Isotropic quantum magnetoresistance and inelastic and spin-orbit relaxation times in cold-deposited cesium and rubidium films

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A method is developed for determining inelastic and spin-orbit relaxation times of electrons from the magnetic field dependence of the resistivity. It is found that the magnetoresistance of films is, to a large extent, governed not by localization corrections to the conductivity but by a new quantum component, which does not depend on the direction of the magnetic field and is a function of $g\mu_B H/kT$. Measurements on the same film in perpendicular and parallel magnetic fields allow one to extract the localization component and to obtain the electron phase coherence times for a wide range of resistivity and of temperature values. The inelastic relaxation frequency varies linearly with temperature and satisfies $2\pi\hbar\tau_i^{-1} \approx 3.5\,kT$ for both cesium and rubidium films regardless of their resistivity. The spin-orbit relaxation frequency in cesium films is found to be independent of temperature, of resistivity, and of electron mean free path. No appreciable spin-orbit scattering is observed in rubidium. A novel isotropic quantum magnetoresistance is studied in detail. Its main features allow one to associate it with electron–electron interaction quantum corrections; however, no quantitative agreement with theory could be obtained. The discrepancy with theory is particularly large for cesium which exhibits a much larger effect with a very different behavior; in particular, the effect is asymptotically linear in weak fields.

Anomalous magnetoresistance is the best known and most studied effect related to quantum corrections to the conductivity. There have been attempts to use quantum magnetoresistance as a measuring probe for phase coherence times of electrons ever since its discovery. The magnitudes of inelastic and spin-orbit relaxation times and their temperature dependence have been estimated from the limiting behavior of quantum magnetoresistance: parabolic and logarithmic field dependence in very weak and very strong fields, respectively. It has been found that inelastic scattering is much stronger in disordered films than in bulk single crystals and that it is a linear function of temperature.

Good agreement between variations obtained theoretically and experimentally has been previously found in the whole magnetic field range studied (Ref. 1 and references given below) only at relatively high temperatures (above liquid-helium temperature) and for sufficiently thick low-resistance ($R \leqslant 100\ \Omega$) films. This has consequently led to the belief that good agreement with theory is possible only in weak fields, $H \ll kT/g\mu_B$, and only far enough from the metal-insulator transition, for $R \ll 2\pi^2\hbar/e$, although theory imposes no such limitations. On the other hand, if the phase coherence times are to be obtained from the magnetoresistance as accurately as possible, it is necessary to work with high-resistance films, since the quantum magnetoresistance amplitude decreases ($\propto R^2$) rapidly with decreasing resistance.

We can relate the observed discrepancies between localization theory and experiments for thin high-resistance films neither to their high resistance nor to their proximity to the metal-insulator transition; we can relate them to their structural inhomogeneity. As a rule, close to the transition, the films are made up of small clusters and their conduction mechanism is complicated. Shal'nikov has proposed and developed a method for the preparation of ultrathin homogenous films of alkali metals[2] by deposition of metal vapors, at a controlled low (270–290 K) temperature, on a substrate with in good thermal contact with a liquid helium bath. The solid angle at the substrate subtended by the evaporation source is large; there are consequently no penumbra and the ensuing film homogeneity is better than the one obtained with a point evaporator. Conduction in films prepared by this method start at $\approx 10^{-10}\ \Omega^{-1}$ for an average thickness of ≈ 0.7 of a monoatomic layer; conduction becomes metallic for films of one monolayer; then their resistance is ≈ 80 kΩ.

Therefore, films of alkali metals prepared by Shal'nikov's method are the thinnest films with metallic conduction known at present. As can be estimated, these films start percolating prior to the transition to metallic conduction for coverages smaller than a monoatomic layer;[3] the calculated percolation cluster length is larger than the phase coherence length of electrons over the whole metallic regime, and such films should be homogenous for localized magnetoresistance down to the metal-insulator transition. Consequently, we have attempted[3] to study the magnetoresistance of these films in detail in order to, once good agreement with theory is obtained for thick films, approach the metal-insulator transition as closely as possible.

It unexpectedly turned out that the experimental curves for the magnetoresistance of cesium films[3] could not be described by the localization theory of noninteracting electrons[4] for any film thickness. The search for the possible sources of this discrepancy carried out in Ref. 3 has led to agreement between theory and experiment only for some range of the thickness and for small magnetic fields. However, discrepancies of the same order of magnitude as the effect remain at high fields, and they increase as $g\mu_B H/kT$ as the temperature decreases. It has been assumed that a new quantum magnetoresistance effect, associated with the elec-

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tron–electron interaction, gives rise to these discrepancies.

The observation of the new effect as the difference between the calculated, with two fitting parameters (inelastic and spin-orbit scattering), and the experimental curves was not very convincing. Furthermore, the values obtained for the phase coherence times of the scattered electrons differed from the ones predicted theoretically by factors of tens and of hundreds. If the assumption of Ref. 3 is right, one can propose a method for the direct observation of the new magnetoresistance as well as for a more precise determination of the scattering parameters. The magnetoresistance component coming from electron–electron interactions should be isotropic with respect to the magnetic field,[5] whereas the localization contribution is sensitive only to the perpendicular component of the field.[4] Therefore, if measurements are carried out on the same sample in perpendicular and parallel fields, one should observe a negative magnetoresistance in a perpendicular field and a positive one, which varies as $g\mu_B H/kT$, in a parallel field. They should be of the same order of magnitude. Their difference should be well approximated by the localization relation [Eq. (1)], and it should give the magnitudes of inelastic and spin-orbit scattering of electrons.[4]

Results from the first experiments on cesium films, using two magnetic field orientations,[6] have indeed fulfilled this expectation. We report in this paper results of detailed studies of phase coherence times in cesium and rubidium films and of the new quantum magnetoresistance.

EXPERIMENTAL METHOD

The procedure we used for the preparation of films, for measurements and for calculations, is similar to the one described previously.[2,3] A glass ampoule, which was outgassed for several hours at temperature of 400 °C, was filled with a small amount of a purified alkali metal and sealed. The metal inside the ampoule was sublimated on an evaporation source: a platinum disk 16 mm in diameter placed at a distance of 5 mm from a polished glass substrate 9 mm in diameter with soldered platinum current leads (Fig. 1). During the preparation of the film and in the course of measurements the ampoule was immersed in liquid helium, which ensured that the vacuum was 10^{-12} torr. Experiments were performed at substrate temperatures between 4.2 K and 1.5 K. Evaporation took place from the solid phase at temperatures below -5 °C and 10 °C for Cs and Rb, respectively. The vapor pressures of the metal is therefore low. In addition, the substrate was kept at liquid helium temperatures. Consequently, a homogeneous film was obtained. The large diameter of the evaporation source, located close to the substrate, prevented penumbra and improved homogeneity. The perpendicular magnetic field, up to 5 T, was produced by a superconducting solenoid and the parallel magnetic field, up to 3 T, was produced by a superconducting Helmholtz coil (Fig. 1). The apparatus, mounted on a sliding rod, could be moved from the solenoid to the coil within a helium bath. The measurements and the data analysis were carried out with an IBM AT computer.

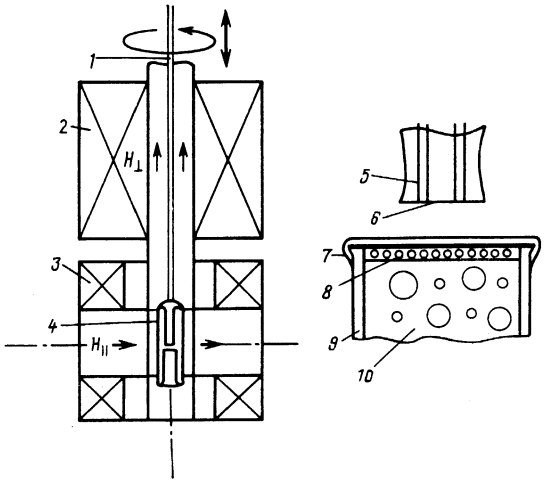

FIG. 1. Overall view of set-up: 1) sliding rod, 2) superconducting solenoid which produces field perpendicular to the substrate surface, 3) superconducting Helmholtz coil for a parallel field, 4) position of the apparatus for measurements in a parallel field. Substrate: 5) platinum contacts, 6) polished surface. Evaporator: 7) Alkali metal layer, 8) heater, 9) glass body, 10) styrofoam thermal insulation.

Freshly deposited cesium and rubidium films 5 to 100 Å thick were studied in a temperature range of 4.2–1.2 K. The most reliable results were obtained in the resistance range 10^4–10^2 Ω. The resistivity of thinner films changes rapidly with temperature, and the accuracy of the measurements was affected by the temperature instability of the helium bath. The magnetoresistance of thicker films is too small and the accuracy of the measurements was not sufficient for the mathematical analysis that follows.

THE LOCALIZATION COMPONENT OF THE MAGNETORESISTANCE

Figures 2 and 3 show the measured magnetoresistance of cesium and rubidium films.

As expected, perpendicular magnetic fields give rise to a negative magnetoresistance, whereas parallel fields give rise to a positive one (Fig. 2). The difference between the two magnetoresistance curves is approximately given (see Fig. 3, where the calculated curve fits the experimental curve 3) by the equation for the localization of noninteracting electrons.[4] Accordingly, the dependence of the conductance $G \equiv 2\pi^2\hbar/e^2R$ on the magnetic field perpendicular to the plane of the film is given by

$$\delta G_{loc}(H_\perp) \equiv G(H_\perp) - G(0)$$
$$= \ln \frac{H_\perp H_3^{1/2}}{H^{3/2}} + \frac{3}{2}\Psi\left(\frac{1}{2}+\frac{H_2}{H_\perp}\right) - \frac{1}{2}\Psi\left(\frac{1}{2}+\frac{H_3}{H_\perp}\right), \quad (1)$$

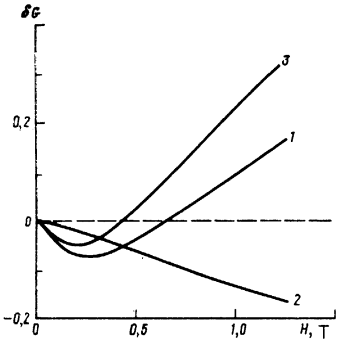

FIG. 2. Magnetoconductance of a 9.5 Å thick cesium film with a resistance of 1033 Ω at a temperature of 1.34 K: 1) in a perpendicular magnetic field, 2) in a parallel magnetic field, 3) the difference between curves 1 and 2. An approximation of this difference, using Eq. (1), nearly matches curve 3.

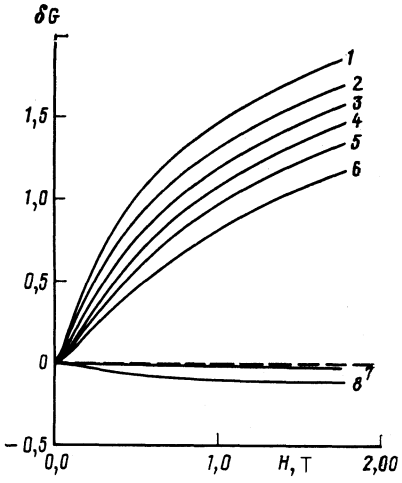

FIG. 3. Magnetoconductance of a 9.5 Å thick cesium film with a resistance of 906 Ω in a perpendicular magnetic field for the following temperatures: 1) 1.28 K, 2) 1.96 K, 3) 2.51 K, 4) 2.91 K, 5) 3.38 K, 6) 4.11 K; in a parallel field: 7) 3.39 K and 8) 1.28 K.

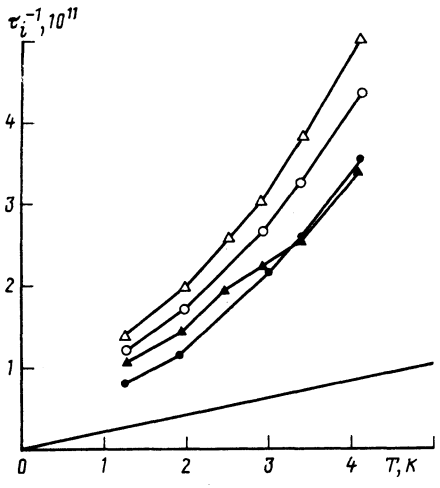

FIG. 4. Temperature variation of the inelastic scattering frequency for cesium films with a resistance of: ○) 2390 Ω, △) 1027 Ω, ●) 294 Ω, ▲) 93 Ω. The solid straight line corresponds to $2\pi\hbar\tau^{-1} = kT$.

where $\Psi(x)$ is the digamma function (logarithmic derivative $\Gamma'(x)/\Gamma(x)$ of the Γ-function), $H_2 = H_i + 4/3H_{so}$ and $H_3 = H_i + 2H_s$ are characteristic values of the magnetic field, and the subindices i, so, and s correspond to inelastic, spin-orbit, and spin scattering, respectively. Good agreement with theory allows us to determine values of H_i and of H_{so}, which are reliable over the whole temperature range and for values of the film resistance up to several kiloohms. Deviations of ten percent show up in films with resistance values as low as (10–20) kΩ and the error in the scattering parameters determined increases sharply. Nevertheless, the values obtained agree with the values extrapolated from the low-resistance region. Therefore, the localization equation (1) works well at low temperatures and strong magnetic fields $g\mu_B H \approx kT$, up to the metal-insulator transition, and can be used as the basic method for the determination of the phase coherence times of electrons.

The value of H_s (corresponding to spin-flip scattering by magnetic impurities) in all the films studied, for both cesium and rubidium, was found to vanish within experimental errors (the corresponding concentration of spin scattering centers is less than 10^{-5}). This result was to be expected since the preparation method[2,3] used produces alkali metal films, which ought to be free of magnetic impurities.

INELASTIC SCATTERING

The temperature variation of the inelastic-scattering frequency is shown in Figs. 4 and 5. The error in the determination of τ_i and the scatter of the values obtained is smaller for rubidium films (in Fig. 5, the largest error occurs for the lowest curve) because only one fitting parameter H_i is to be determined, since there is no spin-orbit interaction. Within the error limits, the inelastic scattering frequency is linear with temperature, in agreement with results of other studies of disordered films. The data obtained for cesium films are less accurate (Fig. 4). Here, as for rubidium films, τ_i^{-1} varies almost linearly with temperature.

However, the proportionality coefficient is quite different from the theoretically predicted one. The calculated inelastic scattering frequency, $\tau_i^{-1} = \hbar c/4eDH_i$ (D is the diffusion coefficient), shows that the linear variations for cesium and rubidium films of different thickness are not very different,

$$2\pi\hbar\tau_i^{-1} = CkT, \tag{2}$$

where the coefficient C varies in the ranges 3.4–5.1 and 2.4–3.5 for cesium and rubidium films, respectively; thus, within the error limits, C is independent of resistance and its value for these two metals is almost the same. On the other hand, existing theory[7] predicts that this coefficient depends strongly on the resistance of the film,

$$2\pi\tau_i^{-1} = kT\frac{e^2R}{2\pi\hbar}\ln\frac{\pi\hbar}{e^2R}, \tag{3}$$

which, for $R = 10^2$ Ω films, gives a value as low as ≈ 0.007.

The scatter of the data in the figure suggest that the determination of C gives at least the right order of magnitude. The observed strong inelastic scattering, $\propto kT$, follow

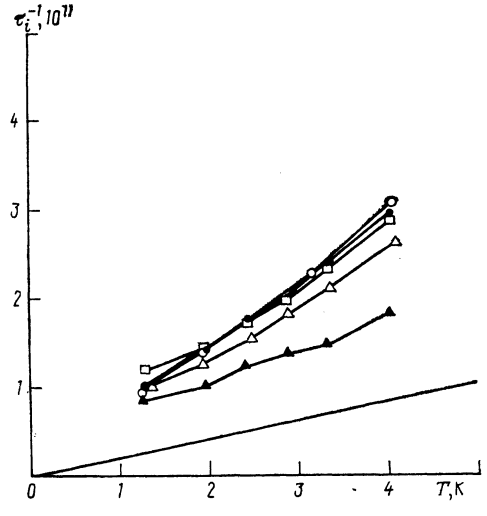

FIG. 5. Temperature variation of the inelastic scattering frequency for rubidium films with a resistance of ●) 11 651 Ω, ○) 3561 Ω, △) 931 Ω, □) 342 Ω, ▲) 86 Ω.

neither from the procedure used nor from alkali metal properties. It has been observed in other studies for other metals. For instance, Bergmann has described his inelastic scattering data for cold-deposited gold films with resistance of $\approx 100\ \Omega$ by the relation $\tau_i^{-1} \propto T^{1.4}$. In Ref. 2 we reported measurements on gold films performed in order to check the validity of the method used to determine the scattering parameters. The films were prepared similarly, with the same resistance, but a wider temperature range (1.2–80 K) was covered. The experimental values obtained turned out to be very close to Bergmann's data. The best fit, for temperatures in the liquid-helium to liquid-nitrogen range, was obtained for $\tau_i^{-1} = 3.11 \cdot 10^{10} T^{1.14}$. However, an equally good fit is obtained for the same data with a linear temperature dependence with a proportionality coefficient for kT in the range 1.6–2.4. This is also true for Bergmann's results.

We cannot propose a specific mechanism for inelastic scattering in disordered thin films which is linear in temperature and leads to a value of C of order unity. On the other hand, the fact that C is independent of resistance must be accepted to be experimentally established.

SPIN-ORBIT SCATTERING

The shape of the magnetoresistance curves as well as their sign suggest that spin-orbit scattering is weaker than inelastic scattering in cesium and especially in rubidium films. On the other hand, it is generally accepted[8] that there is a large spin-orbit scattering probability $(\alpha Z)^4$, where α is the fine-structure constant and Z is the atomic number, for each elastic electron scattering process. Therefore, τ_{so}^{-1} should be larger than τ_i^{-1} by a factor of several tens for cesium films; in addition, τ_{so}^{-1} should depend strongly on the elastic mean free path of electrons, that is, on the film resistance. As follows from Fig. 6, spin-orbit scattering for cesium films does not depend on film resistance and is weaker or about the same as inelastic scattering. To within experimental errors it does not depend on temperature, as is expected for this range of temperature and of magnetic fields.

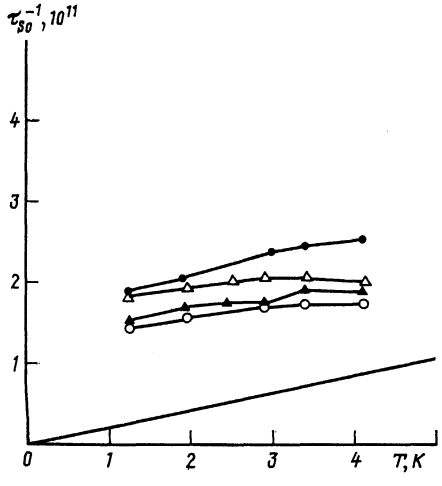

FIG. 6. Temperature variation of the spin-orbit scattering frequency for cesium films with a resistance of ○) 2390 Ω, △) 1027 Ω, ●) 294 Ω, ▲) 93 Ω.

Since the rubidium atomic number ($Z = 85$) is smaller than that of cesium ($Z = 133$) one can expect that spin-orbit scattering, which is proportional to the fourth power of Z, will be very small in rubidium films. In fact, it was found to be below our sensitivity threshold. A simultaneous determination of the two fitting parameters H_i and H_{so} in Eq. (1) yields the same absolute error for both of them. In addition, as Bergmann has already pointed out,[1] the separation of these parameters is difficult when H_i is larger than H_{so} (which corresponds to monotonically increasing magnetoresistance curves). After checking that fits of the magnetoresistance curves for all rubidium films give the magnitudes of spin-orbit scattering uniformly distributed about zero, we recalculated all data with only one fitting parameter, H_i. This yields a determination of inelastic scattering which is more accurate for rubidium films than for cesium films (Figs. 4 and 5) and allows us to analyze the results for rubidium films for values fo the resistance of up to 20 kΩ.

SPIN-ORBIT SCATTERING IN THE PRESENCE OF SURFACE ELECTRON SCATTERING

The fact that spin-orbit scattering is small in the films studied is not, in our opinion, a peculiarity of alkali metals. It is probably related to the method of film preparation, which gives films of much better structural homogeneity than other cold-deposited films. It also assures that spin-orbit scattering is due to the film matrix metal, since there are hardly any impurity atoms. The variation of the resistance with thickness[2,3] shows that the mean free path in alkali metal films is large and, for large thicknesses, goes to $l_\infty \approx 300$ Å. Therefore, for a film thickness of several tens of angstroms, there is no scattering by defects or impurities inside the film, only by the surface.

The probability for spin-orbit scattering for specular reflection should be $(v_F/c)^4$ times smaller[9] than the probability $(\alpha Z)^4$ for diffusive reflection (v_F is the Fermi velocity and c is the velocity of light). For an ideal film with parallel surfaces, specular reflection does not contribute to resistance. The spatial variations of the thickness of our films is of the same order of magnitude as their thickness; consequently, high-resistance films are not like planes but rather like conducting nets. Then, surface reflection gives rise to an additional resistance and can give no spin-orbit scattering. This argument has allowed us to propose previously[3] that the observed smallness of spin-orbit interaction is related to the fact that all scattering occurs at the surface. The fact that τ_{so}^{-1} is independent of resistance shows directly that surface scattering does not contribute to spin-orbit interactions.

It is more difficult to suggest what kind of bulk scattering is related to the observed spin-orbit interaction. In Ref. 6, where cesium film results are reported, it is assumed that this interaction occurs by scattering on rubidium impurities, since only rubidium could be present in cesium films in appreciable concentrations. Stronger spin-orbit scattering should be observed in rubidium films when the impurities are cesium atoms. Cesium evaporates more easily than rubidium, and, consequently, higher impurity concentrations are expected in rubidium films; furthermore, the scattering cross section $(\alpha Z)^4$ for cesium is larger. The opposite is observed experimentally, which indicates that scattering comes from atoms of deposited metal (native defects like

vacancies, dislocations, etc.). In addition, not all native defects contribute to spin-orbit scattering. For mean free paths l in the range 5 to 30 Å (Fig. 6), the distance $l_{so} = v_F \tau_{so}$ traversed by an electron between two spin-flips is independent of l, and $l_{so} \approx 40\,000$ Å. Since $(\alpha Z)^4 \approx 40$ for cesium, spin-orbit scattering takes place every ≈ 1000 Å, which is larger than the native defect scattering length $l_\infty \approx 300$ Å. However, this may be an overestimation of the spin-orbit scattering cross section, which goes as $(\alpha Z)^4$.

Absence of appreciable interactions between electrons and atoms in the glass substrate is an additional feature of surface scattering which is not too obvious. In the opposite case, even if there were no magnetic impurities in the film there would be some spin scattering, but no such scattering is observed in the films studied. At liquid-helium temperature magnetic impurities with a concentration $\approx 10^{-5}$ have a significant effect on the magnetoresistance curves. The chemical glass which makes up the substrate contains iron in concentrations of a tenth of a percent and in thin films half of the scattering processes comes from reflection of electrons from the substrate. The same can be stated for the observed absence of spin-orbit scattering at the film-substrate interface. Although the substrate glass contains impurities of both heavy (such as lead) and light elements, none of them contribute to spin-orbit scattering.

It is known that the quantum magnetoresistance of films changes strongly if a small amount of a magnetic metal[10] or of a metal with a large atomic number[11] is evaporated on the film. For example, 10^{-3} of a monoatomic layer of iron on a Mg film of a thickness of 86 Å leads to $\tau_s^{-1} \approx 2.1 \cdot 10^{11}$. In such evaporation, impurity atoms condense with a large adsorption energy. Therefore, they probably do not appear as a external agent for the film material, in contrast to substrate atoms. Further experimental studies are needed in order to understand why atoms in the film are completely decoupled from atoms in the substrate.

MAGNETORESISTANCE IN A PARALLEL FIELD

In this paper, we concentrate on measurements in fields parallel to the plane of the substrate. It is the first direct study of isotropic quantum magnetoresistance which shows that it is related to quantum corrections associated with electron–electron interactions; however the experimental results are not as clear-cut as the theoretically predictions.

The correction to the conductance of a metal in a magnetic field arising from the interaction of an electron and a hole with a total spin $j = 1$ is[5]

$$\delta G_{int}(H,T) = \frac{\lambda}{2(\hbar D)^{d/2-1}} \int_0^\infty d\omega \, \frac{\partial^2(\omega \operatorname{cth}(\omega/2T))}{\partial \omega^2} \cdot [\varphi_d(\omega + g\mu_B H) + \varphi_d|\omega - g\mu_B H| - 2\varphi_d(\omega)], \quad (4)$$

where, for dimensions $d = 1$, 2 and 3,

$$\varphi_1(\omega) = \pi\left(\frac{2}{\omega}\right)^{1/2}, \quad \varphi_2(\omega) = \ln \omega, \quad \varphi_3(\omega) = \pi\left(\frac{\omega}{2}\right)^{1/2}. \quad (5)$$

For two dimensions, Eq. (4) becomes a function of a ratio of the magnetic field to the temperature, $h = g\mu_B H/kT$, and it no longer depends on the diffusion coefficient,

$$\delta G_{int}(h) = \frac{\lambda}{2} \int_0^\infty dx \, \frac{\partial^2(x \operatorname{cth} x)}{\partial x^2} \ln\left|1 - \frac{h^2}{x^2}\right|. \quad (6)$$

The amplitude λ of the effect can be estimated from the electron density and the interatomic distance, assuming one conduction electron per atom. Once the Fermi momentum p_F and the Coulomb screening inverse radius $\varkappa$ are estimated, it is possible to find the screening factor[12] $F = (2/x^2)\ln(1 + x^2)$, where $x = 2p_F/\hbar\varkappa$, and, thus, to determine λ (Ref. 13),

$$\lambda = \frac{32}{3} \frac{1 + {}^3/_4 F - (1 + {}^1/_2 F)^{3/2}}{F}. \quad (7)$$

This procedure gives $F \approx 0.2$ and $\lambda \approx -0.2$ for both cesium and rubidium. Therefore, in two dimensions the magnetoresistance in a parallel field should be a function of $g\mu_B H/kT$ as in Eq. (6), irrespective of film resistance, and should differ negligibly from cesium to rubidium.

The basic experimental results are exhibited in Figs. 7 and 8, which show how the magnetoresistance varies with temperature in a parallel field, and in Figs. 9 and 10, where the magnetoconductance is plotted as a function of $g\mu_B H/kT$ for cesium and rubidium films of various thicknesses. For rubidium films the magnetoresistance in a parallel field is significantly smaller than the one coming from localization, but they are of the same order of magnitude for cesium (Figs. 2 and 3). A comparision between Fig. 9 and Fig. 10 shows that the data for cesium and rubidium differ sharply in magnitude as well as in shape. This is an unexpect-

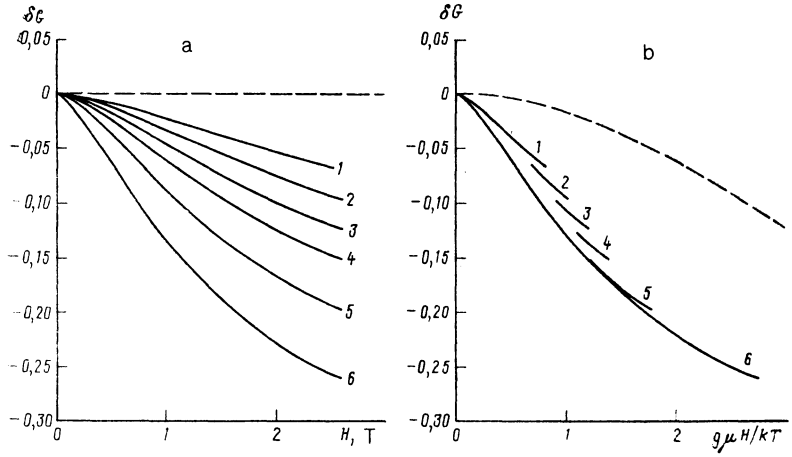

FIG. 7. a) Variation of the conductance with a parallel magnetic field for a 9.8 Å thick cesium film with a resistance of 906 Ω at temperatures of 1) 4.16 K, 2) 3.45 K, 3) 2.93 K, 4) 2.54 K, 5) 1.98 K, 6) 1.27 K; b) the same data as a function of $g\mu_B H/kT$; the dashed line corresponds to Eq. (6) with $\lambda = -0.2$.

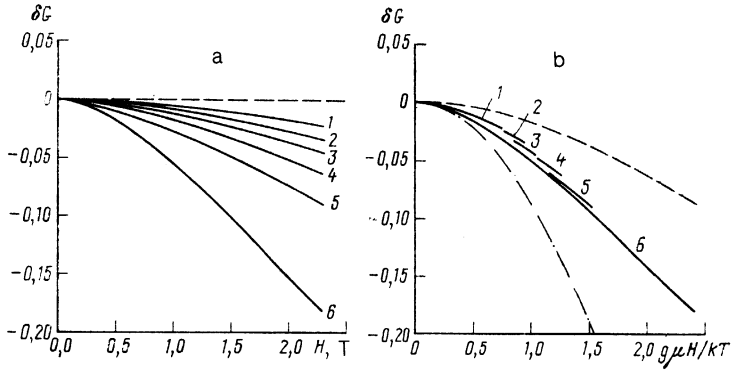

FIG. 8. a) Variation of the conductance with parallel magnetic field for a 5.3 Å thick rubidium film with a resistance of 10 720 Ω at temperatures of 1) 4.14 K, 2) 3.36 K, 3) 3.02 K, 4) 2.45 K, 5) 2.05 K, 6) 1.27 K; b) the same data as a function of $g\mu_B H/kT$; the dashed and dashed-pointed lines correspond to Eq. (6) with $\lambda = -0.2$ and $\lambda = -1$, respectively.

ed result, since cesium and rubidium are nearly identical with respect to Coulomb interactions.

LOCALIZATION COMPONENT OF THE MAGNETORESISTANCE IN A PARALLEL FIELD

The curves shown in Figs. 9 and 10 are for $T = 1.2$ K; in fact, each curve stands for a family of curves obtained at various temperatures which coincide in the variables of these figures, as shown in Figs. 7 and 8. These curves do not fully coincide for cesium films and the magnetoresistance changes its sign in rubidium films, which indicates that there is a localization contribution to the magnetoresistance measured in a parallel field. This cannot be simply trapping of perpendicular components, accounted for by the inaccurate orientation of the film with respect to the magnetic field. An additional degree of freedom associated with rotation about the vertical axis (Fig. 1) allowed us to rotate the plane of the film, which is tilted away slightly from the horizontal plane, for maximum effect in a parallel field. However, this produced no significant changes in the curves.

A perpendicular component probably arises from microscopic irregularities of the substrate surface. Obviously, the glass substrates we used, with high quality polishing, do not exhibit optical inhomogeneities; however, they do exhibit irregularities of hundreds of angstroms, which are comparable to the film thickness, covered by several monoatomic layers of metal. If the phase-coherence length is much larger than the irregularities, as it happens in our case, the total phase shift vanishes to lowest order. However, in first order the localization term enters into the magnetoresistance in a parallel field. The magnetoresistance is therefore some function of the magnetoresistances in Eq. (1) and in Eq. (4)

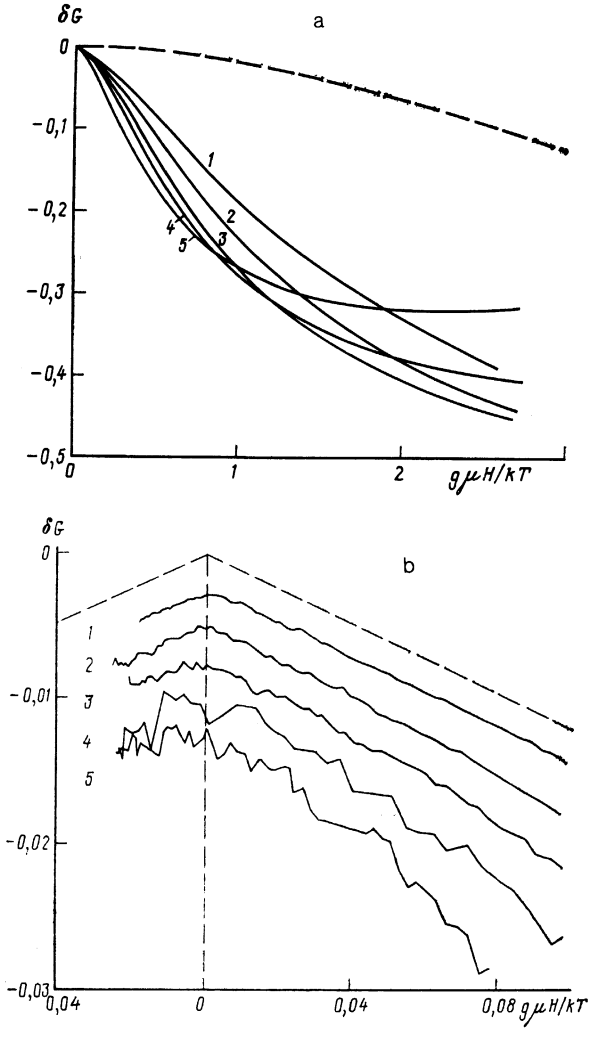

FIG. 9. (a) Magnetoconductance in a parallel magnetic field at a temperature of 1.3 K for cesium films with a resistance of 1) 3488 Ω, 2) 1438 Ω, 3) 687 Ω, 4) 337 Ω, 5) 166 Ω; the dashed curve corresponds to Eq. (6) with $\lambda = -0.2$. (b) curves shown in Fig. 9a near the coordinate origin; the dashed line corresponds to $\delta G = 0.12|g\mu_B H/kT|$. The curves are shifted along the vertical axis.

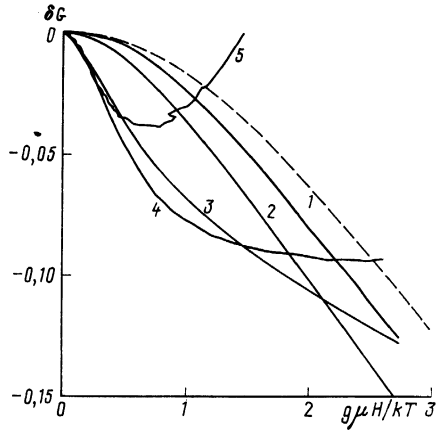

FIG. 10. Magnetoconductance in a parallel magnetic field at a temperature of 1.3 K for rubidium films with a resistance 1) 24 248 Ω, 2) 10 382 Ω, 3) 2304 Ω, 4) 1054 Ω, 5) 234 Ω; the dashed curve corresponds to Eq. (6) with $\lambda = -0.2$.

$$\delta G(H, T)=F(\delta G_{int}(H, T), \delta G_{loc}(H, H_i(T), H_{so})). \quad (8)$$

As our data analysis shows, this function is not simply a sum of Eq. (1) and Eq. (4) with some weight α (the angle of intersection): $\delta G(H) \neq \delta G_{\text{int}}(H) + \delta G_{\text{loc}}(\alpha H)$. Qualitatively, one can see it from the fact that the magnetoresistance curves for cesium films in the variables (δG, $g\mu_B H/kT$) do not move apart with decreasing temperature but approach each other (Fig. 9).

For rubidium films the curves $\delta G(g\mu_B H/kT)$ coincide much better than for cesium films (Figs. 7b and 8b). However, this does not indicate that the localization component does not contribute in a parallel field; on the contrary, it follows by inspection of Fig. 10 that this component plays an important role, particularly in strong fields and in thick films. The agreement of the curves $\delta G(g\mu_B H/kT)$ follows from the absence of spin-orbit scattering in rubidium films ($H_{so}=0$) and from the linear temperature dependence of H_i; consequently, the localization component of the magnetoresistance, Eq. (1), also becomes a function of $g\mu_B H/kT$ as well as all the magnetoresistance components in a parallel field.[8]

Therefore, the magnetoresistance $\delta G(H_{\|})$ we measured in a parallel field is not purely its isotropic part. Consequently, the localization component of the magnetoresistance, $\delta G_{\text{loc}}(H)$, which is obtained as a difference between $\delta G(H_{\|})$ and $\delta G(H_{\perp})$, is not determined accurately; this stands out most clearly for low-resistance rubidium films. As a whole, these inaccuracies are small and cannot, in principle, affect the observed pattern for most films. Obviously, one would like to find a method for completely separating the magnetoresistance into localization and isotropic parts, and we hope that further theoretical and experimental work will allow one to elaborate such a procedure. However, this could lead only to a more accurate determination of details. The basic properties of isotropic magnetoresistance are determined directly by measurements in a parallel field without additional mathematical analysis.

CESIUM FILMS

There is no quantitative agreement between experiment and theory, either for cesium or for rubidium films, but the deviations are not equal for these two cases. Cesium films, for which the effect in a longitudinal field is larger, were studied in more detail. Near the metal-insulator transition, the isotropic component of the magnetoresistance becomes even more localized; as a consequence, the magnetoresistance of thin cesium films in a perpendicalar field changes its sign with decreasing temperature and remains positive at all values of magnetic field.

Beside the unexpected large value, tens of times higher than estimated,[7] the magnetoresistance curves for cesium films (Fig. 9) differ sharply from the theoretical relation (6) in several respects: first, their shape is different and cannot be approximated by Eq. (6) for any value of the parameter λ; second, they are different for films of different thickness. The amplitude of the effect increases with decreasing resistance and an increasing diffusion coefficient, as expected for less than two dimensions. In principle, it is possible that the thinnest films are sufficiently close to the percolation conduction threshold, which leads to a quasi-one-dimensional treatment of the correction to the conductivity associated with the electron–electron interaction.[4] A transition to the two-dimensional regime takes place as films become thicker. The dimensionality of the corresponding regime can be found by constructing the curve of the magnetoresistance amplitude at fixed magnetic field versus the diffusion coefficient D; that was done for small fields ($0.2\ \text{T} < H < 1\ \text{T}$) where the localization component is not too large.

We did not find any dimensionality transition. It turns out that the magnetoresistance increases with increasing thickness proportionally to D,[0.33–0.34] consistently with a dimensionality value of $d=4/3$, for the whole range of magnetic fields values considered. Since the characteristic electron–electron interaction lengths are much smaller than the localization length, localization effects can be two-dimensional, but electron–electron interactions take place in a lower dimension (of the percolation cluster). This could explain the large amplitude of the effect and its dependence on D. However, as our estimations show, this situation is only possible for a narrow range of small thickness, whereas a D dependence is observed for all films (Fig. 9).

The behavior of the corrections to the magnetoresistance coming from electron–electron interactions should be determined[5] by the thermal length $l_T=(\hbar D/T)^{1/2}$ which is nearly equal to the phase-coherence length we measured (Fig. 4). Anyway, the percolation length, which exhibits a strong power dependence on film thickness, cannot lie between these two lengths in a wide range of values of the thickness. Here further experimental and theoretical studies are needed. There is no method for accurately distinguishing the localization and isotropic components of the magnetoresistance, and the procedure used is insufficiently accurate for the determination of the film dimensionality.

Comparison of the complicated experimental variations exhibited in Fig. 9 with the awkward equation (6) only underscores the lack of agreement. In order to reach some conclusions from the experimental data, it is necessary to make simplifications, and, for instance, to study the asymptotic behavior. In the logarithmic asymptotic region (strong magnetic fields) the localization component of the magnetoresistance is slightly too large. Consequently, we mainly explore the region of small fields, where, in an arbitrarily wide range, a parabolic asymptotic form of Eq. (6) should be observed,

$$\delta G(h)=6\frac{h^2}{\pi^2}\left[\zeta(3)-\frac{5}{3}\frac{h^2}{\pi^2}\zeta(5)+\ldots\right], \quad (9)$$

where ζ is the Riemann function[1)] $\zeta(3)=1.202$.

Instead of a quadratic dependence for small magnetic fields $H<0.1T$, the magnetoresistance is found to be linear in the field, $\delta G=|0.12h|$ (Fig. 9a) which is the most unexpected result reported in this paper. To within experimental errors (the minimal error for high resistances is for the upper curve in Fig. 9a), a parabolic shape is not observed anywhere. The observed linear dependence of the magnetoresistance is a quantum effect, since the change in the conductance

$$\delta G \equiv \frac{\delta R}{R^2}\frac{2\pi^2\hbar}{e^2}$$

is the same, to within experimental errors, for films of differ-

ent thickness (Fig. 9a). It is also a function of $g\mu_B H/kT$, as follows from the coincidence of the curves, obtained at different temperatures, for the same film.

There are, to our knowledge, neither theoretical predictions nor experimental observations of a linear quantum magnetoresistance. A somewhat similar result has been reported in Ref. 14, where computer simulations of the magnetoresistance in the region of activated conductivity, $R > 2\pi^2\hbar/e^2$, have shown a region of linear dependence of the magnetoresistance on the magnetic field, but in the region we studied, $R \ll 2\pi^2\hbar/e^2$, the high-resistance contribution ought to be shorted out by metallic conductivity.

RUBIDIUM FILMS

In rubidium films, the theoretical expression comes closer to the experimentally observed amplitude and to the shape of the magnetoresistance curves in a parallel field (Fig. 10). For high-resistance films, the observed variation is nearly parabolic, as it should be for this range of fields. Since the measurements on rubidium films were carried out when the linear magnetoresistance in cesium films was already known, special attention was paid to the region of small magnetic fields. No linear dependence was observed for high-resistance films. For films of several kiloohms (curves 3 and 4 in Fig. 10), a cusp is observed at $H = 0$, similar to the one in Fig. 9 but significantly less pronounced. The linear part of the magnetoresistance is only a few times larger than the scatter of the experimental data. However, this effect ought be observable also for rubidium films, but the amplitude should be as small as the instrumental sensitivity. We do not understand why the linear magnetoresistance is not observed in rubidium, since the film structure and the electronic properties of both metals ought to be similar.

As shown in Fig. 10, the measured amplitude of the magnetoresistance of rubidium films in a parallel field increases considerably with decreasing resistance. In fact, the amplitude increases even faster, since it is partially balanced by the localization component of the magnetoresistance, which increases rapidly in low-resistance films. The small isotropic magnetoresistance in rubidium and the large localization component, which comes about because there is no spin-orbit scattering, do not allow us to draw any qualitative conclusions, a method for accurately resolving these contributions is needed.

CONCLUSIONS

In summary, our study of cesium and rubidium films show that there is a new, previously unobserved, component of the magnetoresistance in these films. It is related to quantum corrections to the conductivity, since the quantity,

$$\frac{\delta R}{R^2}\frac{2\pi^2\hbar}{e^2}$$

has approximately the same value, of order unity, for all films; in addition, it is isotropic with respect to the field, since the positive magnetoresistance, directly observed in a parallel field, is in agreement with the results obtained in a perpendicular field; finally, it depends on the spin of the conduction electrons, since it is a function of $g\mu_B H/kT$. The properties listed above allow us to state that the corrections to the conductivity found are related to electron-electron interactions. The quantitative discrepancies with theory and the large magnitude of the effect in cesium films remain unexplained.

It is also possible to draw consistent conclusions about the linear asymptotic dependence of the magnetoresistance observed in cesium films. The nondimensional variables found for it,

$$G \equiv 2\pi^2\hbar/e^2R, \quad h \equiv g\mu_B H/kT$$

(see Fig. 9, where each curve represents the family of curves obtained at different temperatures) indicate that this asymptotic behavior is related to quantum corrections to the conductivity and to the electron spin. It is independent of the direction of the magnetic field. This can be observed directly: the same cusp at the origin of the curve is observed both in a parallel and in a perpendicular field.

Further studies are needed of the conditions for the observation of the isotropic magnetoresistance, specially, of the requirements on film purity with respect to spin and spin-orbit scatterers. Quantum corrections coming from electron–electron interaction are very effectively suppressed by minute quantities of magnetic impurities in the film or by appreciable spin-orbit interactions. Then the relation given by Eq. (6) will be only fulfilled in strong fields,[4]

$$g\mu_B H \gg {}^4/_3\hbar\,(\tau_s^{-1}+\tau_{so}^{-1}). \tag{10}$$

The above condition is very strict and it explains why the effect is not observed for other metallic films. For most metals studied, it is very difficult to meet the low levels of magnetic impurities required, whereas such purity is easily achieved in alkali metals. In rubidium, for which there is no spin-orbit interaction, condition (10) holds. However, in cesium, there is spin-orbit scattering, as shown by the analysis of the localization component. This scattering, even if it were small and related to scattering centers which do not contribute to the resistivity, would nevertheless be sufficient to destroy the isotropic quantum resistance or to cut down its magnitude considerably. Instead, the amplitude of the isotropic quantum resistance in cesium is much larger than the one estimated theoretically and than the one observed in rubidium flims In small fields, where Eq. (6) is not applicable because condition (10) is not fullfilled, the magnetoresistance coming from electron–electron interaction may not disappear completely, but its behavior may change; in particular, its dependence may become linear.

The fundamental result reported in this paper is not the discovery of a new quantum magnetoresistance but the development of a method, which allows one to determine electron phase coherence times, once the localization component of the magnetoresistance is extracted in its pure form. It can be considered as established that the frequency of inelastic scattering is independent of resistance, that its value is close to $kT/\hbar$, that spin-orbit interaction is independent of the mean free path, and that it takes place at much larger distances. These results can be used to determine the temperature and dimensions at which microelectronic elements, based on quantum interference effects, work.

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