# Suppression of transition radiation by a magnetic field

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The role which the curvature of the path traced out by the relativistic particle plays in the generation of transition radiation in a randomly inhomogeneous medium is discussed. The nature and properties of the transition radiation from a particle moving in a magnetic field are shown to be quite different from those in the case of rectilinear motion. The emission by high-energy particles, with $\gamma > \omega_p/\omega_B$, is strongly suppressed. This suppression is so strong that it sharply reduces the total amount of energy radiated and weakens the dependence of the radiated power on the particle energy.

**1.** Several aspects of the emission of electromagnetic waves by ultrarelativistic particles do not depend on the properties of the particular external field or the particular medium in which these particles are moving.[1] The characteristic distance $l_c$ over which the emission occurs (the radiation formation zone) is considerably greater than $c/\omega$, where $c$ is the velocity of light, and $\omega$ the emission frequency. If the path traced out by the relativistic particle is in some sense only slightly different from rectilinear (more on this point below), the size of the formation zone can be determined by requiring that the wave and the particle "become separated spatially" by a distance $c/\omega$ over the length $l_c$. Noting that the phase velocity of the waves in a plasma is different from $c$, we find

$$l_c \approx \frac{2c}{\omega}\gamma^2\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right)^{-1}, \tag{1}$$

where $\gamma = \mathscr{E}/Mc^2$ is the Lorentz factor of the particle, $\mathscr{E}$ and $M$ are the energy and mass of the particle, and $\omega_p$ is the plasma frequency. Most of the energy emitted at the frequency $\omega$ goes into a narrow cone around the direction of the particle velocity, within an angle

$$\theta_c \approx \left(\gamma^{-2}+\frac{\omega_p^2}{\omega^2}\right)^{1/2}. \tag{2}$$

The deviation of the path traced out by the particle from rectilinear is important[1] if the angle $\alpha'$ by which the direction of the particle velocity changes over the distance $l_c$ is greater than $\theta_c$:

$$\alpha' > \theta_c.$$

In this case the size of the formation zone is smaller than (1) and is the same as the length over which the direction of the particle velocity changes by an angle $\theta_c$.

Transition radiation by a relativistic particle moving in a randomly inhomogeneous medium or crossing an interface between two media arises even when the charge is in uniform rectilinear motion, since transition radiation is the result of a coherent superposition of the emission by the individual particles of the medium, which are excited by the virtual field of the passing particle.[2] Under actual conditions, however, the path of the particle is often different from rectilinear. This difference may be caused by Coulomb collisions of the relativistic particle with the particles of the medium and also by external electromagnetic fields.

Let us take a qualitative look at how the curvature of the particle path should affect the formation of the transition radiation of this particle. Transition radiation is known to be a resonant process.[2] By this we mean that the emission at the frequency $\omega$ arises from the interaction of the field of the relativistic particle with harmonics of the spectrum of irregularities of the medium for which $\tilde{k}^{-1} \sim l_c(\omega)$. Let us assume that the curvature of the path is such that the direction of the particle velocity changes by an angle $\theta_c$ over a distance $l' \ll l_c(\omega)$. Since the emission is concentrated along the direction of the particle velocity, the emission is thus formed over a short distance $l'$, and the emission from subsequent parts of the path will be in a different direction. In this case, a charge having traversed a path $l' \ll \tilde{k}^{-1} \sim l_c(\omega)$, will clearly interact less effectively with the density inhomogeneities of the medium with length scales $\tilde{k}^{-1} \sim l_c(\omega)$, with which the transition radiation at the frequency $\omega$ is associated. This radiation should thus be strongly suppressed. A similar effect is known in bremsstrahlung, when the multiple scattering of a relativistic particle by Coulomb centers in a medium is taken into account (see, for example, Ref. 3). The effect is called the "Landau-Pomeranchuk effect" in that case.

We are interested here in the transition radiation by a particle which is moving through a plasma with random variations in the electron density. This plasma is in a uniform magnetic field **B**. A situation of this sort is typical of a variety of astrophysical entities and also under laboratory conditions. In a magnetic field, we have[1] $l' = l_s = Mc^2/qB = c/\omega_B$, where $\omega_B = qB/Mc$ is the gyrofrequency of the particle. Since $l_c(\omega)$ reaches its greatest value at $\omega = \omega_p\gamma$,

$$l_c(\omega_p\gamma) = c\gamma/\omega_p,$$

the condition for suppression of the transition radiation by the magnetic field reduces to

$$\gamma \gg \omega_p/\omega_B. \tag{3}$$

In this region of parameter values, the suppression of the transition radiation also changes the behavior of the total radiated energy as a function of the energy of the relativistic particle.

The effect of a strong magnetic field on the dielectric constant of a plasma was considered in Ref. 2, but the charge was assumed to be in rectilinear motion along the field. In the present paper, we ignore the effect of the magnetic field

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on the dispersion of the medium. In a magnetized plasma, we can use the formula[2,4]

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2} \quad (4)$$

if the electron gyrofrequency $\omega_{Be}=eB/mc$ is small in comparison with the frequency $\omega>\omega_p$:

$$\omega_{Be}/\omega\ll 1. \quad (5)$$

The role played by the magnetic field in the path of the particle is important (Sec. 2) if $\gamma\gtrsim\omega_p/\omega_{Be}$ holds for electrons or $\gamma\gtrsim\omega_p/\omega_{Bp}$ for protons. With $\gamma\gg 1$, this condition is compatible with condition (5).

**2.** Let us examine the transition radiation by one relativistic particle moving through a turbulent magnetized plasma which has small-scale variations of the electron density. These variations might be produced by magnetosonic waves or shock waves, as well as by plasma waves and ion acoustic waves. We assume that the turbulence is statistically isotropic and homogeneous in space and time. We assume that the spectrum of irregularities can be described by a power law, as in Ref. 4:

$$|\delta N|_{\mathbf{k}}^2=\frac{\nu-1}{4\pi}\frac{\langle\Delta N^2\rangle k_0^{\nu-1}}{k^{\nu+2}} \qquad k\geqslant k_0, \quad (6)$$

where $\langle\Delta N^2\rangle$ is the mean square value of the irregularities, and $k_0=2\pi/L_0$, where $L_0$ is their dominant length scale.

According to the theory derived in Ref. 2, to find the intensity of the transition radiation in a plasma with random variations we need to calculate the Fourier component of the plasma current, $\mathbf{j}_{\mathbf{k},\omega}^m$, which is bilinear in the electric field of the relativistic particle, $\mathbf{E}_{\mathbf{k},\omega}^{q,t}$ and in the amplitude of the plasma density fluctuations $\delta N_{\omega,\mathbf{k}}$:

$$\mathbf{j}_{\mathbf{k},\omega}^m=\frac{ie^2}{m\omega}\int\mathbf{E}_{\omega-\omega',\mathbf{k}-\mathbf{k}'}^{q,t}\,\delta N_{\omega',\mathbf{k}'}\,d\omega'\,d\mathbf{k}'. \quad (7)$$

Here $e$ and $m$ are the charge and mass of an electron (in contrast with the $q$ and $M$ of the particle). Only the virtual part of the electric field of the moving particle should be considered in this expression. The real part (i.e., the synchrotron radiation field) also interacts with the fluctuations, but this interaction results in a scattering of the radiation, not an emission of transition radiation. The energy emitted at a given frequency $\omega$ in a given direction $n$ is expressed in terms of $\mathbf{j}_{\mathbf{k},\omega}^m$ by

$$\mathscr{E}_{\mathbf{n},\omega}=(2\pi)^6\frac{\omega^2}{c^3}\langle|[\mathbf{n}\mathbf{j}_{\mathbf{k},\omega}^m]|^2\rangle, \quad (8)$$

where the angle brackets mean an average over the fluctuation spectrum. The field of the charge is related to its current by

$$E_{i,\mathbf{k},\omega}^{q,t}=G_{ij}^t(\omega,\mathbf{k})j_{j,\mathbf{k},\omega}^q, \quad (9)$$

where $G_{ij}^t(\omega,\mathbf{k})$ is the transverse Green's function, given by

$$G_{ij}^t(\omega,\mathbf{k})=\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\frac{4\pi i\omega}{c^2k^2-\omega^2\varepsilon(\omega)}, \quad (10)$$

and the particle current is expressed in terms of the particle path $\mathbf{r}(t)$ and the velocity $\mathbf{v}(t)$:

$$\mathbf{j}_{\mathbf{k},\omega}^q=q\int_{-\infty}^{\infty}\mathbf{v}(t)\exp[-i\mathbf{k}\mathbf{r}(t)+i\omega t]\frac{dt}{(2\pi)^4}. \quad (11)$$

Substituting (10) and (11) into (9), then substituting (9) into (7), then substituting (7) into (8), and then taking an average over the random phases by means of the relation

$$\langle\delta N_{\mathbf{k}',\omega'}\,\delta N_{\mathbf{k}'',\omega''}^*\rangle=|\delta N|_{\mathbf{k}'}^2\delta(\omega')\delta(\omega'')\delta(\mathbf{k}'-\mathbf{k}''), \quad (12)$$

we find

$$\mathscr{E}_{\mathbf{n},\omega}^m=\frac{4q^2e^4}{m^2c^3\omega^2\varepsilon^2(\omega)}\int d\mathbf{k}'\int_{-\infty}^{\infty}dt$$

$$\times\mathrm{Re}\int_{-\infty}^{\infty}d\tau\frac{|\delta N|_{\mathbf{k}'}^2\exp(i\omega\tau-i(\mathbf{k}-\mathbf{k}')[\mathbf{r}(t+\tau)-\mathbf{r}(t)])}{[1-(\mathbf{k}-\mathbf{k}')^2c^2/\omega^2\varepsilon(\omega)]^2}$$

$$\times\left\{[\mathbf{n}\mathbf{v}(t)]+\frac{[\mathbf{n}\mathbf{k}']((\mathbf{k}-\mathbf{k}')\mathbf{v}(t))}{(\mathbf{k}-\mathbf{k}')^2}\right\}$$

$$\times\left\{[\mathbf{n}\mathbf{v}(t+\tau)]+\frac{[\mathbf{n}\mathbf{k}']((\mathbf{k}-\mathbf{k}')\mathbf{v}(t+\tau))}{(\mathbf{k}-\mathbf{k}')^2}\right\}. \quad (13)$$

The second terms in the braces can be ignored here, since they are small quantities of higher order than $[\mathbf{n}\mathbf{v}]$.

The particle path in a magnetic field (a helix) and the velocity of the particle can be written

$$\mathbf{r}(\tau)=\mathbf{r}_0+\mathbf{b}v_\parallel\tau+\frac{\mathbf{v}_\perp}{\Omega}\sin\Omega\tau+\frac{[\mathbf{b}\mathbf{v}_\perp]}{\Omega}(1-\cos\Omega\tau),$$
$$\mathbf{v}(\tau)=\mathbf{v}_0+\mathbf{v}_\perp(1-\cos\Omega\tau)-[\mathbf{b}\mathbf{v}_\perp]\sin\Omega\tau, \quad (14)$$

where $\mathbf{b}=\mathbf{B}/B$, $\Omega=\omega_B/\gamma$, and the $\|$ and $\perp$ mean the longitudinal and transverse components, respectively, of the velocity vector with respect to the direction $\mathbf{b}$.

Since the emission by the ultrarelativistic particle occurs over a distance over which the direction of the velocity changes by the small angle $\theta_c$ in (2), and since this distance is short in comparison with the radius of curvature of the path (correspondingly, $\omega_{B\perp}\tau\lesssim 1$), the arguments of the sines and cosines in (14) are small: $\Omega\tau\ll 1$ for $\gamma\gg 1$. It is thus legitimate to expand the trigonometric functions in power series in the small quantity $(\Omega\tau)$. In order to correctly deal with the deviation of the particle path from rectilinearity, however, we should retain the terms in expansion (14) which follow the linear terms, as is usually done in a study of synchrotron radiation:[5]

$$\mathbf{v}(t)=\mathbf{n}v\left(1-\frac{\theta^2}{2}\right)+\boldsymbol{\theta}v,$$

$$\mathbf{v}(t+\tau)=\mathbf{n}v\left(1-\frac{\theta^2}{2}\right)+\boldsymbol{\theta}v+\Delta\mathbf{v}(\tau),$$

$$\Delta\mathbf{v}(\tau)=[\mathbf{b}[\mathbf{v}\mathbf{b}]](1-\cos\Omega\tau)-[\mathbf{b}\mathbf{v}]\sin\Omega\tau\approx v[\mathbf{n}\boldsymbol{\Omega}]\tau, \quad (15)$$

$$\mathbf{r}(t+\tau)-\mathbf{r}(t)\approx\mathbf{v}(t)\tau-v[\mathbf{n}\boldsymbol{\Omega}]\frac{\tau^2}{2}+v[\boldsymbol{\Omega}\boldsymbol{\theta}]\frac{\tau^2}{2}-v[\boldsymbol{\Omega}[\mathbf{n}\boldsymbol{\Omega}]]\frac{\tau^3}{6},$$

where $v=|\mathbf{v}|\approx c$.

Figure 1 clarifies the meaning of the vector $\boldsymbol{\theta}$, which is equal in magnitude to the angle between the emission direction $\mathbf{n}$ and the velocity vector $\mathbf{v}(t)$. When the emission is observed along a fixed direction, one detects brief distinct pulses, just as one would in the case of synchrotron radiation.[5] In other words, the corresponding emission intensity is a function of the time $t$. The intensity of the emission into the total solid angle (i.e., the energy loss of the particle on

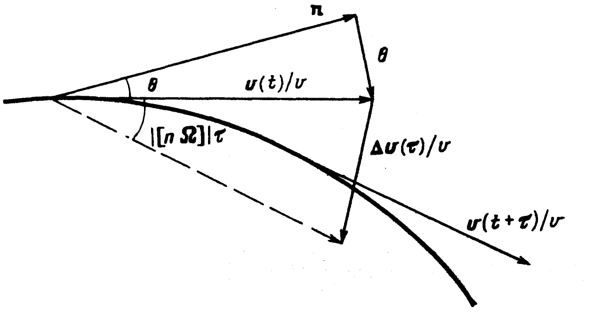


FIG. 1. Meaning of the vectors $\boldsymbol\theta$, $\mathbf{n}$, $\mathbf{v}/v$, and $\Delta\mathbf{v}/v$ and of the angles $\theta$ and $|[\mathbf{n}\boldsymbol\Omega]|\tau$.

transition radiation per unit time), however, is independent of $t$. It is thus possible to evaluate the outer integral over $dt$ in (13) and hence to go over to the intensity of the emission into the total solid angle (i.e., to the energy radiated per unit time, since the total emission energy over an infinite time would be infinite in this case):

$$I_\omega^m = \frac{2\pi q^2 e^4}{m^2 c^3}\int do \int \frac{d\mu\, dk' |\delta N|_{\mathbf{k}'}^2}{\mu^2}\,\mathrm{Re}\int_{-\infty}^{\infty} d\tau([\mathbf{n}\boldsymbol\theta]^2+\boldsymbol\theta[\mathbf{n}\boldsymbol\Omega]\tau)$$
$$\times\exp\left\{i[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]\tau - i\omega\boldsymbol\theta[\mathbf{n}\boldsymbol\Omega]\frac{\tau^2}{2}+i\omega\frac{[\mathbf{n}\boldsymbol\Omega]^2\tau^3}{6}\right\}, \quad (16)$$

where $\mu = \cos\vartheta = \mathbf{k}'\mathbf{k}/k'k$.

Because the emission by the relativistic particle is distinguished by its pronounced directionality, the integration over the solid angle $do = \sin\beta d\beta d\varphi$ can be replaced by an integration over the two-dimensional vector $d\boldsymbol\theta = d\theta_x d\theta_y$. Because the corresponding expressions converge rapidly, this integration can be carried out between infinite limits, from $-\infty$ to $+\infty$. We can thus put the origin for the scale of the angle $\theta$ in (15) at a certain instantaneous value of $\mathbf{v}(t)$, because a change in variables of the type $\boldsymbol\theta = \boldsymbol\theta' + \Delta\boldsymbol\theta(t)$ does not alter the range of integration over the angle. A replacement of this type, $\boldsymbol\theta \to \boldsymbol\theta' + [\mathbf{n}\boldsymbol\Omega]\tau$, is conveniently carried out in order to integrate (16) over the time $\tau$. Expression (16) then becomes (we are omitting the prime)

$$I_\omega^m = \frac{2\pi q^2 e^4}{m^2 c^3}\int d\boldsymbol\theta \int \frac{d\mu\, dk' |\delta N|_{\mathbf{k}'}^2}{\mu^2}$$
$$\times\,\mathrm{Re}\int_{-\infty}^{\infty} d\tau\left(\theta^2+2\boldsymbol\theta[\mathbf{n}\boldsymbol\Omega]\tau+\frac{3}{4}[\mathbf{n}\boldsymbol\Omega]^2\tau^2\right)$$
$$\times\exp\left\{\frac{i\omega\tau}{2}\left(\gamma^{-2}+\frac{\omega_p^2}{\omega^2}+\theta^2+\frac{2\mathbf{k}'\mathbf{v}}{\omega}\right)+\frac{i\omega\omega_{B\perp}^2\tau^3}{24\gamma^2}\right\}. \quad (17)$$

The inner integral can be expressed in terms of an Airy function. Switching to the dimensionless variable $x = (\omega\omega_{B\perp}^2/\gamma^2)^{1/3}(\tau/2)$ and using $\mathrm{Ai}''(\xi) = \xi\,\mathrm{Ai}(\xi)$ [the function $\mathrm{Ai}(\xi)$ is normalized], we find

$$I_\omega^m = \frac{8\pi^2 q^2 e^4}{\omega m^2 c^3}\left(\frac{\omega\gamma}{\omega_{B\perp}}\right)^{2/3}\int d\boldsymbol\theta \int \frac{d\mu\, dk' |\delta N|_{\mathbf{k}'}^2}{\mu^2}$$
$$\times\left[\theta^2-3\left(\frac{\omega_{B\perp}}{\omega\gamma}\right)^{2/3}\xi\right]\mathrm{Ai}(\xi), \quad (18)$$

where

$$\xi = \frac{\gamma^{-2}+\omega_p^2/\omega^2+\theta^2+2\mathbf{k}'\mathbf{n}c/\omega}{(\omega_{B\perp}/\omega\gamma)^{2/3}}. \quad (19)$$

The integration in (18) should be carried out over a region of parameter values in which the relation $\xi \leqslant 0$ holds. This relation corresponds to the virtual part of the electric field of the particle. Expression (18) allows us to take the limit of a zero magnetic field, $\omega_{B\perp}\to 0$. Since

$$\lim_{b\to 0}\frac{\mathrm{Ai}(a/b)}{b} = \delta(a), \quad (20)$$

the integrals in (18) can be evaluated easily. The second term in square brackets contributes nothing. Evaluating the outer integral over $d\theta$, and using (6), we find (in agreement with Ref. 2) the following expression for the spectrum and angular distribution of the transition radiation emitted by a particle in rectilinear motion:

$$I_{\mathbf{n},\omega}^{t} = \frac{8\pi(\nu-1)q^2e^4\langle\Delta N^2\rangle}{\nu c m^2\omega^3}\left(\frac{2k_0 c}{\omega}\right)^{\nu-1}\frac{\theta^2}{(\gamma^{-2}+\theta^2+\omega_p^2/\omega^2)^{\nu+2}}. \quad (21)$$

In the general case $\mathbf{B}\neq 0$, an evaluation of the outer integral over $d\theta$ in (18) does not give us the intensity of the emission in a given direction. The formal reason is that the angle $\theta$ depends on the times $t$ and $\tau$ (there is no such dependence in the case $\mathbf{B} = 0$). The physical reason is the continuous variation in the direction of the particle velocity. One consequence of this change in the nature of the particle motion is that it is not legitimate to introduce the concept of "exactly forward" emission ($\theta = 0$) in this case. Correspondingly, the intensity of the emission does not vanish in any direction, in contrast with the results in (21).

It is thus convenient to evaluate the integral over $d\theta$ in (18) for a certain time and to call the corresponding quantity the spectrum and angular distribution of the intensity, so that we can compare the resulting expressions with (21). In view of the comments above, no confusion should result.

Substituting (6) into (18), and using the condition $\xi\leqslant 0$, which fixes the limits on the integrals, we can write (18) in the form

$$I_{\mathbf{n},\omega}^{m} = \frac{2\pi(\nu-1)q^2e^4k_0^{\nu-1}\langle\Delta N^2\rangle}{\omega m^2c^3}\left(\frac{\omega\gamma}{\omega_{B\perp}}\right)^{2/3}\int_{k_{min}}^{\infty}\frac{dk'}{(k')^{\nu+2}}\int_{-1}^{-k_{min}/k'}\frac{d\mu}{\mu^2}$$
$$\times\left[\theta^2-3\left(\frac{\omega_{B\perp}}{\omega\gamma}\right)^{2/3}\xi\right]\mathrm{Ai}(\xi), \quad (22)$$

where

$$k_{min}(\theta) = \frac{\omega}{2c}\left(\gamma^{-2}+\theta^2+\frac{\omega_p^2}{\omega^2}\right). \quad (23)$$

It is difficult to analyze expression (22) in the general case, since it is necessary to evaluate integrals of combinations of an Airy function with power functions between finite limits. Nevertheless, it is possible to find asymptotic expressions which give a good description of the emission spectrum in certain frequency intervals.

We first consider the frequency region in which the condition $\gamma^{-2}+\omega_p^2/\omega^2 \gg (\omega_{B\perp}/\omega\gamma)^{2/3}$ holds i.e., the intervals

$$\omega \ll \omega_* = \omega_p\left(\frac{\omega_p\gamma}{\omega_{B\perp}}\right)^{1/2} \quad (24)$$

and

$$\omega \gg \omega_{B\perp}\gamma^2. \tag{25}$$

In this case the curvature of the particle path plays a minor role, and the Airy function can be replaced by a $\delta$-function. However, the finite width of this function must be taken into consideration in the second term in square brackets in (22). The reason this is necessary is that in the limit $\theta \to 0$ the second term in square brackets in (22) is the leading term. Because the function $(\omega\gamma/\omega_{B\perp})^{2/3}\,\mathrm{Ai}(\xi)$ has finite width, $\xi$ is not equal to zero and instead varies over an interval $-1 \lesssim \xi \leqslant 0$. For a rough estimate we can set (for example) $\xi = -1/3$; doing so, we find

$$I_{\mathbf{n},\omega}^{2} = \frac{8\pi(\nu-1)q^2e^4\langle\Delta N^2\rangle}{\nu c m^2\omega^3}\left(\frac{2k_0c}{\omega}\right)^{\nu-1}\frac{\theta^2+(\omega_{B\perp}/\omega\gamma)^{2/3}}{(\gamma^{-2}+\theta^2+\omega_p^2/\omega^2)^{\nu+2}}. \tag{26}$$

This expression differs from (21) in having an additional term $(\omega_{B\perp}/\omega\gamma)^{2/3}$ in the numerator. According to the discussion above, this term stems from the nonlocal nature of the emission process, i.e., from the circumstance that the emission comes from a finite arc of the particle path, over which the direction of the particle velocity changes. As will be seen below, however, this term contributes little to the total emission intensity, over all angles, in the frequency intervals defined by condition (24) or (25).

In the other case $\gamma^{-2} + \omega_p^2/\omega^2 \ll (\omega_{B\perp}/\omega\gamma)^{2/3}$ or

$$\omega_* \ll \omega \ll \omega_{B\perp}\gamma^2, \tag{27}$$

the argument of the Airy function is effectively small, $|\xi| \ll 1$, because of the rapid convergence of the interval over $dk'$. It thus becomes possible to expand the function $\mathrm{Ai}(\xi)$ in a series, in which only the first term need be retained:

$$\mathrm{Ai}(\xi) \approx \mathrm{Ai}(0) = \frac{1}{3^{2/3}\Gamma(2/3)}.$$

The integration in (22) over $d\mu$ and $dk'$ then presents no difficulties; it yields

$$I_{\mathbf{n},\omega}^{3} = \frac{24\pi(\nu-1)q^2e^4\langle\Delta N^2\rangle \mathrm{Ai}(0)}{\nu^2(\nu+1)cm^2\omega^3}\frac{(2k_0c/\omega)^{\nu-1}(\omega\gamma/\omega_{B\perp})^{2/3}}{(\gamma^{-2}+\theta^2+\omega_p^2/\omega^2)^{\nu}} \times\left(1+\frac{\nu\theta^2}{3(\gamma^{-2}+\theta^2+\omega_p^2/\omega^2)}\right). \tag{28}$$

Our ability to replace the Airy function $\mathrm{Ai}(\xi)$ by $\mathrm{Ai}(0)$ signifies an important change in the nature of the transition radiation. In particular, there is no resonant interaction with fluctuations (such an interaction resulted from the $\delta$-function in the case of a particle in rectilinear motion).

It can be seen from (28) that the emission intensity falls off with increasing magnetic field, i.e., $I_{\mathbf{n},\omega}^{3} \propto \omega_{B\perp}^{-2/3}$; i.e., the transition radiation is suppressed by a magnetic field.

Shown for comparison in Fig. 2 are the asymptotic spectra in (26) and (28) and the corresponding curve found through numerical integration of exact expression (22). The difference between these curves is seen to be slight, and in region (27) expressions (22) and (28) agree within 10%. Also shown in this figure is the spectrum (21) for the case of zero magnetic field, $\omega_{B\perp} = 0$.

For real experiments with beams of relativistic particles,[6] and also in the interpretation of the radio emission from astrophysical sources,[7] the topic of greatest interest is the intensity of the transition radiation into the total solid angle. It is convenient to carry out the integration over the vector $\boldsymbol{\theta}$ in polar coordinates. The magnitude of the angle, $\theta = |\boldsymbol{\theta}|$, then runs over the semi-infinite interval from 0 to $+\infty$. As a result we find

$$I_{\omega}^{2} = \frac{8\pi^2(\nu-1)}{\nu^2(\nu+1)}\frac{q^2e^4\langle\Delta N^2\rangle}{cm^2\omega^3}\left(\frac{2k_0c}{\omega}\right)^{\nu-1} \times\left(\gamma^{-2}+\frac{\omega_p^2}{\omega^2}\right)^{-\nu}\left[1+\frac{\nu(\omega_{B\perp}/\omega\gamma)^{2/3}}{\gamma^{-2}+\omega_p^2/\omega^2}\right]. \tag{29}$$

In regions (24) and (25), where (29) is valid, the second term in square brackets is small in comparison with unity and can be ignored. The magnetic field is thus unimportant in these frequency intervals, and the emission spectrum is essentially the same as that in the case of a rectilinear motion of the particle. At the frequencies in (24) the spectrum of the transition radiation is $I_\omega^2 \propto \omega^{\nu-2}$, while at frequencies (25) it is $I_\omega^2 \propto \omega^{-\nu-2}$ and weak in comparison with synchrotron radiation.

In the intermediate frequency region in (27) the magnetic field is important. After integrating (28) over angles, we find

$$I_{\omega}^{3} = \frac{16\pi^2(2\nu+1)\mathrm{Ai}(0)}{\nu^2(\nu+1)^2}\frac{q^2e^4\langle\Delta N^2\rangle}{cm^2\omega^3} \times\left(\frac{\omega\gamma}{\omega_{B\perp}}\right)^{2/3}\left[\frac{\omega}{2k_0c}\left(\gamma^{-2}+\frac{\omega_p^2}{\omega^2}\right)\right]^{1-\nu}. \tag{30}$$

In this region the intensity of the transition radiation falls off rapidly with increasing frequency: $I_\omega^3 \propto \omega^{\nu-10/3}$ and $I_\omega^3 \propto \omega^{-\nu-4/3}$. When there is a magnetic field, the transition radiation is thus strongly suppressed at frequencies $\omega \gtrsim \omega_*$. This effect is observed if $\omega_* < \omega_p\gamma$; this condition is the same

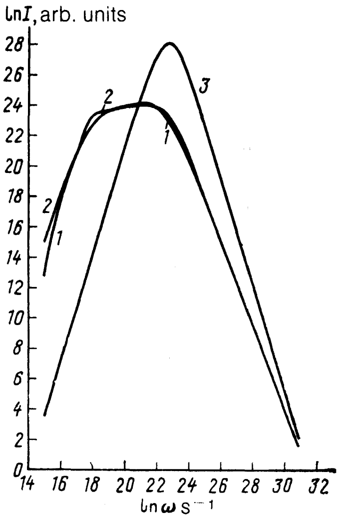


FIG. 2. Spectrum and angular distribution of the transition radiation for the parameter values $\gamma = 10^6$, $\theta = 10^{-6}$, $\omega_p = 10^4$ s$^{-1}$, $\omega_p/\omega_{B\perp} = 10^2$, and $\nu = 1.5$. 1—Results of a numerical calculation based on (22); 2—approximation of expression (22) by (26) and (28); 3—results in the absence of a magnetic field ($\omega_{B\perp} = 0$).

as (3). The meaning here is that the curvature of the paths of high-energy particles in a magnetic field completely changes the nature of the transition radiation by these particles, suppressing this radiation at those frequencies at which most of the energy would be radiated in the absence of a field. In addition, there is an increase in the characteristic angle at which most of the radiation is emitted (although this angle remains small in comparison with unity). Substituting $\omega_*$ into (2), we find

$$\theta_{cB} \approx \left(\frac{\omega_{B\perp}}{\omega_p \gamma}\right)^{1/2}. \tag{31}$$

Figure 3 shows the spectra of the transition radiation for two relativistic particles, with $\gamma_1 \ll \omega_p/\omega_{B\perp}$ and $\gamma_2 \gg \omega_p/\omega_{B\perp}$.

A particle moving in a magnetic field generates synchrotron radiation.[1] The generation of this radiation in a turbulent plasma was studied in detail in Ref. 4. This type of radiation is exponentially weak in a plasma because of a density effect at frequencies $\omega < \omega_p\gamma$, while the transition radiation is generated most intensely at specifically these frequencies. The transition radiation is suppressed for $\omega \gtrsim \omega_*$ and $\omega_* < \omega_p\gamma$. The synchrotron radiation by heavy particles (in particular, protons) is very weak because of the large mass of the particles. In this case the transition radiation may outweigh the synchrotron radiation even at frequencies $\omega > \omega_p\gamma$, since the intensity of the transition radiation does not depend on the mass of the emitting particles. In the latter case the transition radiation also determines the total radiative loss in a randomly inhomogeneous plasma. In each case (for electrons and protons), the transition radiation is suppressed by the magnetic field at those frequencies at which transition radiation is the dominant type of radiation.

A few comments are in order regarding the astrophysical implications of this effect. In typical radio sources the emission comes from electrons with a power-law energy spectrum[8] $N(E) \propto E^{-\xi}$. In this case the synchrotron radiation is strongly suppressed at frequencies $\omega < \omega_p^2/\omega_B$. The suppression of the transition radiation is seen at frequencies $\omega > \omega_p^2/\omega_B$; at lower frequencies there can be an effective generation of transition radiation. It thus becomes legitimate to use the standard expressions for the transition radiation from particles in rectilinear motion in interpreting the

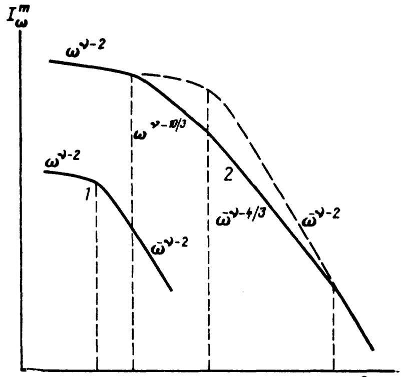


FIG. 3. Spectra of the transition radiation for two particles. 1—Particle width $\gamma_1 \ll \omega_p/\omega_{B\perp}$; 2—particle width $\gamma_2 \gg \omega_p/\omega_{B\perp}$. The dashed line shows the emission spectrum of the second of these particles in the absence of a magnetic field.

low-frequency part of the observed radio spectra at[7] $\omega < \omega_p^2/\omega_B$. In general, however, in sufficiently dense objects ($\omega_p/\omega_B \gg 10^2$) and in highly inhomogeneous objects ($\langle \Delta N^2 \rangle / N^2 \sim 1$), the transition radiation can outweigh the synchrotron radiation at frequencies $\omega \sim \omega_p^2/\omega_B$. If this is the case, and if the particles also have quasimonoenergetic distributions, the suppression of the transition radiation by a magnetic field might be observed under astrophysical conditions.

Finally, we consider the intensity of the transition radiation integrated over all frequencies:

$$I_{tot}^m = \int_{\omega_p}^{\infty} I_\omega^m \, d\omega. \tag{32}$$

In the absence of a magnetic field, this integral would be dominated by the frequency region $\omega \lesssim \omega_p\gamma$, since at higher frequencies the intensity $I_\omega^m$ falls off rapidly. Correspondingly, we have

$$I_{tot}^m \approx \int_{\omega_p}^{\omega_p\gamma} I_\omega^2 \, d\omega \propto \gamma^{\nu-1}. \tag{33}$$

If, on the other hand, the magnetic field is not zero, and if the condition $\gamma > \omega_p/\omega_{B\perp}$ holds, then the transition radiation begins to fade away rapidly even at frequencies $\omega \gtrsim \omega_* = \omega_p(\omega_p\gamma/\omega_{B\perp})^{1/2}$ ($\omega_* < \omega_p\gamma$). In this case, the dependence of the total radiated energy (per unit time) on the energy of the particle becomes weaker:

$$I_{tot}^m \approx \int_{\omega_p}^{\omega_*} I_\omega^2 \, d\omega \propto \gamma^{(\nu-1)/2}. \tag{34}$$

The reason for this change is that (as was mentioned earlier) the transition radiation is suppressed in a magnetic field under condition (3) at those frequencies at which most of the energy would be radiated in the case $\mathbf{B} = 0$.

If a system has sharp boundaries (e.g., if the system is a stack of plates—a case of interest for the problem of transition-radiation detectors of heavy particles[2,6]), we have $\nu = 2$, and instead of the linear dependence $I_{tot}^m \propto \gamma$ at $\gamma > \omega_p/\omega_{B\perp}$ we find a square-root dependence $I_{tot}^m \propto \gamma^{1/2}$. Although the effects of magnetic fields can be eliminated in the laboratory, one should consider how the transition radiation would be affected by the scattering of the relativistic particle by the atoms of the plates. This scattering would lead to a similar effect (one should bear in mind that a suppression of radiation might also be advantageous, e.g., in the acceleration of heavy particles to extremely high energies in a plasma). The effect of multiple scattering on the emission of resonant radiation (i.e., the emission in a medium with a sinusoidally modulated dielectric constant) was studied by Ter-Mikaélyan.[9] In connection with experiments on transition radiation in foam plastics[6] and in other media with random fluctuations, however, it is worthwhile to solve the problem of how multiple scattering affects the characteristics of the emission in such media. There is also obvious interest in an experimental study of transition radiation both in magnetized plasmas and in condensed media under conditions such that the effects which we have been discussing here (which stem from the curvature of the particle path) are important.

**3.** We can draw the following conclusions from this analysis.

a) The curvature of the path traced out by a relativistic particle in a magnetic field results in a suppression of the transition radiation at frequencies $\omega \gtrsim \omega_*$. For electrons, the value of $\omega_*$ is determined by the electron cyclotron frequency $\omega_{Be\perp}$, while for ions it is determined by the ion cyclotron frequency $\omega_{Bi\perp} = (m/M_i) z \omega_{Be\perp}$.

b) A dip appears at frequencies $\omega_* < \omega < \omega_p \gamma$ in the overall spectrum of the emission by a electron moving through a turbulent magnetized plasma (the spectrum of the radiation containing the transition radiation and the synchrotron radiation). The reason is that both synchrotron radiation and transition radiation are suppressed in this region (by, respectively, a density effect and a magnetic field).

c) The dependence of the total intensity of the transition radiation, $I_{\rm tot}^m$, on the energy of the particle weakens at $\gamma > \omega_p / \omega_{B\perp}$.

d) The directionality of the transition radiation is degraded, although the characteristic angle in (31) remains small in comparison with unity.

e) The effects discussed here are important even in fairly weak magnetic fields, $\omega_{B\perp} \ll \omega_p$. Under the opposite inequality $\omega_{B\perp} \gtrsim \omega_p$, these effects are even more obvious. It should be kept in mind, however, that the magnetic field affects both the path of the particle and the dispersive properties of the plasma.

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[1] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Nauka, Moscow, 1973 (Pergamon, New York, 1976).
[2] V. L. Ginzburg and V. N. Tsytovich, *Transition Radiation and Transition Scattering*, Nauka, Moscow, 1984.
[3] M. L. Ter-Mikalian, *High-Energy Electromagnetic Processes in Condensed Media*, Izd. Akad. Nauk Arm. SSR, Erevan, 1969 (Wiley, New York, 1972).
[4] I. N. Toptygin and G. D. Fleishman, Astrophys. Space Sci. **132**, 213 (1987).
[5] V. N. Baĭer, V. M. Katkov, and V. S. Fadin, *Radiation by Relativistic Electrons*, Atomizdat, Moscow, 1973.
[6] M. P. Lorikyan, R. A. Sardaryan, and K. N. Shikhlyarov, Izv. Akad. Nauk Arm. SSR **24**, 159, 252 (1989).
[7] G. D. Fleĭshman, Astron. Zh. **66**, 932 (1989) [Sov. Astron. **33**, 482 (1989)].
[8] V. L. Ginzburg and S. I. Syrovatskiĭ, *Origin of Cosmic Rays*, Pergamon, Oxford, 1964 (Russ. transl., Izd. Akad. Nauk USSR, Moscow, 1963).
[9] M. L. Ter-Mikaélyan, Zh. Eksp. Teor. Fiz. **8**, 100 (1968) [JETP Lett. **8**, 61 (1968)].

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