# Domain-wall drift in an oscillating magnetic field

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(Submitted 10 February 1990; resubmitted 1 June 1990)
Zh. Eksp. Teor. Fiz. **98**, 1345–1353 (October 1990)

A theory is developed for the drift of 180-degree Bloch- and Néel-type domain walls in an external oscillating magnetic field of arbitrary frequency. The dependence of the drift velocity on the frequency and polarization of the oscillating field, on the topological charge of the domain wall, and on the direction of the magnetization-vector rotation in the wall is determined.

## 1. INTRODUCTION

Domain-wall (DW) drift in a ferromagnet constitutes the onset of a constant DW velocity in an external magnetic field that is homogeneous in space and oscillates in time. The first to call attention to the possibility of drift motion of a solitary DW were Schlöman and Milne,[1] who considered the case when the frequency $\omega$ of the external field considerably exceeds the frequency $\omega_r$ of the homogeneous ferromagnetic resonance. The approach proposed in Ref. 1 was subsequently extended to the general case of an arbitrary external-field frequency.[2] Schlömann and Milne attributed the DW drift to the action of a constant effective pressure due to the difference in the ferromagnet energy density on the two sides of the DW. It was assumed that the pressure applied to the DW is equal to the difference, averaged over the field oscillations, of these energy densities.

Another approach[3,4] to the description of the DW drift, is based on the equations of motion, in a spherical coordinate frame, that specify the direction of the magnetization vector. The region in which this theory is valid is bounded by the condition $\omega \gg \omega_r$. It is obviously possible go outside this bound, in the case of a small-amplitude external field, by resorting to perturbation theory. It is clear, however, that standard perturbation theory cannot describe DW drift for solitons,[5] for in this theory the time derivatives of the parameters of the unperturbed solution of the equation of motion for the magnetization, particularly the DW velocity, are assumed to be linear in the amplitude of the oscillating field, whereas the DW drift is at least second-order in the field amplitude.

The DW drift induced by an oscillating field was experimentally observed in the form of translational motion of the stripe domain structure.[6,7] These experimental data, unfortunately, are insufficient for a detailed comparison with theoretical results.

Drift of a magnetic topological soliton of another type, a Bloch line, induced by an external low-amplitude oscillating field, was investigated in Refs. 8 and 9. Iordanskii and Marchenko[9] have proposed a theory of such a drift and have shown that the drift velocity is proportional to the square of the external-field amplitude.

In contrast to Refs. 1–4, no restrictions were imposed in Ref. 9 on the frequency of the magnetic field, but its amplitude was assumed to be low. Our present calculation procedure is close to that of Ref. 9 and has two basic purposes: 1) to develop a systematic asymptotic procedure for describing the dynamics of a magnetic soliton [of a 180-degree DW of the Bloch (Néel) type] in a weak oscillating field of arbitrary frequency and polarization; 2) to demonstrate the possibility of a directed drift of a system of such DW in an oscillating field polarized in the plane of rotation of the magnetization vector in the DW.

Even though a Bloch (Néel) DW is as a rule a rather idealized model of real DW, the advantage offered by the possibility of investigating its dynamics in detail justifies, from our viewpoint, the formulation of this problem.

## 2. BASIC EQUATIONS

We consider the dynamics of the magnetization vector $\mathbf{M}$ in an oscillating magnetic field $\mathbf{H}_1(t)$ by starting from the phenomenological Landau–Lifshitz equation[10] with a relaxation term in the Gilbert form

$$\dot{\mathbf{M}} = -\gamma[\mathbf{MH}] + (\alpha/M)[\mathbf{M}\dot{\mathbf{M}}]. \tag{1}$$

Here $\mathbf{M} = \mathbf{M}(y,t)$, $\dot{\mathbf{M}} = \partial\mathbf{M}/\partial t$, $M = |\mathbf{M}|$, $\gamma$ is the gyromagnetic ratio and $\alpha$ is the damping parameter. Assuming that the energy of a two-axis ferromagnet is of the form

$$W = \int_{-\infty}^{\infty} \left[\frac{1}{2}\varepsilon\mathbf{M}'^2 - \frac{1}{2}\beta M_z^2 + \frac{1}{2}\varkappa M_y^2 - \mathbf{MH}_1\right] dy, \tag{2}$$

we write for the magnetic field $\mathbf{H} = \mathbf{H}(y,t)$, connected with $W$ by the relation $\mathbf{H} = -\delta W/\delta\mathbf{M}$, the expression

$$\mathbf{H} = \varepsilon\mathbf{M}'' + \beta M_z\mathbf{e}_z - \varkappa M_y\mathbf{e}_y + \mathbf{H}_1, \tag{3}$$

where $\varepsilon$ is the exchange constant, $\beta > 0$ and $\varkappa$ are the anisotropy constants, $\mathbf{M}' = \partial\mathbf{M}/\partial y$, and $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors along the corresponding axes of the Cartesian coordinate frame $xyz$.

In the ground state (at $\mathbf{H}_1 = 0$) the magnetization distribution $M^{(0)}(y)$, which satisfies the condition

$$[\mathbf{M}^{(0)}\mathbf{H}^{(0)}] = 0 \tag{4}$$

and the boundary conditions $\mathbf{M}^{(0)}(\pm\infty) = \mp\delta M\mathbf{e}_z$ ($\delta = \pm 1$ is the DW topological charge[11]) corresponding to an 180-degree DW, can have the structure of a Bloch or Néel DW, depending on the sign of $\varkappa$. Thus, according to Eq. 2 the magnetization vector should undergo rotation in the $xz$ plane, i.e., in the DW plane (Bloch-type DW) if $\varkappa > 0$, and in the $yz$ plane (Néel-type DW) if we have $\varkappa < 0$ and $\beta > |\varkappa|$. It is known then that in the case of a Bloch DW we have

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$$\mathbf{M}^{(0)}(y)=\mathbf{M}_B(y)=\mathbf{e}_x\rho M\sin\theta_B(y)+\mathbf{e}_z\delta M\cos\theta_B(y),$$
$$\sin\theta_B(y)=\operatorname{sech}(y/\Delta),\quad \cos\theta_B(y)=-\operatorname{th}(y/\Delta) \tag{5}$$

($\Delta=(\varepsilon/\beta)^{1/2}$) is the DW-width parameter and $\rho=\pm1$ indicates the direction of the magnetization-vector rotation in the DW), while in the case of a Néel DW

$$\mathbf{M}^{(0)}(y)=\mathbf{M}_N(y)=\mathbf{e}_y\rho M\sin\theta_N(y)+\mathbf{e}_z\delta M\cos\theta_N(y).$$

The expressions for $\sin\theta_N(y)$ and $\cos\theta_N(y)$ are of the same form as in the case of a Bloch DW, but $\beta$ must be replaced by $\beta-|\varkappa|$.

### 3. PERTURBATION THEORY

We express the dynamic distribution of the magnetization in an oscillating magnetic field in the form

$$\mathbf{M}(y,t)=\vec{\mathscr{M}}(y-Y(t))+\mathbf{m}(y-Y(t),t), \tag{6}$$

which takes explicit account of the possibility of drift motion of the DW. The functions $\vec{\mathscr{M}}(y)$ and $\mathbf{m}(y,t)$ which describe the magnetization distribution in a reference frame connected with the DW, as well as the function $Y(t)$ which has the meaning of the DW coordinate, must satisfy the equation

$$\dot{\mathbf{m}}-(\vec{\mathscr{M}}'+\mathbf{m}')\dot{Y}=-\gamma[(\vec{\mathscr{M}}+\mathbf{m}),(\vec{\mathscr{H}}+\mathbf{h})]$$
$$+(\alpha/M)[(\vec{\mathscr{M}}+\mathbf{m}),(\dot{\mathbf{m}}-(\vec{\mathscr{M}}'+\mathbf{m}')\dot{Y})], \tag{7}$$

which follows from (1). In accordance with (3), we have here

$$\vec{\mathscr{H}}=\varepsilon\vec{\mathscr{M}}''+\beta\mathscr{M}_z\mathbf{e}_z-\varkappa\mathscr{M}_y\mathbf{e}_y, \tag{8}$$
$$\mathbf{h}=\varepsilon\mathbf{m}''+\beta m_z\mathbf{e}_z-\varkappa m_y\mathbf{e}_y+\mathbf{H}_1.$$

The functions $\vec{\mathscr{M}}(y)$, $\mathbf{m}(y,t)$ and $\dot{Y}(t)$ are not uniquely defined by Eq. (7), so that in addition to the condition

$$\vec{\mathscr{M}}^2+2\vec{\mathscr{M}}\mathbf{m}+\mathbf{m}^2=M^2, \tag{9}$$

which follows from the integral of motion ($\mathbf{M}^2=\text{const}$) of Eq. (1), they must be subject to a number of additional constraints. Considering, to be specific, the case of a Bloch DW, we assume that $\vec{\mathscr{M}}(y)$, $\mathbf{m}(y,t)$, and $\dot{Y}(t)$ can be represented in a weak oscillating magnetic field by the series

$$\vec{\mathscr{M}}(y)=\mathbf{M}_B(y)+\vec{\mathscr{M}}_2(y)+\dots,$$
$$\mathbf{m}(y,t)=\mathbf{m}_1(y,t)+\mathbf{m}_2(y,t)+\dots,$$
$$\dot{Y}(t)=V+\dot{u}_1(t)+\dot{u}_2(t)+\dots \tag{10}$$

with zero mean values of $\mathbf{m}_n(y,t)$ and $\dot{u}_n(t)$

$$\langle\mathbf{m}_n\rangle=0,\quad \langle\dot{u}_n\rangle=0,\quad n=1,2,\dots \tag{11}$$

Here $V=V_2+V_3+\dots$ is the DW drift velocity, the numerical subscripts indicate the order relative to the amplitude of the external-field amplitude, and the angular brackets denote averaging over the period of the field oscillations. We have left out of the expansions of $\vec{\mathscr{M}}(y)$ and $V$ the terms $\vec{\mathscr{M}}_1(y)$ and $V_1$ linear in the field, since it is clear beforehand that $\vec{\mathscr{M}}_1(y)=V_1=0$. On the basis of (10), (11), and the series

$$\vec{\mathscr{H}}(y)=\mathbf{H}_B(y)+\vec{\mathscr{H}}_2(y)+\dots,$$
$$\mathbf{h}(y,t)=\mathbf{h}_1(y,t)+\mathbf{h}_2(y,t)+\dots,\qquad \langle\mathbf{h}_n\rangle=0, \tag{12}$$

which follow from (8) and (10), a solution of (7) can in principle be obtained to any order of perturbation theory. Confining ourselves to the principal terms of the asymptotic expansions of the oscillating ($\mathbf{m}_1$, $\dot{u}_1$) and averaged ($\vec{\mathscr{M}}_2$, $V_2$) variables, we write for the linear approximation of (7)

$$\dot{\mathbf{m}}_1-\dot{u}_1\mathbf{M}_B'=-\gamma\{[\mathbf{M}_B\mathbf{h}_1]+[\mathbf{m}_1\mathbf{H}_B]\}$$
$$+(\alpha/M)\{[\mathbf{M}_B\dot{\mathbf{m}}_1]-\dot{u}_1[\mathbf{M}_B\mathbf{M}_B']\} \tag{13}$$

and for the quadratic approximation averaged over the field oscillations

$$V_2\mathbf{M}_B'+\langle\dot{u}_1\mathbf{m}_1'\rangle=\gamma\{[\mathbf{M}_B\vec{\mathscr{H}}_2]+[\vec{\mathscr{M}}_2\mathbf{H}_B]$$
$$+\langle[\mathbf{m}_1\mathbf{h}_1]\rangle\}+(\alpha/M)\{V_2[\mathbf{M}_B\mathbf{M}_B']$$
$$+\langle\dot{u}_1[\mathbf{m}_1\mathbf{M}_B']\rangle+\langle\dot{u}_1[\mathbf{M}_B\mathbf{m}_1']\rangle-\langle[\mathbf{m}_1\dot{\mathbf{m}}_1]\rangle\}. \tag{14}$$

The conduction that the length (9) of the magnetization vector be constant requires that the vectors $\mathbf{m}_1$ and $\vec{\mathscr{M}}_2$ satisfy also the relations

$$\mathbf{M}_B\mathbf{m}_1=0,\qquad 2\mathbf{M}_B\vec{\mathscr{M}}_2+\langle\mathbf{m}_1^2\rangle=0. \tag{15}$$

To simplify the analysis that follows, it is convenient to express Eqs. (13) and (14) in a local coordinate frame with basis vectors $\mathbf{e}_\perp(y)=[\mathbf{e}_y\mathbf{e}_\parallel(y)]$, $\mathbf{e}_e$, $\mathbf{e}_\parallel(y)=\mathbf{M}_B(y)/M$. Recognizing that according to (15)

$$\mathbf{m}=m_\perp\mathbf{e}_\perp+m_y\mathbf{e}_y,\quad \vec{\mathscr{M}}=\mathscr{M}_\perp\mathbf{e}_\perp+\mathscr{M}_y\mathbf{e}_y-(\langle\mathbf{m}^2\rangle/2M)\mathbf{e}_\parallel$$

(the subscripts of $\mathbf{m}_1$, $\dot{u}_1$, $\vec{\mathscr{M}}_2$ and $V_2$ will be omitted from now on), Eq. (13) leads to a system of equations for $\tilde{m}_\perp(\xi)$ and $\tilde{m}_y(\xi)$:

$$(\hat{L}-ip\alpha)\tilde{m}_\perp(\xi)+ip\tilde{m}_y(\xi)=-ip\alpha M\tilde{u}\delta\rho\sin\theta_B(\xi)/\Delta$$
$$+[\rho\tilde{H}_z\sin\theta_B(\xi)-\delta\tilde{H}_x\cos\theta_B(\xi)]/\beta, \tag{16}$$
$$(\hat{L}-ip\alpha-q)\tilde{m}_y(\xi)-ip\tilde{m}_\perp(\xi)=-ipM\tilde{u}\delta\rho\sin\theta_B(\xi)/\Delta-\tilde{H}_y/\beta,$$

while Eq. (14) breaks up into independent equations for $\mathscr{M}_\perp(\xi)$ and $\mathscr{M}_y(\xi)$:

$$\omega_r\Delta\hat{L}\mathscr{M}_\perp(\xi)=-\alpha VM\delta\rho\sin\theta_B(\xi)+\langle\gamma\Delta m_\perp(\xi,t)h_\parallel(\xi,t)$$
$$+\dot{u}(t)m_y'(\xi,t)-\alpha\dot{u}(t)m_\perp'(\xi,t)$$
$$+2\beta\gamma\Delta\delta\rho\mathbf{m}(\xi,t)\mathbf{m}'(\xi,t)\sin\theta_B(\xi)\rangle, \tag{17}$$

$$\omega_r\Delta(\hat{L}-q)\mathscr{M}_y(\xi)=-VM\delta\rho\sin\theta_B(\xi)+\langle\gamma\Delta m_y(\xi,t)h_\parallel(\xi,t)$$
$$-\dot{u}(t)m_\perp'(\xi,t)-\alpha\dot{u}(t)m_y'(\xi,t)\rangle. \tag{18}$$

Here $\xi=y/\Delta$, $q=\varkappa/\beta$, $p=\omega/\omega_r$, $\omega_r=\beta\gamma M$, $\hat{L}=d^2/d\xi^2-\cos2\theta_B(\xi)$, $\mathbf{m}(\xi,t)=\operatorname{Re}\tilde{\mathbf{m}}(\xi)\exp(i\omega t)$, $u(t)=\operatorname{Re}\tilde{u}\exp(i\omega t)$, $\mathbf{H}_1(t)=\operatorname{Re}\tilde{\mathbf{H}}\exp(i\omega t)$.

In accordance with the physical meaning of the problem, the solution of Eqs. (16)–(18) must be sought in a class of smooth functions $\psi(\xi)$ that obey the condition $\psi'(\xi)\to0$ as $\xi\to\pm\infty$. This circumstance yields, even without solving the equations, a general equation for a DW drift velocity in a biaxial ferromagnet, using only the conditions that Eqs. (17) and (18) have solutions in this class of functions.

Equations (17) and (18) contain a negative-definite self-adjoint operator $\hat{L}$ having a zero eigenvalue corresponding to a uniform shift of the DW. For $q>0$ the operator $\hat{L}-q$ has no zero eigenvalues and therefore, according to the Fredholm alternative, Eq. (18) has a solution for any value of $V$. Equation (17), on the other hand, can be solved only if its right-hand side is orthogonal to the function $\theta_B'(\xi)=\sin\theta_B(\xi)$, which is the kernel of the operator $\hat{L}$.

Obviously, to satisfy this condition the expression for the DW drift velocity must be of the form

$$V=\frac{\delta\rho}{2\alpha M}\int_{-\infty}^{\infty}\theta_B'\langle\gamma\Delta m_\perp h_\parallel+\dot u m_y'-\alpha\dot u m_\perp'$$
$$+2\gamma\beta\Delta\delta\rho\mathbf{m}\mathbf{m}'\sin\theta_B\rangle\,d\xi. \tag{19}$$

In the case of a uniaxial ferromagnet ($q=0$), however, the solubility conditions of Eqs. (17) and (18) are found to be incompatible, and it becomes incorrect to describe the magnetization in an oscillating field on the basis of (10) and (11). If, however, we include in the energy of the uniaxial ferromagnet also the magnetostatic energy whose density, since the problem is spatially one-dimensional, equals $2\pi M_y^2$, the total energy takes the form given by Eq. (2) with $\varkappa=4\pi$, and we return to the case of a biaxial ferromagnet.

We proceed now to solve the system (16) and obtain the DW drift velocity in an oscillating field polarized in the DW plane and in the plane perpendicular to the easy-magnetization axis.

## 4. OSCILLATING FIELD POLARIZED IN THE DW PLANE ($\tilde{H}_y=0$)

For $\tilde H_y=0$ we seek the solution of the system (16) in the form

$$\begin{pmatrix}\tilde m_\perp\\ \tilde m_y\end{pmatrix}=A\begin{pmatrix}\sin\theta_B\\ \cos\theta_B\end{pmatrix}. \tag{20}$$

Substituting (20) in (16) and using the linear independence of the functions $\sin\theta_B(\xi)$ and $\cos\theta_B(\xi)$, we obtain for the elements $a_{ij}$ of the matrix $A$ the expressions

$$a_{11}=\delta\rho M\frac{\tilde u}{\Delta}-\frac{p\alpha-iq}{\beta p[q\alpha+ip(1+\alpha^2)]}\rho\tilde H_z,\quad a_{12}=\frac{1+q+ip\alpha}{\beta d_1d_2}\delta\tilde H_x,$$
$$a_{21}=\frac{1}{\beta[q\alpha+ip(1+\alpha^2)]}\rho\tilde H_z,\qquad a_{22}=-\frac{ip}{\beta d_1d_2}\delta\tilde H_x, \tag{21}$$

where

$$d_{1,2}=1+{}^1/_2q+ip\alpha\pm[({}^1/_2q)^2+p^2]^{1/2}.$$

It follows from (21) that $\tilde{\mathbf m}(\xi)$, meaning also $\mathbf m(\xi,t)$, depends on $\tilde u$. Since the only restriction on the values assumed by $\tilde u$ is the inequality $|a_{11}|\ll M$, satisfaction of which calls for the condition $|\mathbf m|\ll M$, it is obvious that a unique choice of $\tilde u$ is possible. By finding the variation of $\mathbf M(\xi,t)$ due to the variation of $\tilde u$ we easily verify that in the approximation linear in the oscillating field this ambiguity is not reflected in the distribution of the magnetization, so that the manner used to choose $\tilde u$ has no particular significance. If, for example, we demand that the condition $a_{11}=0$, hold, we get

$$\tilde u=\Delta\delta\frac{p\alpha-iq}{\beta p[q\alpha+ip(1+\alpha^2)]}\frac{\tilde H_z}{M} \tag{22}$$

and the function $u(t)$ will satisfy the equation[12]

$$\frac{1+\alpha^2}{\gamma^2\varkappa\Delta}\ddot u+\frac{M\alpha}{\gamma\Delta}\dot u=\delta MH_{1z}+\frac{\alpha}{\gamma\varkappa}\delta\dot H_{1z}, \tag{23}$$

which describes the linear dynamics of the DW in the adiabatic approximation.

Using relations (20) and (21), and recognizing also that

$$h_\parallel=-2\beta\delta\rho\sin\theta_B(m_\perp'+m_\perp\cos\theta_B)+\rho H_{1x}\sin\theta_B+\delta H_{1z}\cos\theta_B, \tag{24}$$

by means of elementary integration in (19) we get for the drift velocity of a solitary DW in an oscillating ($p\neq0$) field polarized in the DW plane the expression

$$V=V_d\delta\rho\,\mathrm{Re}\,S, \tag{25}$$

which is invariant, as expected, to the choice of $\tilde u$. Here $V_d=\pi\gamma\Delta M/4\beta$,

$$S=\frac{q^2+q+p^2(1+\alpha^2)-ip\alpha}{d_1d_2[q\alpha-ip(1+\alpha^2)]}\frac{\tilde H_x\tilde H_z^*}{M^2}. \tag{26}$$

It follows from (25), in particular, that a stripe domain structure can also drift if the directions of rotation of the vector $\mathbf M_B$ in neighboring DW are suitably matched. Indeed, since the topological charges of any two neighboring DW are of opposite sign, the DW velocities will, in accordance with (25), be equal only if the magnetization rotations in them are oppositely directed.

We present also the principal terms of the expansion of $V$ in powers of $p$ for $p\gg1$ and $p\ll1$. If $p\gg1$, then

$$V=V_d\frac{\delta\rho}{p(1+\alpha^2)}\mathrm{Im}\frac{\tilde H_x\tilde H_z^*}{M^2} \tag{27}$$

in an elliptically polarized field and

$$V=V_d\frac{\delta\rho\alpha\sin2\varphi}{2p^2(1+\alpha^2)^2}\left|\frac{\tilde H}{M}\right|^2 \tag{28}$$

in the case of linear polarization ($\tilde H_x=|\tilde H|\sin\varphi$, $\tilde H_z=|\tilde H|\cos\varphi$). In the other limiting case, when the frequency of the oscillating elliptically polarized field satisfies the condition $p\ll1$, we have

$$V=V_d\frac{\delta\rho}{\alpha}\mathrm{Re}\frac{\tilde H_x\tilde H_z^*}{M^2}. \tag{29}$$

## 5. OSCILLATING FIELD POLARIZED IN THE *XY* PLANE ($\tilde{H}_z=0$)

We transform the system (16) at $\tilde H_z=0$ into

$$(\hat L-ip\alpha)m_\perp^{(1)}+ipm_y^{(1)}=0,$$
$$(\hat L-ip\alpha-q)m_y^{(1)}-ipm_\perp^{(1)}=-\tilde H_y/\beta, \tag{30}$$

by putting

$$\tilde m_\perp(\xi)=a_{12}\cos\theta_B(\xi)+m_\perp^{(1)}(\xi),$$
$$\tilde m_y(\xi)=a_{22}\cos\theta_B(\xi)+m_y^{(1)}(\xi). \tag{31}$$

We expand the solution of Eqs. (30) in terms of the eigenfunctions of the operator $\hat L$:

$$m_{\perp,y}^{(1)}(\xi)=b_{\perp,y}\psi_B(\xi)+\int_{-\infty}^{\infty}f_{\perp,y}(k)\psi_k(\xi)\,dk, \tag{32}$$

the complete set of which is known to consist of one localized state

$$\psi_B(\xi)=\frac{1}{\sqrt2}\sin\theta_B(\xi),$$

corresponding to a zero eigenvalue, and of the continuum modes

$$\psi_k(\xi)=\frac{1}{(2\pi)^{1/2}}\frac{e^{ik\xi}}{(1+k^2)^{1/2}}[ik+\cos\theta_B(\xi)]$$

with eigenvalues $-(1+k^2)$. Using the conditions of normalization and completeness of the eigenfunctions[13]

$$\int_{-\infty}^{\infty}\psi_i(\xi)\psi_j^*(\xi)\,d\xi=\begin{cases}\delta(k-k') & \text{for } i=k,\ j=k'\\ \delta_{ij} & \text{in the remaining cases,}\end{cases}$$

$$\int_{-\infty}^{\infty}\psi_k(\xi)\psi_k^*(\xi')\,dk+\psi_B(\xi)\psi_B(\xi')=\delta(\xi-\xi') \tag{33}$$

[$\delta(k)$ is the Dirac delta function and $\delta_{ij}$ is the Kronecker symbol], we obtain from Eqs. (30) the coefficients of the expansion in (32):

$$\begin{pmatrix}b_\perp\\ b_y\end{pmatrix}=\frac{\pi\tilde{H}_y}{\sqrt{2}\,\beta}\frac{1}{q\alpha+ip(1+\alpha^2)}\begin{pmatrix}1\\ \alpha\end{pmatrix},$$

$$\begin{pmatrix}f_\perp(k)\\ f_y(k)\end{pmatrix}=\frac{\tilde{H}_y}{\beta(k^2+d_1)(k^2+d_2)}\int_{-\infty}^{\infty}\psi_k^*(\xi')\,d\xi'\begin{pmatrix}ip\\ 1+ip\alpha+k^2\end{pmatrix}. \tag{34}$$

Recognizing finally that

$$\int_{-\infty}^{\infty}\psi_k^*(\xi')\,d\xi'=\left(\frac{2}{\pi}\right)^{1/2}\frac{i}{(1+k^2)^{1/2}}\left[\frac{\pi}{2\,\mathrm{sh}(\pi k/2)}-\lim_{c\to\infty}\frac{\cos(kc)}{k}\right],$$

we can represent the solution of Eqs. (16) for $\tilde{H}_z=0$, on the basis of (31), (32), and (34), in the form

$$\tilde{m}_\perp(\xi)=\delta\tilde{H}_x\frac{1+q+ip\alpha}{\beta d_1 d_2}\cos\theta_B(\xi)+\frac{\pi\tilde{H}_y}{2\beta}\frac{\sin\theta_B(\xi)}{q\alpha+ip(1+\alpha^2)}-\frac{p\tilde{H}_y}{2\beta}\int_{-\infty}^{\infty}\frac{e^{ik\xi}(ik-\mathrm{th}\,\xi)}{(k^2+d_1)(k^2+d_2)(k^2+1)\,\mathrm{sh}(\pi k/2)}dk,$$

$$\tilde{m}_y(\xi)=-\delta\tilde{H}_x\frac{ip}{\beta d_1 d_2}\cos\theta_B(\xi)+\frac{\pi\tilde{H}_y}{2\beta}\frac{\alpha\sin\theta_B(\xi)}{q\alpha+ip(1+\alpha^2)}+i\frac{\tilde{H}_y}{2\beta}\int_{-\infty}^{\infty}\frac{e^{ik\xi}(1+ip\alpha+k^2)(ik-\mathrm{th}\,\xi)}{(k^2+d_1)(k^2+d_2)(k^2+1)\,\mathrm{sh}(\pi k/2)}dk. \tag{35}$$

Calculation, with the aid of (35), of the DW drift velocity (19) in an oscillating field polarized in a plane perpendicular to the easy-magnetization axis (the $z$ axis) yields

$$V=V_d\delta\,\mathrm{Re}\,T, \tag{36}$$

where

$$T=\frac{ip}{2\alpha d_1 d_2}\left\{\frac{\pi}{2}\frac{\alpha(2+q)+ip(1+\alpha^2)}{q\alpha-ip(1+\alpha^2)}+\int_0^{\infty}\frac{k^2[k^4+k^2(2+q)+d_1d_2]}{(k^2+d_1^*)(k^2+d_2^*)(k^2+1)\,\mathrm{sh}^2(\pi k/2)}dk\right\}\frac{\tilde{H}_x\tilde{H}_y^*}{M^2}. \tag{37}$$

To analyze the asymptotic behavior of $V$ for $p\gg1$ and $p\ll1$, using the identity

$$k^4+k^2(2+q)+d_1d_2=(k^2+d_1^*)(k^2+d_2^*)+2ip\alpha(k^2+2+q)$$

and evaluating the integral

$$\int_0^{\infty}\frac{k^2dk}{(k^2+1)\,\mathrm{sh}^2(\pi k/2)}=\frac{\pi}{2}-\frac{4}{\pi}$$

it is convenient to transform (37) into

$$T=\frac{ip}{2\alpha d_1 d_2}\left\{\pi\frac{\alpha(1+q)}{q\alpha-ip(1+\alpha^2)}-\frac{4}{\pi}+2ip\alpha\int_0^{\infty}\frac{k^2(k^2+2+q)}{(k^2+d_1^*)(k^2+d_2^*)(k^2+1)\,\mathrm{sh}^2(\pi k/2)}dk\right\}\frac{\tilde{H}_x\tilde{H}_y^*}{M^2}. \tag{38}$$

On the basis of (38) we obtain for the DW drift velocity in the case $p\gg1$ the expression

$$V=V_d\frac{2\delta}{\pi p\alpha(1+\alpha^2)}\mathrm{Im}\frac{\tilde{H}_x^*\tilde{H}_y}{M^2}, \tag{39}$$

which agrees with that obtained in Ref. 3, while for $p\ll1$ we have

$$V=V_d\frac{2p\delta}{\pi\alpha}\left[\frac{\pi^2}{4q}-\frac{1}{1+q}\right]\mathrm{Im}\frac{\tilde{H}_x^*\tilde{H}_y}{M^2}. \tag{40}$$

As already noted in the Introduction, it was assumed in the DW drift description of Refs. 1 and 2 that the effective pressure acting on the DW is equal to the difference, averaged over the field oscillations, of the energy densities of the ferromagnet on the two sides of the DW. The present results, however, do not confirm this assumption, according to which the DW drift velocity for $p\gg1$ and $\alpha\ll1$ should exceed (39) by a factor of two, and be of order $p^3$ for $p\ll1$ (Ref. 2).

To conclude the analysis of the DW drift, we present an expression for the drift velocity of a Bloch DW in an oscillating field of arbitrary polarization

$$V=V_d\delta\,\mathrm{Re}(\rho S+T). \tag{41}$$

The same equation can also be used to determine the drift velocity of a Néel DW, by replacing in the expressions for $S$ and $T$, $\varkappa$ by $|\varkappa|$, $\beta$ by $\beta-|\varkappa|$, $\tilde{H}_x$ by $\tilde{H}_y$ and $\tilde{H}_y$ by $-\tilde{H}_x$.

We note in conclusion that among the most promising objects for experimental verification of the conclusions of the proposed theory are apparently long and narrow yttrium iron garnet specimens, in which solitary 180-degree DW have been observed.[14]


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Translated by J. G. Adashko