Nonlinear microwave response of YBaCuO in a critical state

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The nonlinear microwave response of YBaCuO near the critical temperature was investigated experimentally. A nonlinearity mechanism based on the interaction of the superconducting current with free or bound vortices and capable of describing the experimental results is considered.

INTRODUCTION

A model permitting calculation of microwave absorption in high-temperature superconductors, and primarily the dependence of the absorption intensity on the applied magnetic field, was proposed in Refs. 1 and 2. The Portis model starts from the assumption that the absorption is due to vortices weakly bound to the pinning centers, or simply free. Their number is proportional to the depinning force, i.e., to the magnetic-field gradient dH/dz.

In the present paper we show, on the basis of an investigation of second-harmonic generation, that this model explains naturally the magnetic-field dependence of the nonlinear response of YBaCuO in the microwave band.

Higher harmonics produced when a microwave is reflected from the surface of a superconductor were observed only at temperatures below critical, $T \leqslant T_c$. Examples of the temperature dependences of the second-harmonic power $P_{2\omega}$ are shown in Fig. 1.

Far from T_c, the second ($P_{2\omega}$, Refs. 3–5) or third ($P_{3\omega}$, Ref. 3) harmonics were observed in ceramic YBaCuO samples. The nonliner signals in single crystals, under the same conditions, are too weak to be observable in practice. The temperature dependence and the intensity of the harmonics at $T < T_c$ vary, depending on the technique used to produce the ceramic.

A weak external magnetic field $H < 100$ Oe suppresses the $P_{2\omega}$ signal far from T_c, in which case $P_{2\omega}$ has a monotonic dependence on the level of the wave power P_ω incident on the sample. This behavior of a nonlinear signal in weak external and alternating fields is in accord with the Josephson generation mechanism.[3,4,6]

At $H > 100$ Oe and $T < T_c$, the signal $P_{2\omega}$ is independent of the external field so long as H increases (or decreases) monotonically. This fact is explained in Ref. 5 on the basis of Bean's critical-state model.

The signals $P_{2\omega}(T)$ and $P_{3\omega}(T)$ take near T_c the form of maxima and are observed in both ceramic and single-crystal YBaCuO (see Fig. 1).

As shown in Ref. 7, at triple the frequency the value of P_3 of a crystal at $T \approx T_c$ is independent of the magnetic field and is governed by the nonstationary behavior induced in the order parameter by the high-frequency field.

We investigate in the present paper the behavior of the generation of the $P_{2\omega}$ signal near T_c.

EXPERIMENT

The experiment consisted of irradiating a YBaCuO sample by an electromagnetic wave with pulse power P_ω and frequency $\omega/2\pi = 9.4$ GHz and measuring the dependences of the second harmonic of the reflected signal on various external parameters.

The experimental procedure is described in detail in Ref. 8. We recall only some details of importance to the present experiments. A ceramic or single-crystal YBaCuO sample (we used the same single crystal as in Ref. 7) was placed on the bottom of a bimodal cylindrical cavity tuned simultaneously to the frequencies ω and 2ω, in the region of an antinode of an alternating magnetic field H_ω. The maximum amplitude $H_\sim$ of this field on the sample of this surface could be varied from 0.5 to 25 Oe. To produce an external field $\mathbf{H} \| \mathbf{H}_\omega$ ($H < 200$ Oe) we used a Helmholtz system with the earth's field cancelled out.

The $P_{2\omega}(P_\omega)$ dependence was measured with polarization attenuators in the waveguide channels, and a phase shifter was used to measure the variation of the wave generated in the sample when the field H was varied.

The sample temperature was measured with a germanium thermometer soldered to an outer surface of the cavity. It was possible to measure the temperature accurate to ± 0.01 K. The temperature dependence of $P_{2\omega}$ is shown in Fig. 1. A signal $P_{2\omega} \neq 0$ is obtained only at $T < T_c$ or $T \lesssim T_c$. All the subsequent measurements were made at a temperature $T \approx T_c$ corresponding to the peak of the maximum on the $P_{2\omega}(T)$ curve.

Figure 2 shows the dependence of $P_{2\omega}$ on the power P_ω of the microwave incident on a YBaCuO single crystal. A similar dependence was obtained also for a ceramic sample. A quadratic regime $P_{2\omega} \propto P_\omega^2$ obtains in the entire range of P_ω, which differs in principle from the nonmonotonic $P_{2\omega}(P_\omega)$ dependence in the ceramic at $T < T_c$.

Near T_c, the dependence of $P_{2\omega}$ on a weak external magnetic field (Fig. 3) also differs from the form of the function $P_{2\omega}(H)$ at $T < T_c$. The $P_{2\omega}(H)$ curve in Fig. 3 exhibits hysteresis for both a single crystal and a ceramic.

The sample was cooled beforehand in the absence of a field ($H = 0$) and $P_{2\omega}$ signal was zero. The value of $P_{2\omega}$ increases rapidly with increase of the field and saturates already at $H \approx 5$ Oe. The $P_{2\omega}$ level remains practically unchanged with further increase of the magnetic field. At the point A (Fig. 3) for $H = H_0$ the direction of the field evolution is reversed. At this instant $P_{2\omega}$ drops abruptly, after which it decreases smoothly almost to zero, and begins to increase only in a very weak field. With further variation of the field in the interval $-H_0 < H < H_0$ the form of the $P_{2\omega}(H)$ curve remains unchanged and symmetric about the vertical axis $H = 0$.

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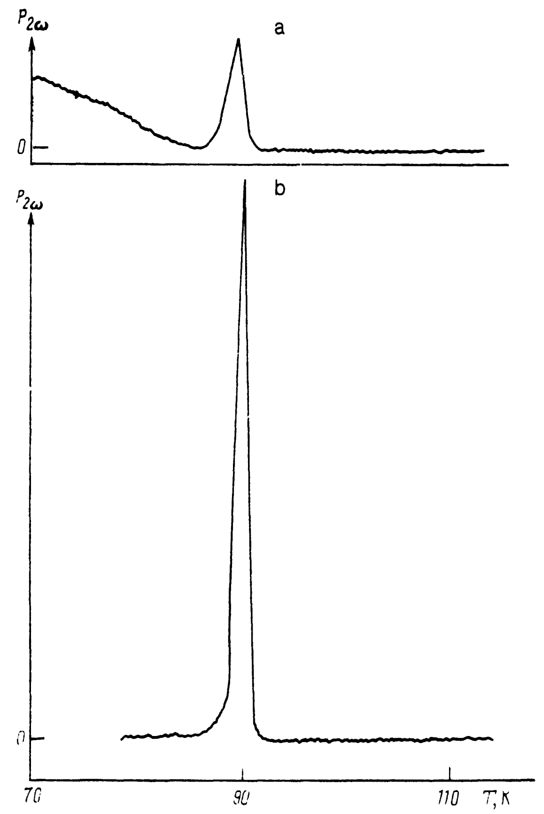

FIG. 1. Dependences of the second-harmonic power $P_{2\omega}$ on the temperature for ceramic (a) and single-crystal (b) samples of YBaCuO. Frequency $\omega/2\pi = 9.4$ GHz, external magnetic field $H = 20$ Oe, the maximum amplitude of the alternating magnetic field $\mathbf{H}_\omega \| \mathbf{H}$ on the sample surface is $H_\sim = 18$ Oe (a) and 15 Oe (b).

Measurements of the phase of the $P_{2\omega}(H)$ signal have shown that it does not change in the entire range $-H_0 < H < H_0$.

The microwave response of a superconductor at double the frequency near T_c is thus the same for ceramic and single-crystal YBaCuO samples, and its onset is apparently not connected with the Josephson generation mechanism that acts in weak fields at $T < T_c$.

SECOND-HARMONIC GENERATION IN THE PORTIS MODEL

To explain the experimental results we shall use the model proposed by Portis *et al.*,[1,2] which describes weak-field nonresonant absorption by high-temperature super-

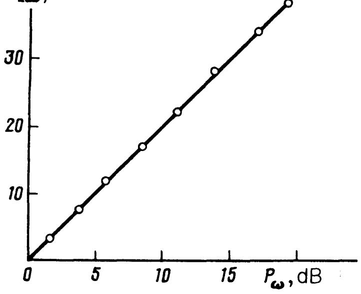

FIG. 2. $P_{2\omega}(P_\omega)$ dependence in single crystal YBaCuO at $T \approx T_c$; $\mathbf{H} \| \mathbf{H}_\omega$, $H = 15$ Oe and $\omega/2\pi = 9.4$ GHz.

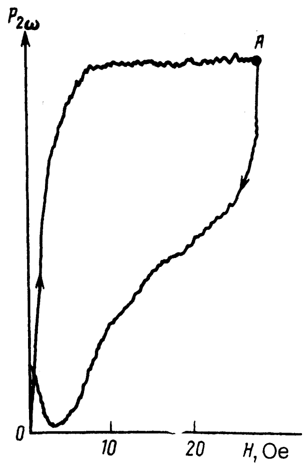

FIG. 3. $P_{2\omega}(H)$ dependence in single-crystal YBaCuO for $T \approx T_c$; $H_\sim = 10$ Oe and $\omega/2\pi = 9.4$ GHz. The direction of the current variation was reversed at the point A.

conductors in the critical state.

According to this model, the microwave response is formed when a superconducting current interacts with free or weakly bound vortices. Pinned vortices do not contribute to the absorption. The vortex equation of motion is

$$M\frac{dV_\omega}{dt}+\eta V_\omega+k\xi=\frac{1}{c}j_\omega\varphi_0. \tag{1}$$

Here M is the vortex mass per unit length, η a damping constant describing the "viscosity," k the elastic restoring force coefficient, j_ω the density of the superconducting current, ξ the displacement, and φ_0 the flux quantum.

Let the incident wave have a time dependences $\propto \exp(-i\omega t)$. The amplitude of the vortex velocity is then

$$V_\omega=\frac{j_\omega\varphi_0}{c}\left(-i\omega M+\eta+i\frac{k}{\omega}\right)^{-1} \tag{2}$$

A moving vortex produces an electric field of frequency

$$E_\omega^\varphi=-V_\omega fH/c, \tag{3}$$

where f is the fraction of the free or weakly bound vortices. $f \approx 0.1$ in the experiments of Ref. 1.

The field E_ω^φ jointly with the applied microwave field $\mathbf{E}_\omega$ are determined in turn by a vector potential $\mathbf{A}_\omega$ and the current $\mathbf{j}_\omega$ self consistently via the London equation:

$$\mathbf{j}_\omega=-\frac{c\mathbf{A}_\omega}{4\pi\lambda_{mw}^2},\qquad \frac{d\mathbf{j}_\omega}{dt}=\frac{c^2(\mathbf{E}_\omega+\mathbf{E}_\omega^\varphi)}{4\pi\lambda_{mw}^2}, \tag{4}$$

where λ_{mw} is the depth of penetration of the microwave field.

From the Maxwell equations, using the connection of the current j_ω with the field E_ω in (4), an expression can be found for the surface impedance.[1]

Allowance for the wave magnetic field H_ω adds to the spectrum of the electric field produced by the vortex a second harmonic of the fundamental frequency

$$E_{2\omega}^\varphi\propto fH_\omega V_\omega. \tag{5}$$

From the London equation (4) we get an expression for the "nonlinear source" (Ref. 10) at double the frequency:

$$\mathbf{j}_{2\omega}{}^{nl}=ic^2\mathbf{E}_{2\omega}{}^*/(8\omega\pi\lambda_{mw}{}^2). \quad (6)$$

This source determines the radiated power recorded in the experiment

$$P_{2\omega}\propto\left|\int_0^\infty \mathbf{j}_{2\omega}{}^{nl}(z)\mathbf{E}_{2\omega}(z)\,dz\right|^2, \quad (7)$$

where $\mathbf{E}_{2\omega}(z)$ describes in the linear approximation the dependence of the field at double the frequency on the coordinate z directed into the interior of the superconductor.

It follows from (5) that the nonlinear current (6) is proportional to the fraction of the free (or weakly bound) vortices, which is determined by the modulus of the gradient of the field H. The free-vortex density averaged with allowance for the penetration of the microwave field, is equal to

$$\langle f\rangle\propto\int_0^\infty\left|\frac{dH}{dz}\right|\exp\left(-\frac{3z}{\lambda_{mw}}\right)dz \quad (8)$$

and varies with the distribution of the field $H(z)$ in the sample.

The following is assumed in the Portis model:

1. An increasing applied magnetic field $\mathbf{H}_0$ parallel to the sample surface penetrates into the bulk exponentially $H(z)=H_0\exp(-z/\lambda)$ (λ is the penetration depth of the static field) all the way to $H^*_{c1}=4\pi j_c\lambda/c$ (j_c is the critical current).

2. With further increase of H_0, the field gradient, equal to $4\pi j_c/c=H^*_{c1}/\lambda$, remains constant on the sample surface (the Bean critical state). The field decreases linearly

$$H(z)=H_0-zH_{c1}{}^*/\lambda$$

at $z<z_1$ and exponentially

$$H(z)=H_0\exp[-(z-z_1)/\lambda]$$

at $z>z_1$, where $z_1=\lambda(H_0-H^*_{c1})/H^*_{c1}$.

3. If the applied magnetic field begins to decrease after reaching a certain maximum H_m, the $H(z)$ dependence assumes according to Ref. 11 in the interval $H_m-2H^*_{c1}\leqslant H_0\leqslant H_m$ the form

$$H(z)=H_m-(H_m-H_0)\exp(-z/\lambda)-zH_{c1}{}^*/\lambda,$$

so that

$$\left|\frac{dH}{dz}\right|=\frac{1}{\lambda}\left[H_{c1}{}^*-(H_m-H_0)\exp\left(-\frac{z}{\lambda}\right)\right]$$

at $H_m-H^*_{c1}<H_0\leqslant H_m$, while at $H_m-2H^*_{c1}\leqslant H_0\leqslant H_m-H^*_{c1}$ we have

$$\left|\frac{dH}{dz}\right|=\begin{cases}(H_m-H_0)\exp(-z/\lambda)/\lambda-H_{c1}{}^*/\lambda, & 0\leqslant z\leqslant z_2\\ H_{c1}{}^*/\lambda-(H_m-H_0)\exp(-z/\lambda)/\lambda, & z>z_2\end{cases},$$

where $z_2=\lambda\ln[(H_m-H_0)/H^*_{c1}]$.

4. In fields $H_0<H_m-2H^*_{c1}$ an inverse critical state develops on the sample surface, with a field distribution

$$H(z)=\begin{cases}H_0+zH_{c1}{}^*/\lambda, & z\leqslant z_3\\ H_m-2H_{c1}{}^*\exp[-(z-z_3)/\lambda]-(z-z_3)H_{c1}{}^*/\lambda, & z>z_3\end{cases},$$

where $z_3=\lambda(H_m/H^*_{c1}-H_0/H^*_{c1}-2)$.

The expressions given for $H(z)$ make it easy to calculate from (8) the values of f in the entire range of the external magnetic field.

COMPARISON WITH EXPERIMENT

The measured $P_{2\omega}(H)$, $P_{2\omega}(P_\omega)$, and $P_{2\omega}(T)$ dependences are determined by the integral (7). Both factors under the integral sign vary with the applied magnetic field. In contrast to the second-harmonic intensity, the impedance in a zero field has a finite value.[12] In sufficiently weak fields the linear electrodynamic characteristics of YBaCuO can therefore be regarded as independent of the field. Consequently, the principle dependence of the observed signal $P_{2\omega}$ on the external field is determined by the function $\mathbf{j}^{nl}_{2\omega}(z,H)$ in (7). With allowance for (5) and (6) it can be assumed that in weak fields

$$P_{2\omega}\propto P_\omega{}^2\langle f\rangle^2. \quad (9)$$

This conclusion is confirmed by measurements of the harmonic-signal phase. According to (9), the phase of $P_{2\omega}$ is independent of the magnetic field. In strong fields, where the generation mechanism of Ref. 5 differs from that considered by us, the phases in the forward and backward directions of the field variation directions differ by π. The proportionality in (9) of the radiated power $P_{2\omega}$ to the square of the incident power is also confirmed by experiment (Fig. 2).

We consider now the $P_{2\omega}(H)$ plot in Fig. 3. The rapid growth and saturation of the observed signal with increase of the applied field correspond to the critical current produced in the skin layer. When the field-variation direction is reversed, the current decreases in magnitude without sign reversal, and this decreases strongly the signal $P_{2\omega}$.

This part of the $P_{2\omega}(H)$ curve is well described by relation (9), in which the function $\langle f\rangle$ was calculated using Eq. (8) with the field distribution $H(z)$ indicated in Items 1–3 of the preceding section. The only fitting parameter is in this case the ratio $\lambda_{\rm mw}/\lambda$. For the curve of Fig. 2, the best agreement is reached at $\lambda_{\rm mw}/\lambda\approx1.5$.

The last part of the curve, obtained with decrease of the applied field (to the left of the arrow on Fig. 3), corresponds to establishment of an inverse critical state, when the current reverses sign in the skin layer. It cannot be described with the aid of Eq. (9).

Note that an attempt to account fully for the form of the hysteresis loop is hardly expedient, since account must be taken of the actual shape of the sample.

As to the temperature dependence of $P_{2\omega}$ in a YBaCuO single crystal (Fig. 1b), it is determined primarily by the strong dependence of the impedance on the temperature. When the temperature is lowered below T_c, the impedance decreases abruptly, and with it the region of wave interaction in the superconductor—the integration region in (7). In a YBaCuO single crystal the impedance is 50 times smaller than in the ceramic (at $T\approx77$ K) (Ref. 13) making the nonlinear effects in the signal crystal practically unobservable far from T_c.

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