Resonance phenomena in a two-dimensional electron system with a periodically modulated density

V. B. Shikin

Institute of Solid-State Physics, Academy of Sciences of the USSR, Chernogolovka, Moscow Province

(Submitted 4 July 1990)
Zh. Eksp. Teor. Fiz. **98**, 2086–2097 (December 1990)

An analysis is made of various resonance phenomena in a two-dimensional electron system with a periodically modulated density. It is shown that a homogeneous exciting field can excite both cyclotron and plasma resonances. It is also shown that the ratio of the cyclotron and plasma resonance amplitudes gives information on the degree of modulation of the density of a two-dimensional electron system. The problem of the shift $\Delta\omega_c$ of a cyclotron resonance line is discussed. The reasons for the excitation of a resonance at twice the cyclotron frequency are considered for the case when the occupation factor ν tends to zero.

Excitation of a two-dimensional ($2D$) electron system with a periodically modulated[1)] density $n(x)$ by the application of a homogeneous electric field $E_x(t) = E_0\exp(i\omega t)$ oriented in the plane of the $2D$ system reveals a whole series of resonances of different origins.[1-3] They include harmonics of the plasma resonance ω_p, the cyclotron resonance ω_c in the presence of a magnetic field H normal to the plane of the $2D$ system, the double cyclotron resonance ω_{2c}, etc. Figure 1 shows schematically the behavior of these resonances as a function of a gate or control voltage V_g, which can alter the average electron density and the degree of its modulation. Resonances along the line ABC represent excitation of magnetoplasma waves (fundamental harmonic). The line DD_1 represents the cyclotron resonance and the line EF represents the double cyclotron resonance. There are also other peaks whose presence is not as important. The cases plotted in Figs. 1a–1c illustrate the sequence of events observed on gradual increase in the magnetic field intensity. For example, intersection of the lines AB and EF gives rise to a strong interaction between the double cyclotron and plasma modes, which lifts the degeneracy at the point where these lines intersect. We can clearly see also a tendency for an increase in the shift $\Delta\omega_c$ of the position of the cyclotron resonance line as the magnetic field intensity increases. The point B on the line ABC corresponds to the loss of continuity in the distribution of electrons [manifested by the appearance of zeros of the function $n(x)$ and formation of a system of quasi-one-dimensional electron channels in the range of voltages $V_g > V_g^*$].

Interpretation of the resonances shown schematically in Fig. 1 has been attempted in a number of papers, some of which[4-6] rely particularly on the experiments reported in Refs. 1–3. Other treatments[7-10] are of general nature, which can be applied also to the case of interest to us. The nonmonotonic dependence of the plasma frequency ω_p on V_g is explained fully in Refs. 4–6. The unresolved problem is the question whether the plasma frequency ω_p $(H\to 0, V_g \to V_g^*)$ is finite, as would follow from the observations reported in Refs. 1–3, or whether it should vanish, as predicted by calculations carried out within the classical hydrodynamic approximation framework.[4,6] The problem of the shift $\Delta\omega_c$ of a cyclotron resonance line of a $2D$ electron system with a periodically modulated density is considered in Refs. 7 and 8. Finally, the factors facilitating excitation of double cyclotron resonance in a periodically perturbed $2D$ system are discussed in Refs. 9 and 10. However, it should be pointed out that the results of Refs. 7–10 must be "fitted" to the conditions of Refs. 1–3.

Our aim is a critical analysis of the available theoretical results on resonance phenomena in a periodically modulated $2D$ electron system. We propose a fairly simple (but not entirely rigorous) procedure for estimating the degree of modulation of the electron density on the basis of the available experimental data on the cyclotron and plasma resonances, and we obtain an explicit expression for the screened periodic potential $\tilde{V}(x)$ which perturbs the motion of one electron in the $2D$ system. We use this information to obtain specific information on the shift $\Delta\omega_c$ that follows from predictions of

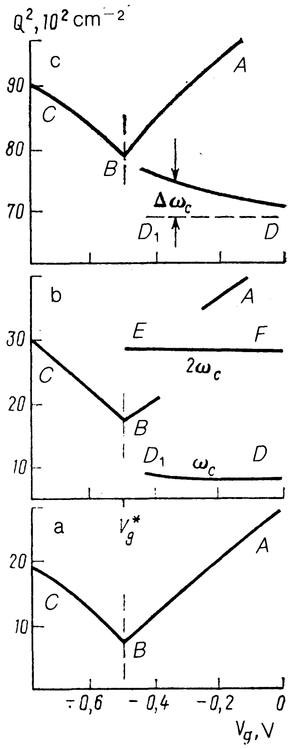

FIG. 1. Schematic representation of the behavior of various resonances as functions of the gate voltage V_g applied in the presence of different magnetic fields: a) $H = 0$; b) $H = 2.2$ T; c) $H = 6.4$ T. The line labeled ABC is a plasma resonance; DD_1 is a cyclotron resonance; EF is a double cyclotron resonance. The frequency ω is given by the expression $\omega = 2\pi c Q$, where c is the velocity of light. The dashed line DD_1 in Fig. 1c represents the unperturbed cyclotron frequency.

 0038-5646/90/121172-07$03.00

Refs. 7 and 8 for the situation described in Refs. 1–3. We consider coupling between the cyclotron and plasma motion in an excited 2D electron system with a periodically modulated density, and we propose a solution of the problem of the double cyclotron resonance in the limit when the population of the fundamental Landau level is low. Comments on the results obtained are given in the section headed Conclusions.

1. CLASSICAL ANALYSIS

A. The first question which needs to be discussed is how to describe a perturbation of the electron density in a 2D system. In principle, we can solve the relevant electrostatic problem of the distribution of the density in the 2D electron system when a periodically bent metal screen kept at a potential V_g is located in the vicinity of this system. However, the experiments reported in Refs. 1 and 2 show that modulation of the electron density in a 2D system also occurs even when $V_g = 0$. Therefore, we assume that an equilibrium density in a 2D electron system is set up in advance and is given by

$$n(x)=n_0+n_1\sin kx,\quad k=2\pi/a,\quad \delta=n_1/n_0<{}^1/_2, \tag{1}$$

where a is the perturbation period and δ is the degree of modulation. The dependence of the coefficients n_0 and n_1 on V_g is found directly from the experimental data[1,2] by a method which we shall discuss at the end of the next subsection (**B**).

B. The classical solution of the problem of excitation of a 2D modulated electron system at zero absolute temperature by a homogeneous electric field $E_x(t) = E_0\exp(i\omega t)$ reduces to the analysis of the following system of equations:

$$(\omega_c^2-\omega^2)v_x=\frac{i\omega e}{m^*}\left(\frac{d\varphi}{dx}+E_0\right),\quad \omega_c=\frac{eH}{m^*c}, \tag{2}$$

$$i\omega\delta n+\frac{d}{dx}[n(x)v_x]=0, \tag{3}$$

$$\Delta\varphi=0, \tag{4}$$

$$\left.\frac{\partial\varphi}{\partial z}\right|_{+0}-\left.\frac{\partial\varphi}{\partial z}\right|_{-0}=\frac{4\pi e}{\varkappa}\tilde n. \tag{5}$$

Here, $n(x)$ is the equilibrium density of electrons from Eq. (1); $\tilde n$ and φ are oscillations of the density and potential on excitation of plasma oscillations in the system; $v(x)$ is the local density of electrons; ω and ω_c is the frequency of oscillations and the cyclotron frequency in a magnetic field H; m^* is the effective mass of an electron; $\varkappa$ is the permittivity of the medium surrounding the investigated 2D electron system; and E_0 is the amplitude of the exciting oscillatory electric field. The condition (5) represents the dynamics of the system in the absence of screening electrodes.

If we assume that

$$\tilde n=\tilde n_1\cos kx,\quad v(x)=v_0+v_1\sin kx,\quad \varphi(x,z)=\varphi_1e^{\pm kz}\cos kx, \tag{6}$$

and use the smallness of the degree of modulation $\delta = n_1/n_0 \ll 1$ and, consequently, the smallness of $v_1/v_0 \ll 1$, we can reduce Eqs. (1)–(5) to equations for v_0 and v_1:

$$\omega_c^2-\omega^2=\frac{i\omega e}{m^*}E_0, \tag{7}$$

$$(\omega_c^2+\omega_p^2-\omega^2)v_1=-\frac{2\pi e^2n_1k}{\varkappa m^*}v_0, \tag{8}$$

where

$$\omega_p^2=\frac{2\pi e^2n_0}{\varkappa m^*}k.$$

It follows from Eqs. (7) and (8) that the homogeneous field E_0 excites two types of resonances: the usual cyclotron resonance at a frequency

$$\omega=\omega_c \tag{9}$$

and a magnetoplasma resonance at a frequency

$$\omega_p^2(H)=\omega_p^2+\omega_c^2. \tag{10}$$

In the range $|V_g|>|V_g^*|$ there is no homogeneous motion of electrons at the velocity v_0 and it is not possible to observe any "pure" cyclotron resonance.

The result of Eq. (10) provides a qualitatively correct description of the dependence of the magnetoplasma resonance frequency on the voltage V_g. In fact, the average density n_0 of electrons in the investigated 2D system decreases as V_g increases. This results in the experimentally observed[1-3] reduction in $\omega_p(V_g)$ as a function of V_g in the range $|V_g|<|V_g^*|$. This problem is discussed in greater detail in Refs. 4 and 6. In the cyclotron resonance case the classical response given by Eq. (9), $\omega_c = \text{const}$, identified by the dashed line in Fig. 1c, is insufficient to interpret the available data. Moreover, the hydrodynamic approach fails to predict also the resonance at the frequency $\omega = 2\omega_c$.

Explanation of the observed deviations of the cyclotron resonance goes beyond the above hydrodynamic description. Nevertheless, the classical solution of Eqs. (7) and (8) is very useful for analyzing the details of the experimental data. In particular, the solution makes it possible to estimate the degree of modulation of the 2D electron density from the experimentally determined amplitudes of the cyclotron and plasma resonances T_c and T_p [Eq. (11) applies to estimates of all the Fourier components of the expansion of the density $n(x)$ described by Eq. (1)]:

$$T_p/T_c=n_1/n_0. \tag{11}$$

Figure 2 shows the relevant information on the ratios of n_1/n_0 for various values of H based on the experimental results of Ref. 1.

The definition of Eq. (11) requires some comment.

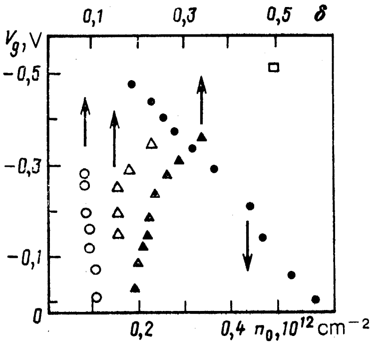

FIG. 2. Dependences of the following quantities on V_g: ●) average density n_0 [deduced from the dependence ω_p on V_g given by Eq. (8)]; ○), ▲), △) degree of modulation $\delta = n_1/n_0$ in fields H = 2.2, 2.9, and 6.4, respectively. The arrows identify the scale used. The point □ represents the position of quasi-one-dimensional channels.

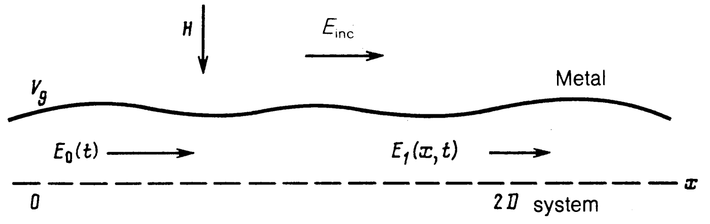

FIG. 3. Schematic representation of the relative positions of the $2D$ electron system and the metal gate electrode.

This is necessary because the proposed solution describing the excitation of a $2D$ electron system postulates that the exciting factor is simply a homogeneous electric field E_0. In fact, only the electromagnetic field incident on the gate electrode is homogeneous along the direction x; the electrode is bent periodically along the x direction as shown in Fig. 3. The existence of this geometric perturbation by the gate electrode results in modulation of the equilibrium density in the $2D$ system when a gate voltage V_g is applied between the electrode and the $2D$ system. However, this geometric modulation of the electrode shape leads unavoidably to the appearance in the transmitted wave not only of a homogeneous component E_0 of the electric field, but also of a coordinate-dependent correction $E_1(x,t) = E_1 \sin kx \exp(i\omega t)$, which participates together with E_0 in the excitations of the $2D$ system (Fig. 3). The properties of this additional field, which influences in particular the relationship (11), is discussed in the Appendix. Its influence on Eq. (11) can be ignored for $n_0 \ll \langle n_m \rangle$, where $\langle n_m \rangle$ is the average $2D$ density of electrons in the gate electrode, which is assumed to be satisfied.

2. QUANTUM CHARACTERISTICS OF THE CYCLOTRON RESONANCE

A. A quantum theory of the cyclotron resonance covering in principle the effects of interest to us was developed by Chaplik[7] and by Aĭzin and Volkov.[8] If the one-electron motion is perturbed by a one-dimensional potential

$$\tilde V(x) = \sum_m \tilde V_m \sin k_m x, \qquad \tilde V_m \ll \hbar\omega_c, \tag{12}$$

the electron spectrum in a magnetic field becomes

$$\varepsilon_l(q) = \sum_m \varepsilon_{lm}(q),$$
$$\varepsilon_{lm}(q) = \hbar\omega_c(l+{}^1/_2) + \tilde V_m L_l({}^1/_2 l_H^2 k_m^2) \times \exp(-{}^1/_4 l_H^2 k_m^2)\sin(l_H^2 q k_m). \tag{13}$$

Here, $l_H^2 = c\hbar/eH$ is the magnetic length, $L_l(x)$ are the Laguerre polynomials, $k_m = mk$, and $x_0 = eH/c\hbar q$ is the coordinate along the cyclotron orbit axis.

The appearance of magnetic minibands whose width depends on the Landau level number disturbs the initially equidistant nature of the electron spectrum in a magnetic field and thus complicates in particular the cyclotron resonance line profile. According to Ref. 8, when an exciting electric field is directed normally to the modulation axis and the $0 \to 1$ transition is excited, the real part of the conductivity of our $2D$ system is

$$\mathrm{Re}\,\sigma_{xx}(\omega) = \frac{e^2\omega_c^2}{4\pi\omega}\sum_m \frac{\nu_0}{[(\Delta_{01}^m)^2 - \hbar^2(\omega-\omega_c)^2]^{1/2}}, \tag{14}$$

where

$$\nu_0 = \pi l_H^2 n_0, \quad \Delta_{01}^m = \Delta_0^m - \Delta_1^m,$$
$$\Delta_l^m = \tilde V_m L_l({}^1/_2 l_H^2 k_m^2)\exp(-{}^1/_4 l_H^2 k_m^2).$$

Each term in the sum of Eq. (14) makes a contribution to the conductivity in the form of a double-humped fork with maxima at the following frequencies:

$$\omega = \omega_c \pm \Delta_{01}^m/\hbar. \tag{15}$$

Therefore, the problem of the cyclotron resonance line profile reduces to an estimate of the Fourier components $\tilde V_m$ of the screened potential.

B. The problem of the screening properties of an inhomogeneous $2D$ electron system in a magnetic field has been discussed quite thoroughly in the literature, mainly in connection with the behavior of the electron-density of states under the conditions of the quantum Hall effect. A qualitative analysis demonstrating the strong influence the screening properties of a $2D$ system in a magnetic field exert on the density of states of the magnetized $2D$ system can be found in the work of Luryĭ[11] and of Shklovskiĭ and Éfros.[12] Descriptions of the screening properties of the magnetized $2D$ system based on numerical calculations in the final stage were proposed by Labbe[13] and by Gerhardts and his colleagues.[14] In spite of the large number of papers on this topic, quite general methods for solving the nonlinear problem have not yet been proposed. We therefore briefly discuss the problem of nonlinear screening of the potential $V(x)$ in the usual random-phase approximation under the specific conditions of the experiments described in Refs. 1–3. The structure of the potential $V(x)$ is not known with the exception of information on its periodicity and the period a of the perturbation of the gate electrode. The explicit form of $V(x)$ is probably not very important in the case of qualitative treatments of the behavior of the cyclotron resonance. However, monochromaticity of this potential [i.e., the existence of just one harmonic in the expansion of Eq. (12)] is also unlikely.

An electron system of this kind largely screens the potential $V(x)$. For example, in classical electrostatics in the absence of a magnetic field such screening is complete, because the classical condition for equilibrium of a $2D$ system is

$$V(x) + e\varphi(x) = \mathrm{const}, \tag{16}$$

where $\varphi(x)$ is the screening potential.

Under quantum conditions (low temperatures, strong magnetic fields, reduction in the period of the one-dimensional potential, etc.) the condition of Eq. (16) is no longer obeyed and we have

$$\tilde V(x) = V(x) + e\varphi(x) \neq \mathrm{const}, \tag{17}$$

so that the one-electron motion in the $2D$ system begins to be disturbed by the potential $\tilde{V}(x)$. The relationship (17) then couples the initial potential $V(x)$, the screening potential $e\varphi$, and the effective perturbing field $\tilde{V}(x)$ of Eq. (12), which occurs in the theory of Refs. 7 and 8. Assuming that the combination $\tilde{V}(x)$ of Eq. (17) is small in the sense that

$$\tilde{V}(x) \ll \hbar\omega_c, \tag{17a}$$

we can solve the problem of deformation of the one-electron spectrum using perturbation theory, which gives the result of Eq. (13) for $\varepsilon_l(q)$ or, in the limit $kl_H \ll 1$, which applies to the situation in Refs. 1 and 2, the corresponding result is

$$\varepsilon_l(x) \approx \hbar\omega_c(l+{}^1/_2)+\tilde{V}(x), \quad \tilde{V}(x)=\sum_m \tilde{V}_m \sin k_m x. \tag{18}$$

This representation of the electron spectrum is typical of the Thomas–Fermi approximation. Its use simplifies greatly the subsequent treatment. Moreover, the Thomas–Fermi approximation is not subject to the condition (17a). The amplitude of the potential $\tilde{V}(x)$ can be arbitrary relative to the energy $\hbar\omega_c$.

The next step is determination of the equilibrium electron density in terms of the perturbed electron spectrum. In the Thomas–Fermi approximation of Eq. (18), we have

$$n(x)=\frac{g}{2\pi l_H^2}\sum_l \left\{\exp\left[\frac{\varepsilon_l+\tilde{V}(x)-\mu}{T}\right]+1\right\}^{-1}, \tag{19}$$

where

$$\varepsilon_l = \hbar\omega_c(l+{}^1/_2), \quad g=2,$$

where T is the absolute temperature and μ is the position of the chemical potential. Substituting the definition of $n(x)$ of Eq. (19) into the Poisson equation, we obtain a nonlinear equation for $\varphi(x)$, which is the main equation in the theory of screening of a magnetized $2D$ system. A fairly simple solution of this equation is possible only in the linear approximation[13,14] if

$$\tilde{V}(x) < T. \tag{20}$$

Unfortunately, the limiting case represented by Eq. (20) is insufficient to consider all the details of the cyclotron resonance problems that arise from the experimental data reported in Refs. 1–3.

When the relationship between $\tilde{V}(x)$ and T is arbitrary, we can estimate the screened potential $\tilde{V}(x)$ using the following considerations. As pointed out earlier in comments relating to Eq. (11), it should be possible to estimate experimentally the degree of modulation of the electron density as a function of V_g. Assuming therefore that the electron density $n(x)$ is described by Eq. (1) and that the ratio $\delta = n_1/n_0$ is given by Eq. (11), and also bearing in mind that the information on δ can be obtained, as in Ref. 2, in strong magnetic fields when the occupation factor is $\nu = \pi l_H^2 n_0 \lesssim 1$, we represent $\tilde{V}(x)$ as a function of $n(x)$ using the definition of $n(x)$ given by Eq. (19):

$$\tilde{V}(x)=\mu-\frac{\hbar\omega_c}{2}+T\ln\left[\frac{1}{\pi l_H^2 n(x)}-1\right], \tag{21}$$

where

$$n(x)=\left(1+\frac{T_p}{T_c}\sin kx\right)n_0, \quad \pi l_H^2 n(x)<1.$$

Data on n_0 and δ as functions of V_g are presented graphically in Fig. 2.

Knowing $\tilde{V}(x)$, we can now calculate the components $\tilde{V}_m$:

$$\tilde{V}_m=\frac{1}{a}\int_0^a \tilde{V}(x)\sin k_m x\,dx, \tag{22}$$

i.e., we can obtain information on the value of Δ_l^m from Eq. (14). Obviously, in calculating $\tilde{V}_m$ described by Eq. (22) the actual dependence of μ on V_g is not important, because by the definition of $\tilde{V}(x)$ in Eq. (21) the combination $\mu - 1/2\hbar\omega_c$ drops out of the integral of Eq. (22). When several Landau levels are filled, this particular simplification is lost.

The coefficients of $\tilde{V}_m$ described by Eq. (22) are calculated and the subsequent summation in the definition of σ_{xx} of Eq. (14) is performed numerically, because the structure of $\tilde{V}(x)$ of Eq. (21) is fairly complex. Nevertheless, it is obvious that the screened potential $\tilde{V}(x)$ of the magnetized $2D$ system is sensitive to V_g, because the intensity $n(x)$ changes under the action of $\tilde{V}_g$. Moreover, the degree of screening depends on the magnetic field intensity. In the definition of $\tilde{V}(x)$ of Eq. (21) this is manifested by the fact that, for example, if $\pi l_H^2 n_0 \to 1/2$, then the potential $\tilde{V}(x)$ is minimal, but it rises strongly as $n_0 \to 0$. Such nonmonotonic behavior of the degree of screening was naturally known earlier (see Refs. 11–14).

In commenting on the result given by Eq. (21) it is interesting to note the difference between the structure $\tilde{V}(x)$ of a magnetized $2D$ system and the corresponding quantity for a $2D$ system in the absence of a magnetic field. If the Thomas–Fermi approximation is valid, it then follows from Ref. 15 that

$$\tilde{V}(x)\,|_{H=0}=\mu-\frac{\pi\hbar^2}{2m^*}n(x). \tag{23}$$

The definition $\tilde{V}(x)$ of Eq. (23) naturally generalizes the equilibrium condition of Eq. (16). Comparison of the definitions of $\tilde{V}(x)$ of Eqs. (21) and (23) shows that in the former case the potential $\tilde{V}(x)$ may differ from a constant only for $T \neq 0$, whereas in the latter case the temperature is unimportant and it disappears completely from the definition of $\tilde{V}(x)$ given by Eq. (23).

C. Using the definitions of $\tilde{V}(x)$ given by Eq. (21) and of $\sigma_{xx}(\omega)$ given by Eq. (14), as well as the experimental data on the behavior of a cyclotron resonance line, we can analyze the capabilities of the theory of Refs. 7 and 8 in explaining the shift $\Delta\omega_c$. The experiments reported in Refs. 1 and 2 demonstrate that $\Delta\omega_c$ increases and there is a corresponding reduction in the electron mass m^* as the magnetic field increases. For example, according to Refs. 1 and 2, for $V_g = -0.2$ V and $H = 2.19$, 2.88, 6.4, or 9.87 T, the cyclotron mass assumes respectively the values $m^*/m_e = (7.2;\ 7.1;\ 6.99;\ 6.97)\cdot 10^{-2}$, where m_e is the mass of a free electron. However, the theory does not predict a reduction in $\Delta\omega_c$ with increasing H. In fact, the potential $\tilde{V}(x)$ of Eq. (21) increases no faster than logarithmically when H is increased. The shift $\Delta\omega_c = \Delta_{01}/\hbar$ of Eq. (15) is characterized by the fact that in addition to $\tilde{V}_1$ there is a factor $L_0(x) - L_1(x) \simeq x$ where $x = 1/2k^2 l_H^2 \ll 1$

($a=5\times10^{-5}$ cm, $l_H\approx10^{-6}$ cm when $H=5$ T and $kl_H\equiv 2\pi l_H/a=1.26\cdot10^{-1}$). Consequently, the combination described by

$$\Delta_{01}={}^1/_2\tilde{V}_1k^2l_H^2,\quad l_H^2\propto 1/H \tag{24}$$

is a decreasing function of the magnetic field, which disagrees qualitatively with the observations of Refs. 1 and 2.

Further analysis of the cyclotron resonance data for a $2D$ system with a modulated density must include a discussion of the nonparabolicity effects which influence the cyclotron resonance line position also in the case of homogeneous $2D$ systems.[16,17] However, the band nonparabolicity exhibited by GaAs is known to increase the effective cyclotron mass of an electron when the applied magnetic field is increased, i.e., allowance for this effect simply magnifies the difficulties encountered in explaining the observed behavior of the cyclotron mass in the experiments reported in Refs. 1 and 2.

3. COUPLING OF THE CYCLOTRON AND PLASMA MOTION

Qualitative difficulties in applying the formalism of the treatments in Refs. 7 and 8 encountered in explaining the shift $\Delta\omega_c$ observed experimentally[1-3] make it necessary to look for additional ways to explain $\Delta\omega_c$. One such explanation may be the coupling between the cyclotron and plasma motions which occurs when a $2D$ electron system is excited with a periodically modulated density. An investigation of this effect, carried out below on the basis of the classical equations of motion, shows that such a coupling does indeed exist. Consequently, the general picture of the influence of a periodic perturbation on the one-electron motion that follows from the discussions in Refs. 7 and 8 must be modified in the case of $2D$ electron systems with a finite density.

A. We can solve the problem by a number of refinements in Eqs. (2)–(5). However, it is initially desirable to consider the auxiliary problem of the excitation of a cyclotron resonance in the classical one-electron problem in the presence of an additional periodic potential $\tilde{V}(x)$, which disturbs electron motion. Its solution makes it possible in particular to reproduce the results of a quantum analysis in the limiting case $kl_H\ll1$. The initial equations of motion are

$$m^*\dot{v}_x=\frac{e}{c}v_yH+\tilde{V}'(x)+eE_x,\quad m^*v_y=-\frac{e}{c}v_xH. \tag{25}$$

Obviously, if the amplitude $\xi(t)$ of the cyclotron motion of an electron is sufficiently small, so that

$$x=x_0+\xi(t),\quad \xi\ll x_0\leqslant a,\quad v_x=\dot{\xi},\quad v_y=(i\omega_c/\omega)v_x, \tag{26}$$

the nonlinear system of equations (25) can be simplified to

$$(\omega_c^2-\omega^2)v_x=\frac{i\omega}{m^*}\left(\tilde{V}''|_{x=x_0}\frac{v_x}{i\omega}+eE_0\right) \tag{27}$$

or

$$\left[\omega_c^2+\frac{1}{m^*}V''(x_0)-\omega^2\right]v_x=\frac{i\omega e}{m^*}E_0. \tag{27a}$$

Here x_0 is the center of the electron orbit.

The cyclotron frequency ω_c is then shifted by an amount

$$\Delta\omega_c={}^1/_2V''(x_0)/m^*\omega_c, \tag{28}$$

which depends on the coordinate x_0. The shift $\Delta\omega_c$ is largest and positive at minima of the potential $\tilde{V}(x)$ and negative at its maxima. In reality an electron should be in the vicinity of one of the minima of the potential $\tilde{V}(x)$ where $\tilde{V}(x)\approx V_0+1/2Kx^4$, so that in the one-electron problem the shift $\Delta\omega_c$ is positive:

$$\Delta\omega_c={}^1/_2\omega_0^2/\omega_c,\quad \omega_0^2=K/m^*.$$

The result given by Eq. (28) for $\Delta\omega_c$ reproduces the predictions of the quantum theory described by Eq. (24) in the limiting case $kl_H\ll1$, which makes it possible to provide a simple classical explanation of this effect. The cyclotron frequency shifts because in the vicinity of extrema of the potential $\tilde{V}(x)$ the effective characteristic frequency of an electron is the combination $\omega^2=\omega_c^2\pm\omega_0^2$, which is typical of the motion of a magnetized electron in an additional parabolic potential $V=V_0+1/2Kx^2$.

B. We now consider the possibility of the appearance of a shift $\Delta\omega_c$ of the form Eq. (28) in a many-electron problem. We must begin with a discussion of the equilibrium properties of a $2D$ electron system perturbed by a potential $V(x)$. If $T\neq0$, then

$$\frac{m^*}{\tau}v_x=V'(x)+e\varphi'(x), \tag{29}$$

$$j(x)=Dn'(x)+n(x)v_x,\quad \mu_0=eD/T=e\tau/m^*, \tag{29a}$$

where $\varphi(x)$ is the equilibrium electrostatic potential of a system of electrons which screens the external potential $V(x)$, τ is the characteristic momentum relaxation time, and D is the diffusion coefficient related to the mobility μ_0 of an electron by the Einstein relationship.

At absolute zero the condition for an equilibrium $j(x)=0$ reduces to the requirement [by analogy with Eq. (16)]

$$\tilde{V}'(x)=V'(x)+e\varphi'(x)=0, \tag{30}$$

i.e., the Coulomb field of electrons screens the external potential $V(x)$ entirely. In this case the dynamic equation of motion is given by Eq. (2).

For $T\neq0$ it follows from Eqs. (29) and (29a) that the requirement $j(x)=0$ is equivalent to

$$Tn'/n+V'(x)+e\varphi'(x)=0. \tag{31}$$

In other words, the combination

$$\tilde{V}'(x)=V'(x)+e\varphi'(x)=-Tn'(x)/n(x) \tag{32}$$

differs from zero and it may occur in the equation of motion as an additional periodic potential $\tilde{V}(x)$, exactly as in Eq. (25). It is interesting to note that the classical potential $V(x)$ of Eq. (32), which perturbs the one-electron motion, is analogous to the quantum definition $\tilde{V}(x)$ of Eq. (21) in the limiting case $\nu=\pi l_H^2n_0\ll1$.

The equation of motion replacing Eq. (2) is now

$$(\omega_c^2-\omega^2)v_x=\frac{i\omega}{m^*}\left(-\frac{v_x}{i\omega}Tk^2\frac{n_1}{n_0}\sin kx+eE_0\right),\quad \frac{n_1}{n_0}\ll1. \tag{33}$$

Then, by analogy with Eqs. (26) and (27) and assuming, as before, that

$$\tilde{n}=\tilde{n}_1\cos kx,\quad v(x)=v_0+v_1\sin kx,\quad \varphi=\varphi e^{\pm kz}\cos kx,$$

we find that Eqs. (7) and (8) become

$$(\omega_c^2-\omega^2)v_0=\frac{i\omega e}{m^*}E_0-T\frac{k^2n_1}{m^*n_0}\langle\sin^2 kx\rangle v_1,$$

$$(\omega_c^2+\omega_p^2-\omega^2)v_1=-\frac{n_1}{n_0}\omega_p^2v_0\left(1-\frac{k^2T}{2m^*\omega_p^2}\right), \quad (34)$$

$$\omega_p^2=2\pi e^2n_0/\varkappa m^*, \quad \langle\sin^2 kx\rangle={}^1/_2.$$

It follows from the system (21) that when a periodically modulated electron system is excited we can expect coupling between the plasma mode and the cyclotron motion, which leads to renormalization of the cyclotron ω_c and plasma ω_p frequencies:

$$\tilde\omega_c^2=\omega_c^2+\Delta, \quad \tilde\omega_p^2=\omega_p^2-\Delta,$$

$$\Delta\omega_c=\frac{1}{2}\frac{\Delta}{\omega_c}, \quad \Delta=T\frac{k^2}{m^*}\left(\frac{n_1}{n_0}\right)^2\left(1-\frac{k^2T}{2m^*\omega_p^2}\right). \quad (35)$$

Unfortunately, the shift $\Delta\omega_c$ based on the definition of $\tilde\omega_c$ of Eq. (35) decreases, as before [see the definition of $\Delta\omega_c$ given by Eq. (28)], when the magnetic field is increased, so that we cannot identify this shift with that observed and reported in Refs. 1–3. On the other hand, it is obvious that the self-consistent shift $\Delta\omega_c$ of Eq. (35) considered for a system of interacting electrons differs considerably from an analogous shift of Eq. (28) in the one-electron problem and in particular we no longer have the splitting of the cyclotron resonance line given by Eq. (15). The result of Eq. (35) shows directly that the cyclotron resonance line shift should be $\Delta/\omega_c > 0$.

4. RESONANCE AT TWICE THE CYCLOTRON FREQUENCY

The experiments reported in Ref. 1 demonstrated a tendency for the ratio T_{2c}/T_p to be independent of the average density n_0 of a $2D$ electron system when n_0 is reduced (here, T_{2c} and T_p are the amplitudes of the double cyclotron resonance and of the plasma resonance, respectively). This behavior of the ratio T_{2c}/T_p is difficult to explain on the basis of the current ideas on the cause of the double cyclotron resonance, when the relative amplitude T_{2c}/T_p is governed by nonlocal effects in a degenerate magnetized $2D$ system:[9,10]

$$\frac{T_{2c}}{T_p}\propto\frac{k^2v_F^2}{\omega_c^2}, \quad \frac{m^*v_F^2}{2}=\frac{\pi\hbar^2}{m^*}n_0. \quad (36)$$

Here, v_F is the Fermi velocity in a $2D$ electron system, n_0 is the average density of electrons, and k is the wave number representing the spatial variation of the exciting electric field. Clearly, the ratio T_{2c}/T_p of Eq. (36) should decrease as n_0 is reduced, which was not observed experimentally[1] at relatively low values of n_0.

In the absence of the dependence of T_{2c}/T_p on n_0 the double cyclotron resonance may appear because of direct electron transitions at the frequency $2\omega_c$ caused by a spatially varying hf electric field with a potential $\tilde V(x) = \tilde V_0 \cos kx$. The corresponding matrix element [generalizing the definition of Eq. (13)],

$$\langle lq|\tilde V_0 \cos kx|l_1q_1\rangle,$$

differs from zero for arbitrary indices ll_1, which label the Landau levels. The relative amplitude of this double resonance is then controlled by the ratio

$$T_{2c}/T_p\propto(kl_H)^4, \quad (37)$$

which is independent of n_0.

CONCLUSIONS

We now draw some conclusions. A qualitative analysis has been given of the experimental data on the excitation of various resonances in a periodically modulated $2D$ electron system. These resonances are of interest not only in connection with the problems in the physics of collective phenomena in $2D$ electron systems, but also in connection with the development of new diagnostic possibilities in studies of spatially inhomogeneous electron systems. The following statements and results are relevant to the physics of collective phenomena in $2D$ charged systems:

a) The excitation of a weakly modulated $2D$ electron system is basically due to a homogeneous electric field $E(t) = E_0e^{i\omega t}$, whose presence may excite both the cyclotron and plasma resonances.

b) The plasma resonance frequency decreases with V_g because of a reduction in the average density n_0 of the $2D$ electron system. The behavior of these resonances as a function of V_g can be used to determine the dependence of the average density of the $2D$ electron system on V_g, as is done in Fig. 2 on the basis of the data reported in Ref. 1.

c) A method for reconstructing the effective potential $\tilde V(x)$, which quantizes the one-electron motion in a modulated $2D$ system, has been proposed. It involves the use of Eq. (21) for $\tilde V(x)$, where the information on the electron density $n(x)$ is obtained from Fig. 2 and from Eq. (11).

d) The explicit form of $\tilde V(x)$ of Eq. (21) has been used to analyze the problem of interpreting the observed cyclotron resonance shift $\Delta\omega_c$ on the basis of the theory of Refs. 7 and 8; however, the results are negative because the theory predicts a reduction in the shift on increase in the magnetic field, whereas the experiments demonstrate the opposite trend. Moreover, the problem is not solved either by the modification proposed in Sec. 4, which demonstrates the existence of a coupling between the cyclotron and plasma motion in a periodically modulated $2D$ system.

e) The cause of the resonance at double the cyclotron frequency have been analyzed when the occupancy factor ν is small. The proposed mechanism for exciting this resonance is independent of the occupancy factor and should therefore be dominant in the limit $\nu\to 0$.

In the opinion of the present author a feature of interest from the point of view of diagnostics of $2D$ inhomogeneous electron systems is the ability to estimate the degree of modulation of the electron density from the amplitudes of the cyclotron T_c and plasma T_p resonances [see Eq. (11) and the comments following it]. The above results show that modulation of the electron density in the experiments reported in Refs. 1 and 2 is governed not only by the presence of V_g, but also by additional factors associated with the preliminary modification of the $2D$ system. Consequently, the observed modulation is fundamentally different from those obtained on the basis of the estimates relying on the geometric shape of the gate electrode and on the potential difference V_g.

APPENDIX

We consider a $2D$ electron system covered by a metal gate electrode characterized by weak corrugations (Fig. 3). If a gate voltage V_g is applied between the conducting planes, the two-dimensional electron density $n_m(x)$ in the metal is modulated so that

$$n_m(x)=\langle n_m\rangle+n_m^{(1)}\sin kx,\quad k=2\pi/a. \tag{A1}$$

It is assumed that the average electron density $\langle n_m\rangle$ in the metal plate is much higher than the average density n_0 of Eq. (1), which applies to the $2D$ electron system located below the gate electrode.

An incident electromagnetic wave excites a current j_x in the metal. It is natural to assume that this current satisfies the condition div $j_x = 0$. If j_x is represented in the form

$$j_x=(\langle n_m\rangle+n_m^{(1)}\sin kx)(v_0+v_1\sin kx)=\text{const}, \tag{A2}$$

where v_0 and v_1 are the amplitudes of the average v_0 and oscillatory v_1 electron velocities in a metal plate, we find that to first order in $v_1/v_0 \ll 1$, the results are

$$\langle n_m\rangle v_0=\text{const},\qquad v_1=-v_0 n_m^{(1)}/\langle n_m\rangle. \tag{A3}$$

The velocities v_0 and v_1 correspond to electric fields E_0 and E_1:

$$eE_0=i\omega m^* v_0\quad E_1(x)=E_1\sin kx,\ E_1=-E_0 n_m^{(1)}/\langle n_m\rangle, \tag{A4}$$

which apply inside and outside the metal plate and which act, in particular, on the system of the $2D$ electrons below the plate. Therefore, Eq. (2) should in fact contain not only the field E_0, but also the oscillatory corrections $E_1 = E_1 \sin kx$. Applying now the whole procedure of calculations similar to that used in connection with Eqs. (2) and (8) above, we find that instead of Eq. (8), we now have

$$v_1=\frac{i\omega e}{m^*}E_0\left(\frac{n_m^{(1)}}{\langle n_m\rangle}+\frac{n_1}{n_0}\right)\Big/(\omega_c^2+\omega_p^2-\omega^2). \tag{A5}$$

Obviously, the definition of v_1 given by Eq. (A5) is identical with v_1 of Eq. (8) if

$$n_1\geqslant n_m^{(1)},\quad n_0\ll\langle n_m\rangle. \tag{A6}$$

These two inequalities are assumed to be obeyed. The first of them should generally reduce to an equality if modulation of the electron density in the investigated $2D$ system is simply related to the action of the gate potential V_g. The inequality $n_0 \ll \langle n_m\rangle$ actually represents the experimental situation: the average electron density in the gate electrode should be considerably higher than the average density of our $2D$ electron system so that in the range of frequencies of interest to us the incident electromagnetic wave does not excite intrinsic plasma frequencies of the gate electrode plate.

[1)] A $2D$ electron system was formed in the experiments described in Refs. 1–3 employing a single heterojunction made of GaAs and using a gate electrode with a periodic corrugation (Fig. 3). The data on the average density of electrons in the $2D$ system and on the degree of its modulation are given in Fig. 2. A typical temperature was $T\approx 2$ K. The modulation period was $a\approx 5\times 10^{-5}$ cm.

[1] W. Hansen, Thesis, Hamburg University (1987).

[2] J. P. Kotthaus, W. Hansen, W. Pohlmann, M. Wassermeier, and K. Ploog, Surf. Sci. **196**, 600 (1988).

[3] T. Demel, D. Heitmann, and P. Grambow, *Proc. NATO Advanced Research Workshop on Spectroscopy of Semiconductor Microstructures, Venice, 1989* (ed. by G. Fasol, A. Fasolino, and P. Lugli), Plenum Press, New York (1989), p. 75. [NATO Advanced Study Institute Series B, Physics, Vol. 206].

[4] V. Cataudella and V. M. Ramaglia, Phys. Rev. B **38**, 1828 (1988).

[5] V. B. Shikin, T. Demel, and D. Heitmann, Zh. Eksp. Teor. Fiz. **96**, 1406 (1989) [Sov. Phys. JETP **69**, 797 (1989)]; *Abstracts of Papers presented at Eighth Intern. Conf. on Electronic Properties of Two-Dimensional Systems, Grenoble, 1989*, p. 519.

[6] S. V. Meshkov, J. Phys. C (in press).

[7] A. V. Chaplik, Solid State Commun. **53**, 539 (1985).

[8] G. R. Aĭzin and V. A. Volkov, Preprint No. 10(428) [in Russian], Institute of Radio Engineering and Electronics, Academy of Sciences of the USSR, Moscow (1985); G. R. Aĭzin, Thesis for Candidate's Degree [in Russian], Institute of Radio Engineering and Electronics, Academy of Sciences of the USSR, Moscow (1985).

[9] A. V. Chaplik and D. Heitmann, J. Phys. C **18**, 3357 (1985).

[10] E. Batke, D. Heitmann, and C. W. Tu, Phys. Rev. B **34**, 6951 (1986).

[11] S. Luryi, *High Magnetic Fields in Semiconductor Physics* (Proc. Intern. Conf., Wurzburg, 1986, ed. by G. Landwehr), Springer Verlag, Berlin (1987), p. 16 [Springer Series in Solid State Sciences, Vol. 71].

[12] B. I. Shklovskiĭ and A. L. Éfros, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 520 (1986) [JETP Lett. **44**, 669 (1986)].

[13] J. Labbe, Phys. Rev. B **35**, 1373 (1987).

[14] U. Wulf, V. Gudmundsson, and R. R. Gerhardts, Phys. Rev. B **38**, 4218 (1988).

[15] V. Shikin, Pis'ma Zh. Eksp. Teor. Fiz. **50**, 150 (1989) [JETP Lett. **50**, 167 (1989)].

[16] F. Thiele, U. Merkt, J. P. Kotthaus *et al.*, Solid State Commun. **62**, 841 (1987).

[17] K. Ensslin, D. Heitmann, H. Sigg, and K. Ploog, Phys. Rev. B **36**, 8177 (1987).

Translated by A. Tybulewicz