Radiative corrections to the motion of an electron in a plane-wave electromagnetic field

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Using the method of dispersion relations we obtain the amplitude for elastic scattering, classified by the spin polarization states of the electron, in an arbitrary plane-wave electromagnetic field. We consider radiative corrections to the motion of the electron in a field formed by the superposition of a constant crossed field and a plane electromagnetic wave with elliptic polarization. We determine the anomalous magnetic and electric moments of the electron in such a field.

1. INTRODUCTION

Considerable success has been achieved recently in the field of theoretical studies of radiative corrections to the motion of particles in external electromagnetic fields. Of particular interest is the study of the dependence of radiative effects on the parameters of the external field, which can be easily varied by the experimenter.

In this work we consider radiative corrections to the motion of an electron in an arbitrary plane-wave electromagnetic field, whose vector potential can be expressed as:

$$A_\mu = A_\mu(\xi),\ \xi = (n_\mu x^\mu),\ n^2 = (n^\nu A_\nu) = 0. \tag{1}$$

Based on general expressions obtained for the radiation probability, classified according to the polarization spin states of the electron in the field (1), we find by means of dispersion relations the elastic scattering amplitude of the electron. The method of dispersion relations, proposed in Ref. 1 and rigorously grounded in quantum field theory in Ref. 2, is one of the main methods of theoretical investigations of the interaction of elementary particles.

The development of the method of dispersion relations in application to the study of radiative corrections to the motion of an electron in a constant crossed field was given in Ref. 3 and extended in Ref. 4 to the case when the external field is a superposition of a constant uniform magnetic field and a plane circularly polarized electromagnetic wave.

One of the most important special cases of the field (1) is the superposition of a plane elliptically polarized electromagnetic wave and a constant crossed field ($\mathbf{E}_0 \perp \mathbf{H}_0$, $|\mathbf{E}_0| = |\mathbf{H}_0|$). The vector potential for such a field can be written in the following form:[5]

$$\mathbf{A}(\xi) = a\xi(\mathbf{e}_1 \sin\beta + \mathbf{e}_2 \cos\beta) - (A_0/\omega)(\mathbf{e}_1 \cos\psi \sin\omega\xi - \mathbf{e}_2 \sin\psi \cos\omega\xi). \tag{2}$$

The angle $\psi(-\pi \leqslant 2\psi \leqslant \pi)$ characterizes the polarization of the wave. Here the condition $2\psi = 0, \pi$ corresponds to a linearly polarized wave, and $4\psi = g\pi$, $g = \pm 1$, corresponds to right (for $g = 1$) and left (for $g = -1$) circular polarization. In Eq. (2) $\mathbf{e}_i$ ($i = 1, 2$) is a unit vector in the plane perpendicular to the wave propagation vector $\mathbf{n}$:

$$\mathbf{e}_1\mathbf{e}_2 = \mathbf{e}_1\mathbf{n} = \mathbf{e}_2\mathbf{n} = 0,\ \mathbf{n}^2 = \mathbf{e}_1^2 = \mathbf{e}_2^2 = 1,$$

β is the angle of inclination of the principal axes of the ellipse with respect to the vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ of the crossed field of intensity a, ω is the frequency, A_0 is the amplitude of the wave.

The study of various processes in a field, which is the superposition of constant and varying fields, is of considerable interest, since in this type field effects are possible similar to those that occur separately in constant and varying fields, as well as qualitatively new ones.

The presence of the constant crossed field in the superposition (2) makes it possible to correctly separate in the expression for the elastic scattering amplitude of the electron two terms, linearly dependent on the electron polarization ζ and proportional to $\zeta\cdot\mathbf{H}_0$ and $\zeta\cdot\mathbf{E}_0$, and interpret them in the following as the interaction energy of an anomalous magnetic moment of the electron with the magnetic field $\mathbf{H}_0$ and an anomalous electric moment of the electron with the electric field $\mathbf{E}_0$. The anomalous moments turn out to be complicated functions of all the paramters that characterize the superposition (2) as well as of the electron energy; the anomalous electric moment appears in the field (2) provided $\sin 2\beta \neq 0$ and provided an electromagnetic wave with noncircular polarization ($4\psi \neq g\pi$) is present.

An anomalous electric moment of the electron, induced by a constant field ($\mathbf{E}\cdot\mathbf{H} \neq 0$), was studied in Ref. 6, while an anomalous electric moment of the neutrino was studied in Refs. 5 and 7. The obtained mass shift, divided between the polarization states of the electron, permits one to follow the contribution to the correction in the electron mass due to electron transitions into intermediate states without and with electron spin flip.

2. ELECTRON RADIATION PROBABILITY

The probability of emission of a photon with 4-momentum $\varkappa_\mu$ and polarization e_μ with the electron going from the state Ψ_q to the stateo $\Psi_{q'}$, referred to unit volume V and unit time T, is given by the formula

$$w = \frac{1}{TV} \int \frac{d^3\varkappa}{(2\pi)^3} \frac{d^3 q'}{(2\pi)^3} \left| \frac{e}{(2\varkappa_0)^{1/2}} \int \bar{\Psi}_{q'} (e_\mu \gamma^\mu) \Psi_q e^{-i\varkappa x} dx \right|^2. \tag{3}$$

Substituting into (3) the electron wave functions obtained by Volkov[8] and expanding the functions of nx in a Fourier integral, we obtain after integration over x a four-dimensional δ-function, with the help of which we remove the integration over $d^3\varkappa$ and ds_1 after evaluation of the trace of the Pauli

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matrices (s_1 is the variable of integration in the Fourier integral):

$$d^3\varkappa ds_1\delta^{(4)}(q-q'-\varkappa+ns_1)\to ds_1\delta(q_0-q_0'-\varkappa_0+s_1)\to\varkappa_0(n\varkappa)^{-1}.$$

For the integration over $\mathbf{q}'$ we have introduced the following variables:

$$t=\frac{n\varkappa}{nq},\quad \mathbf{x}_\perp=\frac{\mathbf{q}_\perp'}{nq'}-\frac{\mathbf{q}_\perp}{nq}-\frac{t\mathbf{B}}{nq},\qquad \mathbf{B}=\frac{e}{v}\int_{u-v/2}^{u+v/2}\mathbf{A}(\xi)d\xi,$$

$$\xi_1=u+v/2,\ \xi_2=u-v/2,\ d\xi_1 d\xi_2 d^3q'|=dudvdtd\mathbf{x}_\perp q_0'(nq)^2(1+t)^{-3}.$$

The integration over the variables $\mathbf{x}_\perp=(x_1,x_2)$ reduces to Gaussians.

After performing the indicated integrations in (3) we obtain for the photon emission probability by the electron in the field (1) the following expression:

$$w=-i\frac{\alpha m^2}{4\pi q_0}\int_{-\infty}^{\infty}\frac{du}{2\pi L_\xi}\int_{-\infty}^{\infty}\frac{dv}{v}\int_0^{\infty}\frac{t^2\,dt}{(1+t)^3}(Ne^{i\eta}-N_0e^{i\eta_0}),\tag{4}$$

where

$$N=\delta_{\zeta\zeta'}\left\{2\frac{1+t}{t^2}+(\boldsymbol{\zeta}\mathbf{n})(\boldsymbol{\zeta},\mathbf{n}-\mathbf{F}_1-\mathbf{F}_2)+\left(1+(\boldsymbol{\zeta}\mathbf{n})^2+4\frac{1+t}{t^2}\right)\mathbf{F}^2-i\left(1+\frac{2}{t}\right)[(\mathbf{F}[\boldsymbol{\zeta}\mathbf{n}])+4(\boldsymbol{\zeta}[\mathbf{F}_1\mathbf{F}_2])]\right\}$$

$$+\delta_{\zeta,-\zeta'}[\boldsymbol{\zeta}_\perp^2\mathbf{F}^2-(\boldsymbol{\zeta}\mathbf{n})(\boldsymbol{\zeta},\mathbf{n}-\mathbf{F}_1-\mathbf{F}_2)+i(\mathbf{F}[\boldsymbol{\zeta}\mathbf{n}])],$$

$$\eta=\frac{m^2vt}{2nq}-\frac{tv\mathbf{B}^2}{2nq}+\frac{e^2t}{2nq}\int_{u-v/2}^{u+v/2}\mathbf{A}^2(\xi)d\xi,$$

$$\eta_0=\eta(A=0),\qquad N_0=N(A=0),$$

$$\alpha=\frac{e^2}{4\pi},$$

$$\mathbf{F}=\mathbf{F}_1-\mathbf{F}_2,\qquad \mathbf{F}_\nu=\frac{1}{2m}(e\mathbf{A}_\nu-\mathbf{B}),$$

$$A_\nu=A\left(u-(-1)^\nu\frac{v}{2}\right),\qquad \nu=1,2,\qquad L_\xi=\delta(0)$$

$\delta_{\mu\nu}$ is the Kronecker symbol, and $\boldsymbol{\zeta}=\zeta\mathbf{l}(\zeta=\pm1)$ characterizes the two possible orientations of the electron spin component along the unit vector $\mathbf{l}$.[9] The terms proportional to $\delta_{\zeta,\zeta'}$ describe emission without spin flip while the terms proportional to $\delta_{\zeta,-\zeta'}$—with spin flip of the electron.

We note that the dependence of the emission probability with electron spin flip ($\propto\delta_{\zeta,-\zeta'}$) on ζ is an indication of the fact that in an arbitrary plane-wave field as a result of photon emission the electron may become polarized under certain conditions ($\mathbf{F}\cdot\boldsymbol{\zeta}\times\mathbf{n}\neq0$). Since the radiation by the electrons is treated in perturbation theory, but the exact solutions of the Dirac equation are used for the unperturbed wave functions, the resultant Eq. (4) describes the probability of the process under study with the external arbitrary plane-wave electromagnetic field taken into account exactly.

3. ELECTRON ELASTIC SCATTERING AMPLITUDE

According to the optical theorem, the probability of radiation by the electron in an external field determines the imaginary part of the elastic scattering amplitude in this field. Consequently, from a knowledge of the radiation probability we can reconstruct the real part of the amplitude by means of dispersion relations.

The mathematical basis of dispersion relations is the integral Cauchy formula,[2] which establishes a connection between the real and imaginary parts of a function of a complex variable. In our case the role of the integration variable z' will be played by the parameter $(nq)^{-1}$. The dependence of the radiation probability (4) on this parameter [$nq=q_0-q_3$, p_0 is the quasi-energy, $q_3=\mathbf{n}\cdot\mathbf{q}$, $\mathbf{n}=(0,0,1)$, $\mathbf{q}$ is the quasi-momentum of the electron] is contained only in the exponent so that in the calculation of the real part of the amplitude we need the integrals[10]

$$\int_{-\infty}^{\infty}\frac{dz'}{z'-z}(\sin z'\tau,\cos z'\tau)=\pi(\cos z\tau,-\sin z\tau).\tag{5}$$

Taking into account that the imaginary part of the elastic scattering amplitude T is connected to the total radiation probability w by the relation $2\,\mathrm{Im}\,T=w$, and making use of Eqs. (4) and (5), we obtain after evaluating the real part of T the following expression for the elastic scattering amplitude of the electron in the field (2):

$$T=\mathrm{Re}\,T+i\,\mathrm{Im}\,T=-\frac{\alpha m^2}{4\pi q_0}\int_{-\infty}^{\infty}\frac{du}{2\pi L_\xi}\int_0^{\infty}\frac{dv}{v}\int_0^{\infty}\frac{t^2\,dt}{(1+t)^3}(N^+e^{-i\eta}-N_0e^{-i\eta_0}),\tag{6}$$

where N^+ stands for the complex conjugate of N.

In this manner the existence of electron radiation (4) in an arbitrary plane-wave electromagnetic field (1) leads to elastic scattering, whose amplitude, classified by polarization states, is determined by Eq. (6). Performing in (6) the summation over the electron polarizations in the final state (ζ') we obtain an expression for the electron elastic scattering amplitude, polarized in the initial state, which could also be obtained by evaluating the electron mass operator in the field (1).[11,12] Substituting (2) into (6) and making use of the familiar expansion of a plane wave in terms of Bessel functions we obtain for the elastic scattering amplitude in the field (2), after needed transformations and integration over the variable u, the following expression:

$$T=\frac{\alpha m^2}{4\pi q_0}\int_0^{\infty}\frac{d\tau}{\tau}\int_0^{\infty}\frac{t^2\,dt}{(1+t)^3}\left\{N_0e^{-i\eta_0}-\sum_{s=-\infty}^{\infty}i^se^{i\varphi}J_s(x_1)J_{2s}(x_2)\times[(R_1-R_2)\delta_{\zeta,\zeta'}+R_2\delta_{\zeta,-\zeta'}]\right\},$$

$$R_1=2\frac{1+t}{t^2}+2\left(1+2\frac{1+t}{t^2}\right)G_1-\frac{2}{t}G_2-hb(\boldsymbol{\zeta}\mathbf{n})\sin2\psi\times\left(1+\frac{2}{t}\right)\left[ib\sin x+\frac{\tau}{cz^{1/2}}\left(L_{2s}-\frac{2is}{x_2}\sin2\beta\,\mathrm{ctg}\,2\psi\right)\right],$$

$$R_2=G_2-(\boldsymbol{\zeta}\mathbf{n})^2+\boldsymbol{\zeta}_\perp^2G_1-\frac{bh(\boldsymbol{\zeta}\mathbf{n})}{2ca}\left[(\boldsymbol{\zeta}\mathbf{H}_0)\sin2\psi\left(iL_{2s}+2\frac{s}{x_2}\sin2\beta\,\mathrm{ctg}\,2\psi\right)-4\frac{c^2s}{x_2}(\boldsymbol{\zeta}\mathbf{E}_0)\right],\tag{7}$$

$$G_1=\frac{\tau^2}{z}+\frac{1}{2}b^2\sin^2 x(1-iL_s\cos 2\psi)-2i\frac{c\tau}{z^{1/2}}bL_{2s}\sin x,$$

$$G_2=\frac{(\zeta\mathbf{H}_0)}{a}\left(\frac{i\tau}{z^{1/2}}+cbL_{2s}\sin x\right)+\frac{b}{2c}\sin x\sin 2\psi$$

$$\times\left(2\frac{is}{x_2}+L_{2s}\sin 2\beta\operatorname{ctg} 2\psi\right)\frac{(\zeta\mathbf{E}_0)}{a},$$

$$\rho=2s\theta-\frac{\tau^3}{3}-\tau z-\frac{1}{2}b^2\tau z\left(1-\frac{\sin^2 x}{x^2}\right),\qquad \eta_0=\tau z,$$

$$x=\tau\frac{\lambda z}{2t},\quad z=\left(\frac{t}{\chi}\right)^{2/3},\quad \chi=\frac{ea}{m^3}(nq),$$

$$h=\cos x-\frac{1}{x}\sin x,\quad \lambda=2\frac{\omega}{m^2}(nq),$$

$$\theta=\operatorname{arctg}(\operatorname{tg}\beta\operatorname{ctg}\psi),\quad L_m=\frac{J_m'}{J_m},\quad b=\frac{eA_0}{m\omega},$$

$$c=\left[\frac{1}{2}(1-\cos 2\beta\cos 2\psi)\right]^{1/2},$$

$$x_1=b^2\frac{ht}{\lambda}\sin x\cos 2\psi,\qquad x_2=4bc\frac{\tau ht}{\lambda z^{1/2}},$$

$J_m(x_m)$ and $J'_m(x_m)$ is a Bessel function and its derivative.

The sum over s in (7) is lifted if in the superposition (2) a circularly polarized wave is present, since for $4\psi=\pi g$, $x_1=0$, $J_s(0)=\delta_{s,0}$.

Equation (7) permits the study of the interesting question on the contributions to the correction to the electron mass in the field (2) caused by transitions of the electron to intermediate states without and with spin flip (terms proportional to $\delta_{\zeta,\zeta'}$ and $\delta_{\zeta,-\zeta'}$ respectively).

For $b=0$ we obtain from (7) after summation over ζ' the amplitude for elastic scattering of an electron in a constant crossed field,[13] and for $a=0$ we have the amplitude for elastic scattering in the field of a plane electromagnetic wave[14] (see also Ref. 4). For $\beta=0$ the summation over ζ' in (7) gives results obtained previously in Ref. 12.

4. ANOMALOUS MOMENTS OF THE ELECTRON

The amplitude for elastic scattering of the electron in an external field determines its mass change Δm in that field: $\Delta m=-q_0T/m$. We shall represent Δm in the form $\Delta m_1+\Delta m_2$, where in Δm_2 only terms proportional to $\zeta\cdot\mathbf{E}_0$ and $\zeta\cdot\mathbf{H}_0$ appear. We shall write in the rest frame $\Delta m_2=-\mu_1\zeta\cdot\mathbf{H}_0-\mu_2\zeta\cdot\mathbf{E}_0$. The addition Δm_2 to the mass of the electron in the external field (2) results from the interaction of the anomalous magnetic moment of the electron with the constant magnetic field and the anomalous electric moment with the constant electric field.

It therefore follows from (7) that the anomalous moments of the electron μ_ν (μ_1—magnetic, μ_2—electric), due to second order radiative effects and interaction with the external electromagnetic field (2), are determined by the expression

$$\mu_\nu=\mu_0\frac{\alpha}{2\pi}\operatorname{Re}\int_0^\infty\frac{d\tau}{\tau}\int_0^\infty\frac{t^2\,dt}{\chi(1+t)^3}\sum_{s=-\infty}^{\infty}i^s e^{i\rho}J_s(x_1)$$

$$\times J_{2s}(x_2)\Phi_\nu\left[\left(1+\frac{2}{t}\right)\delta_{\zeta,\zeta'}-\delta_{\zeta,-\zeta'}\right],$$

$$\Phi_1=\frac{i\tau}{z^{1/2}}+cbL_{2s}\sin x,$$

$$\Phi_2=\frac{b}{2c}\sin x\sin 2\psi\left(2i\frac{s}{x_2}+L_{2s}\sin 2\beta\operatorname{ctg} 2\psi\right).\tag{8}$$

This expression is a rather involved function of all the parameters that characterize the superposition (2), namely: the frequencies, amplitudes, and polarizations of the electromagnetic wave, and the intensity of the constant crossed field, and also the electron parameters. If $\beta=0;\ \pm\pi/2;\ \pm\pi;\ldots$ [the condition of "orthogonality" of the crossed and wave fields in the superposition (2)], and if a plane wave of circular polarization is present in the superposition (2), then the anomalous electric moment vanishes. Indeed, for $4\psi=\pi g$, $x_1=0$, $J_s(0)=\delta_{s,0}$, $J_{2s'}(0)=0$, $L_{2s}=0$ and $\Phi_2=0$.

We consider next several special cases corresponding to limiting values of the external field (2). For a plane wave of weak intensity $b\ll l$ and for low intensity of the constant crossed field $\chi\ll 1$, after expansion in (7) keeping terms proportional to χ^0, χ^1, χ^2, b^0, b^1, b^2, χb^2, and ignoring terms proportional to χ^2b^2 we can express the electron elastic scattering amplitude in the following form [$\zeta=(\zeta_1,\zeta_2,0)$]:

$$T=-\frac{m}{q_0}(\Delta m_a+\Delta m_b)+\frac{1}{2}i(w_a+w_b).\tag{9}$$

Here

$$\Delta m_a=\frac{4\alpha m}{3\pi}\chi^2\left[\left(C+\ln\frac{3^{1/2}}{\chi}-\frac{63}{32}\right)\delta_{\zeta,\zeta'}\right.$$

$$\left.-\frac{3}{32}\delta_{\zeta,-\zeta'}\right]+\frac{nq}{q_0}\mu_1{}^a\quad(C=0{,}577\ldots)$$

is the change in the electron mass due to the constant crossed field

$$w_a=\frac{5\alpha m^2}{4\cdot 3^{1/2}q_0}\chi\left\{\left[1-\frac{8\cdot 3^{1/2}}{15}\chi-\frac{3\chi}{10}\frac{(\zeta\mathbf{H}_0)}{a}\right]\delta_{\zeta,\zeta'}\right.$$

$$\left.+\frac{3}{8}\chi^2\left[1-\frac{8\cdot 3^{1/2}}{15}\frac{(\zeta\mathbf{H}_0)}{a}\right]\delta_{\zeta,-\zeta'}\right\}$$

is the electron radiation probability in the constant crossed field, and

$$\mu_1{}^a=\frac{\alpha}{2\pi}\mu_0\left\{\frac{3}{2}\left[1-\frac{20}{3}\chi^2\left(C+\ln\frac{3^{1/2}}{\chi}-\frac{59}{20}\right)\right]\delta_{\zeta,\zeta'}\right.$$

$$\left.-\frac{1}{2}\left[1+4\chi^2\left(\ln\frac{3^{1/2}}{\chi}+C-\frac{15}{4}\right)\right]\delta_{\zeta,-\zeta'}\right\}$$

is the anomalous magnetic moment in the constant crossed field.

It follows then that in a constant crossed field the contribution to the mass correction due to electron transitions into an intermediate state without spin flip is larger than the contribution from transitions with spin flip for

$$C+\ln\frac{3^{1/2}}{\chi}>\frac{15}{8}.$$

The following notation was introduced in Eq. (9):

Δm_b, the electron mass change due to the electromagnetic wave with the effect of the constant field taken into account, is of the form

$$\Delta m_b = -\frac{\alpha m}{4\pi} b^2 (P_1 \delta_{\zeta,\zeta'} + P_2 \delta_{\zeta,-\zeta'}) + \frac{nq}{m} [\mu_1{}^b (\zeta \mathbf{H}_0) + \mu_2{}^b (\zeta \mathbf{E}_0)],$$

$$P_1 = \frac{1}{\lambda} g_2 + \frac{\lambda^2 - 16}{8\lambda^2} g_1 + \frac{8 - 7\lambda^2}{8(\lambda^2 - 1)} - \frac{\lambda^4 + 5\lambda^2}{8(\lambda^2 - 1)^2} \ln \lambda,$$

$$P_2 = \frac{1}{8} g_1 + \frac{\lambda^2}{8(\lambda^2 - 1)} + \frac{\lambda^2 (3\lambda^2 - 1)}{8(\lambda^2 - 1)^2} \ln \lambda;$$

w_b, the electron radiation probability in the field of the plane electromagnetic wave with effects of the constant field taken into account, is given by the relation

$$w_b = \frac{\alpha m^2}{8 q_0} b^2 (N_1 \delta_{\zeta,\zeta'} + N_2 \delta_{\zeta,-\zeta'}),$$

$$N_1 = \frac{1}{4} - \frac{4}{\lambda} + \frac{3\lambda^2 + 2\lambda - 2}{8(\lambda + 1)^2} - \left(\frac{1}{4} - \frac{2}{\lambda} - \frac{4}{\lambda^2} \right) \ln (1 + \lambda)$$
$$+ \chi \frac{\cos 2\psi}{\lambda a} \left[\frac{2\lambda^2 + 3\lambda + 2}{2(\lambda + 1)^2} - \frac{1}{\lambda} \ln (1 + \lambda) \right] [(\zeta \mathbf{H}_0) \cos 2\beta - (\zeta \mathbf{E}_0) \sin 2\beta].$$

$$N_2 = -\frac{\lambda (3\lambda + 2)}{8(1 + \lambda)^2} + \frac{1}{2} \ln (1 + \lambda) - \chi \frac{\cos 2\psi}{\lambda a} \left[\frac{2\lambda^2 + 9\lambda + 6}{2(1 + \lambda)^2} - \frac{3}{\lambda} \ln (1 + \lambda) \right] [(\zeta \mathbf{H}_0) \cos 2\beta - (\zeta \mathbf{E}_0) \sin 2\beta];$$

μ_1^b, the addition to the anomalous magnetic moment of the electron due to the presence in the superposition (2) of the electromagnetic wave, is given by

$$\mu_1{}^b = \frac{\alpha}{2\pi} \mu_0 b^2 (f_1 \delta_{\zeta,\zeta'} + f_2 \delta_{\zeta,-\zeta'}), \qquad \mu_0 = \frac{e}{2m},$$

$$f_1 = \frac{\lambda^2 (3\lambda^2 - 2)}{2(\lambda^2 - 1)^2} - \frac{\lambda^2 (\lambda^2 - 5)}{(\lambda^2 - 1)^3} \ln \lambda - \cos 2\beta \cos 2\psi \left[\frac{1}{\lambda^2} g_1 - \frac{3\lambda^2 + 1}{2(\lambda^2 - 1)^2} + \frac{5\lambda^2 - 1}{(\lambda^2 - 1)^3} \ln \lambda \right],$$

$$f_2 = -\frac{\lambda^2 (\lambda^2 + 3)}{2(\lambda^2 - 1)^2} + \frac{\lambda^2 (3\lambda^2 + 1)}{(\lambda^2 - 1)^3} \ln \lambda + \cos 2\beta \cos 2\psi \left[\frac{3}{\lambda^2} g_1 + \frac{7\lambda^2 - 3}{2(\lambda^2 - 1)^2} - \frac{8\lambda^4 - 7\lambda^2 + 3}{(\lambda^2 - 1)^3} \ln \lambda \right],$$

μ_2^b, the anomalous electric moment induced by the plane wave is given by

$$\mu_2{}^b = \frac{1}{2} \frac{d}{d\beta} \mu_1{}^b = \frac{\alpha}{2\pi} \mu_0 b^2 \sin 2\beta \cos 2\psi \left[\left(\frac{1}{\lambda^2} g_1 - \frac{3\lambda^2 + 1}{2(\lambda^2 - 1)^2} + \frac{5\lambda^2 - 1}{(\lambda^2 - 1)^3} \ln \lambda \right) \delta_{\zeta,\zeta'} - \left(\frac{3}{\lambda^2} g_1 + \frac{7\lambda^2 - 3}{2(\lambda^2 - 1)^2} - \frac{8\lambda^4 - 7\lambda^2 + 3}{(\lambda^2 - 1)^3} \ln \lambda \right) \delta_{\zeta,-\zeta'} \right].$$

The quantities $g\nu (\nu = 1,2)$ are determined by the Spence functions $F(x)$:

$$g_1 = \pi^2/3 + F(1 - \lambda) + F(1 + \lambda), \quad g_2 = F(1 - \lambda) - F(1 + \lambda),$$

$$F(x) = \int_0^x \frac{dt}{t} \ln |1 + t|.$$

We give the values of the anomalous electron moments in the limiting cases $\lambda \ll 1$ and $\lambda \gg 1$.

1. $\lambda \ll 1$.

$$\mu_1{}^b = \mu_0 \frac{\alpha \lambda^2}{4\pi} b^2 \ln \frac{1}{\lambda} \left(1 - \frac{1}{2} \cos 2\beta \cos 2\psi \right) (5\delta_{\zeta,\zeta'} + \delta_{\zeta,-\zeta'}),$$

$$\mu_2{}^b = \frac{1}{2} \frac{d}{d\beta} \mu_1{}^b = \mu_0 \frac{\alpha \lambda^2}{8\pi} b^2 \ln \frac{1}{\lambda} \sin 2\beta \cos 2\psi (5\delta_{\zeta,\zeta'} + \delta_{\zeta,-\zeta'}).$$

We note that these formulas are asymptotic and valid in the region where $\ln(1/\lambda) \gg 1$.

2. $\lambda \gg 1$.

$$\mu_1{}^b = \mu_0 \frac{\alpha b^2}{2\pi} \left\{ \left[\frac{3}{2} - \frac{\ln \lambda}{\lambda^2} + \frac{\ln^2 \lambda}{\lambda^2} \cos 2\psi \cos 2\beta \right] \delta_{\zeta,\zeta'} + \frac{3 \ln \lambda}{\lambda^2} \left[\ln \lambda - \frac{8}{3} \right] \cos 2\psi \cos 2\beta \delta_{\zeta,-\zeta'} \right\},$$

$$\mu_2{}^b = \frac{1}{2} \frac{d}{d\beta} \mu_1{}^b$$

$$= -\mu_0 \frac{\alpha b^2}{2\pi \lambda^2} \ln \lambda [\ln \lambda \delta_{\zeta,\zeta'} + (3 \ln \lambda - 8) \delta_{\zeta,-\zeta'}] \sin 2\beta \cos 2\psi.$$

In the other limiting case, when $b \ll 1$, $\chi \gg 1$, the elastic scattering amplitude is given by the expression $(\zeta \cdot \mathbf{n} \neq 0)$

$$T = -\frac{m}{q_0} \Delta m + \frac{i}{2} w,$$

$$\Delta m = \frac{\alpha m}{54 \cdot 3^{1/2}} (3\chi)^{2/3} \Gamma\left(\frac{2}{3}\right) \left(1 + \frac{\chi_2{}^2}{\chi^2}\right) \tau_2 + \frac{nq}{m} [\mu_1 (\zeta \mathbf{H}_0) + \mu_2 (\zeta \mathbf{E}_0)],$$

$$\mu_1 = \alpha \mu_0 \frac{(3\chi)^{-2/3}}{9 \cdot 3^{1/2}} \Gamma\left(\frac{1}{3}\right) \left(1 - \frac{\chi_1{}^2}{\chi^2}\right) \tau_2,$$

$$\chi_\nu{}^2 = \frac{\lambda^2 b^2}{24} \left(1 - \frac{2\nu}{3} C^2\right), \quad \nu = 1, 2, \tag{10}$$

$$\mu_2 = -\frac{3}{2} \frac{d}{d\beta} \mu_1 = -\alpha \mu_0 \lambda^2 b^2 \frac{(3\chi)^{-8/3}}{24 \cdot 3^{1/2}} \Gamma\left(\frac{1}{3}\right) \tau_2 \sin 2\beta \cos 2\psi,$$

$$w = \frac{\alpha m^2}{27 q_0} \Gamma\left(\frac{2}{3}\right) (3\chi)^{2/3} \left\{ \tau_1 \left(1 + \frac{\chi_2{}^2}{\chi^2}\right) + \tau_2 (3\chi)^{-1/3} \times \frac{\Gamma(1/3)}{\Gamma(2/3)} \left[\frac{(\zeta \mathbf{H}_0)}{a} \left(1 - \frac{\chi_1{}^2}{\chi^2}\right) - \frac{\lambda^2 b^2 (\zeta \mathbf{E}_0)}{24 a \chi^2} \sin 2\beta \cos 2\psi \right] \right\},$$

$$\tau_1 = [13 + (\zeta \mathbf{n})^2] \delta_{\zeta,\zeta'} + \zeta_\perp{}^2 \delta_{\zeta,-\zeta'}, \quad \tau_2 = {}^7/_2 \delta_{\zeta,\zeta'} - {}^5/_2 \delta_{\zeta,-\zeta'}.$$

Thus, with increasing intensity of the constant crossed field the anomalous electron moments in the field (2) tend to zero for $b \ll 1$.

We also call attention to the oscillatory dependence of the anomalous electric moment of the electron on the angle β (β characterizes the relative orientation of the fields). This serves to emphasize the fact that the anomalous electric moment is induced exclusively by the field of the electromagnetic wave and its appearance is connnected to the violation of axial symmetry for an elliptically polarized wave.

We note that for $\chi \gg 1$ ($b \ll 1$) the anomalous magnetic and electric moments of the electron depend on a power of χ—the characteristic parameter of the constant crossed field—with $\mu_1 \propto \chi^{-2/3}$ and $\mu_2 \propto \chi^{-8/3}$.

These calculations show also that in the region of limiting values of the parameters b and χ, expanding up to terms of order $b^2\chi$, accurate up to a constant, the anomalous electric moment can be obtained as the product of the anomalous magnetic moment and the angle β. For $\chi \ll 1$ this constant equals 1/2, for $\chi \gg 1$ it equals $-3/2$. However, if higher-order terms are included in the expansion this relation is violated

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[1]M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).

[2]N. N. Bogolyubov, B. V. Medvedev, and M. K. Polivanov, *Problems in Dispersion Relations* [in Russian] (GIFML, Moscow, 1958); N. N. Bogolyubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Wiley, New York, 1980) [Russ. original, Nauka, Moscow, 1976].

[3]V. I. Ritus, Zh. Eksp. Teor. Fiz. **57**, 2176 (1969) [Sov. Phys. JETP **30**, 1181 (1970)].

[4]V. N. Rodionov, I. M. Ternov, and V. R. Khalilov, Zh. Eksp. Teor. Fiz. **71**, 871 (1976) [Sov. Phys. JETP **44** 459 (1976)].

[5]V. Ch. Zhukovskii and A. V. Kurilin, Izv. Vyssh. Uchebn. Zaved. Fiz. **1**, 76 (1989).

[6]V. I. Ritus, Zh. Eksp. Teor. Fiz. **75**, 1560 (1978) [Sov. Phys. JETP **48**, 788 (1978)].

[7]A. V. Borisov, V. Ch. Zhukovskii, A. V. Kurilin, and A. I. Ternov, Yad. Fiz. **41**, 743 (1985) [Sov. J. Nucl. Phys. **41**, 473 (1985)].

[8]D. M. Wolkow, Z. Phys. **94**, 250 (1935); D. M. Volkov, Zh. Eksp. Teor. Fiz. **7**, 1286 (1937).

[9]I. M. Ternov, V. G. Bagrov, and A. M. Khapaev, Ann. der Phys. **22**, 25 (1968).

[10]I. S. Gradshtein and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 1980) [Russ. original, Nauka, Moscow, 1971].

[11]Yu. I. Klimenko and O. S. Pavlova, Yad. Fiz. **29**, 167 (1979) [Sov. J. Nucl. Phys. **29**, 84 (1979)].

[12]V. G. Bagrov, Yu. I. Klimenko, and O. S. Pavlova, Yad. Fiz. **31**, 806 (1980) [Sov. J. Nucl. Phys. **31**, 417 (1980)].

[13]V. I. Ritus, Tr. Fiz. Inst. Akad. Nauk SSSR **111**, 3 (1979).

[14]V. N. Baĭer, V. M. Katkov, A. I. Mil'shtein, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **69**, 784 (1975) [Sov. Phys. JETP **42**, 400 (1975)].

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