Effect of Coulomb collisions on the space-time echo in a plasma

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The attenuation of the space-time echo in a plasma as the result of collisional relaxation of Van Kampen waves is analyzed. The Fokker-Planck operator, the Lenard-Bernstein operator, and the τ approximation are considered as collision integrals. It is possible to determine the dynamic friction and diffusion coefficients in velocity space which appear in the Fokker-Planck operator. It is also possible to determine the effective electron collision rate which appears in the two other model collision integrals. A comparison of the experimental echo characteristics with the theoretical predictions reveals the validity of the various model collision integrals for a real plasma.

1. INTRODUCTION

The echo effect in a plasma stems from the phase memory of particles, which retains information about an applied perturbation in the form of Van Kampen waves, which are rapid oscillations of the distribution function. Karpman[1] has shown that small-scale perturbations of this sort can be smoothed out by Coulomb collisions which involve small-angle scattering of charged particles and which are of diffusive nature. In other words, Coulomb collisions with a relatively small momentum transfer erase the phase memory of the particles and thus attenuate the echo.

The effect of Coulomb collisions and of the microscopic turbulence described by Fokker-Planck collision integrals on the temporal and spatial echoes was studied theoretically in Refs. 2–9. It was shown there that Coulomb collisions sharpen the echo, since the tails on the echo signal are attenuated more rapidly than the central part of the signal. It was also shown that diffusion in velocity space leads to a loss of the phase memory of the particle, i.e., to a reduction of the echo intensity. It was demonstrated that the plasma echo can be utilized to measure the time scale of diffusive damping and thus the diffusion coefficient and autocorrelation coefficient of electric field fluctuations.

The theoretical work has stimulated corresponding experiments, both in the range of ion waves[10–12] and in the range of electron waves.[13–15] The results of these experiments agree well with the theory. For example, Jensen *et al.*[13] studied the electron diffusion in velocity space which arises from the presence of a microscopic turbulence in a plasma. They obtained an experimental value for the diffusion coefficient which figures in quasilinear theory. The experimental method which makes use of the attenuation of an echo as a result of diffusion has demonstrated a surprising sensitivity: The measured mean free path was found to be 10^3 times as long as the experimental apparatus. The electron diffusion coefficient in velocity space which appears in the Fokker-Planck operator has been determined either from spatial Fourier components of the echo signal[14,15] or from the dispersion relation between the wavelength and frequency of the echo[15] which was found through separate measurements of the phase and amplitude of a signal. Such measurements are convenient when the structure of the echo signal can be determined quantitatively. That requirement, however, seriously limits the use of the echo method to study Coulomb collisions in several cases, e.g., in a solid-state plasma.

In the present paper we show that when the space-time echo[16] is used to study collisions and weak turbulence in a plasma one need measure only the amplitude of the echo signal at a fixed point. This simplification circumvents the obstacle which we just mentioned, and it also goes a long way toward simplifying the process of analyzing the experimental data to determine the electron diffusion coefficient in velocity space. First, a Fokker-Planck collision integral is used to describe the effect of Coulomb collisions on the space-time echo (a brief report of these results was published in Ref. 17). Lenard–Bernstein and τ-approximation model operators are then used for this purpose. A comparison of the experimental characteristics of the echo signal with the theoretical predictions reveals whether a given collision model is suitable for describing a plasma of interest.

2. FORMULATION OF THE PROBLEM; LINEAR APPROXIMATION

An echo in a plasma, which is the result of a nonlinear wave-particle interaction, can be observed most conveniently if the length scale of the Landau damping of external perturbations is short in comparison with the distance between the sources and also in comparison with the length scale of collisional damping. Nonlinear wave-wave interactions, which are capable of masking the echo signal in the case of a weak Landau damping, are then inconsequential, and the self-consistent field can be ignored.

We consider a homogeneous electron plasma with infinitely heavy ions. Restricting the discussion to one-dimensional waves, we start with the kinetic equation for the electron distribution function $f(x, v, t)$:

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}-\frac{e}{m}E\frac{\partial f}{\partial v}=\mathrm{St}\{f\}, \tag{1}$$

where the electric field $E(x, t)$ is the field of the external perturbations, $\mathrm{St}\{f\}$ is a collision integral, and e and m are the charge and mass of an electron. As mentioned earlier, Van Kampen waves, which store information about external perturbations, are damped particularly effectively as a result of Coulomb collisions of electrons characterized by a small change in momentum and thus deflections through small angles. The effect of such collisions is described quantitative-

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ly by the Fokker-Planck collision integral

$$\mathrm{St}\{f\}=-\frac{\partial}{\partial v}(D_1f)+\frac{\partial^2}{\partial v^2}(D_2f), \qquad (2)$$

where D_1 is the dynamic friction coefficient, and D_2 is the diffusion coefficient in velocity space.

We consider the space-time evolution of the response of the distribution function to an external perturbation

$$E(x,t)=\Phi_1\delta(x)\exp(i\omega_1t)\,[\theta(t)-\theta(t-\tau)]. \qquad (3)$$

Here Φ_1 is the amplitude of the potential perturbation, which is a high-frequency pulse of modulated frequency ω_1 with an envelope which is square (in time) with a length τ. This pulse is applied to the plasma at the time $t=0$ at the point $x=0$; $\theta(t)$ is the unit step function. Assuming that the external perturbation is small, we solve Eq. (1) with collision integral (2) perturbatively. We write the electron distribution function as a series

$$f=f_0+f^{(1)}+f^{(2)}+\ldots,$$

where f_0 is the unperturbed distribution function, and $f^{(1)}$ and $f^{(2)}$ are the corrections linear and quadratic in the external perturbation. Assuming that the coefficients D_1 and D_2 in the Fokker–Planck collision integral are small, so that the right side of Eq. (1) is small in comparison with the last term on the left side near the point $x=0$ [where the external perturbation (3) has not yet been damped by the Landau mechanism], we use the method developed in Ref. 5. In the collisionless limit, a solution of Eq. (1) for a δ-function linear perturbation of the distribution function

$$f_\omega^{(1)}(x,v)=\int dt\exp(i\omega t)f^{(1)}(x,v,t)$$

is

$$f_\omega^{(1)}(x,v)=i\frac{e\Phi_1}{m}\frac{\theta(v)}{v}\frac{\partial f_0}{\partial v}\frac{1-\exp[i(\omega+\omega_1)\tau]}{\omega+\omega_1+i0}\exp\left(i\frac{\omega}{v}x\right). \qquad (4)$$

The function $\theta(v)$ here reflects the circumstance that solution (4) is valid for particles with positive velocities; the infinitesimal increment $i0$ specifies the way the pole is circumvented.

In a region without a macroscopic electric field, the function $f_\omega^{(1)}$ satisfies the equation

$$-i\omega f_\omega^{(1)}+v\frac{\partial f_\omega^{(1)}}{\partial x}=-\frac{\partial}{\partial v}(D_1f_\omega^{(1)})+\frac{\partial^2}{\partial v^2}(D_2f_\omega^{(1)}), \qquad (5)$$

supplemented by the boundary condition which makes the solution of Eq. (5) go over to the collisionless solution (4) at small values of x. Under the assumption that the coefficients D_1 and D_2 are small, the derivatives $\partial f_\omega^{(1)}/\partial v$ on the right side of Eq. (5) can be replaced by their approximations found by differentiating the most rapidly varying factor:

$$\frac{\partial f_\omega^{(1)}}{\partial v}\approx-i\frac{\omega x}{v^2}f_\omega^{(1)}.$$

Finally, taking inverse Fourier time transforms, we find the following expression for the linear response of the distribution function to external perturbation (3) (this expression is valid over distances $x>k_L^{-1}$, where k_L is the coefficient of spatial Landau damping):

$$f^{(1)}(x,v,t)=\frac{e\Phi_1}{m}\frac{\theta(v)\theta(x)}{v}\frac{\partial f_0}{\partial v}\times\left[\theta\left(t-\frac{x}{v}\right)-\theta\left(t-\frac{x}{v}-\tau\right)\right]\cdot\exp\left[i\omega_1\left(t-\frac{x}{v}\right)-i\frac{\omega_1}{v^3}\int_0^x D_1y\,dy-\frac{\omega_1^2}{v^5}\int_0^x D_2y^2\,dy\right]. \qquad (6)$$

The problem of the effect of Coulomb collisions on the temporal and spatial echoes was solved exactly in Ref. 18. In the limit of low collision rates, the exact result becomes the expression which was found in Ref. 5 by joining the collisionless solution with the solution of the ballistic equation (5). The legitimacy of the approximate method was thus confirmed.

3. SPACE-TIME ECHO

For an echo response to arise, the phase evolution in (6) must be inverted. In the problem at hand, that of the space-time echo, this inversion is achieved by applying a second external perturbation pulse, at the point $x=l$, at the time $t=T$. The spatial and temporal intervals between the first and second pulses must exceed the length scale k_L^{-1} and the time scale γ_L^{-1} of Landau damping: $k_Ll>1$ and $\gamma_LT>1$. The assumption (used above) of low collision rates requires the conditions $k_cl<1$ and $\gamma_cT<1$, where k_c^{-1} and γ_c^{-1} are respectively the length scale and time scale of the collisional damping. By analogy with (3), we specify the second external perturbation pulse to be of the form

$$E(x,t)=\Phi_2\delta(x-l)\exp(-i\omega_2t)\,[\theta(t-T)-\theta(t-T-\tau)]. \qquad (7)$$

The application of this pulse to the plasma gives rise to both a linear response of the distribution function, at the frequency ω_2 [this response is analogous to the response (6)], and the nonlinear modulation of the perturbation (6), which is described by Eq. (1) in second-order perturbation theory. The nonlinear response of the electron distribution function to the sequential application of the external perturbations (3) and (7) is found from the kinetic equation, as in the derivation of the linear response (6). The result is

$$f^{(2)}(x,v,t)=i\frac{e^2\Phi_1\Phi_2\omega_1l}{m^2}\frac{\theta(v)\theta(x-l)}{v^4}\frac{\partial f_0}{\partial v}\times\exp\left[-i\omega_3\left(t-\frac{x-l'}{v}\right)-i\frac{\omega_1}{v^3}\int_0^l D_1x\,dx-\frac{\omega_1^2}{v^5}\int_0^l D_2x^2\,dx+i\frac{\omega_3}{v^3}\int_l^x D_1(y-l')\,dy-\frac{\omega_3^2}{v^5}\int_l^x D_2(y-l')^2\,dy\right]\cdot\left\{\left[\theta\left(t-\frac{x}{v}\right)-\theta\left(t-\frac{x-l}{v}-T-\tau\right)\right]\right.$$

$$\times\left[\theta\left(v-\frac{l}{T+\tau}\right)-\theta\left(v-\frac{l}{T}\right)\right]$$
$$+\left[\theta\left(t-\frac{x-l}{v}-T\right)-\theta\left(t-\frac{x}{v}-\tau\right)\right]$$
$$\times\left[\theta\left(v-\frac{l}{T}\right)-\theta\left(v-\frac{l}{T-\tau}\right)\right]\Bigg\}. \tag{8}$$

Here $\omega_3=\omega_2-\omega_1$; $l'=l\omega_2/\omega_3$; and the function $\theta(x-l)$ reflects the conditions that the echo signal arise only in the region $x>l$. Since the coefficients D_1 and D_2 are small, we can replace the upper limit of the integration in the last two terms of the exponential factor in (8) by l'.

Substituting (8) into the Poisson equation, we find the electric field of the echo signal to be

$$E^{(2)}(x,t)=i\frac{4\pi e}{\omega_3}\int dv\,v f^{(2)}(x,v,t). \tag{9}$$

Because of the rapidly oscillating function of the velocity in (8) $-\exp[i\omega_3(x-l_3)/v]$—the integral in (9) vanishes throughout space except near the point $x=l_e$. It is not difficult to show that we have

$$l_e=l'+\frac{\omega_1}{v^2\omega_3}\int_0^l D_1 x\,dx-\frac{1}{v^2}\int_l^{l'} D_1(x-l')\,dx. \tag{10}$$

Restricting the analysis to the central part of the echo signal at the point $x=l_e$, we can carry out the integration over velocity in (9) with the help of the mean value theorem, since the difference between two unit step functions with approximately equal arguments lying near the point $v_0=l/T$, under the condition $\tau\ll T$, is retained in the integrand. In addition, in the case of a damping due to Coulomb collisions the coefficients D_1 and D_2 are independent of the spatial coordinate and the time, although this condition may not be satisfied if the damping is caused by microscopic turbulence.[5] In the case of Coulomb collisions we thus have

$$l_e=l'\left(1+\frac{D_1\omega_1 T^2}{2\omega_3 l}\right). \tag{11}$$

As a result, expression (9) for the electric field of the echo response in its central part, $x=l_e$, becomes

$$E^{(2)}(x=l_e,\ t)=A\exp(-i\omega_3 t)E(t), \tag{12}$$

where

$$E(t)=\frac{\omega_3}{\omega_1\tau}\left(t-\frac{\omega_2}{\omega_3}T+\frac{\omega_1}{\omega_3}\tau\right)$$
$$\times\left[\theta\left(t-\frac{\omega_2}{\omega_3}T+\frac{\omega_1}{\omega_3}\tau\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T\right)\right]$$
$$+\left[\theta\left(t-\frac{\omega_2}{\omega_3}T\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T-\tau\right)\right]+\frac{\omega_3}{\omega_1\tau}\left(t-\frac{\omega_2}{\omega_3}T-\frac{\omega_2}{\omega_3}\tau\right)$$
$$\cdot\left[\theta\left(t-\frac{\omega_2}{\omega_3}T-\tau\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T-\frac{\omega_2}{\omega_3}\tau\right)\right], \tag{13}$$

$$A=-\frac{4\pi e^3\Phi_1\Phi_2\omega_1\tau}{m^2\omega_3 v_0}\left.\frac{\partial f_0}{\partial v}\right|_{v=v_0}\cdot\exp\left(-\frac{D_2 l^3\omega_1^2\omega_2}{3v_0^5\omega_3}\right). \tag{14}$$

The echo response in (12) is thus a high-frequency pulse with a modulated frequency ω_3, a trapezoidal envelope $E(t)$, and an amplitude A. The coefficients D_1 and D_2, which depend on the particle velocity, in expressions (11) and (14) are of the form $D_1=D_1(v_0)$ and $D_2=D_2(v_0)$.

It can be seen from (14) that the value of the electron diffusion coefficient in velocity space, $D_2(v_0)$, can be found from the behavior of the amplitude of the space-time echo signal as a function of the delay time T or the distance (l) between the external perturbations which excite the echo. It is first necessary to determine the value of the derivative $\partial f_0/\partial v$ from the echo signal obtained over a distance l small enough that the effect of collisions on the echo amplitude is negligible.[16] We have made no assumptions regarding the shape of the unperturbed electron distribution function $f_0(v)$, which may in general be non-Maxwellian. In addition, the dynamic friction coefficient $D_1(v_0)$ can be determined from the shift of the maximum of the echo signal, $x=l_e$, with respect to the point $x=l'$ predicted by the collisionless theory, with the help of (11).

4. THE LENARD-BERNSTEIN COLLISION INTEGRAL AND THE τ APPROXIMATION

In studies of how collisions of charged particles accompanied by small-angle deflections affect the distribution function of these particles, the exact Coulomb collision integral is frequently replaced by a model collision term of the Fokker-Planck type which was proposed by Lenard and Bernstein:[19]

$$\mathrm{St}\{f\}=\nu\frac{\partial}{\partial v}\left(vf+v_{Te}^2\frac{\partial f}{\partial v}\right). \tag{15}$$

Here ν is the effective collision rate, and $v_{Te}=(T_e/m)^{1/2}$ is the electron thermal velocity. The Lenard-Bernstein collision integral in (15) retains two extremely important properties of the Coulomb collision integral: It is of a diffusive nature, and it vanishes when a Maxwellian distribution function is substituted into it.

Using the method outlined above to find the asymptotic solution of kinetic equation (1), we find the linear response of the distribution function to the external perturbation (3) for the collision integral (15):

$$f^{(1)}(x,v,t)=\frac{e\Phi_1}{m}\frac{\theta(v)\theta(x)}{v}\frac{\partial f_0}{\partial v}$$
$$\times\exp\left[i\omega_1\left(t-\frac{x}{v}\right)+i\frac{\nu\omega_1 x^2}{2v^2}-\frac{\nu\omega_1^2 v_{Te}^2 x^3}{3v^5}\right]$$
$$\cdot\left[\theta\left(t-\frac{x}{v}\right)-\theta\left(t-\frac{x}{v}-\tau\right)\right]. \tag{16}$$

The electric field of the echo signal is described again in this case by expression (12), but its maximum is at the point $x=l_e'$, where

$$l_e'=l'\left(1-\frac{\nu T\omega_1}{2\omega_3}\right). \tag{17}$$

Its amplitude is

$$A=-\frac{4\pi e^3\Phi_1\Phi_2\omega_1\tau}{m^2\omega_3 v_0}\left.\frac{\partial f_0}{\partial v}\right|_{v=v_0}\cdot\exp\left(-\frac{\nu v_{Te}^2 l^3\omega_1^2\omega_2}{3v_0^5\omega_3}\right). \tag{18}$$

The effective collision rate $\nu=\nu(v_0)$ can evidently be found from the T or l dependence of the maximum amplitude of the echo response, with the help of the method proposed in the preceding section of this paper for determining the diffusion coefficient in velocity space.

Finally, we consider a very simple model collision integral which is frequently used in estimating electron collisions with neutral particles:

$$\mathrm{St}\{f\} = -\frac{f-f_0}{\tau_0}, \tag{19}$$

where $\tau_0 = \nu_0^{-1}$ is the the mean free time of the electrons. This is the so-called τ approximation in kinetic theory.[20] Although this model does not conserve charge it does describe the disruption of the phase of the ordered motion of particles in a Van Kampen wave as a result of collisions. The kinetic equation (1) with the collision integral (19) can be solved exactly in first- and second-order perturbation theory. The linear response of the distribution function to perturbation (3) is

$$f^{(1)}(x,v,t) = \frac{e\Phi_1}{m}\frac{\theta(v)\theta(x)}{v}\frac{\partial f_0}{\partial v}\exp\left[i\omega_1\left(t-\frac{x}{v}\right)-\frac{\nu_0 x}{v}\right] \cdot\left[\theta\left(t-\frac{x}{v}\right)-\theta\left(t-\frac{x}{v}-\tau\right)\right]. \tag{20}$$

The nonlinear response of the electric field to the sequential application of pulses (3) and (7) is described (as in the two preceding cases) by (12), in which $l_e \equiv l'$, and the amplitude of the echo signal is

$$A = -\frac{4\pi e^3\Phi_1\Phi_2\omega_1\tau}{m^2\omega_3 v_0}\frac{\partial f_0}{\partial v}\bigg|_{v=v_0}\cdot\exp\left(-\nu_0\frac{\omega_2}{\omega_3}T\right). \tag{21}$$

A comparison of this expression with the result of the collisionless approximation[16] shows that the effect of a collision integral like (19) on the space-time echo is to reduce the signal amplitude. This reduction is described by the exponential factor $\exp(-\nu_0 T\omega_2/\omega_3)$.

Collision operators of the diffusive type like (2) and (15) lead to the same functional dependence of the amplitude of the space-time echo signal on l and T, while this dependence is qualitatively different for the τ-approximation model (19).

5. DISPERSION PROPERTIES OF THE PLASMA

The ballistic approximation, which we used above, involves ignoring the dispersion properties of the plasma. That simplification is justified if the frequencies of the external perturbations and of the echo signal lie outside the range of natural plasma waves. In the opposite case, a resonance between the high-frequency driving force and collective plasma oscillations gives rise to a substantial change in the echo amplitude, as was shown in Ref. 21. In order to deal with the effect of the collective properties of the plasma in a study of Coulomb collisions by means of the space-time echo effect, we need to treat the field $E(x, t)$ in Eq. (1) as a self-consistent electric field satisfying the Poisson equation

$$\frac{\partial E}{\partial x} = -4\pi e\int dv f + 4\pi\rho. \tag{22}$$

Here $\rho(x, t)$ is the density of the external charge, which we specify as follows, by analogy with the external perturbations in (3) and (7):

$$\rho(x,t) = \rho_1\delta(x/r_D)\exp(i\omega_1 t)[\theta(t)-\theta(t-\tau)] + \rho_2\delta[(x-l)/r_D]\exp(-i\omega_2 t)[\theta(t-T)-\theta(t-T-\tau)], \tag{23}$$

where r_D is the electron Debye length.

Solving the system of equations (1), (22) with the collision integral (2) by the method of successive approximations, under the assumption that the coefficients $D_1(v)$ and $D_2(v)$ are small, we find that the nonlinear echo response of the electric field to external perturbations (23) has the form (12), with an amplitude

$$A = -\frac{(4\pi e)^3\rho_1\rho_2 r_D^2\tau v_0}{m^2\omega_2\omega_3}\frac{\partial f_0}{\partial v}\bigg|_{v=v_0}\cdot\exp\left(-\frac{D_2 l^3\omega_1^2\omega_2}{3v_0^5\omega_3}\right) \cdot\varepsilon^{-1}\left(-\frac{\omega_1}{v_0},-\omega_1\right)\varepsilon^{-1}\left(\frac{\omega_2}{v_0},\omega_2\right)\varepsilon^{-1}\left(\frac{\omega_3}{v_0},\omega_3\right), \tag{24}$$

where $\varepsilon(k,\omega)$ is the ordinary dielectric constant of an electron plasma, which describes electrostatic waves:

$$\varepsilon(k,\omega) = 1 + \frac{4\pi e^2}{mk}\int\frac{dv}{\omega-kv+i0}\frac{\partial f_0}{\partial v}. \tag{25}$$

All three dielectric constants in (24) describe waves with the same phase velocity v_0. This is because the space-time echo signal is generated exclusively by those electrons whose velocities are close to v_0.

For a quantitative estimate of the amplitude of the echo signal, we replace the dielectric constants in (24) by their values calculated from (25) when a Maxwellian distribution function is used as f_0. This approximation is legitimate since f_0 is inside the integral in (25), while the dielectric constants (25) themselves are inside a logarithm in the expression for the diffusion coefficient D_2. We can thus write

$$\varepsilon\left(\frac{\omega}{v_0},\omega\right) = 1 + \frac{v_0^2}{r_D^2\omega^2}\left[1-J_+\left(\frac{v_0}{v_{Te}}\right)\right], \tag{26}$$

where $J_+(x)$ is the plasma dispersion function.

It follows from (26) that when the coefficient D_2 is determined with the help of the space-time echo effect the features associated with the plasma dispersion may be manifested only at high velocities, $v_0 \gg v_{Te}$, and then only if the frequency of the external perturbation or that of the echo falls in the range of natural plasma wave frequencies. A resonance peak appears on the curve of the echo signal amplitude versus v_0 in this case. This peak lies near the point at which v_0 coincides with the phase velocity of the plasma wave with the given frequency. The amplitude of the peak at this point can be found from (9), where the value of the integral over the velocity is determined by the pole which stems from the corresponding dielectric constant.

6. CONCLUSION

The space-time echo is a time-dependent phenomenon and thus difficult to study experimentally. This difficulty can be countered, however, by simplifying the process of analyzing and interpreting the data to find the characteristics of the Coulomb collisions. Specifically, the measurements of the electron diffusion coefficient in velocity space which were carried out in Refs. 14 and 15 were based on a determination of the shape of the spatial echo signal and a subsequent calculation of the Fourier harmonics of the envelope of this signal. That method would be difficult to apply to (for example) a solid-state plasma. The space-time echo effect, whose signal is generated by electrons with a given velocity $v_0 = l/T$, makes it possible to eliminate the determination of the spatial shape of the envelope and to simply measure the v_0 dependence of the amplitude of the echo signal at the point of its maximum. This dependence can be used along with (14) or (24) to find the diffusion coefficient $D_2(v_0)$ in velocity space, which figures in the Fokker-Planck collision integral. Alternatively, one can use (18) and (21) to determine the effective collision rate ν in the Lenard-Bernstein collision integral and in the τ approximation. The method proposed here for measuring electron collision rates can be used for a solid-state plasma, by working from the theoretical and experimental studies in Refs. 22–24 of the spatial echo in a metal plasma.

The validity of various models for ion collisions was evaluated in Refs. 10–12 by comparing experimental results on the spatial echo in the range of ion frequencies with the predictions of corresponding theories. The results of Refs. 10 and 11, on a highly ionized plasma, agree well with the theory of Coulomb collisions described by the Fokker-Planck collision operator. The effect of collisions of ions with neutral atoms, on the other hand, turned out to be negligible. The opposite situation arose in the experiments of Ref. 12: The attenuation of the echo amplitude was described well by the effect of ion collisions with neutral atoms in the τ approximation, while no manifestations of ion-ion diffusion collisions were seen.

The results of the present study imply that, in principle, one can use the l and T dependence of the echo amplitude and also the spatial position of the maximum of the echo signal to evaluate the validity of a given model collision integral for studying a particular plasma. It follows from (11) that because of the dynamic electron friction which is characteristic of the Fokker-Planck operator the maximum of the echo signal lies further from the sources of the external perturbations than the point $x = l'$, predicted by the collisionless theory. This shift can be determined in absolute value from the coefficient $D_1(v_0)$. For the Lenard–Bernstein operator, which is also of a diffusive nature, the echo maximum shifts away from l' in the opposite direction. An additional possibility for testing the Lenard-Bernstein model stems from the circumstance that the shift and the attenuation of the echo maximum are determined by the same effective collision rate. In the τ approximation, collisions do not shift the maximum of the echo signal away from the point $x = l'$.

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