Photon statistics in two-quantum collective optical nutation

N. A. Enaki

Applied Physics Institute, Moldavian Academy of Sciences
(Submitted 21 November 1989; resubmitted 23 April 1990)
Zh. Eksp. Teor. Fiz. **98**, 783–796 (September 1990)

We investigate the photon statistics in two-quantum collective nutation of a concentrated system of atoms. We show that, in the course of the nutation, the fluctuations of the in-phase and quadrature components of the electric field (EMF) intensity with an external laser field oscillate in antiphase. The EMF fluctuations are then periodically compressed along the in-phase or quadrature component of the EMF. We investigate also theoretically the conditions under which two-photon collective nutation of the atom system sets in. We obtain the critical number of atoms for which collective light-scattering processes stop the nutation.

1. INTRODUCTION

Quantum fluctuations and generation of incoherent electromagnetic fields in two-photon and multiphoton processes have recently been the subject of a number of theoretical[1–5] and experimental[6,7] studies. Two-photon absorption of initially coherent light leads to antibunching of the photon in a laser beam, whereas two-photon emission is accompanied by generation of bunched pairs of photons (biphotons) in the light beam. These processes make the quantum fluctuations of the number of photons larger when the light is absorbed, and smaller when it is generated, compared with the analogous fluctuations in a coherent electromagnetic field (EMF).

Interest attaches also to the conditions under which the system's excited atoms that enter into two-photon resonance with an external coherent EMF are capable of converting the generation of antibunched photons into that of bunched ones. It was shown in Ref. 2 that if the resonance levels of the atoms are in thermal balance, when the number of excited emitters is one-sixth of the total number of atoms participating in the two-photon interaction with the external coherent field, the generation of antibunched photons turns into that of bunched ones. On the other hand, it has been shown in Refs. 3 and 4 that for two-photon processes in the weak single-mode field of a microcavity the inversion of a single atom alternately vanishes and is restored in time. This alternating process of absorption and reradiation of the photons is transferred directly to the photon-number fluctuations.

It must be emphasized that the cited studies neglected the collective behavior of the atoms upon absorption and reradiation of photon pairs. It was assumed that the distance between the system atoms exceeds the laser-field wavelength,[2,5] or else the number of excited atoms was neglected compared with the number of ground-state atoms participating in the two-photon absorption process.[1]

In contrast to the cited references, the present paper deals with the conditions under which an ensemble of atoms goes over into a regime of collective two-photon nutation (CTN), and with the behavior of the quantum fluctuations of the EMF in the course of the nutation. To simplify this problem, we consider a concentrated system of Λ-type atoms that enters into two-photon resonance with an external coherent EMF relative to the forbidden transition $|1\rangle - |2\rangle$, where $|1\rangle$ and $|2\rangle$ are the ground and first excited states (see the figure). The excited state $|3\rangle$ does not enter into resonance with the external field. It is shown that the collective excitation of the ensemble of atoms is accompanied by a decrease of the fluctuations of the EMF density, while a phased transition of atoms to the ground state is accompanied by an increase of these fluctuations. In other words, in one period of the CTN the replacement of the generation of bunched photon pairs by that of antibunched ones depends on the sign of the rate of change of the population difference and, just as in the case of thermal balance between the levels $|1\rangle$ and $|2\rangle$ (see Ref. 2), it does not depend on the number of excited atoms.

It is also shown in the present paper that coherent two-photon excitation of an ensemble of atoms increases the probability of collective two-photon spontaneous decay of the excited emitters. This stops the optical nutation of the population difference of the atoms when a definite critical number of atoms is reached (see Ref. 29). This collective effect depends on the atom density and cannot be described by the previously assumed two-photon models of optical notation of single atoms (see Refs. 3 and 4 and the citations therein).

For a more detailed study of the properties of photon reradiation we investigate in the present paper also the quantum fluctuations of two noncommuting quadrature components $\mathscr{E}_x$ and $\mathscr{E}_y$ of the EMF vector ($\mathscr{E}_x$ is the amplitudes of the in-phase component of the EMF intensity vector with the external coherent field, and $\mathscr{E}_y$ is the amplitude of the quadrature component, lagging the in-phase component by $\pi/2$). In an in-phase transition of the emitters from the excited state $|2\rangle$ to the ground state $|1\rangle$, the quantum fluctuations of the component $\mathscr{E}_x$ increase substantially, while the fluctuations of $\mathscr{E}_y$ become lower than the vacuum values. When a system of atoms is excited, on the contrary, the fluctuations of the in-phase component decrease while those of the quadrature component increase.

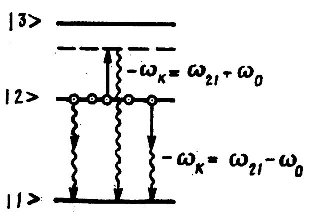

FIG. 1. Possible processes of spontaneous transition of excited atoms to the ground state $|1\rangle$. The straight lines denote photons of the external coherent EMF, and the wavy ones spontaneously created photons.

 0038-5646/90/090435-08$03.00

The mathematical method proposed in the present article permits the EMF operators and correlators to be expressed in terms of the operators and correlators of an atomic subsystem. A similar method is used to investigate single-photon resonant processes[8] but is not traditional for two-photon ones.

The plan of the article is the following. In the second section we represent the fluctuations of the EMF density and of the components $\mathscr{E}_x$ and $\mathscr{E}_y$ in terms of correlators of an atomic subsystem. In Sec. 3 we derive a control equation for the density matrix of the atomic subsystem in two-photon interaction of the emitters with an external coherent EMF. This equation is used in Sec. 4 to investigate both the time dependences of the atomic-subsystem correlators and the behavior of the density fluctuations of the EMF and of the components $\mathscr{E}_x$ and $\mathscr{E}_y$. The Conclusion deals with the possibility of experimentally observing collective two-photon nutation.

2. REPRESENTATION OF PHOTON-SUBSYSTEM CORRELATORS IN TERMS OF ATOMIC-SUBSYSTEM CORRELATORS

The time dependence of the quantum fluctuations of the EMF density in two-photon interaction of an atomic subsystem with an external EMF can be investigated with the aid of the correlation functions [9, 10]

$$\Lambda(\mathbf{r}, t, \tau)=R_2(\mathbf{r}, t, \tau)-|R_1(\mathbf{r}, t, \tau)|^2, \tag{1}$$

where

$$R_1(\mathbf{r}, t, \tau)=\langle\mathscr{E}^+(\mathbf{r}, t)\mathscr{E}^-(\mathbf{r}, t-\tau)\rangle,$$
$$R_2(\mathbf{r}, t, \tau)=\langle\mathscr{E}^+(\mathbf{r}, t)\mathscr{E}^+(\mathbf{r}, t-\xi)\mathscr{E}^-(\mathbf{r}, t-\tau)\mathscr{E}^-(\mathbf{r}, t)\rangle, \tag{1a}$$

with $R_1(\mathbf{r},t,\tau=0)$ the EMF density at the observation point $\mathbf{r}$, and $R_2(\mathbf{r},t,\tau)$ the correlation between the EMF densities at the instants of time t and $t-\tau$,

$$\mathscr{E}^+(\mathbf{r},t)=i\sum_k g_k^e a_k^+ e^{-i(\mathbf{k},\mathbf{r})}, \quad \mathscr{E}^-(\mathbf{r},t)=(\mathscr{E}^+(\mathbf{r},t))^+, \tag{2}$$

a_k^+ and a_k are the creation and annihilation operators for photons with momentum $\hbar k$, energy $\hbar\omega_k$, and polarization λ, $(\vec{\mathscr{E}}(\mathbf{r},t),\mathbf{e}) = \mathscr{E}^+(\mathbf{r},t) + \mathscr{E}^-(\mathbf{r},t)$ is the projection of the EMF intensity $\vec{\mathscr{E}}(\mathbf{r}, t)$ on the direction of the polarization vector $\mathbf{e}$ of the external coherent field,

$$g_k^e=(2\pi\hbar\omega_k/V)^{1/2}(\mathbf{e}_\lambda\mathbf{e}),$$

$\mathbf{e}_\lambda$ is the photon polarization vector, $\lambda = 1$ or 2, and V is the EMF quantization volume.

The operators in Eq. (1a) are written in the Heisenberg representation, and the averaging is over the states $\psi(t=0) = |A\rangle \otimes |\Phi\rangle$, of a system of noninteracting atoms with the EMF where $|A\rangle$ is the wave function of the atomic subsystem and $|\Phi\rangle$ the wave function of the free EMF. We note also that the external single-mode laser field is in the coherent state

$$|\alpha\rangle_{k_0}=\exp\{\alpha a_{k_0}^+-\alpha^* a_{k_0}\}|0\rangle_{k_0},$$

and the remaining EMF modes are in the vacuum state

$$\prod_{k\neq k_0}|0\rangle_k.$$

Therefore

$$|\Phi\rangle=\exp\{\alpha a_{k_0}^+-\alpha^* a_{k_0}\}\prod_k|0\rangle_k.$$

Interest attaches also to formation of a squeezed state of the EMF in two-photon interaction of a system of atoms with an external laser field. To this end, assuming that arg $\alpha = \pi/2$, we consider the EMF quadrature operators[11,12]

$$\mathscr{E}_x(r, t)=\tilde{\mathscr{E}}^+(\mathbf{r}, t)+\tilde{\mathscr{E}}^-(\mathbf{r}, t),$$
$$\mathscr{E}_y(\mathbf{r}, t)=i[\tilde{\mathscr{E}}^+(\mathbf{r}, t)-\tilde{\mathscr{E}}^-(\mathbf{r}, t)], \tag{3}$$

whose commutation relation is

$$[\mathscr{E}_x(\mathbf{r}, t), \mathscr{E}_y(\mathbf{r}, t')]=iC(t-t').$$

Here

$$\tilde{\mathscr{E}}^+(\mathbf{r},t)=\mathscr{E}^+(\mathbf{r},t)\exp\{i(\mathbf{k}_0\mathbf{r})-i\omega_0 t\}, \quad C(t-t')=\sum_k(g_k^e)^2\cos[(\omega_k-\omega_0)(t-t')].$$

The component of the EMF intensity along the polarization of the external laser field is expressed in terms of the operators $\mathscr{E}_x$ and $\mathscr{E}_y$ as follows:

$$\mathscr{E}(\mathbf{r}, t)=\mathscr{E}_x\cos[\omega_0 t-(\mathbf{k}_0\mathbf{r})]+\mathscr{E}_y(\mathbf{r}, t)\sin[\omega_0 t-(\mathbf{k}_0\mathbf{r})]. \tag{3a}$$

It follows from (3) and (3a) that for a laser field that does not interact with the system atoms the mean value of the operator $\mathscr{E}_x$ coincides with the mean value of the amplitude of the external coherent field, and the mean value of the operator $\mathscr{E}_y$ is zero. Consequently, $\mathscr{E}_x$ and $\mathscr{E}_y$ describe respectively the behaviors of the amplitudes of the in-phase and quadrature components of the EMF intensity. The fluctuations of the components $\mathscr{E}_x$ and $\mathscr{E}_y$, averaged over the coherent or vacuum state of the EMF, are equal:

$$\frac{\langle(\Delta\mathscr{E}_x)^2\rangle}{C(0)}=\frac{\langle(\Delta\mathscr{E}_y)^2\rangle}{C(0)}=1.$$

Here $\Delta\mathscr{E}_i = \mathscr{E}_i - \langle\mathscr{E}_i\rangle$, and $i = x, y$. The criterion for the formation of a squeezed state of the EMF in an interaction of a coherent field with atoms is determined by the inequality $\langle(\Delta\mathscr{E}_x)^2\rangle/C(0) < 1$ or $\langle(\Delta\mathscr{E}_y)^2\rangle/C(0) < 1$. Here

$$\frac{\langle(\Delta\mathscr{E}_x)^2\rangle\langle(\Delta\mathscr{E}_y)^2\rangle}{C^2(0)}\geqslant 1.$$

The deviation of the fluctuations of the components $\mathscr{E}_x$ and $\mathscr{E}_y$ from the fluctuations of the same components in a coherent or vacuum field can be taken into account with the aid of the correlation functions

$$M_i(t, \tau)=\langle:\mathscr{E}_i(t)\mathscr{E}_i(t-\tau):\rangle-\langle\mathscr{E}_i(t)\rangle\langle\mathscr{E}_i(t-\tau)\rangle, \quad i=x, y, \tag{4}$$

where $:f(t):$ denotes normal ordering of the EMF operators. If $M_x < 0$ or $M_y < 0$ the fluctuations of the EMF are squeezed along the component $\mathscr{E}_x$ or $\mathscr{E}_y$, respectively.

Upon substitution of (2) in (1) and (4), the functions $\Lambda(\mathbf{r}, t, \tau)$, $M_x(t,\tau)$ and $M_y(t,\tau)$ take the form

$$\Lambda(\mathbf{r}, t, \tau)=\sum_{k_i(i=1,\ldots,4)}\prod_{j=1}^{4}g_{k_j}^e\exp[i((\mathbf{k}_1+\mathbf{k}_3-\mathbf{k}_2-\mathbf{k}_4)\mathbf{r})]$$
$$\times[\langle a_{k_1}^+(t)a_{k_3}^+(t-\tau)a_{k_4}(t-\tau)a_{k_2}(t)\rangle$$
$$-\langle a_{k_1}^+(t)a_{k_4}(t-\tau)\rangle\langle a_{k_3}^+(t-\tau)a_{k_2}(t)\rangle], \tag{5a}$$

$$M_x(t,\tau)=-\Phi_1(t,\tau)+\Phi_2(t,\tau), \quad (5b)$$

$$M_y(t,\tau)=\Phi_1(t,\tau)+\Phi_2(t,\tau), \quad (5c)$$

where

$$\Phi_1(t,\tau)=\sum_{k_1,k_2} g_{k_1}{}^e g_{k_2}{}^e\{\exp[-2i\omega_0 t+i\omega_0\tau-i((\mathbf{k}_1+\mathbf{k}_2-2\mathbf{k}_0)\mathbf{r})]$$
$$\times[\langle a_{k_1}{}^+(t)a_{k_2}{}^+(t-\tau)\rangle-\langle a_{k_1}{}^+(t)\rangle\langle a_{k_2}{}^+(t-\tau)\rangle]+\text{H.c.}\}, \quad (6a)$$

$$\Phi_2(t,\tau)=\sum_{k_1,k_2} g_{k_1}{}^e g_{k_2}{}^e\{\exp[-i\omega_0\tau+i(k_1-k_2,r)][\langle a_{k_1}{}^+(t)a_{k_2}(t-\tau)\rangle-\langle a_{k_1}{}^+(t)\rangle\langle a_{k_2}(t-\tau)\rangle]+\text{H.c.}\}. \quad (6b)$$

We shall express below a_k^+ and a_k in terms of the operators of the atomic subsystem and the operators of the external laser field. To this end we consider the Hamiltonian N of Λ-type three-level atoms in an external coherent field that enters into two-photon resonance with the dipole-forbidden transition $|1\rangle - |2\rangle$:

$$H=H_0+H_i,$$

$$H_0=\sum_{\alpha=1}^{3}\sum_{j=1}^{N}\hbar\omega_\alpha U_{j\alpha}{}^\alpha+\sum_k\hbar\omega_k a_k{}^+a_k, \quad (7)$$

$$H_i=\sum_{j=1}^{N}\sum_{\beta=1}^{2}(\mathbf{d}_{3\beta}\vec{\mathscr{E}}(\mathbf{r}))\{U_{j\beta}{}^3+U_{j3}{}^\beta\},$$

where $\hbar\omega_\alpha$ is the energy of the α level, $d_{3\beta}$ is the dipole moment of the transition between the second excited state $|3\rangle$ and the state $|\beta\rangle$ ($\beta = 1, 2$), and $U_{j\beta}^3$ is the corresponding matrix of the transition between states $|3\rangle$ and $|\beta\rangle$ of the jth atom. The operators $U_{j\beta}^\alpha$ satisfy the commutation relations

$$[U_{j\beta}{}^\alpha, U_{l\alpha'}{}^{\beta'}]=\delta_{j,l}\{\delta_{\beta,\beta'}U_{j\alpha'}{}^\alpha-\delta_{\alpha,\alpha'}U_{j\beta}{}^{\beta'}\}.$$

Taking the Hamiltonian (7) into account, we can obtain the following expressions for the Heisenberg operators a_k^+ and $a_k(t)$:

$$a_k{}^+(t)=a_k{}^{+e}(t)+a_k{}^{+s}(t),\quad a_k(t)=a_k{}^e(t)+a_k{}^s(t), \quad (8a)$$

where

$$a_k{}^{+s}(t)=\sum_{\gamma=1}^{2}\frac{(\mathbf{g}_k\mathbf{d}_{3\gamma})}{\hbar}\int_0^t d\tau\, G_k{}^*(\tau)[U_\gamma{}^3(t-\tau)+U_3{}^\gamma(t-\tau)] \quad (8b)$$

is the part of the operator $a_k^+(t)$ which is connected with the emission and absorption processes of the atomic subsystem,

$$G^*(\tau)=\exp(i\omega_k\tau),\quad a_k{}^e(t)=a_k{}^e\exp(-i\omega_k t),$$
$$a_k{}^e(t)|\Phi\rangle=\delta_{k,k_0}\alpha\exp(-i\omega_k t)|\Phi\rangle.$$

In the derivtion of (8b) it was assumed that the dimensions of the atomic subsystem are smaller than the laser-field wavelength $\lambda_0 = 2\pi/k_0$. We consider therefore in (8b) and below the collective operators of the atomic subsystem

$$U_\beta{}^\alpha(t)=\sum_{j=1}^{N}U_{j\beta}{}^\alpha(t).$$

Following substitution of (8) in (5) we must exclude the fluctuating parts of the operators a_k^e and a_k^{+e}. We formulate for this purpose a lemma similar to that proposed in Ref. 13.

If $B(t_1)$ is an atomic-subsystem operator corresponding to the instant of time t_1, then the fluctuating part of the operator $a_k^{+e}(t)$ to the right of the operator $B(t_1)$ under the mean-value sign is represented in terms of the atomic-subsystem operators as follows:

$$\langle B(t_1)(a_k{}^{+s}(t)+a_k{}^{+e}(t))\ldots\rangle=\exp[i\omega_k(t-t_1)]\langle[a_k{}^{+s}(t_1),B(t_1)]\ldots\rangle+\langle B(t_1)a_k{}^{+s}(t)\ldots\rangle$$
$$+\alpha^*\langle B(t_1)\ldots\rangle\exp(i\omega_k t)\delta_{k,k_0} \quad (9)$$

(the proof of this relation is similar to the one proposed in Ref. 13).

With allowance for (9), the correlators in (6a) take the form

$$\langle a_{k_1}{}^+(t)a_{k_2}{}^+(t-\tau)\rangle-\langle a_{k_1}{}^+(t)\rangle\langle a_{k_2}{}^+(t-\tau)\rangle$$
$$=\sum_{\alpha,\beta=1}^{2}\frac{(\mathbf{g}_{k_1}\mathbf{d}_{3\alpha})(\mathbf{g}_{k_2}\mathbf{d}_{3\beta})}{\hbar^2}\left\{\int_0^t d\tau_1\int_0^{t-\tau_1}d\tau_2\, G_{k_1}{}^*(\tau_1)\right.$$
$$\times G_{k_2}{}^*(\tau_2)\exp[-i\omega_{k_2}(\tau-\tau_1)]\langle[U_\beta{}^3(t-\tau_1-\tau_2)+\text{H.c.},$$
$$U_\alpha{}^3(t-\tau_1)+\text{э. c.}]_-\rangle+\int_0^t d\tau_1\int_0^{t-\tau}d\tau_2\, G_{k_1}{}^*(\tau_1)G_{k_2}{}^*(\tau_2)\cdot$$
$$\times\{\langle(U_\alpha{}^3(t-\tau_1)+\text{H.c.})(U_\beta{}^3(t-\tau-\tau_2)+\text{H.c.})\rangle$$
$$-\langle(U_\alpha{}^3(t-\tau_1)+\text{H.c.})\rangle$$
$$\times\langle(U_\beta{}^3(t-\tau-\tau_2)+\text{H.c.})\rangle\}\} \quad (10)$$

The main contribution to (10) is made by the first term, obtained after commuting the operators. The second term makes a nonzero contribution compared with the first term in the higher order of the expansion in the smallness parameter $H_i/\hbar\omega_{3\alpha}$. Therefore, neglecting the retardation in the slow part of the operators in the integral with respect to τ_2

$$U_\beta{}^3(t-\tau_1-\tau_2)\approx\tilde{U}_\beta{}^3(t-\tau_1)\exp\{i\omega_{3\beta}(t-\tau_1-\tau_2)\},$$

we obtain the following expression for the correlators (10):

$$\langle a_{k_1}{}^+(t)a_{k_2}{}^+(t-\tau)\rangle-\langle a_{k_1}{}^+(t)\rangle\langle a_{k_2}{}^+(t-\tau)\rangle$$
$$=d_{31}d_{23}g_{k_1}g_{k_2}\frac{1}{\hbar}\int_0^t d\tau_1\int_0^{t-\tau_1}d\tau_2$$
$$\times\{\exp[i(\omega_{32}+\omega_{k_2})\tau_2]-\exp[i(\omega_{k_2}-\omega_{31})\tau_2]\}$$
$$\times\exp[i(\omega_{k_1}+\omega_{k_2}-\omega_{21})\tau_1]$$
$$\times\exp[i(\omega_{21}t-\omega_{k_2}\tau)]\langle\tilde{U}_1{}^2(t-\tau_1)\rangle, \quad (11)$$

where

$$g_k=(\mathbf{g}_k,\mathbf{n}),\quad \mathbf{n}-\mathbf{d}_{3\alpha}/d_{3\alpha},\quad \alpha=1,2,\quad \omega_{\gamma\delta}=\omega_\gamma-\omega_\delta.$$

The contribution Φ_2 to the right-hand side of (5b) is much smaller than Φ_1. This can be verified by substituting the solution (8) in Eq. (6b).

Thus, after substituting (11) in (6a) and integrating over k_1, k_2, τ_1, and τ_2 we obtain the following expressions for M_x and M_y:

$$M_x(t,\tau)=i\{\mu(\tau)\langle\tilde{U}_2{}^1(t-r/c)\rangle-\mu^*(\tau)\langle\tilde{U}_1{}^2(t-r/c)\rangle\},$$
$$M_y(t,\tau)=-M_x(t,\tau), \quad (12)$$

where we have for $k_0r\gg 1$

$$\mu(\tau)=\frac{d_{23}d_{31}(\mathbf{en})^2[1-(\mathbf{mn})^2]^2(\omega_{31}+\omega_{32})}{2\pi c^4r^2}$$
$$\times\int_0^{\omega_{21}} dx\frac{x^2(\omega_{21}-x)^2}{(\omega_{31}-x)(\omega_{32}+x)}\exp\{-i(\omega_0-x)\tau+i\omega_{21}(r-z)/c\},$$

$z=(\mathbf{r}\cdot\mathbf{k}_0)/k_0$ is the component of $\mathbf{r}$ along the wave vector of the external coherent EMF, $\mathbf{m}=\mathbf{r}/r$, and c is the speed of light.

We obtain similarly, after substituting (8) in (5a) and using the lemma (9), the following expression for $\Lambda(\mathbf{r},t,\tau)$:

$$\Lambda(\mathbf{r},t,\tau)=\sum_{k_i(i=1,\ldots,4)}\prod_{j=1}^{4}g^*_{k_j}\exp\{-i(\mathbf{k}_1+\mathbf{k}_3-\mathbf{k}_2-\mathbf{k}_4)\mathbf{r}\}$$
$$\times\{{}^1/{}_2\langle B^+_{k_1,k_3}(t,\tau)B_{k_2,k_4}(t,\tau)\rangle+\alpha^2\exp[-2i\omega_0t$$
$$+i\omega_0\tau]\delta_{k_2,k_0}\delta_{k_4,k_0}\langle B^+_{k_1,k_3}(t,\tau)\rangle$$
$$+\alpha\langle B^+_{k_1,k_3}(t,\tau)[a^s_{k_4}(t-\tau)\delta_{k_2,k_0}+a^s_{k_2}(t)$$
$$\times\exp(i\omega_0\tau)\delta_{k_4,k_0}]\rangle\exp(-i\omega_0t)\}+\text{H.c.},\qquad(13)$$

where

$$B^+_{k_1,k_3}(t,\tau)=\frac{g_{k_1}g_{k_3}d_{13}d_{32}(\omega_{31}+\omega_{32})\exp[i(\omega_{21}t-\omega_{k_3}\tau)]}{\hbar^2(\omega_{31}-\omega_{k_3})(\omega_{32}+\omega_{k_3})}$$
$$\times\int_0^t d\tau_1\exp[i(\omega_{k_1}+\omega_{k_3}-\omega_{21})\tau]U_1^2(t-\tau),$$
$$B_{k_1,k_3}(t,\tau)=[B^+_{k_1,k_3}(t,\tau)]^+.$$

The first two terms in (13) are expressed in terms of the transition operators between the levels $|1\rangle$ and $|2\rangle$. It is more difficult to express the terms $\langle B^+_{k_1,k_3}(t,\tau)a^s_{k_4}(t-\tau)\rangle$ and $\langle B^+_{k_1,k_3}(t,\tau)a^s_{k_2}(t)\rangle$ in terms of the two-photon transition operators U_2^1 and U_1^2. To this end we express first the operators $a^s_{k_4}(t-\tau)$ in terms of $a^s_{k_2}(t)$. This is possible if we represent in the integral with respect to τ in (8b) the solution of the Heisenberg equation for the operators $U^3_\gamma(t-\tau)$ and $U^\gamma_3(t-\tau)$ in the form:

$$U_\gamma^3(t-\tau)=U_\gamma^3(t)e^{-i\omega_{3\gamma}\tau}$$
$$-\frac{i}{\hbar}\sum_{\beta=1}^{2}\int_0^\tau d\tau_1e^{-i\omega_{3\gamma}\tau_1}(\mathbf{d}_{3\beta},\vec{\mathscr{E}}(t-\tau-\tau_1))$$
$$\times[U_\gamma^\beta(t-\tau+\tau_1)-\delta_{\beta,\gamma}U_3^3(t-\tau+\tau_1)].$$

Next, following the procedure indicated above for excluding the Boson operators, we find that

$$\langle B^+_{k_1,k_3}(t,\tau)a_{k_2}{}^s(t)\rangle e^{-i\omega_0(t-\tau)}\approx\alpha^*\langle B^+_{k_1,k_3}(t,\tau)B_{k_0,k_2}(t,\tau)\rangle,$$
$$\langle B^+_{k_1,k_3}(t,\tau)a_{k_4}{}^s(t-\tau)\rangle e^{-i\omega_0t}\approx\alpha^*\langle B^+_{k_1,k_3}(t,\tau)B_{k_0,k_2}(t,\tau)\rangle.$$

The expression for $\Lambda(\mathbf{r},t,\tau)$ takes, upon integration over time and summation over the wave vectors of Eq. (13), the form

$$\Lambda(\mathbf{r},t,\tau)=E_0^2M_x(t,\tau)/4+[\mu(\tau)\mu^*(\tau)$$
$$+\mu(\tau)\mu_0^*+\mu^*(\tau)\mu_0]\langle U_1^2(t-r/c)U_2^1(t-r/c)\rangle,\qquad(14)$$

where

$$\mu_0=i\frac{d_{13}d_{32}(\mathbf{en})^2[1-(\mathbf{mn})^2]^2}{4(\omega_{31}-\omega_0)c^3\hbar}L(\mathbf{r}),$$
$$L(r)=\frac{c}{r\omega_0}\exp\{i\omega_0(r-z)/c\},$$
$$k_0r\gg1,\qquad E_0=2|\alpha|\left(\frac{2\pi\hbar\omega_0}{V}\right)^{1/2}$$

is the mean amplitude of the external coherent EMF.

It follows from (12) and (14) that when the EMF interacts with the atoms of the system the photons are created and annihilated in pairs. The time dependence of the EMF fluctuations depends strongly on the kinetics of the population difference of levels $|1\rangle$ and $|2\rangle$. We shall therefore investigate below the behavior of the atomic subsystem in an external coherent field.

3. CONTROL EQUATION FOR THE ATOMIC-SUBSYSTEM DENSITY MATRIX

A chain of equations describing the possibility of a transition of an atomic ensemble into a two-photon superradiance regime was obtained directly in Ref. 13 for the atomic operators from the correlation functions by the method of excluding the EMF boson operators. The elimination of the boson operators from the chain of equations is, however, somewhat difficult, since this procedure must be employed in each new chain of equations for the atomic-subsystem correlators. It is therefore of interest to obtain a control equation for the density matrix of an atomic subsystem from which the EMF boson operators have already been excluded. We derive below, by the method of projection operators, a control equation for the density matrix of an atomic subsystem in an external coherent field in two-photon resonance with the $|1\rangle\to|2\rangle$ transition.

The equations for the density matrix of the total "atoms + field" system take in the interaction representation the form

$$i\frac{\partial\rho(t)}{\partial t}=\frac{1}{\hbar}[\check{H}_i(t),\rho(t)],\qquad(15)$$

where

$$\check{H}_i=\exp\left\{\frac{i}{\hbar}H_0t\right\}H_i\exp\left\{-\frac{i}{\hbar}H_0t\right\}.$$

Let $\mathscr{P}$ be the projection operator for the complete density matrix $\rho(t)$ in the basis of a free EMF:

$$\rho_s(t)=\mathscr{P}\rho(t),\quad\rho_b(t)=\bar{\mathscr{P}}\rho(t),$$

where ρ_s is the slow part of the density matrix ρ, ρ_b is its rapidly-oscillating part, and $\bar{\mathscr{P}}=1-\mathscr{P}$ The operator $\mathscr{P}$ has the properties $\mathscr{P}^2=\mathscr{P}$ and $\mathscr{P}\bar{\mathscr{P}}=0$. The equations for the matrices ρ_s and ρ_b are

$$\frac{\partial\rho_s(t)}{\partial t}=-i\lambda\mathscr{P}L_i(\rho_s(t)+\rho_b(t)),\qquad(16a)$$
$$\frac{\partial\rho_b(t)}{\partial t}=-i\lambda\bar{\mathscr{P}}L_i(\rho_s(t)+\rho_b(t)),\qquad(16b)$$

where

$$\lambda L_i(t)=[\check{H}_i(t),\ldots]/\hbar.$$

Following the known[14] procedure for eliminating the rapidly oscillating part of a density matrix, we integrate Eq. (16b) with respect to ρ_b and substitute the resultant solution in (16a), which takes then the form

$$\frac{\partial\rho_s(t)}{\partial t}=-\lambda^2\mathscr{P}\int_0^t d\tau L_i(t)U(t,t-\tau)L_i(t-\tau)\rho_s(t-\tau),\quad(17)$$

where

$$U(t,t-\tau)=T\exp\left\{-i\lambda\bar{\mathscr{P}}\int_{t-\tau}^{t}d\tau_1 L_i(\tau_1)\right\}.$$

Recognizing that for $t=0$ an ensemble of atoms does not interact with an EMF, we define the operator $\mathscr{P}$ as

$$\mathscr{P}=|\Phi\rangle\langle\Phi|\otimes\langle\langle \mathrm{Sp}_R\{\ldots\}\rangle\rangle.$$

Here $|\Phi\rangle\langle\Phi|$ is the density matrix of a free EMF. The averaging $\langle\langle\ldots\rangle\rangle$ is over the period of the dipole-system oscillations, and the trace $Sp_R\{\ldots\}$ is taken over the EMF states. With allowance for the foregoing, it can be shown that

$$\mathscr{P}H_i(t)\mathscr{P}=0,\quad \rho_s(t)=|\Phi\rangle\langle\Phi|\otimes W(t),$$

where $W(t)$ is the density matrix of the atomic subsystem.

Confining ourselves to the second order of the expansion in the small parameter $\lambda(\rho_s(t-\tau)\approx\rho_s^0(t)$, $U(t,t-\tau)=1)$, we obtain after averaging over the photon subsystem and over the period of the system-dipole oscillations the following equation for $\rho_s^0(t)$:

$$\frac{\partial\rho_s^0(t)}{\partial t}=-\frac{i}{\hbar}[H_i^{ef},\rho_s^0(t)].\tag{18}$$

Here

$$H_i^{ef}=\frac{\hbar\Omega_{ef}}{2}(U_1^2+U_2^1),\tag{19}$$

where

$$\Omega_{ef}=d_{13}d_{32}(\mathbf{E}_0\mathbf{n})^2/2\hbar^2(\omega_{31}-\omega_0)$$

is the Rabi two-photon frequency.

The quantum fluctuations of the system in two-photon processes can be taken into account in fourth order of the expansion in the small parameter λ in the right-hand side of (17). To this end we represent the evolution operator $U(t, t-\tau)$ and the density matrix $\rho_s(t-\tau)$ in the form

$$U(t,t-\tau)\approx 1-\lambda i\bar{\mathscr{P}}\int_{t-\tau}^{t}d\tau_1 L_i(\tau_1)+(i\lambda)^2\times\int_{t-\tau}^{t}d\tau_1\int_{t-\tau}^{\tau_1}d\tau_2\,\bar{\mathscr{P}}L_i(\tau_1)\bar{\mathscr{P}}L_i(\tau_2),\tag{20a}$$

$$\rho_s(t-\tau)=\rho_s(t)+\lambda^2\mathscr{P}\int_0^{\tau}d\tau_1 L_i(t-\tau_1)\times\int_0^{t-\tau_1}d\tau_2\,L_i(t-\tau_1-\tau_2)\rho_s(t-\tau_1-\tau_2).\tag{20b}$$

Upon substitution of (20) in (17) the equation for $\rho_s(t)$ takes the form

$$\frac{\partial\rho_s(t)}{\partial t}=-\lambda^4\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2\int_0^{\tau_2}d\tau_3\,\mathscr{P}L_i(t)L_i(\tau_1)L_i(\tau_2)L_i(\tau_3)\rho_s(\tau_3).\tag{21}$$

We average next, taking to explicit form of the operator $\mathscr{P}$ into account, Eq. (21) over the EMF states. Neglecting thus the retardation in the slow part $\rho_s(t)$ of the density matrix and separating the basic diagrams of the transition we obtain the following control equation for the atomic-subsystem density matrix:

$$\frac{\partial W_a(t)}{\partial t}=-\frac{i}{\hbar}[H_i^{ef},W_a(t)]-{}^1/_2\left\{\left(\frac{1}{\tau_\Phi}-i\Delta\right)[U_1^2,U_2^1W_a(t)]+\mathrm{H.c.}\right\},\tag{22}$$

where

$$\frac{1}{\tau_\Phi}=\left(\frac{1}{\tau_0}-i\Delta_0\right)+\left(\frac{1}{\tau_\varphi}-i\Delta_\varphi\right),\tag{23}$$

$$\frac{1}{\tau_0}-i\Delta_0=4\sum_{k_1}\sum_{k_2}\frac{ig_{k_1}^2g_{k_2}^2d_{13}^2d_{32}^2(\omega_{31}+\omega_{32})\zeta(\omega_{21}-\omega_{k_1}-\omega_{k_2})}{\hbar^4(\omega_{31}-\omega_{k_1})^2(\omega_{32}+\omega_{k_1})},$$

$$\frac{1}{\tau_\varphi}-i\Delta_\varphi=\sum_k\frac{g_k^2(\mathbf{E}_0\mathbf{n})^2d_{13}^2d_{32}^2(\omega_{31}+\omega_{32})^2}{2\hbar^4}\times\left\{\frac{i\zeta(\omega_{21}-\omega_0-\omega_k)}{(\omega_{31}-\omega_0)^2(\omega_{32}-\omega_0)^2}+\frac{i\zeta(\omega_{21}+\omega_0-\omega_k)}{(\omega_{31}+\omega_0)^2(\omega_{32}-\omega_0)^2}\right\},$$

τ_0 is the two-photon spontaneous-decay time of the level $|2\rangle$, τ_φ is the spontaneous-decay time stimulated by the external coherent EF, with photon scattering at the frequencies $\omega_k=\omega_{21}\pm\omega_0$ and

$$i\zeta=ip/x+\pi\delta(x).$$

Here p and δ denote, as usual, integration in the sense of the principal value and a delta function, respectively.

The first term of (22) describes two-photon excitation of atoms and their transition into the ground state by the external laser field. The second term takes into account the polarization dissipation of atoms interacting with vacuum fluctuations of the EMF. The symmetry of Eq. (22) with respect to transitions between states $|1\rangle$ and $|2\rangle$ does not differ from that of the similar equation obtained for dipole-allowed two-level systems.[8] In our situation, however, the polarization dissipation of the atom ensemble is influenced not only by two-photon spontaneous decays, but also by scattering of a coherent EMF with emission of photons at frequencies $\omega_k=\omega\pm{}_{21}\omega_0$. In the absence of an external EMF, H_i^{ef} and $\tau_\varphi^{-1}-i\Delta_\varphi$ are equal to zero, and Eq. (22) describes two-photon superradiance.[13]

4. BEHAVIOR OF QUANTUM FLUCTUATIONS OF AN EMF IN THE COURSE OF NUTATION

Taking (22) into account, we obtain the following set of equations for the atomic-subsystem correlation function:

$$\frac{dR}{dt}=-v_c+v_i,$$
$$\frac{dv_c}{dt}=-\frac{v_c-v_i}{\tau_\Phi}+\frac{2}{\tau_\Phi}\left(1-\frac{1}{N}\right)(v_c-v_i)R,\tag{24}$$
$$\frac{dv_i}{dt}=-\frac{v_i}{2\tau_\Phi}+\frac{1}{\tau_\Phi}\left(1-\frac{1}{N}\right)Rv_i-\Omega_{ef}^2R,$$

where $v_c=\tau_\varphi^{-1}\langle U_1^2(t)U_2^1(t)\rangle$ is the rate of the collective spontaneous transitions between levels $|1\rangle$ and $|2\rangle$, $v_i=\frac{1}{2}i\Omega_{ef}(\langle\tilde{U}_2^1(t)\rangle-\langle\tilde{U}_1^2(t)\rangle)$ is the rate of the induced transitions, and $2R=\langle U_2^2(t)\rangle-\langle U_1^1(t)\rangle$ is the population difference of levels $|1\rangle$ and $|2\rangle$. The system (24) was derived using a semiclassical correlator-separation method:

$$\langle U_1^2(U_2^2-U_1^1)U_2^1\rangle=-\langle U_1^2U_2^1\rangle+2\langle U_1^2U_2^1\rangle R(1-1/N),$$
$$\langle U_1^2(U_2^2-U_1^1)\rangle=-\langle U_1^2\rangle+2\langle U_1^2\rangle R(1-1/N). \quad (25)$$

We have neglected in (25) the quantum fluctuations of the operators U_1^1 and U_2^2. It can be shown in analogy with the procedure used for one-photon transitions[15,16] that for $N\gg 1$ the fluctuations of the operators U_2^2 and U_1^1 are smaller than the mean values of the same operators. In one-photon collective processes the approximation (25) is equivalent to neglecting the quantum fluctuations of the EMF density. In our problem, neglect of the quantum fluctuations of the above operators is equivalent to neglect of the fluctuations of the operator of the squared EMF density at the observation point **r**. In other words, the separated correlators (25) describe with good accuracy the time behavior of the EMF-density fluctuations and of the quadratures $\mathscr{E}_x$ and $\mathscr{E}_y$.

Let us consider the time dependence of a quasi-two-level system over times shorter the relaxation time τ_Φ. If τ_Φ/N and Ω_{ef}^{-1} are much shorter than τ_Φ, the Bloch vector

$$R^2+\tau_\Phi v_c=N^2/4$$

is conserved in the two-photon transition.

In this case the system (24) takes the form

$$\frac{dx}{d\theta}=x^2-1+y,$$
$$\frac{dy}{d\theta}=x(y-\beta^2), \quad (26)$$

where

$$x=2R/N,\quad y=4\tau_R v_i/N,\quad \theta=t/2\tau_R,\quad \beta=2\tau_R\Omega_{ef},$$

and $\tau_R=\tau_\Phi/N$ is the time of the collective spontaneous transitions in the system.

The isolated singular points of the system (26) on the x,y plane are $A(0, 1)$, $B(\nu,\beta^2)$ and $C(-\nu,\beta^2)$, where $\nu=(1-\beta^2)^{1/2}$. The point A corresponds to a stable limit cycle, B is an unstable singular point, and C corresponds to a stable state. All the system trajectories emerge from B and enter into C. With allowance for the initial conditions $\dot{x}(0)=\pm 1$ and $y(0)=0$ we obtain the following equation for the atom-system population difference:

$$\left(\frac{dx}{d\theta}-x^2+1\right)^2=\beta^2(1-x^2). \quad (27)$$

As follows from (26) and (27), all three points exist on the phase plane xy and the system trajectory is not closed. In this situation it is impossible to invert coherently a system of atoms by an external laser field, since the collective spontaneous processes are too large. The evolution of a system of atoms from the ground state $x(0)=-1$ is described by the solution

$$x(\theta)=-\frac{\beta^2-\{1-\nu\,\mathrm{cth}[\nu(\theta+\theta_{01})/2]\}^2}{\beta^2+\{1-\nu\,\mathrm{cth}[\nu(\theta+\theta_{01})/2]\}^2},\quad \beta<1, \quad (28a)$$

where $\theta_{01}=2\nu^{-1}\,\mathrm{arccoth}(\nu^{-1})$. For $\nu\theta\gg 1$ the Bloch vector stops at the point $C(-\nu,\beta^2)$ for which the rate of the induced transitions is equal to the rate of the collective spontaneous transitions.

As the parameter β is increased, the points B and C on the phase plane xy tend to the point A, with which they merge at $\beta=1$. This corresponds to the case when the trajectory of the phase plane is closed, and the system of emitters undergoes a transition from aperiodic to periodic two-photon absorption. In this situation the solution of (27) takes the form

$$x(\theta)=-\frac{\beta^2-\{1+\xi\,\mathrm{tg}[\xi(\theta-\theta_{02})/2]\}^2}{\beta^2+\{1+\xi\,\mathrm{tg}[\xi(\theta-\theta_{02})/2]\}^2},\quad \beta>1 \quad (28b)$$

where $\theta_{02}=2\xi^{-1}\,\mathrm{arctg}\,\xi^{-1}$ and $\xi=(\beta^2-1)^{1/2}$. For $\beta\gg 1$ the solution (28b) becomes harmonic: $x(\theta)=-\cos(\beta\theta)$.

It follows from (22) and (24) that the probability of induced transition with increase of the EMF intensity the probabilities of induced and collective transitions increase with increase of the EMF strength. For $\tau_\varphi\ll\tau_0$, therefore, the parameter β ceases to depend on the external laser-field intensity, with $\beta=N_c/N$, where

$$N_c=\frac{3c^3\hbar}{d_{13}d_{32}(\omega_{31}+\omega_{32})^3\omega_0^3}\times\left\{\frac{1}{(\omega_{32}+\omega_0)^2(\omega_{31}+\omega_0)^2}+\frac{27}{(\omega_{32}-\omega_0)^2(\omega_{31}+\omega_0)^2}\right\}^{-1} \quad (29)$$

is the critical number of system atoms for which the solution (28a) goes over into (28b). For $N<N_c$ optical nutation of the atom system sets in. When the number of atoms increases, $N>N_c$, the external coherent EF is incapable of inverting the system of emitters. The reason is that the probability of collective scattering of light at the frequencies $\omega_k=\omega_{21}\pm\omega_0$ increases with increase of the intensity of the external laser field.

Note that the spontaneous-decay time of the atoms is constant in single-photon collective processes. In this situation the parameter β is directly proportional to the EMF intensity,[17] so that the solution (28a) is preplaced by (28b) with increase of the external laser-field intensity for any number of atoms in the system. This difference between two-photon and single-photon collective processes is experimentally of great interest.

Using (12), (14), and (28) we obtain the following time dependence of the densities of the EMF fluctuations and of the quadratures $\mathscr{E}_x$ and $\mathscr{E}_y$ at the frequency ω_0:

$$M_x(t,\varepsilon=0)=-\frac{2\hbar^2[1-(\mathrm{em})^2]^2\omega_0^4N^2}{r^2c^4\tau_\Phi E_0^2}\,y(\theta_r)\cos\left[\frac{\omega_{21}}{c}(z-r)\right], \quad (30a)$$

$$M_y(t,\varepsilon=0)=-M_x(t,\varepsilon=0),$$

$$\Lambda(r,t,\varepsilon=0)=\frac{9\hbar[1-(\mathrm{em})^2]^3N^2}{4^3r^3\tau_0}\times\left\{\frac{\hbar\omega_0^2[1-(\mathrm{em})^2]}{c^2r}+\frac{\pi E_0^2}{4\omega_0}\sin\left[\frac{\omega_0}{c}(r-z)\right]\right\}\times\left(\frac{y(\theta_r)}{\beta}\right)^2+\frac{E_0^2M_x(t,\varepsilon=0)}{4}, \quad (30b)$$

$$y(\theta_r)=\begin{cases}\dfrac{2\beta^2\{1-\nu\,\mathrm{cth}[\nu(\theta_r+\theta_{01})/2]\}}{\beta^2+\{1-\nu\,\mathrm{cth}[\nu(\theta_r+\theta_{01})/2]\}^2}, & \beta<1,\\[2ex] \dfrac{2\beta^2\{\xi\,\mathrm{tg}[\xi(\theta_r-\theta_{02})/2]+1\}}{\beta^2+\{\xi\,\mathrm{tg}[\xi(\theta_r-\theta_{02})/2]+1\}^2}, & \beta>1,\end{cases}$$

where

$$\theta_r=(t-r/c)/2\tau_R, \quad k_0r\gg1, \quad A(t,\varepsilon)=\int_{-\infty}^{\infty}d\tau\,A(t,\tau)e^{i\varepsilon\tau},$$

$$A=M_x, M_y, \Lambda.$$

When the distance from the atom system along the laser beam ($r=z$) increases, the EMF density fluctuations is determined mainly by the term $E_0M_x/4$, since the ratio of the first term of (30b) to the second term decreases in inverse proportion to the square of the distance ($\propto z^{-2}$). The reason is that the spontaneously produced photon pairs lose the space-time correlation faster than the biphotons induced by the external field. Note that the contribution of the first term of (30b) becomes substantial when the external laser field intensity is decreased.

Two-photon nutation alters the photon statistics as follows. The collective excitation of the atoms is accompanied by vanishing of photon pairs from the laser beam. This effect leads to anti-bunching of the photon in the reradiated EMF (i.e., $\Lambda(z, t, \varepsilon=0)<0$), the components $\mathscr{E}_x$ become smaller than the vacuum fluctuations of the EMF ($M_x(t, \varepsilon=0)<0$), whereas the fluctuations of $\mathscr{E}_y$ increase substantially ($M_y(t, \varepsilon=0)>0$). The in-phase transition of the atoms from the excited to the ground state is accompanied by generation of a pair of photons in the radiation field. This increases the EMF density fluctuations ($\Lambda(z, t, \varepsilon=0)>0$). The fluctuations of the component in-phase with the external field also increase ($M_x(t, \varepsilon=0)>0$), while the fluctuations of the component $\mathscr{E}_y$ become smaller than the vacuum fluctuations of the EMF ($M_y(t, \varepsilon=0)<0$).

The vacuum fluctuations at the frequency of the external EMF are equal to

$$\tilde{C}(\varepsilon=0)=4\omega_0^3/3c^3,$$

where $\tilde{C}(\varepsilon)$ is the Fourier transform of the function $C(t-t')$ [see Eq. (3)]. We can now estimate the degree of compression of the EMF in the course of nutation

$$\eta_i=1+\frac{M_i(t,\varepsilon=0)}{\tilde{C}(\varepsilon=0)}, \quad i=x, y.$$

With allowance for (30a) we obtain

$$\eta_{x(y)}=1\mp\frac{3Nd_{13}d_{32}\omega_0}{2\hbar z^2c(\omega_{31}-\omega_0)}\frac{y(\theta_r)}{\beta}.$$

Since the maximum and minimum values of the function $y(\theta)$ are equal respectively to β and $-\beta$, the maximum compression of the EMF is reached after each half-period of the nutation, and is equal to

$$1-3Nd_{13}d_{32}\omega_0/2\hbar z^2c(\omega_{31}-\omega_0).$$

For $N>N_c$ the function y tends to β^2. Note also that the collective light-scattering processes play an important role at $N\sim N_c$. If $N>N_c$ the collective scattering processes suppress the coherent excitation of the atoms, and the EMF fluctuations remain compressed along the in-phase component $\mathscr{E}_x$. In the nutation process (i.e., at $N<N_c$) the time t_{12} of transition of the atoms from the ground to the excited state is longer than the transition time t_{21} of the atoms from the excited to the ground state. Using the solution (28b), these times can be easily obtained:

$$t_{12}=\Omega_{ef}^{-1}(\pi+2\,\mathrm{arctg}\,\xi^{-1}),$$

$$t_{21}=\Omega_{ef}^{1}(\pi-2\,\mathrm{arctg}\,\xi^{-1}).$$

If $N\ll N_c$, the collective scattering processes are weak compared with the induced ones, and the functions M_x execute harmonic oscillations.

5. CONCLUSION

The described behavior of a system of emitters can be experimentally implemented by passing laser pulses through a layer of a dense system of atoms (or an atom beam) of thickness smaller than the laser-field wavelength. It is well known that collisions of atoms with one another, as well as inhomogeneous broadening of energy levels, leads to loss of phase coherence of the oscillations of the dipoles of a system. Denoting by T_2 the corresponding damping time of the polarization of the medium ($T_2<\tau_\Phi$), the conditions for observing collective two-photon nutation become more stringent: Ω_{ef}^{-1}, $\tau_\Phi/N\ll T_2$. The rectangular laser-pulse durations must then be shorter than or of the order of T_2.

The atomic medium employed can consist of helium-like or hydrogen-like atoms undergoing two-photon resonance relative to a dipole-forbidden transition $2^1S_0\to{}^1S_0$ or $2^2S_{1/2}\to{}^1S_{1/2}$. Interest attaches also to the case when the intermediate energy level $|3\rangle$ is located between the resonant levels $|1\rangle$ and $|2\rangle$ and does not enter into single-photon resonance with the external field (the ladder model). The ladderlike arrangement of the energy levels in Rb atoms was used for experimental implementation of the two-photon micromaser transition $40S_{1/2}\to39S_{1/2}$ (Ref. 7). The level $39P_{3/2}$ turn out to be virtual in this case. A similar model was used to observe in Na atoms a two-photon dynamic Stark effect for the transition $37P_{1/2}\to36P_{1/2}$ with intermediate state $37S_{1/2}$ (Ref. 18).

To investigate collective two-photon nutation in ladder three-level systems it is useful to place the atoms in a microcavity or a microcell. The tuning and parameters of the microcavity are chosen such that the single-photon spontaneous decay of the excited state $|2\rangle$ via the state $|3\rangle$ be small compared with the two-photon one (see Ref. 7). The theory presented above describes then qualitatively also the behavior of similar three-level concentrated systems in a coherent single-mode field of a microcavity.

Collective interaction of three-level atoms located at a distance shorter than λ_0 in extended media can become the primary agent for stimulated scattering of light at the frequencies $\omega_{21}\pm\omega_0$. Naturally, an important role is played also in in such a situation by the spactiotemporal synchronism of the generated photons.

[1] C. M. Caves and B. L. Schumaker, Phys. Rev. A **31**, 3068 (1985).
[2] S. Carusotto, Optica Acta **27**, 1567 (1980).
[3] R. R. Puri and G. S. Agraval, Phys. Rev. A **37**, 3879 (1988).
[4] M. Bruno, J. M. Raimond, and S. Haroche, *ibid.* A **35**, 154 (1987).
[5] B. A. Capron, D. A. Holm, and M. Sargent, *ibid.* A **35**, 3888 (1987).
[6] B. Nicolaus and D. Z. Zhang, Phys. Rev. Lett. **47**, 171 (1981).
[7] M. Brune, J. M. Raimond, P. Goy *et al.*, *ibid.* **59**, 1898 (1987).
[8] Q. V. Lawande and S. V. Lawande, Phys. Rev. **38**, 800 (1988).
[9] J. Pevrina, *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, Reidel, 1984.
[10] D. N. Klyshko, *Physical Foundations of Quantum Electronics* [in Russian], Nauka, 1986.
[11] R. Loudon and P. L. Knight, Modern Optics **34**, 759 (1987).
[12] A. Heidmann and S. Reynaud, J. Modern Optics **34**, 923 (1987).

[13] N. A. Enaki, Zh. Eksp. Teor. Fiz. 94, No. 10. 135 (1988) [Sov. Phys. JETP 67, 2033 (1988)].
[14] Ke-Hsueh Li, Phys. Rep. **135**, 1 (1986).
[15] R. Bonifacio, Phys. Rev. A **4**, 302 (1971).
[16] S. V. Lawande, D. C. Khandekar, K. V. Bhagwat, and B. N. Jagtap, Physica A **134**, 598 (1986).
[17] V. I. Rupasov and V. I. Yudson, Kvant. Elektron. (Moscow) **9**, 2179 (1982) [Sov. J. Quantum Electronics **12**, 1415 (1982)].
[18] I. M. Beterov, I. N. Ryabtsev, and I. V. Fateev, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 181 (1988) [JETP Lett. 48, 195 (1988)].

Translated by J. G. Adashko