# Collective annihilation of an electron-hole plasma in a strong magnetic field

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The possibility of the existence of the phenomenon of collective spontaneous annihilation of a dense electron-positron plasma, occurring via coherent acts of single-photon annihilation of electron-positron pairs in a strong magnetic field $B \gtrsim 10^{13}$ G, is pointed out. Collective annihilation develops considerably more rapidly than the known incoherent processes of spontaneous annihilation and collisional relaxation and leads to the generation of powerful coherent $\gamma$ radiation (annihilation super-radiance).

## 1. INTRODUCTION

Recently, the annihilation of an electron-positron ($e^- e^+$) plasma in natural and laboratory conditions has been widely discussed (see, e.g., Refs. 1–3). Despite their extreme nature, the corresponding analysis is necessary for the solution of real physical problems associated with the generation and dynamics of an $e^- e^+$ plasma and its radiation—in particular, on neutron stars (the $e^- e^+$ avalanche in the Ruderman-Sutherland magnetospheric gap),[2] in the vicinity of black holes (including in accretion disks),[3] in the initial stage of the development of the Universe,[4] in accelerators in the coalescence or collision of electron and positron beams,[5] in the collisions of high-energy ions, producing bunches of particles and antiparticles,[6] etc. The annihilation process is of interest not only from a fundamental but also from an applied point of view, e.g., for the production of coherent $\gamma$ radiation (the $\gamma$ laser).[7]

As will be shown below, in a sufficiently dense $e^- e^+$ plasma the annihilation process has a collective, coherent character, and occurs in a time considerably shorter than the time $T$ of the familiar incoherent processes of spontaneous annihilation and collisional relaxation. This highly nonstationary process is characterized by the presence of correlations in a broad frequency band of the $\gamma$ radiation: $\tau^{-1} \gtrsim T^{-1}$, where $\tau$ is the characteristic time scale of the collective-annihilation (CA) pulse. Therefore, it cannot be described in the balance approximation of independent spectral components of the radiation, the width of each of which would be small in comparison with $T^{-1}$. This means that in the analysis of the CA process it is not possible to confine oneself to a traditional quantum-electrodynamic calculation of the cross sections of the elementary acts of annihilation with the corresponding balance equations for the transport of the annihilation-emission intensity.

The situation here is analogous to the collective spontaneous emission (super-radiance) from a sample of excited inverted atoms or molecules with a discrete energy spectrum, familiar in coherent and nonlinear optics (see, e.g., the reviews in Refs. 8–10). At a sufficiently high concentration, the atoms that are excited by the short pumping pulse into an incoherent state on some of the upper energy levels emit, after a certain delay time $t_d$, a coherent pulse of collective spontaneous radiation, the spectral intensity of which exceeds by many orders of magnitude the intensity of the spontaneous emission from the same number of independent atoms, and the duration of which is much shorter than the energy-relaxation and phase-relaxation times in the medium: $\tau \ll T_1, T_2$. This phenomenon is due to the effective mutual phasing of the dipole oscillations of the atoms via the self-consistent field of their emission over time $t < t_d$. The coherence of their emission is thereby ensured up to the time $t_d$ of de-excitation. The description of the formation and propagation of such super-radiance pulses, which have been observed in a number of experiments in the radiofrequency, infrared, and optical ranges, also involved the need to go beyond the limits of the balance approximation traditional in laser physics.

In view of the above considerations, in the present paper we have undertaken an analysis of one of the possible variants of collective annihilation—single-photon CA. Specifically, we have considered the collective spontaneous decay of a relativistic degenerate $e^- e^+$ plasma placed in a strong magnetic field $B$ of the order of the critical field $B_c = m^2c^3/e\hbar \approx 4.4 \times 10^{13}$ G ($m$ and $e$ are the rest mass and charge of the electron, $c$ is the velocity of light in vacuo, and $\hbar$ is Planck's constant). The solution of this problem, on the one hand, makes it possible to elucidate a number of questions in the theory of nonstationary energy release in an annihilating plasma, and, on the other hand, gives an example of an analysis of super-radiance in a medium with a continuous energy spectrum. To examine such processes seems to be a necessary step on the route to obtaining ultrashort pulses in a nonrelativistic dense plasma as well, e.g., in the collective recombination of ions and electrons in a gas discharge, or in the super-radiative annihilation of holes and electrons in a semiconductor, recently observed experimentally.[11] Up to now, however, studies have been made only of the super-radiance from media with a discrete energy spectrum, e.g., in a model of molecules with two energy levels[8–10] or in a system of positronium atoms,[12] and also of the cyclotron super-radiance of electrons rotating about magnetic-field lines with the cyclotron frequency and possessing an energy spectrum with quasi-equal spacing.[10,13]

We use the semiclassical approximation. It is usually valid in the presence of a sufficiently large number of photons, when the electromagnetic field may be regarded as classical and described by the Maxwell equations. The properties of the $e^- e^+$ plasma are described by quantum equations. From these we determine the susceptibility tensor, which specifies the relationship of the mean current density, including the annihilation current, to the electromagnetic field. According to the general approach in the electrodynamics of active media,[10,14–16] first, in Sec. 2, we shall establish properties (the increment, dispersion, and polarization) of normal waves of the $e^- e^+$ plasma near the single-pho-

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ton-annihilation threshold frequency

$$\omega_0=2mc^2/\hbar \sin\theta, \quad (1.1)$$

where $\theta$ is the angle between the wave vector $\mathbf{k}$ and the external magnetic field $\mathbf{B}$ (Fig. 1b). The normal waves of the unmagnetized vacuum and of an $e^- e^+$ plasma in a magnetic field have been investigated in, e.g., Ref. 17. Below, since we are interested only in the resonance region of frequencies near $\omega_0$, we shall simplify considerably the derivation of the susceptibility tensor and the solution of the dispersion equation. This makes it possible to estimate the Hermitian part of the susceptibility for various plasma concentrations $N_e$, and to take into account the concentration dependence of the increment of the extraordinary wave. The expression (2.10) obtained for the increment in the limit of large concentrations $N_e \gg N_e^{(s)}$ [see (2.13) below] coincides with that found in Ref. 17.

Next, in Secs. 3 and 4, the problem of CA in a model with unidirectional propagation of the radiation is solved. A self-similar growth law for the field amplitude in the linear and nonlinear stages of the development of the instability is found. In Sec. 5 we consider the role of effects associated with the angular divergence of the radiation, the geometrical shape of the plasma bunch, and the kinematic flying apart of particles of the plasma bunch. On the basis of the results obtained, we elucidate the energy, spectra, and spatiotemporal characteristics of the CA. The coherent $\gamma$ radiation that arises in a dense bunch of $e^- e^+$ plasma is found to be shorter in duration and considerably greater in spectral intensity than the spontaneous annihilation emission from the same number of independent $e^- e^+$ pairs.

To conclude, we discuss physical situations in which manifestation of the CA effect is possible, and also discuss the role of competing incoherent relaxation processes; a number of open problems are formulated.

## 2. NORMAL WAVES NEAR THE THRESHOLD FOR SINGLE-PHOTON ANNIHILATION

We shall seek unstable normal waves in a uniform magnetized $e^- e^+$ plasma on the basis of a general expression for the four-tensor of its susceptibility, obtained in Ref. 18 to first order in the radiative corrections (with the use of the exact Green functions of the Dirac equation with an external magnetic field) and written in explicitly covariant form in Ref. 19:

$$\Pi^{\mu\nu}(\omega,\mathbf{k})=-\frac{\alpha B}{2\pi B_c}\left(\frac{mc^2}{\hbar}\right)^2\sum_{\substack{\varepsilon,\varepsilon'\\ q,q'}}\int\frac{dp_z}{2\pi}$$
$$\times\frac{(\varepsilon'-\varepsilon)/2+\varepsilon n_q^{\varepsilon}-\varepsilon' n_{q'}^{\varepsilon'}}{\hbar\omega-\varepsilon E_q-\varepsilon' E_{q'}+i0}[\Gamma_{q'q}^{\varepsilon'\varepsilon}(\mathbf{k})]^{\mu}[\Gamma_{qq'}^{\varepsilon\varepsilon'}(\mathbf{k})]^{\nu}. \quad (2.1)$$

This tensor relates the Fourier components of the four-vectors of the induced current density to the potential of the probe field. Here, $\alpha=e^2/\hbar c$ is the fine-structure constant,

$$E_q=[m^2c^4(1+2nB/B_c)+p_z^2c^2]^{1/2}$$

is the modulus of the energy of a Dirac particle in an external field $\mathbf{B}=B\mathbf{z}^0$ (see Fig. 1a), the prime indicates the final state of the particle, $\varepsilon,\varepsilon'=\pm1$ label states with positive and negative energies, and $n_q^{\varepsilon}$ are the occupation numbers of the quantum states $q=\{n,l,\sigma\}$, where $n=l+(\sigma+1)/2$ is the principal quantum number, $l$ is the orbital quantum number, and $\sigma=\pm1$ is the spin quantum number. According to the law of conservation of the $z$ component of the momentum, in (2.1) it is assumed that $\varepsilon' p_z'=\varepsilon p_z-\hbar k_z$. The vector function $[\Gamma_{qq'}^{\varepsilon\varepsilon'}(\mathbf{k})]^{\mu}$ is the spatial Fourier component of the Dirac current density. An explicit expression for it is given in Ref. 19; e.g., for $n=n'=0$, for the $z$ component we have

$$|[\Gamma_{00'}^{\varepsilon\varepsilon'}]^z|^2=\frac{1}{2}\exp\left[-\frac{2B_c}{B}\left(\frac{\hbar k\sin\theta}{2mc}\right)^2\right]$$
$$\times\left(1+\frac{m^2c^4}{\varepsilon\varepsilon' E_0E_{0'}}+\frac{c^2p_zp_z'}{\varepsilon\varepsilon' E_0E_{0'}}\right).$$

Since we are interested in an $e^- e^+$ plasma of high density in a strong magnetic field $B\sim B_c$, we shall confine ourselves for simplicity to the limiting case of complete degeneracy of the electrons and positrons, in the lowest, zeroth Landau level, with equal concentrations

$$N_e=\frac{1}{2\pi^2\lambda_c^3}\frac{B}{B_c}\frac{p_F}{mc}<N_1$$
$$=\left(\frac{B}{B_c}\right)^{3/2}\frac{1}{2^{1/2}\pi^2\lambda_c^3}\approx\left(\frac{B}{B_c}\right)^{3/2}\cdot 1{,}24\cdot10^{30}\,\text{cm}^{-3},$$

where $\lambda_c=\hbar/mc\approx4\times10^{-11}$ cm is the Compton wavelength of the electron. The above inequality implies that the Fermi energy is not too large: $\mu\equiv E_{n=0}(p_z=p_F)<E_{n=1}(p_z=0)$, i.e., for Fermi momentum $p_F<mc(2B/B_c)^{1/2}$, and this permits us to assume that the higher Landau levels are unpopulated (Fig. 1a). Moreover, we neglect the contribution to the susceptibility from the nonzero Landau levels, keeping only the terms with $n, n'=0$ in (2.1). The latter is justified by the narrowness of the CA emission spectrum, which is concentrated near the threshold frequency $\omega_0$ for annihilation of $e^- e^+$ pairs in the zeroth Landau level. (This, it is true, excludes from the analysis the cyclotron resonances, i.e., those values of the magnetic field for which the frequency $\omega_0$ coincides with the frequency of a transition between any of the Landau levels.) It is obvious that the contribution made to the imaginary part of the susceptibility by processes of creation and annihilation of $e^- e^+$ pairs in excited Landau levels is equal to zero at frequencies below the threshold frequency nearest to $\omega_0$, i.e., below

$$\omega_1=mc^2[(1+2B/B_c)^{1/2}+1]/\hbar\sin\theta.$$

After the simplifications made, the integration in (2.1) can be performed to completion. The resulting dielectric-permittivity tensor $\varepsilon_{ij}(\omega,\mathbf{k})=\delta_{ij}+4\pi\chi_{ij}$ $(i,j=x,y,z)$ of the magnetized degenerate $e^- e^+$ plasma differs from the Kronecker symbol $\delta_{ij}$ only by the longitudinal polarizability

$$\chi_{zz}(\omega,\mathbf{k})\equiv\frac{c\Pi_{zz}}{\omega^2}=\frac{2^{1/2}m^5c^9b}{\pi^2\hbar^4\omega^2(\omega^2-c^2k_z^2)\beta}$$
$$\times\left\{-\frac{4\pi i\hbar\omega^3[\eta(p_F-|p_1|)+\eta(p_F-|p_2|)-1]}{mc^2(\omega^2-c^2k_z^2)}\right.$$
$$-\frac{4\hbar k_z\beta}{m^3c^4}(\mu_+-\mu_-)$$
$$\left.+\sum_{g=1}^{2}\frac{(-1)^g\varphi_g\mathscr{L}_g}{[1+(p_g/mc)^2]^{1/2}}\right\},\quad \eta(x)=\begin{cases}1, & x\geq0\\ 0, & x<0\end{cases}, \quad (2.2)$$

where

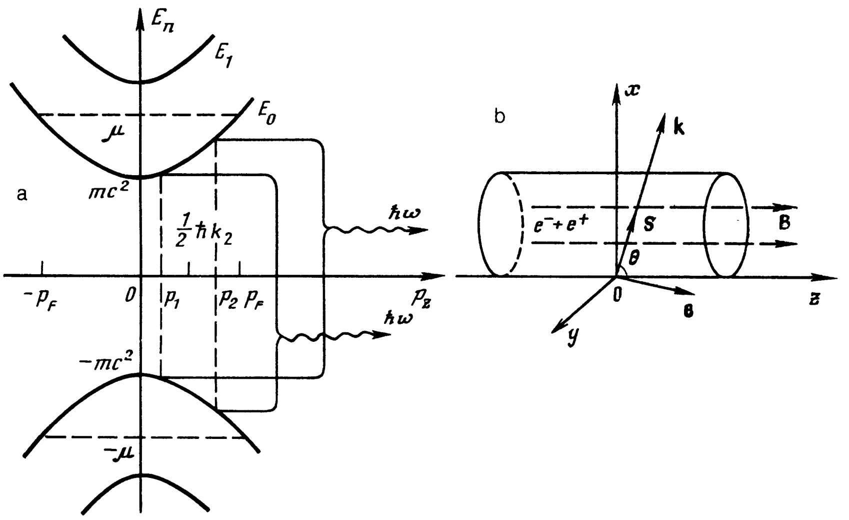


FIG. 1. (a) Energy diagram of single-photon annihilation of Dirac electrons and positrons; (b) geometry of collective annihilation radiation of extraordinary $\gamma$ quanta with wave vector **k** and polarization **e** in a sample of $e^-e^+$ plasma.

$$b=\frac{\alpha}{8\cdot 2^{1/2}}\frac{B}{B_c}\exp\left[-\frac{2B_c}{B}\left(\frac{\hbar k\sin\theta}{2mc}\right)^2\right]$$

$$\varphi_{1,2}=\left(\frac{\hbar\omega}{mc^2}\right)^2+\frac{\hbar k_z(\hbar k_z\pm 2\beta)\omega^2}{m^2c^2(\omega^2-c^2k_z^2)},$$

$$\mathcal{L}_g=\ln\left|\frac{(p_F+p_g)(\hbar k_z-p_F-p_g)[m^2c^2+p_Fp_g+\mu(m^2+p_g^2/c^2)^{1/2}]}{(p_F-p_g)(\hbar k_z+p_F-p_g)[m^2c^2-p_Fp_g+\mu(m^2+p_g^2/c^2)^{1/2}]}\right|$$

$$+\ln\left|\frac{[m^2c^2+(\hbar k_z+p_F)p_g+\mu_+(m^2+p_g^2/c^2)^{1/2}][(m^2c^2+p_g^2)^{1/2}-p_g]}{[m^2c^2+(\hbar k_z-p_F)p_g+\mu_-(m^2+p_g^2/c^2)^{1/2}][(m^2c^2+p_g^2)^{1/2}+p_g]}\right|$$

$$\mu=(m^2c^4+p_F^2c^2)^{1/2},\quad \mu_\pm=[m^2c^4+(\hbar k_z\pm p_F)^2c^2]^{1/2}.$$

Here, the momenta $p_{1,2}$ of the particles of the $e^-e^+$ pair that generate a $\gamma$ quantum with frequency $\omega$ and longitudinal wave number $k_z=k\cos\theta$ appear (Fig. 1a). According to the laws of conservation of energy and momentum, we have

$$p_{1,2}={}^1/_2\hbar k_z\pm\beta,\quad \beta=(\hbar\omega/2c)\,[1-4m^2c^4/\hbar^2(\omega^2-c^2k_z^2)]^{1/2}.$$

As is well known, the most important feature of the polarizability (2.2) is the annihilation resonance: $\chi_{zz}\propto 1/\beta\to\infty$ as $\omega^2\to c^2k_z^2+4m^2c^4/\hbar^2$. It is a root singularity, and is related to the singularity of the density of states of the electrons and positrons near the edge of the forbidden band (cf. the van Hove singularities in solids). In the case $k_z=0$ this is obvious by virtue of the fact that $dp_z/dE_0|_{p_z=0}=\infty$. The case $k_z\neq 0$ can be reduced to the previous case by a Lorentz transformation or can be considered directly, and one can convince oneself that the density of states of $e^-e^+$ pairs that are able to annihilate with emission of a $\gamma$ quantum possessing a fixed longitudinal momentum $\hbar k_z$ has a singularity (Fig. 1a):

$$dp_z/d[E_0(p_z)+E_0(\hbar k_z-p_z)]\,|_{p_z=\hbar k_z/2}=\infty.$$

In such a uniaxial medium, in accordance with the Fresnel equation[14,15]

$$(c^2k^2\delta_{ij}-c^2k_ik_j-\omega^2\varepsilon_{ij})\mathscr{E}_i=0$$

there exist an ordinary and an extraordinary plane normal wave, proportional to $\mathscr{E}\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$. In the ordinary wave the electric-field amplitude $\mathscr{E}\perp\mathbf{k},\mathbf{B}$, and the refractive index $ck/\omega\equiv 1$. The extraordinary wave is polarized elliptically in the **k**, **B** plane (Fig. 1b). Solution of its dispersion equation

$$\omega^2=c^2k^2-4\pi\chi_{zz}(\omega,\mathbf{k})\,(\omega^2-c^2k_z^2)$$

shows that in a degenerate $e^-e^+$ plasma it is unstable and has quasi-transverse polarization, i.e., $\vec{\mathscr{E}}\cdot\mathbf{k}\approx 0$ (since, for this wave,

$$\mathscr{E}_x/\mathscr{E}_z=-\varepsilon_{zz}\operatorname{ctg}\theta,\quad |\varepsilon_{zz}-1|=4\pi|\chi_{zz}(\omega,\mathbf{k})|\ll 1$$

for $\mathbf{k}=\operatorname{Re}\mathbf{k}$).

In the problem of CA as a problem with initial conditions, it is necessary to know the complex frequency $\omega_e(\mathbf{k})=\omega_e'+i\omega_e''$ of this unstable wave as a function of the real wave number $k$ for arbitrary angle $\theta$. Far from the annihilation resonance (1.1), for $|ck-\omega_0|\gg\omega_e''$, $|ck-\omega_e'|$, it is determined by the first correction to the vacuum dispersion law $\omega=ck$:

$$\omega_e(k)\approx ck(1-2\pi\sin^2\theta\chi_{zz}|_{\omega\to ck}),\qquad (2.3)$$
$$\omega_e''(k)\approx 2^{1/2}\omega_0 b\sin^2\theta(k_0/k)^2(k^2/k_0^2-1)^{-1/2},\quad k>k_0=\omega_0/c.$$

This balance increment $\omega_e''$ can also be obtained directly from the well known formula for the probability of single-photon annihilation.[19]

However, the most interesting region— the region of maximum values of the increment—lies near the resonance (1.1), and is not described by the balance approximation (2.3). It is possible here to reduce the dispersion equation approximately to a cubic equation for the detuning from the resonance frequency, of the form

$$\bar{y}^3+R\bar{y}+Q=0,\quad \bar{y}=-i(\omega/\omega_0-1)^{1/2},\qquad (2.4)$$
$$R=k/k_0-1+\Phi b_0\sin^2\theta,\quad Q=b_0\sin^3\theta,\qquad (2.5)$$
$$b_0=\frac{\alpha}{8\cdot 2^{1/2}}\frac{B}{B_c}\exp\left(-\frac{2B_c}{B}\right),\qquad (2.6)$$

if we assume that $b_0\ll 1$ and confine ourselves to the linear terms in the expansion of $\mathcal{L}_g$ from (2.2) in a series in powers of $\beta/mc\ll 1$. In this resonance approximation, $k_z\sim|\omega_0/c|\cos\theta$ and

$$\chi_{zz}'\approx-\frac{b_0}{2\pi}\left\{\Phi(p_F,k_z)+\frac{i[\eta(p_F-|p_1|)+\eta(p_F-|p_2|)-1]\sin\theta}{(\omega/\omega_0-1)^{1/2}}\right\}.\qquad (2.7)$$

Since we shall be interested in the solution of the dispersion equation in the case $p_F \gtrsim |\beta|$, when the imaginary part of (2.7), expressed in terms of the step function $\eta$, is greater than the real dispersion factor $\Phi(p_F, k_z)$, we shall not give its explicit dependence on $k_z$. For transverse propagation, we have

$$\Phi(\theta=\pi/2) = (2/\pi)^{3/2}[2(1+m^2c^2/p_F^2)^{1/2}-1]. \qquad (2.8)$$

The dispersion and increment of the extraordinary wave for wave numbers $k$ in the vicinity of $k_0 = \omega_0/c$ are determined by solving Eq. (2.4):[1)]

$$\bar{y} = -\frac{(u+v)}{2} - \frac{3^{1/2}i(u-v)}{2}, \qquad u,v = \left\{-\frac{Q}{2} \pm \left[\left(\frac{R}{3}\right)^3 + \left(\frac{Q}{2}\right)^2\right]^{1/2}\right\}. \qquad (2.9)$$

It follows from this solution that the increment has a maximum, equal to

$$\frac{\omega''}{\omega_0} \approx \frac{3^{1/2}Q^{2/3}}{2} = \left[\frac{\alpha}{8\cdot 2^{1/2}}\frac{B}{B_c}\exp\left(-\frac{2B_c}{B}\right)\right]^{2/3}\frac{3^{1/2}\sin^2\theta}{2}. \qquad (2.10)$$

The value $\omega_e''(k) \sim \omega'' = \max\omega_e''(k)$ is reached only in a narrow band of wavelengths $|k-k_0|/k_0 \lesssim Q^{2/3} \ll 1$ (see Fig. 2). Within this "resonance" band,

$$\omega_e'(k) \approx \omega_0\left[1 + \frac{1}{2}Q^{2/3}\left(1 + \frac{4R}{3Q^{2/3}}\right)\right], \qquad \omega_e''(k) \approx \omega_0\frac{3^{1/2}}{2}Q^{2/3}\left[1 - \left(\frac{R}{3Q^{2/3}}\right)^2\right], \qquad (2.11)$$

while outside it [more precisely, for $(k-k_0)/k_0 \gg Q^{2/3}$], the balance approximation (2.3) is valid. The instability is most effective for propagation transverse to **B**, i.e., for $\theta = \pi/2$. Figure 3 shows the dependence $\omega''(B/B_c)$; with decrease of the magnetic field $B \ll B_c$ the increment decreases exponentially, while with increase of the field to $B \sim 10^3 B_c$ it reaches a value of the order of $\omega_0$, and, after that, does not correspond to Eq. (2.10). The scale of the spatial coherence, i.e., the maximum length over which the

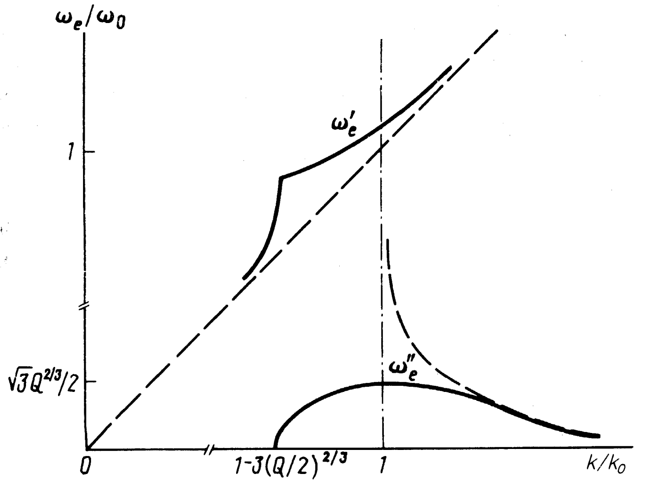


FIG. 2. Dispersion and increment of the unstable extraordinary wave $[\omega_e(\mathbf{k}) = \omega_e' + i\omega_e'']$ near the single-photon annihilation threshold $\omega_0 \equiv ck_0 \equiv 2mc^2/\hbar \sin\theta$: The solid curve is the resonance approximation (2.9)–(2.11), and the dashed curve is the balance approximation (2.3). The curves are plotted for the values $B = B_c$, $\theta = \pi/2$, and $p_F = mc$.

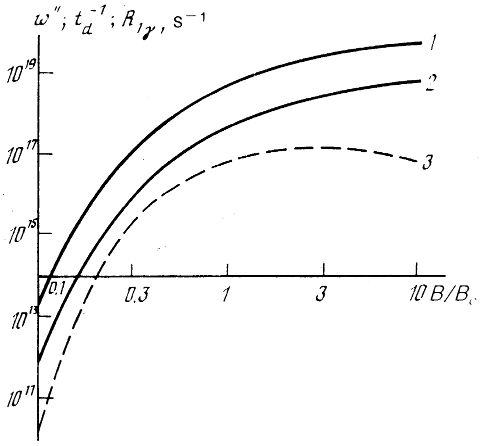


FIG. 3. Increment $\omega''$ (2.10) (curve 1) and inverse delay time $t_d^{-1}$ (3.5) (curve 2) for the CA from a sample of degenerate $e^-e^+$ plasma for $p_F \gtrsim \Delta p_F = (\hbar\omega_0/c)(\omega''/\omega_0)^{1/2}$, $\theta = \pi/2$, $s = L_c$, $\xi_d = 10$, and also the rate $R_{1\gamma}$ (6.1) of spontaneous single-photon annihilation (curve 3) for $N_e = 4\times 10^{28}$ cm$^{-3}$ as a function of the magnitude $B/B_c$ of the magnetic field.

annihilating $e^-e^+$ pairs are still causally coupled by the field of their own radiation, is determined by the so-called cooperative length $L_c = c/\omega''$. For $B = B_c$ and $\theta = \pi/2$ we have $\omega'' \approx 3\times 10^{18}$ sec$^{-1} \ll \omega_0 \approx 2\times 10^{21}$ sec$^{-1}$ and $L_c \sim 1$ Å.

We emphasize that the increment and dispersion depend very weakly on the concentration, while the maximum $\omega''$ does not depend on it at all. These conclusions pertain to a sufficiently dense plasma, when particles fill all states in the range of momenta $p_z$ of width of order $2\beta$ at $|\omega - \omega_0| \sim \omega''/2$, i.e.,

$$\Delta p_F = \left(\frac{\omega''}{\omega_0}\right)^{1/2}\frac{\hbar\omega_0}{c\sin\theta} \qquad (2.12)$$

about the value $\hbar k_z/2$ (Fig. 1a). It is the annihilation of these particles that gives rise to the instability of the extraordinary wave propagating in the specified direction $\theta$. The corresponding concentration that saturates the maximum increment at the level of (2.10) is determined from the condition $p_F \gtrsim (\Delta p_F + \hbar k_z)/2$, i.e.,

$$N_e \gtrsim N_e^{(s)} = N_0\frac{B(\Delta p_F + \hbar k_z)}{B_c mc} = N_0\frac{B}{B_c}\left[\left(\frac{2\hbar\omega''}{mc^2\sin^3\theta}\right)^{1/2} + 2\operatorname{ctg}\theta\right],$$

$$N_0 \equiv \frac{1}{4\pi^2\lambda_c^3} \approx 4\cdot 10^{29}\,\text{cm}^{-3}. \qquad (2.13)$$

Saturation first occurs for the wave propagating transversely to the magnetic field ($\theta = \pi/2$); for $B = B_c/4$ it occurs at $N_e^{(s)} \approx 10^{27}$ cm$^{-3}$, and for $B = B_c$ it occurs at $N_e^{(s)} \approx 4\times 10^{28}$ cm$^{-3}$. This saturation of the increment at a constant level, in contrast to the law $\omega'' \propto N^{1/2}$ familiar for the super-radiance of two-level molecular media, is related, clearly, to the Fermi degeneracy of the $e^-e^+$ plasma. We note that the actual instability arises only above a certain threshold of degeneracy, when the average (over the range $\Delta p_F$) occupation number $n_0$ of the annihilating quantum states satisfies $n_0 \equiv \langle n_q^e(p_z)\rangle > 1/2$ in the vicinity of $p_z \doteq \hbar h_z/2$. In fact, in the general case it is easy to show that the sign of $\omega_c''$ coincides with the sign of the factor $2n_0 - 1$, and the magnitude of $\omega_c''$, both in the balance approximation and in the resonance approximation, is obtained from (2.3) and (2.11), respectively, by the replacement $b \to |2n_0 - 1|b$. At high temperatures $k_B T \gg \mu$ or at low concentrations $N_c \ll N_c^{(s)}$, when $n_0 \ll 1$, absorption due to the creation of $e^- e^+$ pairs from the vacuum suppresses the annihilation instability.

### 3. MODEL OF UNIDIRECTIONAL PROPAGATION OF ANNIHILATION RADIATION

We shall consider the formation of a CA-radiation field

$$e_z(t, s) = {}^1/_2 \mathscr{E}_z(t, s) \exp(-i\omega_0 t + ik_0 s) + \text{c.c.}$$

from spontaneous initial ($t = 0$) fluctuations $\mathscr{E}_0(s)$, and consider its propagation along one direction **s** in a sample of $e^- e^+$ plasma with dimensions $L_y \ll L_x \lesssim L_c \ll L_z$ (Fig. 1b). In the initial, linear stage, when the occupation numbers $n_0 \approx 1$ and the slow complex amplitude $\mathscr{E}_z(t,s)$ is still sufficiently small, its temporal Laplace transform

$$\hat{\mathscr{E}}_z(p, s) = \int_0^\infty \mathscr{E}_z(t, s) \exp(-pt)\, dt$$

obeys the equation

$$[c\partial/\partial s + p - 2\pi i\omega_0 \chi_{zz}(p) \sin^2\theta] \hat{\mathscr{E}}_z(p, s) = \mathscr{E}_0(s). \qquad (3.1)$$

This is also established from the dispersion equation of the unstable extraordinary wave (Sec. 2) with neglect of spatial-dispersion effects and with the assumption that $4\pi|\chi_{zz}(\omega)| \ll 1$. For the polarizability $\chi_{zz}(\omega)$ in (3.1) we confine ourselves to the resonance approximation (2.7), retaining the dependence of $\chi_{zz}^r(p)$ on $p = i(\omega_0 - \omega)$ only in the resonance denominator $\propto 1/p^{1/2}$.

The solution of Eq. (3.1) describes the nonstationary amplification of a packet of plane unstable waves in an annihilating $e^- e^+$ plasma, and is expressed, in the form of the convolution

$$\mathscr{E}_z(t, s) = \int_0^s \mathscr{E}_0(s') D(s - s', t) \frac{ds'}{c},$$

in terms of their Green function

$$D(s-s', t) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \exp\left\{ \frac{(s-s')}{c} [p - 2\pi i\omega_0 \chi_{zz}^r(p) \sin^2\theta] + pt \right\} dp. \qquad (3.2)$$

Expanding the exponential in a series in powers of $p^{-1/2}$ and integrating, we obtain

$$D = \sum_{n=0}^{\infty} \frac{\xi^n \exp[-ik_0(s-s')\Phi b_0]}{\omega_0 \tilde{t}\, \Gamma(n+1) \Gamma(n/2)}$$

$$\xi = \left(\frac{2\omega''}{3^{1/2}\omega_0}\right)^{3/2} \frac{\omega_0(s-s')}{c} (\omega_0 \tilde{t})^{1/2} \exp\left(-\frac{i\pi}{4}\right), \qquad (3.3)$$

$$\tilde{t} = t - \frac{(s-s')}{c}.$$

We neglect the contribution of the dispersion factor $\propto \Phi b_0$ to the exponent, since over lengths $s - s' \lesssim L_c$ it is smaller than unity. Over lengths $s \lesssim L_c$ we can also neglect the retardation ($\tilde{t} \approx t$) and convince ourselves that the principal contribution to the convolution $\mathscr{E}_z(t,s)$ for sufficiently smooth initial conditions ($|d\mathscr{E}_0/ds'|_{s'=0} \ll \xi |\mathscr{E}_0(0)|/s$ is given by the region with the greatest amplification length—the neighborhood of the point $s' = 0$. Then, folding the series (3.3) term by term with the initial field $\mathscr{E}_0(s')$, we find the intermediate (in the linear stage) asymptotic form of the field for $t \to \infty$:

$$\mathscr{E}_z(t, s) \approx \mathscr{E}_0(0) \frac{4\pi s}{3ct} \left(\frac{3}{2\xi}\right)^{1/2} \exp\left[\xi \exp\left(-\frac{i\pi}{6}\right)\right],$$

$$\xi = \left[\frac{(6\cdot 3^{1/2})s}{L_c}\right]^{2/3} (\omega'' t)^{1/3} \gg 1. \qquad (3.4)$$

According to (3.4), the delay time $t_d$ needed for the field to reach its maximum, nonlinear level $\mathscr{E}_m$ [see (4.5) and (4.8) below], and the characteristic duration $\tau$ of the leading front of the CA pulse, determined by the condition

$$\xi|_{t=t_d-\tau} \sim \xi|_{t=t_d} - 1,$$

are equal to

$$t_d = \frac{\xi_d^3}{6\cdot 3^{1/2}\omega''}\left(\frac{s}{L_c}\right)^2, \quad \tau \sim \frac{3t_d}{\xi_d}, \quad \xi_d \approx \frac{2}{3^{1/2}} \ln\left|\frac{\mathscr{E}_m}{\mathscr{E}_0(0)}\right| \gg 1. \qquad (3.5)$$

The corresponding width of the $\gamma$-radiation spectrum ($\Delta\omega \lesssim \tau^{-1}$) turns out to be smaller than the increment $\omega''$ (for $s \sim L_c$, by a logarithmically large factor). This narrowing of the spectrum is a natural consequence of the nonstationary amplification of the radiation in the CA process.[2)] The dependence of the characteristic times on $B/B_c$ for $\xi_d = 10$ is shown in Fig. 3.

We note that the form of the intermediate asymptotic form (3.4) of the CA and the corresponding self-similar variable $\xi \propto s^{2/3} t^{1/3}$ differ substantially from those known for super-radiance of two-level atoms, for which $\xi \propto (st)^{1/2}$. This is connected with the fact that the singularity in the susceptibility of an $e^- e^+$ plasma near annihilation resonance is a root singularity, and not a linear one as it is near the frequency of a quantum transition in atoms. As a result, the dynamics of CA in the nonlinear stage also turns out to be different.

### 4. THE NONLINEAR STAGE

To determine the entire spatiotemporal profile of the pulse of $\gamma$ radiation it is necessary to take into account the nonlinear depletion of the concentration of annihilating $e^- e^+$ pairs. Roughly, by neglecting subtle kinetic effects associated with the redistribution of the particles over the momenta, we may assume that the radiation in a specified direction **s** at frequencies close to $\omega_0$ is generated by annihilation of $e^- e^+$ pairs with momenta $p_z$ in the interval of width $\Delta p_F$ (2.12) about the value $\hbar k_z/2$. Then the change of the averaged (over this range of momenta) occupation number $n_0(E_0)$ of the electron (or positron) states with energy $E_0 \approx \hbar\omega_0/2$ is determined by the energy-conservation law:

$$\Delta N_e \frac{\partial n_0}{\partial t} = \frac{\text{Im}(\mathscr{P}\mathscr{E}_z^*)}{2\hbar}, \qquad \Delta N_e = N_0 \frac{B}{B_c} \frac{\Delta p_F}{mc}. \qquad (4.1)$$

It describes the transformation of the total energy of the $e^- e^+$ pairs of the plasma into energy of $\gamma$ radiation as a result of the work of the field $\mathscr{E}_z$ on the current

$j_z = -i\omega_0\mathscr{P}$ of the annihilating pairs. In the indicated approximation it has been taken into account that the principal factor leading to the generation of radiation in the CA process is the disappearance, rather than the braking, of charges. It is this circumstance that leads to the essential differences of CA from ordinary (e.g., beam) instabilities in a plasma.[20]

In the resonance approximation the Fourier transforms of the polarization $\mathscr{P}$ and field $\mathscr{E}_z$ in the nonlinear stage are connected by the relation

$$\mathscr{P}(p,s) = -[b_0(2n_0-1)\omega_0^{1/2}\sin\theta\exp(i\pi/4)/2\pi p^{1/2}]\mathscr{E}_z(p,s). \quad (4.2)$$

It generalizes (2.7) to the case $n_0 \leqslant 1$ [with neglect of the small nonresonance contribution $\Phi(p_F,k_z)$], and includes the current local value of the occupation number $n_0(t,s)$, since the formation of the polarization response of an $e^- e^+$ plasma to an external influence obviously occurs more rapidly than the evolution of the distribution function of the particles. Going over in (4.2) to the originals, we obtain the constitutive equation in integral form:

$$\mathscr{P}(t,s) = -\frac{b_0}{2\pi}\omega_0^{1/2}\sin\theta\exp\left(\frac{i\pi}{4}\right) \times \int_0^t \frac{[2n_0(t',s)-1]\mathscr{E}_z(t',s)}{[\pi(t-t')]^{1/2}}dt'. \quad (4.3)$$

To analyze the nonlinear stage of CA we must solve Eqs. (4.1) and (4.3) jointly with the truncated wave equation

$$\frac{\partial\mathscr{E}_z}{\partial t} + \frac{c\partial\mathscr{E}_z}{\partial s} = 2\pi i\omega_0\mathscr{P}\sin^2\theta, \quad (4.4)$$

describing the propagation along **s** of a quasi-plane wave in the angular interval

$$\Delta\theta = \sin^2\theta\Delta p_F/mc \equiv 2(\omega''/\omega_0)^{1/2}$$

[it coincides with the diffraction interval $\sim(\lambda_0 L_c)^{1/2}/L_c$ for $s \sim L_c$, determined by the size $(\lambda_0 L_c)^{1/2}$ of the first Fresnel zone; $\lambda_0 = 2\pi/k_0$]. Neglecting the retardation, as in (3.3), we find a self-similar solution of this system

$$\mathscr{E}_z(t,s) = \mathscr{E}_{NL}[(6\cdot 3^{1/2})^{1/3}s/L_c]^{3/2}\mathscr{E}_A(\xi),$$
$$2n_0(t,s)-1 = \Delta n(\xi),\ \mathscr{E}_{NL}^2 = \hbar\omega_0\sin^2\theta\Delta N_e. \quad (4.5)$$

It is described by the ordinary integrodifferential equations

$$\frac{d\mathscr{E}_A}{d\xi} + \frac{9\mathscr{E}_A}{4\xi} = \mathscr{P}_A, \quad \frac{d\Delta n}{d\xi} = -(6\cdot 3^{1/2})^{1/3}\pi^{-1}\operatorname{Re}(\xi^3\mathscr{P}_A\mathscr{E}_A^*), \quad (4.6)$$

$$\mathscr{P}_A = \left(\frac{3}{\pi}\right)^{1/2}\exp\left(-\frac{i\pi}{4}\right)\xi^{-1}\int_0^\xi \frac{\Delta n(x)\mathscr{E}_A(x)x^2\,dx}{(\xi^3-x^3)^{1/2}}. \quad (4.7)$$

For $\xi \gg 1$ in the linear stage, when $\Delta n(\xi) \approx 1$, the solution of (4.6) has the intermediate asymptotic form

$$\mathscr{E}_A(\xi) \approx \mathscr{E}_A(0)\xi^{-9/4}\exp[\xi\exp(-i\pi/6)],$$

which, to within the unimportant pre-exponential factor, can be joined with the asymptotic form (3.4) found above. Therefore, the solution (4.6) and $\Delta n < 1$ is the continuation of the latter into the nonlinear stage. Here, owing to the exponential growth of the field, the initial values of the integral $\mathscr{P}_A$ in (4.6) for $\xi \lesssim 1$ up to the point of joining do not play any role. As a result, we obtain universal profiles of the field $\mathscr{E}_A$ and concentrations $\Delta n$ of $e^- e^+$ pairs as functions of the one self-similar variable $\xi$ (Fig. 4). They describe the spatiotemporal dependence of the rapid growth and decay of the coherent $\gamma$ radiation, confirming the estimates indicated at the end of Sec. 3 for the temporal and spatial scales of the CA. The self-similar solution (3.4), (4.5) has a relativistically covariant form and does not depend on the angle $\theta$ for given transverse dimensions $L_{x,y}$ of the bunch. Therefore, different groups $\Delta p_F(\theta)$ of $e^- e^+$ pairs in the vicinity of each momentum value $p_z = \hbar k_z/2$ generate similar CA pulses (with retardation by a factor of $\gamma = 1/\sin\theta$) with the same maximum amplitude of the $z$-component of the field, equal to $\mathscr{E}_m \approx (s/L_c)^{3/2}\mathscr{E}_{NL}/9$. For $s = L_c$ and $B = B_c$ the value $\mathscr{E}_m \approx 3\times 10^{-4}B_c$.

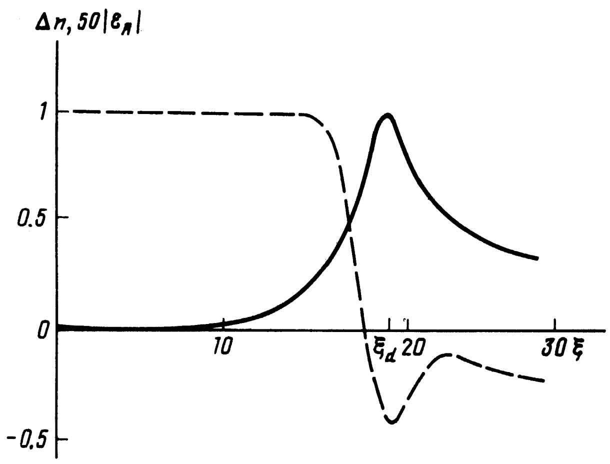


FIG. 4. Self-similar profiles of the field amplitude $|\mathscr{E}_A|$ (the solid curve), and the excess $\Delta n = 2n_0 - 1$ of the number of $e^- e^+$ pairs above the degeneracy threshold (the dashed curve), according to the resonance-approximation equations (4.6) and (4.7). The curves are plotted as function of $\xi \propto s^{2/3}t^{1/3}$ for $\mathscr{E}_A(0) = 10^{-5}$ and $\Delta n(0) = 1$.

The energetics of the process is dictated by the motion of the annihilation discharge front $\xi = \xi_d$ into the interior of a bunch of the $e^- e^+$ plasma. At the maximum of the CA pulse the Poynting vector of the generated quasi-plane wave with field amplitude $\mathscr{E}_z$ is determined by the rate of disappearance of $e^- e^+$ pairs, which is proportional to the discharge-front velocity $|ds/dt| = s/2t \equiv c(3^{1/2}s/L_c\xi_d)^3$:

$$c\mathscr{E}_z^2/8\pi\sin^2\theta \sim \hbar\omega_0\Delta N_e|ds/dt|. \quad (4.8)$$

From this the above-indicated maximum value $\mathscr{E}_z \sim \mathscr{E}_m$ follows immediately. The duration of a CA $\gamma$ pulse (at the 0.5 level) is found to be of the order of the time $t_d$ needed to reach the nonlinear stage of the CA on the boundary of the bunch.

## 5. ALLOWANCE FOR THE DIVERGENCE OF THE RADIATION AND THE FLYING APART OF PARTICLES OF THE $e^-e^+$ PLASMA

The angular divergence of the annihilation $\gamma$ radiation does not substantially alter the pattern of the CA in sufficiently uniform bunches of $e^- e^+$ plasma. The point is that waves propagating in difference directions **s** are incoherent and are amplified independently of each other. Therefore, it should be expected that for sufficiently extended bunches (see footnote 2) the dynamics of the generation of the $\gamma$ radiation will be determined by the self-similar amplification law (3.4), (4.5)–(4.7) along each direction **s** (with diffraction accuracy).

To illustrate what has been said about the angular divergence, we may consider the model linear problem of the annihilation instability of a cylindrical linearly polarized ($\mathbf{e}\|\mathbf{B}$) wave propagating at a right angle to the axis $\mathbf{z}^0\|\mathbf{B}$ of a sample of $e^- e^+$ plasma in the form of a circular cylinder. (the fact that the dominant generation of CA radiation is in the transverse ($\theta\sim\pi/2$) and not in the longitudinal ($\theta\ll 1$) direction is determined by the angular dependence of the increment (2.10): $\omega''\propto\sin\theta$). If we seek the asymptotic (at large distances $r\gg\lambda_0$ from the axis of the cylinder) solution of the inhomogeneous wave equation in the form

$$e(t,r)=\mathscr{E}(t,r)\,[H_m^{(1)}(k_0r)+H_m^{(2)}(k_0r)]\,e^{-i\omega_0 t}\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}, \tag{5.1}$$

then, for the amplitude of the field $\mathscr{E}$ we arrive at the previous asymptotic form (3.4) apart from an unimportant pre-exponential factor and a change of the effective amplification length by an amount $\sim\lambda_0$. Thus, the entire change induced in the spatial structure of the field by the angular divergence is taken into account by the asymptotic form

$$H_m^{(1)}\sim[2/\pi k_0r]^{1/2}\exp(ik_0r)$$

of the Hankel function, and, in essence, the temporal dynamics does not change.

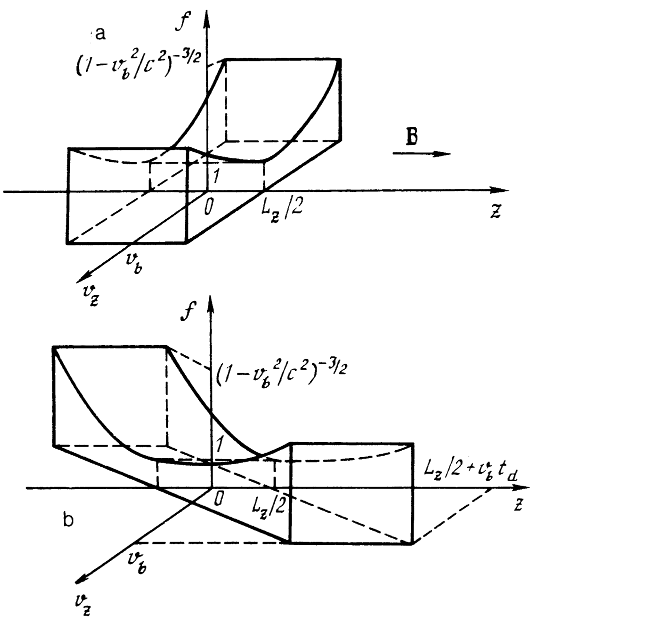


FIG. 5. Spatiotemporal evolution of the distribution function $f(v_z,t,z)$ of the particles of a degenerate $e^- e^+$ plasma in the process of kinematic flying apart along the magnetic field: a) at the initial time $t=0$; b) at the time of the maximum of the annihilation $\gamma$ radiation, i.e., at the delay time $t_d$. The boundary velocity $v_b p_F (m^2+p_F^2/c^2)^{-1/2}$.

The model problems solved above make it possible to present a picture of the development of the CA of an arbitrary three-dimensional bounded bunch of $e^- e^+$ plasma. We need only take it into account that for bunches that are not too extended along the $z$ axis ($L_z\ll ct_d(1+m^2c^2/p_F^2)^{1/2}$ spatiotemporal evolution of the velocity distribution function $f(v_z,t,r)$ occurs as a consequence of the kinematic flying apart of particles along the magnetic field (Fig. 5). Groups of $e^- e^+$ pairs with a spread $\Delta p_F$ (2.12) of momenta about the value $p_z=\hbar k_z/2$ fly away from each other with velocities $v_z=c\cos\theta$, and radiate, at the corresponding times $t_d(\theta)$, practically independently at different spatial points—each at its own frequency $\omega_0=2mc^2/\hbar\sin\theta$ and at its own angle $\theta=\arccos(z/ct_d)$. The complete radiation cone is determined by the range of angles $\theta\in(\theta_{\min},\pi-\theta_{\min})$; the fastest group of $e^- e^+$ pairs, with momenta $p_z\approx p_F$, emits at the angle $\theta_{\min}=\arctan(mc/p_F)$. To summarize, the regions of generation of $\gamma$ radiation with various frequencies and angles is spread out along the $z$ axis (Fig. 6). The CA emission occurs equally from each of the indicated groups of $e^- e^+$ pairs in its own reference frame (moving with velocity $v_z=c\cos\theta$), according to the relativistically covariant self-similar solution (see Sec. 4). For the existence of CA it is necessary only that the longitudinal dimension $L_z$ of the initial bunch be greater than the distance $\Delta L_z$ by which the particles inside the group have flown apart by the time $t_d$ [see (3.5)]:

$$L_z>\Delta L_z=t_d\Delta v_z\equiv 2ct_d\sin\theta\,(\omega''/\omega_0)^{1/2}. \tag{5.2}$$

Then the change of the distribution function inside a given group of $e^- e^+$ pairs as a result of the kinematic flying apart is small, and the pair concentration obeys Eq. (4.1). For

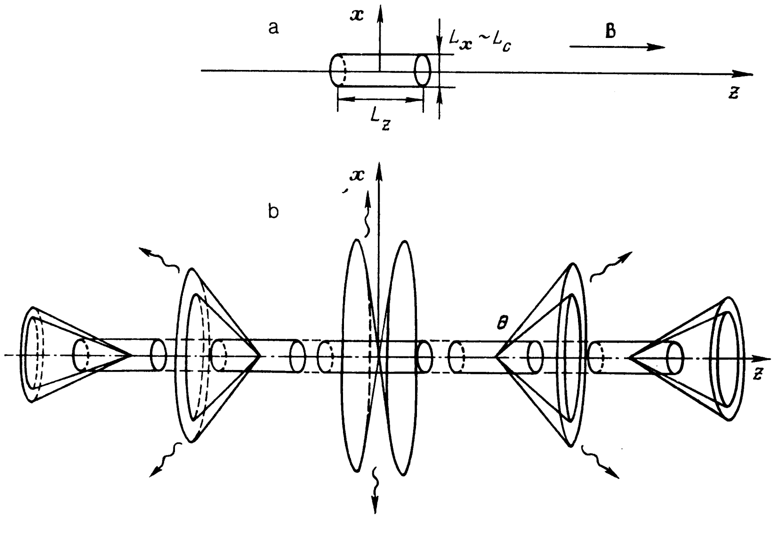


FIG. 6. (a) Initial plasma bunch; (b) geometry of the annihilation radiation, at different angles $\theta$ and on different frequencies $\omega_0=2mc^2/\hbar\sin\theta$, from groups of $e^- e^+$ pairs flying apart from the initial plasma bunch with different longitudinal momenta $p_z=\hbar k_z/2\approx mc\cot\theta$ at the time $t_d$.

$\sin\theta \sim 1$, for $s \gtrsim L_c$, the shortest distance beyond which the CA is suppressed by the flying apart is $\Delta L_z \sim L_c \xi_d (\omega''/\omega_0)^{1/2}$, i.e., of the order of fractions of an Ångstrom.

We estimate the power of the coherent $\gamma$ radiation from each group of $e^- e^+$ pairs with sizes $L_x \sim L_y \sim L_c$ and $L_z \sim \Delta L_z$ by taking into account that in the time $\sim t_d$ approximately half of the $e^- e^+$ pairs of the group annihilate:

$$\Delta W \sim \frac{\hbar\omega_0 \Delta N_e L_c^2 \Delta L_z}{2t_d} = \frac{\mathscr{E}_{NL}{}^2 L_c^2 c}{\sin\theta}\left(\frac{\omega''}{\omega_0}\right)^{1/2}. \quad (5.3)$$

For $B \sim B_c$ and $\sin\theta \sim 1$ we have $\Delta W \sim 10^8$ W. The total CA power of a whole bunch of $e^- e^+$ plasma is greater than the indicated power (5.3) by a factor equal to the number of groups and by a length factor:

$$W \sim \Delta W \frac{2p_F}{\Delta p_F} \frac{L_z}{\Delta L_z}.$$

For $p_F \sim mc$, when $2p_F/\Delta p_F \approx 20$, and $L_z \Delta L_z \approx 5$, we have $W \sim 10^{10}$ W. This value corresponds, for example, to the annihilation of $\sim 10^6$ $e^- e^+$ pairs with concentration $N_e \sim 10^{30}$ cm$^{-3}$ in a volume $\sim 1$ Å$^3$ in a time $t_d \sim 3\times 10^{-18}$ sec.

It can be shown that CA leads to a considerably higher spectral and volume power density of $\gamma$ radiation than does incoherent spontaneous annihilation. For $B \sim B_c$, $s \gtrsim 3L_c$ and $\theta = \pi/2$, their ratio $W_{\omega\Omega}/W^{sp}_{\omega\Omega} \gtrsim 10^2$, and increases both with decrease and with increase of the magnetic field.

## 6. CONCLUDING REMARKS

In conclusion we shall give an assessment of the role of the incoherent processes that compete with the coherent CA, and formulate a number of open problems related to the observation and theoretical description of the phenomenon under consideration. Amongst the competing processes are spontaneous and collisional relaxation, which disrupt the phase of the current of the pairs participating in the CA. These processes are primarily single-photon and two-photon spontaneous annihilation and electron-electron ($e^- e^-$, $e^+ e^+$) and Compton ($\gamma e^-$, $\gamma e^+$) collisions.

We shall estimate the rate of spontaneous single-photon annihilation as the inverse lifetime of a positron (e.g., with momentum $p_z' = 0$) with respect to independent annihilation with all the electrons of a degenerate $e^- e^+$ gas in the lowest Landau level:

$$R_{1\gamma} \equiv \tau_{1\gamma}^{-1} = \frac{eB}{2\pi\hbar c}\int_{-p_F}^{p_F} \frac{c^2 p_z}{E_0(p_z)} \sigma_{1\gamma}(p_z) \frac{dp_z}{2\pi\hbar}$$

$$= \frac{2\alpha mc^2}{\hbar} \exp\left(-\frac{B_c}{B}\right) \int_0^{p_F/mc} \frac{\exp[-(B_c/B)(1+x^2)^{1/2}]\,dx}{(1+x^2)^{1/2}}. \quad (6.1)$$

Here we have used the well known expression for the cross section for single-photon annihilation in the rest frame of the positron:[1]

$$\sigma_{1\gamma}(p_z) = 4\pi^2\alpha\lambda_c^2 \frac{mcB_c}{p_z B} \exp\left\{-\left[1+\left(1+\left(\frac{p_z}{mc}\right)^2\right)^{1/2}\right]\frac{B_c}{B}\right\}. \quad (6.2)$$

As can be seen from Fig. 3, in a wide range of concentrations and magnetic fields spontaneous single-photon annihilation proceeds with a rate (6.1) much lower than the CA rate: $R_{1\gamma} \ll \omega''$. Only at a very high concentration, when $R_{1\gamma} t_d \gtrsim 1$, can spontaneous annihilation lead to some reduction of the total concentration of the $e^- e^+$ plasma by the time $t_d$ of formation of the collective $\gamma$ radiation. However, since $\omega'' \gg R_{1\gamma}$, this will not lead to complete suppression of CA, just as energy relaxation with time $T_1 \gtrsim t_d$ does not suppress the super-radiance of an active medium.[8-10] Qualitatively, this effect can be described in the adiabatic approximation, by introducing a relaxation term $-\tau_{1\gamma}^{-1} n_0(t,s)$ into Eq. (4.1).

The rate of spontaneous two-photon annihilation in strong fields $B \gtrsim B_c/4$, as shown in Ref. 1, is lower than the single-photon rate ($R_{2\gamma} \lesssim R_{1\gamma}$), and for $R_{1\gamma} \ll \omega''$ its role is certainly insignificant. With somewhat lower magnetic fields ($B < B_c/4$) the rate of spontaneous single-photon annihilation naturally decreases in relation to the two-photon rate: $R_{1\gamma} < R_{2\gamma}$; however, for relatively low concentrations, when $\Delta p_F \lesssim p_F \ll mc$, the increment of the single-photon CA can remain greater than the rate of spontaneous two-photon annihilation, so that the inequality $R_{2\gamma} \ll \omega''$ remains valid. Only upon further decrease of the magnetic field ($B < 0.1 B_c$) does the two-photon annihilation process become decisive. We note, however, that in this case too, for a sufficiently high concentration of plasma, even two-photon annihilation can apparently have a collective character.

The estimate given in Ref. 21 for the frequency of electron-electron collisions in a magnetic field $B \sim B_c$ shows that in a wide range of concentrations, up to $p_F \sim mc$, its magnitude is small: $\nu_{ee} \ll t_d^{-1}$. Only above $p_F \gtrsim 10mc$ can this inequality be violated. Furthermore, according to Ref. 22, the cross section for Compton scattering of a $\gamma$ quantum with frequency close to the threshold frequency by an electron in a magnetic field $B \sim B_c$ has a value of the order of a few Thomseon cross sections $\sigma_T \approx 7\times 10^{-25}$ cm$^2$. Extending this estimate to the annihilation $\gamma$ quanta of interest to us, we find that their Compton mean free path $l_\gamma$ in the conditions under consideration is greater than the cooperative length $L_c$, i.e., the scale of the effective amplification of the $\gamma$ radiation. For example, for $N_e \sim 10^{30}$ cm$^{-3}$, we have $l_\gamma \sim 100$ Å $\gg L_c \sim 1$ Å. To summarize, neither of the incoherent processes that we have discussed forbids the realization of single-photon CA.

The question of the observation of the indicated phenomenon remains open. To supplement what was said in the Introduction, we note that the strong magnetic fields $B \sim B_c$ required for this are realistic both for astrophysical conditions (neutron stars) and for terrestrial conditions (collisions of heavy ions[23]). Bunches of $e^- e^+$ plasma of the necessary density $N_e \gtrsim 10^{27}$ cm$^{-3}$ and necessary sizes $s \gtrsim 1$ Å can evidently be formed in collisions of relativistic particles and during their motion in strong accelerating electromagnetic fields, both in the magnetospheres of compact astrophysical objects and in laboratory conditions. Thus, in Ref. 5 there is a discussion of a hypothetical experiment with colliding electron and positron beams, which, after separate acceleration in toroidal chambers, merge and then collapse (pinch). As a result, in the opinion of the author of Ref. 5, who takes only spontaneous annihilation into account, the $e^- e^+$ plasma bunch is compressed to a density of the order of nuclear density ($10^{15}$ g/cm$^4$) and an azimuthal magnetic field with an intensity at the surface of the bunch of $10^{16}$–$10^{17}$ G is generated. The collective process considered in the present article will lead, most probably, to annihilation of the $e^- e^+$-plasma bunch, with emission of coherent $\gamma$ radiation, considerably earlier, when the density is still several orders of magnitude smaller than that indicated by the author of Ref. 5 and the magnetic field is $\sim 10^{13}$ G.

Finally, we indicate a number of questions that have remained outside the scope of this article.

a) The profile of the trailing edge of the pulse of collective $\gamma$ radiation and the law of its re-absorption as it propagates beyond the limits of the annihilating bunch, with allowance for the inhomogeneous and nonstationary $e^- e^+$ plasma that it generates from the vacuum.

b) The role of kinetic effects associated with the redistribution of particles in momentum space.

c) Allowance for the nonzero temperature, the population of the excited Landau levels, and radiative processes of higher orders.

d) The statistical (fluctuation and correlation) effects that arise in the generation and propagation of coherent $\gamma$ radiation in an $e^- e^+$ plasma on the boundary of the region of applicability of the continuous-medium approximation, when the interparticle distances $N_e^{-1/3}$ are greater than or of the order of the wavelength $\lambda_0$.

e) The analysis of analogous CA processes and the generation of coherent $\gamma$ radiation (annihilation super-radiance) in other systems, e.g., the two-photon CA of an $e^- e^+$ plasma, super-radiance in interband electron-hole transitions in semiconductors, in free-to-bound transitions in a recombining electron-ion plasma, etc. As is shown by the above-considered example of single-photon CA of an $e^- e^+$ plasma, the dynamics of the generation of collective spontaneous radiation in such active media with a continuous energy spectrum can differ substantially from the dynamics of super-radiance in media with a discrete energy spectrum.

## 7. CONCLUSIONS

The results obtained permit us to assert the following:

1. There exists a process of collective annihilation of a sufficiently dense [degenerate; see (2.13)] $e^- e^+$ plasma with concentration $N_e \gtrsim 10^{27}$ cm$^{-3}$ in a strong magnetic field $B \gtrsim 10^{13}$ G that occurs via coherent acts of single-photon annihilation of $e^- e^+$ pairs and develops considerably faster than incoherent processes of spontaneous and collisional relaxation.

2. This process is due to instability of the extraordinary normal waves in a narrow band of wave numbers near the threshold wave number $k_0 = 2mc/\hbar \sin\theta$ with maximum increment $\omega''$ (2.10).

3. In the CA process, spontaneous fluctuations of the field and polarization in a bunch of $e^- e^+$ plasma develop into powerful coherent $\gamma$ radiation. In the unidirectional model of propagation the law of increase of the radiation is described by the intermediate asymptotic form (3.4) in the linear stage and by the self-similar solution (4.5)–(4.7) [which is joined to (3.4)] in the nonlinear stage, both of which depend on the single combination $\xi \propto s^{2/3} t^{1/3}$. This specifies the spatial and temporal scales of the process. For example, for $B \sim B_c$ the spatial scale $s$ is of the order of the cooperative length $L_c = c/\omega'' \sim 1$ Å, the CA-delay time $t_d \approx 3 \times 10^{-18}$ sec, and the duration of the leading front of the pulse is $\tau \approx 3t_d/\xi_d$ [see (3.5)]. With decrease of the magnetic field $B$, all the indicated scales $L_c$, $t_d$, and $\tau$ increase significantly.

4. The spatial divergence of the radiation and the geometrical shape of the annihilating $e^- e^+$-plasma bunch do not qualitatively change the above-indicated dynamics of the process, since the decisive factor is the exponential growth of the coherent $\gamma$ radiation along the quasi-one-dimensional (with small diffraction divergence) rays in the direction of the maximum amplification coefficient $\xi$. The latter is determined by the combined action of the factor describing the angular dependence of the increment ($\omega'' \propto \sin\theta$) and the factor $s(\theta)$ describing the geometrical length of the plasma bunch [see (3.4)].

5. Kinematic flying apart of a not too short [see (5.2)] $e^- e^+$ bunch along the magnetic field leads only to the spatial separation of groups of $e^- e^+$ pairs generating coherent $\gamma$ radiation on different frequencies and at different angles to the direction of the magnetic field (Fig. 6), built not to suppression of the CA process itself.

There are two possible subsequent fates of the annihilation $\gamma$ quanta as they propagate in the vacuum beyond the limits of the initial $e^- e^+$-plasma bunch. In those situations where the magnetic field is strongly nonuniform in magnitude and direction (on scales $\sim L_c$ and over times $\sim t_d$), the $\gamma$ quanta radiated outwards turn out to be far, in frequency and direction, from the annihilation resonance of the magnetized vacuum, and can propagate without substantial absorption. A similar variant should arise in the case of localization of the magnetic field near the $e^- e^+$-plasma bunch, e.g., if the magnetic field is produced by internal plasma currents and vanishes as the plasma is annihilated up to the time of generation of the powerful $\gamma$ radiation. The latter pertains, e.g., to situations analogous to the experiment[5] that was discussed in Sec. 6. In such cases the collectively annihilating plasma serves as a source of coherent $\gamma$ radiation. In those situations where the magnetic field is quasi-uniform in space in time, the $\gamma$ quanta will remain close to annihilation resonance and will produce $e^- e^+$ pairs again, at a distance $\sim L_c$ from the initial plasma sample. (Such absorption awaits all the annihilation $\gamma$ radiation emitted in the depth of the plasma sample (see footnote 2) if the same length $s$ is much greater than the cooperative length $L_c$; (cf. the absorption of super-radiance in a two-level active medium.[8-10]) In these conditions the indicated process leads in effect to rapid and effective transfer of electrons and positrons across the magnetic field by way of absorption of $\gamma$ quanta, i.e., to the phenomenon of anomalous transverse diffusion of magnetized plasma. As a result, what is realized is not only kinematic flying apart of the plasma along the magnetic field but also spreading of the plasma across the magnetic field, despite the impossibility of direct motion of the magnetized electrons and positrons across the field **B**. In this case, an external observer will "see" only the burst of incoherent $\gamma$ radiation formed by the $\gamma$ quanta that have emerged from the region of effective pair creation as a result of Compton scattering.

In different concrete situations the ideas that we have described can serve, in our view, as a basis for estimates of the role of CA in a number of other elementary processes involving the dynamics of the energy release, decay, and radiation of a dense cosmic or laboratory $e^- e^+$ plasma in strong magnetic fields.

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[1)] The second [different from (2.9)] complex root (2.4) turns out to be superfluous, i.e., is not a solution of the dispersion equation by virtue of the Landau rule for calculating the integral (2.1). The third, real root of (2.4) corresponds to the excitation of "light-positronium,"[17] and is not important for the annihilation instability.

[2)] The regular solutions considered here and below are valid only over lengths $s$ that do not exceed the length $\sim ct_d \approx L_c \xi_d /2$, i.e., a few cooperative lengths $L_c \sim 1$ Å. Otherwise, over different segments of length $\sim ct_d$, independent CA $\gamma$ pulses will be formed from the quantum noise and will form, in their propagation and absorption in the plasma, an irregular sequence.


[1] G. Wunner, J. Paez, H. Herold, and H. Ruder, Astron. Astrophys. **170**, 179 (1986).
[2] J. Arons, in *101 AIP Conference Proceedings,* edited by M. L. Burns, A. K. Harding, and A. Ramaty (New York, 1983), p. 163.
[3] A. P. Lightman and A. A. Zdziarski, Astrophys. J. **319**, 643 (1987).
[4] S. Rogers and W. B. Thompson, Astrophys. Space Sci. **71**, 257 (1980).
[5] F. Winterberg, Phys. Rev. A **19**, 1356 (1979).
[6] H. Satz, Nature **324**, 116 (1986); **327**, 660 (1987).
[7] V. N. Vysotskiĭ and R. N. Kuz'min, *Gamma Lasers* [in Russian] (Izd. Mosk. Gos. Univ., Moscow, 1989).
[8] M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982).
[9] A. V. Andreev, V. I. Emel'yanov, and Yu. A. Il'inskiĭ, *Cooperative Phenomena in Optics* [in Russian] (Nauka, Moscow, 1988).
[10] V. V. Zheleznyakov, V. V. Kocharovskiĭ, and Vl. V. Kocharovskiĭ, Usp. Fiz. Nauk **159**, 193 (1989) [Sov. Phys. Usp. **32**, 835 (1989)].
[11] É. M. Belenov and P. P. Vasil'ev, Zh. Eksp. Teor. Fiz. **96**, 1629 (1989) [Sov. Phys. JETP. **69**, 922 (1989)].
[12] R. A. Vlasov, O. N. Gadomskiĭ, and V. V. Samartsev, Teor. Mat. Fiz. **79**, 423 (1989) [Theor. Math. Phys. (USSR) **79**, 631 (1989)].
[13] V. V. Zheleznyakov, V. V. Kocharovskiĭ, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **29**, 1095 (1986) [Radiophys. Quantum Electron. **29**, 830 (1986)].
[14] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, transl. of 1st Russian ed. (Pergamon Press, Oxford, 1960) [Russ. original, Nauka, Moscow, 1982].
[15] V. L. Ginzburg, *Theoretical Physics and Astrophysics* (Pergamon Press, Oxford, 1979) [Russ. original, later ed., Nauka, Moscow, 1987].
[16] V. V. Kocharovskii and Vl. V. Kocharovskii, in *Nonlinear Waves*, edited by A. V. Gaponov-Grekhov and M. I. Rabinovich (Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1990).
[17] A. E. Shabad, Tr. Fiz. Inst. Akad. Nauk SSSR **192**, 5 (1988).
[18] G. I. Svetozarova and V. N. Tsytovich, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **5**, 658 (1962).
[19] D. B. Melrose and A. J. Parle, Aust. J. Phys. **36**, 799 (1983).
[20] L. A. Artsimovich and R. Z. Sagdeev, *Plasma Physics for Physicists* [in Russian] (Nauka, Moscow, 1979).
[21] S. H. Langer, Phys. Rev. D **23**, 328 (1981).
[22] H. Herold, Phys. Rev. D **19**, 2868 (1979).
[23] J. Rafelski and B. Müller, Phys. Rev. Lett. **36**, 517 (1976).


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