change in such an approximation during the propagation of the soliton. We note in passing that results analogous to relations (23) and (24) were obtained in Ref. 10 on the basis of a treatment in the Schrödinger representation and the Hartree approximation which also does not take dispersion effects into account.

4. QUASILINEARIZATION WITH RESPECT TO FLUCTUATIONS

Linearization with respect to the fluctuational components ξ and ξ^+ was used also in the prescribed-channel approximation. However, in this section we will obtain more general relations by dispensing with the assumption of invariance of the trajectories. This allows us to take dispersion phenomena into account.

Thus, linearization of Eq. (18) with respect to fluctuations gives

$$\partial\xi/\partial\zeta-(i/2)\partial^2\xi/\partial\tau^2-i[2|\bar{\Psi}|^2\xi+(\bar{\Psi})^2\xi^+]=C(F_\varkappa+F_\gamma). \tag{25}$$

Recall that the regular part is given by Eq. (19), but the criterion of adequacy of the quasilinear equation (25) is fulfillment of condition (20).

The system of equations (19), (25) in fact describes a four-photon parametric process in a prescribed classical pump field, which, however, acquires during the propagation a nonlinear phase advance, i.e., the interaction is nonsynchronous. In this regard it should be noted that an equation analogous to Eq. (25) was previously obtained in Ref. 5, whose right-hand (Langevin) side, also δ-correlated, is of a different nature: it takes neither absorption nor amplification into account, but does account for indeterminacies of the refractive index of the fiber. But the authors of Ref. 5 (and similarly those of Refs. 4 and 6) constructed the rest of their treatment using for the regular component $\bar{\Psi}=\operatorname{sech}\tau$ an incorrect description which did not include the nonlinear phase advance (cf. Eq. (19)). As a result, for small detunings Ω, instead of self-action they obtained synchronous parametric amplification of the fluctuations of exponential character.

An analytic solution even of the quasilinearized equation (25) taking Eq. (29) into account is impossible. Therefore, to calculate the dynamics of the fluctuations during self-action we will consider the simpler case of nonlinear conversion of the spectrum of the initially vacuum modes in the field of an intense monochromatic wave, i.e., a regular soliton, infinite in extent. Here it is of interest to consider not only a single vacuum mode at the frequency of the regular signal (the single-mode case), but also vacuum modes detuned from the carrier frequency (the two-mode case). Each of these variants possesses a specific character. Therefore we will discuss them separately.

5. SINGLE-MODE SELF-ACTION

It is more convenient to analyze the evolution of the fluctuations in the field of a monochromatic wave on the basis of the unnormalized equation (17). We represent the operator $E^{(+)}(t,z)$ in the form of a sum of the classical regular signal and the single-mode quantum fluctuation signal:

$$E^{(+)}(t,z)=A(z)+Ca(z), \tag{26}$$

where $A(z)$ is the complex amplitude of the harmonic wave and $a(z)$ is the annihilation operator of the initially vacuum mode.

We substitute expression (26) into Eq. (17) and the linearized the latter with respect to a and a^+. For the regular part and the fluctuations we then obtain

$$[\partial/\partial z-iq-(\gamma-\varkappa)/2]A=0. \tag{27}$$

$$[\partial/\partial z-(\gamma-\varkappa)/2-i2q]a-iqe^{-i2\theta}a^+=\Gamma_{\Omega\varkappa}+\Gamma_{\Omega\gamma}. \tag{28}$$

Here $q=k\varepsilon_{n1}|A|^2/2\varepsilon_0$, and θ is the phase of the regular component ($A=|A|e^{-i\theta}$).

In what follows we will limit our discussion to the stationary case of propagation of radiation in a waveguide when the amplification completely compensates for the loses: $\gamma=\varkappa$. Here

$$A(z)=A_0e^{i\Psi},\quad A_0\equiv A(z=0)=|A_0|e^{-i\theta_0},\quad \theta=\theta_0-\Psi, \tag{29}$$

where the nonlinear phase is $\Psi=qz$.

We represent the solution of Eq. (28), taking Eqs. (29) into account, in the form

$$a(z)=e^{i\Psi}\left[(1+i\Psi)S+i\Psi e^{-2i\theta_0}S^+-i\Psi\frac{\tilde{S}+\tilde{S}^+e^{-2i\theta_0}}{z}\right]. \tag{30}$$

Here

$$S(z)=a_0+\int_0^z\Gamma(z')e^{-iqz'}dz',$$

$$\tilde{S}(z)=\int_0^z\Gamma(z')e^{-iqz'}dz',$$

$$a_0=a(z=0),\quad \Gamma(z)=\Gamma_{\Omega\varkappa}(z)+\Gamma_{\Omega\gamma}(z).$$

We introduce the quadrature component $\Delta X(z)=a(z)+a^+(z)$ and determine the variance of its fluctuations. As a result we obtain

$$\langle\Delta X^2\rangle=1+2\Psi\gamma/q+2\Psi(1+\Psi\gamma/q)\sin 2\theta+4\Psi^2(1+2\Psi\gamma/3q)\sin^2\theta. \tag{31}$$

Here and below we take the mean phonon numbers in the mode $\langle N_\varkappa\rangle$ and $\langle N_\gamma\rangle$ to be significantly less than unity.

In the absence of amplification and losses ($\gamma=\varkappa=0$) this relation reduces to a well-known result.[9] Thus, self-action leads to the appearance of a squeezed quantum state with suppressed fluctuations of one of the quadratures, however, the amplification and losses lower the efficiency of the squeezing. This matter will be treated in more detail below, in Section 7. What then becomes of the phase and amplitude quantum noise?

Arguments analogous to those already advanced in the derivation of formula (22) allow us to write for the fluctuational component of the phase

$$\Phi(z)=[(1+2i\Psi)e^{i\theta_0}S-(1-2i\Psi)e^{-i\theta_0}S^+-2i\Psi(\tilde{S}e^{i\theta_0}+\tilde{S}^+e^{-i\theta_0})/z]/2i\langle N\rangle^{1/2}, \tag{32}$$

where $\langle N\rangle\approx(|A|/C)^2$ is the mean photon number in the considered mode.

In turn, the variance of the phase fluctuations

$$\langle\Phi^2\rangle=[1+2\Psi\gamma/q+4\Psi^2(1+2\Psi\gamma/3q)]/4\langle N\rangle \tag{33}$$

grows without letup during the propagation of the radiation. Graphs of the dependence of the normalized variance on the magnitude of the nonlinear phase incursion for various values of the relative loss parameter γ/q are shown in Fig. 1.

An analogous picture is observed for the intensity fluctuations. The noise amplitude component of the signal is oriented orthogonal to the phase component (in the phase plane) and is equal to

$$\Delta a(z)=(Se^{i\theta_0}+S^+e^{-i\theta_0})/2. \tag{34}$$

Correspondingly, the variance of the fluctuations of the photon number is

$$\langle\Delta N^2\rangle\approx 4\langle N\rangle\langle\Delta a^2\rangle=(1+2\Psi\gamma/q)\langle N\rangle. \tag{35}$$

However, in contrast to the phase fluctuations the growth of the amplitude fluctuations is due only to the presence of losses and the amplification compensating them. This is perfectly natural, for as a matter of fact the photons nowhere disappear and reappear in a transparent light guide; consequently, their statistics (in the single-mode case) remains unchanged. Note that the growth of the amplitude noise has a purely quantum nature and is absent in the classical description.

6. MULTIMODE SELF-ACTION. FLUCTUATION SPECTRA

Despite the fact that in the initial single-mode signal there are no spectral components with frequencies differing from the carrier frequency ω, vacuum modes with nonzero detunings Ω, which also evolve in the field of an intense wave, are also drawn into the process of nonlinear interaction. As a result, in a dispersive medium an inhomogeneous

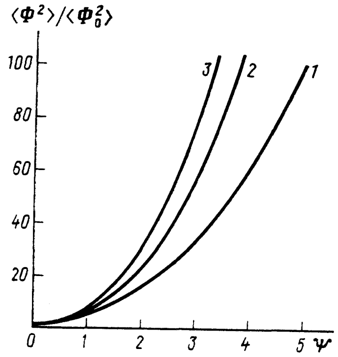

FIG. 1. Plots illustrating the growth of the variance of the phase fluctuations relative to the initial state during propagation, i.e., with growth of the nonlinear phase advance Ψ, constructed for various values of the relative loss parameter γ/q: 0 (*1*), 1/4 (*2*), 1/2 (*3*).

fluctuation spectrum is formed, to the elucidation of which the present section is also devoted.

Let us consider the interaction of the following three modes: an intense harmonic wave with angular carrier frequency ω which we will describe classically, and two initially vacuum modes with frequencies $\omega+\Omega$ and $\omega-\Omega$. Then

$$E^{(+)}(t,z)=A(z)+C(a_\Omega e^{-i\Omega t}+a_{-\Omega}e^{i\Omega t}). \tag{36}$$

We substitute expression (36) into Eq. (17) and linearize with respect to the fluctuational components $a_{\pm\Omega}$ and $a^+_{\pm\Omega}$. As a result, for the regular part we have relation (27), and the fluctuations are described by the equations

$$[\partial/\partial z-i\Omega/u+ig\Omega^2/2-(\gamma-\varkappa)/2-2iq]\,a_\Omega-iqe^{-i2\theta}a_{-\Omega}{}^+=\Gamma_+, \tag{37}$$

$$[\partial/\partial z+i\Omega/u+ig\Omega^2/2-(\gamma-\varkappa)/2-2iq\}a_{-\Omega}-iqe^{-i2\theta}a_\Omega{}^+=\Gamma_-. \tag{38}$$

Here

$$\Gamma_+=\Gamma_{\Omega\varkappa}+\Gamma_{\Omega\gamma},\qquad \Gamma_-=\Gamma_{-\Omega\varkappa}+\Gamma_{-\Omega\gamma}.$$

The employed quasilinear approximation makes it possible to restrict the discussion to only three modes, since the interaction of the initially vacuum modes with detuning eigenfrequencies Ω of different modulus is absent here, and each of the corresponding pairs ($\pm\Omega$) is described independently.

For the case of stationary propagation ($\gamma=\varkappa$) we represent the solution of system (29), (37), (38) in the form

$$a_\Omega(z)=\exp\left[iz\left(\frac{\Omega}{u}+q\right)\right]\left\{\left[\left(\frac{\Pi}{\Psi}+i\Lambda\right)S_{1+}\right.\right.$$
$$\left.+\,i\exp(-2i\theta_0)S_{1-}{}^+\right]\exp(\Pi)$$
$$\left.+\left[\left(\frac{\Pi}{\Psi}-i\Lambda\right)S_{2+}-i\exp(-2i\theta_0)S_{2-}{}^+\right]\exp(-\Pi)\right\}\frac{\Psi}{2\Pi}. \tag{39}$$

Here

$$\Pi=\Psi(1-\Lambda^2)^{1/2},\qquad \Lambda=1-\frac{g\Omega^2}{2q},$$

$$S_{1\pm}=a_{\pm\Omega 0}+\int_0^z\Gamma_\pm(z')\exp\left\{-z'\left[i\left(\frac{\Omega}{u}+q\right)+q(1-\Lambda^2)^{1/2}\right]\right\}dz',$$

$$S_{2\pm}=a_{\pm\Omega 0}+\int_0^z\Gamma_\pm(z')\exp\left\{-z'\left[i\left(\frac{\Omega}{u}+q\right)-q(1-\Lambda^2)^{1/2}\right]\right\}dz',$$

$$a_{\pm\Omega 0}=a_{\pm\Omega}(0).$$

The expression for $a_\Omega(z)$ is obtained by replacing Ω in Eq. (39) by $-\Omega$.

Since the considered fluctuational components possess zero mean amplitudes, to speak of their amplitude or phase noise is without meaning: they are continuous noise. We can characterize it by calculating the variance of the arbitrarily oriented (in the phase plane) quadrature

and also the mean number of noise photons $N_\Omega=a^+_\Omega a_\Omega$. As a result we obtain

$$\langle\Delta X_\Omega{}^2\rangle=\left[\operatorname{ch}2\Pi+\frac{\Psi\gamma}{q\Pi}\operatorname{sh}2\Pi-\Lambda^2\left(1+\frac{2\Psi\gamma}{q}\right)\right]\Psi^2\Pi^{-2}, \tag{40}$$

$$\langle N_\Omega\rangle={}^1/_2\left[\operatorname{ch}2\Pi-1+\frac{\Psi\gamma}{q}\left(\frac{\operatorname{sh}2\Pi}{\Pi}-2\Lambda^2\right)\right]\Psi^2\Pi^{-2}. \tag{41}$$

The first thing that strikes the eyes is the absence of a dependence of the variance of the quadrature in Eq. (40) on φ. This means that the region of indeterminacy in the phase plane is a circle, i.e., no squeezed quantum states appear in the mode with nonzero detuning frequency Ω.

Further, the presence of amplification and losses can only increase the fluctuations since $(\sinh 2\Pi)/\Pi\geqslant 2$ and $|\Lambda|\leqslant 1$. It is particularly easy to see that this fact follows from the corresponding expressions in the limit of small detunings $\Omega\to 0$:

$$\langle\Delta X^2_{\Omega\to 0}\rangle=1+2\Psi^2+2\Psi(\gamma/q)(1+2\Psi^2/3), \tag{42}$$

$$\langle N_{\Omega\to 0}\rangle=\Psi[\Psi+(\gamma/q)(1+2\Psi^2/3)]. \tag{43}$$

However, even in the absence of amplification and losses, quantum noise grows continuously during nonlinear propagation of radiation in a light guide. This fact is illustrated by the graphs of the spectra, shown in Fig. 2. The

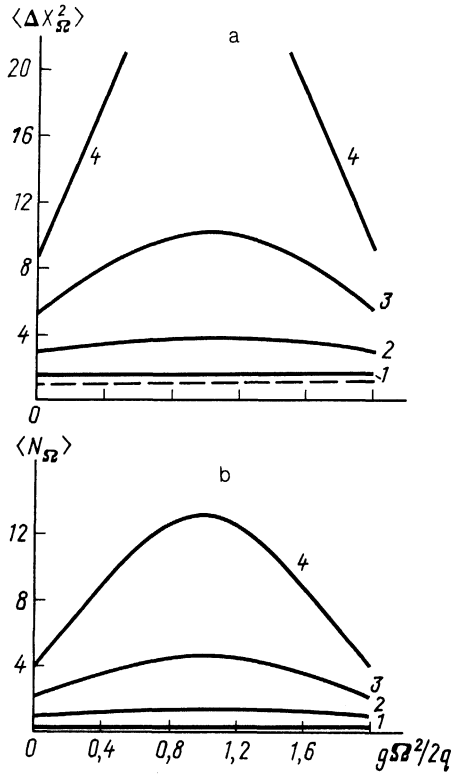

FIG. 2. Evolution of the fluctuation spectra of the quadrature component (a) and of the mean number of noise photons (b) with growth of the nonlinear phase advance $\Psi=1/2$ (*1*), 1 (*2*), 1.5 (*3*), 2 (*4*) in the absence of losses ($\gamma=0$). The dashed straight line corresponds to the vacuum fluctuations ($\Psi=0$).

reason for the increase of the fluctuations is the transfer by pumping of photons from the regular component to the fluctuational component, which follows from an increase of the mean number of noise photons with the growth of Ψ in accordance with Eqs. (41) and (43).

Thus we have shown that quantum fluctuations of all of the spectral components, i.e., of the modes, without exception, grow. Therefore, in a soliton which is a multimode formation it is difficult to expect anything other than growth.

7. SQUEEZED QUANTUM STATES AND MULTIMODE SELF-ACTION. SQUEEZING SPECTRUM

Our discussion would be incomplete without a clarification of the question of the formation of squeezed quantum states during multimode interaction, all the more so since we have already obtained all the relations necessary for this. If this were not the case, on the basis of the results of the two foregoing sections the reader might come to the hasty conclusion that squeezing occurs exclusively in that one mode of the signal that is at the carrier frequency ω. Strictly speaking, this in fact is the case, but in detection at the beat frequency of the regular component of a signal with fluctuational modes there takes place a mixing of the latter, of the form

$$a(\Omega)=(a_\Omega+a_{-\Omega})/2^{1/2}. \tag{44}$$

The annihilation operator $a(\Omega)$ describes the field spectral component which appears upon detection at the frequency Ω in which squeezing is also manifested. This fact was previously noted in Ref. 22, however, the results there pertain to the conversion of classical fluctuations and furthermore in the absence of amplification, losses, and dispersion of the nonlinear medium. The latter circumstance in fact equates the description undertaken by the authors of Ref. 22 to the single-mode case, since the fluctuational components of all the interacting modes, including the intense mode, exist under different conditions, and the efficiency of squeezing for them turns out to be identical, i.e., the squeezing spectrum is homogeneous.

The relations which we obtained in the previous sections make it possible to eliminate these shortcomings and to construct a more general theory of the formation of squeezed states during multimode self-action.

Thus, for a quadrature component of the form $\Delta X(\Omega) = a(\Omega) + a^+(\Omega)$, according to Eqs. (39) and (43), we have

$$\begin{aligned}\langle\Delta X^2(\Omega)\rangle=&(\Psi^2/2\Pi^2)\{[1+(\Pi/\Psi)\sin 2\theta-\Lambda\cos 2\theta]\\ &\times[e^{2\Pi}+(\gamma\Psi/q\Pi)(e^{2\Pi}-1)]+[1-(\Pi/\Psi)\sin 2\theta-\Lambda\cos 2\theta]\\ &\times[e^{-2\Pi}+(\gamma\Psi/q\Pi)(1-e^{-2\Pi})]+2\Lambda(\cos 2\theta-\Lambda)(1+2\Psi\gamma/q)\}.\end{aligned} \tag{45}$$

Here the averaging is carried out over the initial vacuum states of the independent modes with frequencies $\pm\Omega$. In the limit $\Omega\to 0$ we obtain the single-mode approximation (31).

In the absence of nonlinearity, according to Eqs. (45), the variance of the fluctuations of the quadrature component is the same as for the vacuum: $\langle\Delta X^2(\Omega)\rangle = 1$. Self-action leads to the appearance of squeezed states characterized by suppressed quadrature fluctuations: $\langle\Delta X^2(\Omega)\rangle < 1$. However, the evolution of the quantum state during propagation in a dispersive medium causes spectrally inhomogeneous squeezing. With the help of Eqs. (45) the squeezing spectra were calculated for various magnitudes of the phase incursion and the losses. The results are presented in Fig. 3. The initial phase was chosen to be optimal for $\Omega = 0$. In this case it satisfies the relation

$$\theta_0=-\Psi-{}^1/_2\,\mathrm{arctg}\,[(1+\Psi\gamma/q)/\Psi(1+2\Psi\gamma/3q)]. \tag{46}$$

It can be seen from Fig. 3 that an increase of the detuning Ω and/or the dispersion of the group velocity g corresponds to a degradation of the squeezing. The degree of this degradation increases with the growth of the nonlinearity Ψ. Losses, understandably, play an adverse role. This is well taken into account even in the general expression (45): the terms proportional to $\gamma = \varkappa$ are always positive, i.e., as could be expected, the losses and the amplification compensating them can only increase the variance of the quadrature fluctuations, thereby lowering the efficiency of squeezing. This fact follows in an even more apparent way from Eq. (31). From these results it is also clear that for a parameter γ/q (which determines the relative losses) greater than 1/2 one can hardly count on a deep suppression of quantum quadrature noise. This result, which is of practical importance, is illustrated by the graphs in Fig. 4, which present the dependences of the minimum possible variance of the quadrature, i.e., maximal squeezing, on the nonlinear phase incursion Ψ (acquired during propagation) at $\Omega = 0$. Thus, in real media with losses there always exists some optimal interaction length, the exceeding of which leads not to an enhancement of the efficiency of squeezing, but, on the contrary, to its degradation.

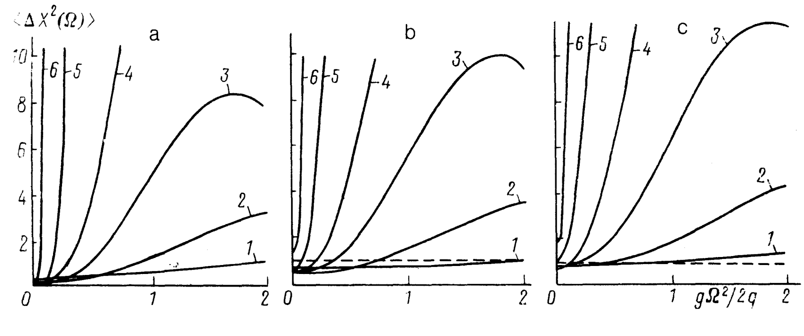

FIG. 3. Evolution of the fluctuation spectra of the quadrature component during nonlinear propagation, i.e., with growth of the nonlinear phase advance, in media with various losses: a) $\gamma/q = 0$, b) 1/4, c) 1/2. The initial phase is optimized for $\Omega = 0$. The dashed straight line corresponds to unitary level of quantum vacuum fluctuations $\Psi = 0.5$ (*1*), 1 (*2*), 1.5 (*3*), 2 (*4*), 3 (*5*), 5 (*6*).

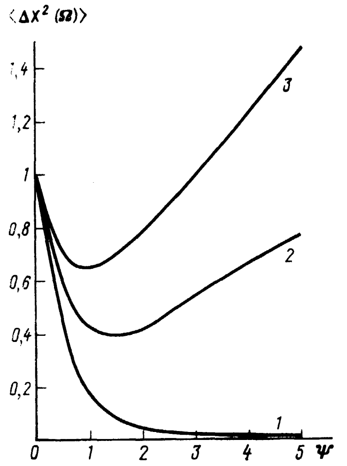

FIG. 4. Plots of the dependences of the limiting values of the variance of the fluctuations of the quadrature component on the magnitude of the nonlinear phase incursion for $\Omega = 0$. The parameter γ/q, which determines the relative losses, was chosen to be equal to 0 (*1*), 1/4 (*2*), 1/2 (*3*). The initial phase θ_0 is optimized as prescribed by Eq. (46).

An explanation of the "unfavorable" role of the variance of the group velocity in the formation of the squeezing spectrum can be most simply offered on the basis of the four-photon treatment of self-action mentioned in Sec. 4. Indeed, an increase in the detuning Ω is accompanied by a corresponding phase detuning, which also leads to an inhomogeneous squeezing spectrum. And the criticality of phase detuning grows with increase of the efficiency of the interaction, i.e., with increase of Ψ. The situation thus has an analogous destructive effect on the squeezing of the diffraction of the modes amplified in the process of parametric amplification.[9,23-25] However, in contrast with parametric amplification, in the case under consideration the condition of phase synchronism is not fulfilled even in the single-mode regime ($\Omega = 0$), a fact which has to do with the nonlinear phase incursion acquired by the regular component. It is precisely for this reason in particular that during self-action exponential amplification of the fluctuational components, which is present in the case of synchronous parametric interaction, is absent.

8. CONCLUSION

Thus, with the help of various approximations and models we have shown that quantum fluctuations of an initially coherent fundamental Schrödinger soliton grow during its nonlinear propagation even in the absence of losses and amplification. The latter only accelerate this growth. And this applies to the phase noise as well as to the amplitude noise. Generally speaking, the tendency towards accumulation is characteristic not only of quantum fluctuations. The same thing, by virtue of the correspondence principle,[26] also takes place in the case of additive stationary δ-correlated classical noise. The reason for such behavior obviously lies concealed within the following. If the soliton "ejects" its initial noise modulation, present only within the limits of the soliton's duration, to the wings and gradually "frees itself," then it is not in a position to free itself from the stationary noise since the "ejection" process is accompanied by a "drifting in" of noise initially present outside the soliton.

It should be borne in mind however that, strictly speaking, the obtained results are valid only in the initial stage of the nonlinear propagation of the initially coherent soliton. What sort of changes take place in what follows? In an ideal transparent light guide obviously the processes of "ejection" and "drifting in" of noise gradually come into balance and the level of quantum fluctuations, unconditionally enhanced relative to the coherent state, stabilizes. Under real conditions of an absorbing fiber with compensating amplification the noise will undoubtedly grow without letup and finally destabilize the soliton. But this belongs to the range of hypotheses, a strict substantiation of which is possible even if only by setting up a computer experiment, e.g., on the basis of Eq. (18) or, at least, the system (19), (21). The conclusion that both the amplitude and phase indeterminacy (the latter is manifested by a gradual spreading of the soliton) increase in the absence of amplification and losses was made in a recent work,[10] based on the Schrödinger picture of the evolution of the quantum state of the soliton, which gives analytic results likewise only in the initial stage of propagation. The assumptions employed by the authors of Ref. 10 are less restrictive than those used here. However, the greater degree of generality of the treatment in Ref. 10 is accompanied by an increase in cumbersomeness and a decrease in the ease of visualization.

We note also a certain distinctive feature of femtosecond solitons which has to do with both the peculiarities of their photon statistics during detection[21] and the necessity of taking account of dispersion terms of third and higher orders, and also with the finite time of the nonlinear response of the medium.[27]

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[2)] The "inversion" of a new Hermitian phase operator was reported recently in Ref. 28.

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