# Effect of spin-orbit coupling on the energy spectrum of a 2D electron system in a tilted magnetic field

Yu. A. Bychkov, V. I. Mel'nikov and É. I. Rashba

*L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR*
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The effect of spin-orbit coupling on the Landau level spectra of a two-dimensional electron gas in a tilted magnetic field is studied. The spin-orbital coupling is described by a Hamiltonian of the form $\alpha[\sigma\cdot\mathbf{k}]\nu$, where $\sigma$ are the Pauli spin matrices, $\mathbf{k}$ is the quasimomentum, and $\nu$ is a normal vector to the 2D layer. The characteristic spin-orbit interaction energy $\Delta$ is related to the constant $\alpha$ by $\Delta = m\alpha^2/2\hbar^2$, where $m$ is the effective mass. For $\gamma = 2(\Delta/\hbar\omega_c)^{1/2} \ll 1$ ($\omega_c$ is the cyclotron frequency) there are two levels $E_{1,2}$ $(H_z, H)$ in each energy interval of width $\hbar\omega_c$. The position of these levels depends on the normal component of the magnetic field $H_z$ and on the value of the total field strength $H$. Therefore, the whole spectrum consists of two branches (ladders) inserted into each other. In the quasiclassical limit, the sum of energies $E_1 + E_2$ is a multiple of $\hbar\omega_c$. The relative positions of the ladders as a function of $H_z$ and $H$ are studied. The case where the levels of one ladder approach the levels of the other ladder and the case where all energy levels of one ladder lie halfway between levels of the other ladder are treated with special attention. The latter case is studied in more detail in connection with the beats of Shubnikov–de Haas oscillations. A mechanism responsible for beat suppression is proposed. Experimental results are discussed.

## 1. INTRODUCTION

The absence of a symmetry center lifts the twofold degeneracy of two-dimensional (2D) electron gases in inversion layers and in heterostructures. The mechanism leading to this lowering of symmetry has been discussed in Ref. 1 and 2 and in the following series of papers. A short review of the first experimental results is given in Ref. 3. Afterward, a series of experiments on Shubnikov–de Haas (SdH) oscillations has been carried out in 2D-layers. The results obtained show clearly the existence of splitting in spectra; they also allow one to measure the magnitude of the splitting. That applies to Si inversion layers[4] as well as to quantum wells in InAs and $In_xGa_{1-x}As$ (Refs. 5,6). Up to now, the measurements of SdH oscillations is the most effective experimental method of studying the spin-orbit (SO) spectrum splitting. In principle, this splitting can also be determined from the magnetic field dependence of the spin-resonance frequency.[1] However, nonparabolic effects[2] become dominant in relatively strong fields, as shown by Dobers *et al.*[7,8] In addition, the beats in the SdH oscillations found by Luo *et al.*[5] and by Das *et al.*[6] provide direct evidence for the existence of two closely spaced Fermi surfaces. Similar oscillations have been extensively studied in 3D systems; a list of examples is given in the review by Seiler.[9]

In magnetospectroscopy of spin levels, the Fang–Stiles technique,[10] which is based on the use of tilted magnetic fields, is very effective. This technique has already been used in studies of the SO interaction. A series of interesting results has been obtained; for example, suppression of beats in a tilted field has been observed.[5b] A theoretical interpretation of the SdH spectra tackles two questions. The first is purely kinetic and concerns the mechanism of electron transport when 2D localization occurs. We assume below that SdH oscillations can be interpreted within the framework of elementary transport theory provided that the oscillation amplitude $\rho_{xx}$ is small compared with the monotonic part of $\rho_{xx}$.

The second question concerns the electron spectrum in a tilted field. The present paper is devoted to this problem. The SO interaction is described by the simple Hamiltonian,

$$\hat{H}_{so} = \alpha[\boldsymbol{\sigma}\mathbf{k}]\boldsymbol{\nu}, \tag{1}$$

where $\mathbf{k}$ is the quasimomentum, $\sigma$ are the Pauli spin matrices, and $\nu$ is a unit vector normal to the 2D layer. For this choice of $\hat{H}_{so}$, exact solutions exist only for $\mathbf{H}$ parallel to $\boldsymbol{\nu}$.[1] The electron energy spectrum in a tilted magnetic field has been studied before by Bychkov.[11] General properties of the spectrum were established there. In particular, it has been shown that levels do not cross in a tilted field. The energy spectrum is studied in more detail below, and results obtained from numerical calculations for currently accepted parameter values are given. Our results show that drastic changes in the spectrum occur when the tilt angle (between $\mathbf{H}$ and $\nu$) is large. In particular, the SO interaction effect on the spectrum decreases when the tangent component of $\mathbf{H}$ increases: the two energy ladders become nearly equally spaced, their step sizes approach each other and converge to the value of the cyclotron frequency $\omega_c$ for $\alpha = 0$. Convergence of ladder step sizes should lead to a significant increase of the beat period; this, in turn, can cause the actual disappearance of beats in the magnetoresistance spectra. Such phenomena have been experimentally observed by Luo *et al.*[5b]

## 2. GENERAL RELATIONS: NORMAL FIELD

In the simplest model with a nondegenerate isotropic spectrum, the Hamiltonian for electrons in magnetic field is given by

$$\hat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)^2 + \frac{\alpha}{\hbar}\left[\boldsymbol{\sigma}\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)\right]\boldsymbol{\nu} + \frac{g}{2}\mu_B(\boldsymbol{\sigma}\mathbf{H}), \tag{2}$$

where $m$, $g$ and $\mu_B$ are the effective mass, $g$-factor and the

  

Bohr magneton, respectively. The constant $\alpha$ in Eq. (1) takes values in an interval $\alpha \approx (1-10)\cdot 10^{-10}$eV·cm for different systems.

We seek the solution of the Schrödinger equation $\hat{H}\Psi = E\Psi$, in the Landau gauge, in spinor form,

$$\psi(x,y) = \exp(ik_y y)\begin{pmatrix} \sum\limits_{n=0}^{\infty} a_n \varphi_n(u) \\ \sum\limits_{m=0}^{\infty} b_m \varphi_m(u) \end{pmatrix}, \qquad (3)$$

where $\varphi_n(x)$ are harmonic oscillator functions, $u = x/l_H + k_y l_H$, and $l_H = (c\hbar/eH_z)^{1/2}$ is the magnetic length. Coefficients $a_n$ and $b_n$ are given by the system of equations

$$(\varepsilon - n - {}^1/_2 - \beta)a_n = \beta b_n \operatorname{tg}\theta + \gamma(n+1)^{1/2} b_{n+1}, \qquad (4)$$

$$(\varepsilon - n - {}^1/_2 + \beta)b_n = \beta a_n \operatorname{tg}\theta + \gamma n^{1/2} a_{n-1},$$

where $\theta$ is the angle between $\mathbf{v}$ and $\mathbf{H}$,

$$\beta = \frac{gm}{4m_0}, \quad \gamma = \left(\frac{4\Delta}{\hbar\omega_c}\right)^{1/2}, \quad \Delta = \frac{m\alpha^2}{2\hbar^2}, \qquad (5)$$

$M_0$ is the free electron mass, and the energy $\varepsilon = E/\hbar\omega_c$ ($\omega_c = eH_z/mc$ is the cyclotron frequency).

Equation (3) can be written in the vector form: $\mathbf{a} = \hat{M}\mathbf{b} = \hat{N}\mathbf{a}$, with the help of Jacobian matrices $\hat{M}$ and $\hat{N}$. Vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{b} = \hat{N}\hat{M}\mathbf{b}, \quad \mathbf{a} = \hat{M}\hat{N}\mathbf{a}. \qquad (6)$$

The spectrum of the system is then determined from the following equation:

$$\det(1 - \hat{N}\hat{M}) = \det(1 - \hat{M}\hat{N}) = 0. \qquad (7)$$

The properties of the energy spectrum, which will be given below, follow from this simple equation.

Explicately, Eqs. (6) are three-term recurrence relations:

$$\alpha_n b_{n-1} - \beta_n b_n + \alpha_{n+1} b_{n+1} = 0, \qquad (8a)$$

$$\alpha_n' a_{n-1} - \beta_n' a_n + \alpha_{n+1}' a_{n+1} = 0, \qquad (8b)$$

where

$$\alpha_n(\varepsilon) = \frac{\beta\gamma n^{1/2}\operatorname{tg}\theta}{\varepsilon - n + {}^1/_2 - \beta},$$

$$\beta_n(\varepsilon) = \varepsilon - n - {}^1/_2 + \beta - \frac{\gamma^2 n}{\varepsilon - n + {}^1/_2 - \beta} - \frac{\beta^2 \operatorname{tg}^2\theta}{\varepsilon - n - {}^1/_2 - \beta}, \qquad (9)$$

and, correspondingly,

$$\alpha_n'(\varepsilon) = \frac{\gamma n^{1/2}\beta\operatorname{tg}\theta}{\varepsilon - n - {}^1/_2 + \beta},$$

$$\beta_n'(\varepsilon) = \varepsilon - n - {}^1/_2 - \beta - \frac{\gamma^2(n+1)}{\varepsilon - n - {}^3/_2 + \beta} - \frac{\beta^2\operatorname{tg}^2\theta}{\varepsilon - n - {}^1/_2 + \beta}. \qquad (10)$$

It follows from Eqs. (9) and (10) that, for $\gamma\theta = 0$, two terms remain in Eq. (8) which can therefore be solved exactly. For $\gamma = 0$,

$$\varepsilon_n^{\pm} = n + {}^1/_2 \pm \beta H/H_z, \qquad (11)$$

whereas for $\theta = 0$,

$$\varepsilon_0 = \delta, \quad \varepsilon_n^{\pm} = n \pm (\delta^2 + \gamma^2 n)^{1/2}, \quad n \geq 1, \qquad (12)$$

where $\delta = 1/2 - \beta$. Both spectra consist of two ladders. For some parameter values, levels of different ladders can meet. For $\gamma\theta \neq 0$, this degeneracy is lifted.[11] When one of the parameters, $\gamma^2$ or $\theta^2$, is small, the spectrum can be found from the equation (linearizing with respect to the small parameter):[11]

$$\beta_n = \alpha_n^2/\beta_{n-1}^{(0)} + \alpha_{n+1}^2/\beta_{n+1}^{(0)} \qquad (13)$$

The small parameter in $\beta_m^{(0)}$ should be set equal to zero.

Configurations close to some energy (in fact, close to the Fermi level $\mu$) where one ladder is displaced by half a step size with respect to the other one are also interesting. If the contribution of each ladder to the SdH oscillations is approximately described by the quasiclassical cosine law, then the contributions from the two ladders cancel each other in this configuration. Consequently, a node appear in the beats. How beats arise for a three-dimensional analogue of the Hamiltonian (2) has been studied previously in de Haas–van Alphen oscillations.[12,13] The condition for the displacement of the ladder by a half a step size is given, for $\mathbf{H}$ parallel to $\mathbf{v}$, by,

$$2\varepsilon_n^{+} = \varepsilon_{n+s}^{-} + \varepsilon_{n+s+1}^{-}, \quad s \geq 0 \ (\text{total}), \qquad (14)$$

which, taking into account Eq. (12), becomes

$$[(2s+1)/4]^2 = \delta^2 + \gamma^2(H_z)n. \qquad (15)$$

when $n \gg 1$ and $\gamma/n^{1/2} \ll 1$. The values of $s$ serve to number the nodes. Equation (15) should be used together with the condition $\varepsilon_n^{+} \approx \mu$, or equivalently,

$$2n + s + 1 \approx 2\pi l_H^2 N; \qquad (16)$$

where $N$ is the concentration of 2D electrons. For high node number values, $s \gg 1$ (but, of course with $s \ll n$), the $\delta^2$ term in Eq. (15), can be neglected. Using Eq. (5) and taking into account $n \approx \mu/\hbar\omega_c$ and the fact that SO band splitting at the Fermi level is $\Delta_{sp} = 2\alpha k_F$, the following equality holds,

$$\Delta_{sp} = 4(\Delta\mu)^{1/2} \qquad (17)$$

From Eq. (15) it is easy to obtain

$$\Delta_{sp}/\hbar\omega_c \approx s + {}^1/_2, \qquad (18)$$

which is the criterion used by Das *et al.*[6] The question of the beat pattern in a tilted magnetic field is considered in Sec. 4.

Perturbation theory [Eq. (13)] can be applied in relatively narrow range, and a numerical study of Eq. (8), which has three parameters, is extremely involved. In addition, the solutions for each ladder depend on their eigenvalues in a complicated way. For this reason we consider, in the next section, a limiting case in which the number of independent parameters is smaller.

## 3. TILTED FIELD–QUASICLASSICAL LIMIT

For the usual parameter values for semiconductors we have, $\gamma \ll 1$. This, however, does not mean that standard perturbation theory in $\gamma$ can be applied since the product $\gamma n^{1/2}$, entering into Eq. (4), is not small. It follows from Eq. (15) that we have $\gamma n^{1/2} \sim s/2 \geq 1$ when there are beats. Under these conditions, the application of the quasiclassical theory allows one to put $n = n_0 + m \approx n_0$ on the right hand side of Eq. (4) when seeking solutions for a state with energy $\varepsilon = n_0 + \varepsilon'(\varepsilon' \sim 1)$. Such a transformation is equivalent to the quasiclassical action-angle method.[14] In this approximation, only two eigenvalues of $\varepsilon'$ which lie in the range $0 \leqslant \varepsilon' \ll 1$ need be considered, since the spectrum is periodic in $\varepsilon$, with period 1. Moreover, $n_0$ and $\gamma$ enter into the equations as one parameter, $\gamma n_0^{1/2}$.

It is convenient to use Eq. (8a) and to redefine the $\alpha_n$ and $\beta_n$ coefficients [Eq. (9)]:

$$\tilde{\alpha}_m(\varepsilon') = \alpha_{n_0+m}(\varepsilon), \quad \tilde{\beta}_m(\varepsilon') = \beta_{n_0+m}(\varepsilon), \quad \varepsilon' = \varepsilon - n_0. \quad (19)$$

In the new definition, the nonzero coefficients of the Jacobian matrix of this system of equations are given by

$$J_{mm} = 1, \quad J_{m,\,m-1} = -\tilde{\alpha}_m/\tilde{\beta}_m, \quad J_{m,\,m+1} = -\tilde{\alpha}_{m+1}/\tilde{\beta}_m. \quad (20)$$

It follows from Eq. (9) that the off-diagonal terms decrease as $m^{-2}$ for $m \gg 1$. Thus, our assumption above, $|m| \ll n_0$, is correct for all relevant values of $m$ since the series converge rapidly (as $m^{-4}$).

In the approximation used here, there is an $\varepsilon' \to -\varepsilon'$ symmetry in the energy spectrum. It is possible to see it by introducing the $\alpha_m'$ and $\beta_m'$ coefficients [by analogy with the procedure used in Eq. (19)] and expressing the determinant for the system of equations (8b) in terms of these coefficients. After a series of purely algebraic operations (shift of $m$ by 1 and an $m \to -m$ transformation) and the replacement $\varepsilon' \to -\varepsilon'$, the transformed determinant becomes equal to the determinant for Eq. (8a), using Eqs. (9) and (10). Taking into account Eq. (7), the above statement of $\varepsilon' \to -\varepsilon'$ symmetry then follows. The periodicity of the spectrum and the $\varepsilon' \to -\varepsilon'$ symmetry imply the $\varepsilon' \to 1 - \varepsilon'$ symmetry.

It follows from Eq. (20) that the determinant of the $J$ matrix has simple poles at the zeros of $\beta_m(\varepsilon')$. This fact allows one, by analogy with the transformation of Hill's determinant in the theory of the Mathieu equation,[15] to construct the following representation of this determinant:

$$\det J = 1 + C_1 \sum_{m=-\infty}^{\infty} (\varepsilon' - m - 1/2 - \beta) f_m^{-1}(\varepsilon') + C_2 \times \sum_{m=-\infty}^{\infty} (\varepsilon' - m + 1/2 - \beta) f_m^{-1}(\varepsilon'), \quad (21)$$

where the function $f_m(\varepsilon)$ is given by

$$f_m(\varepsilon) = [(\varepsilon - m - 1/2)^2 - \beta^2](\varepsilon - m + 1/2 - \beta) - \beta^2 \operatorname{tg}^2\theta(\varepsilon - m + 1/2 - \beta) - \gamma^2 n_0(\varepsilon - m - 1/2 - \beta). \quad (22)$$

The last two terms in Eq. (21) have the same poles as det $J$. In the complex plane, they both converge to zero as $|\varepsilon'| \to \infty$, whereas det $J \to 1$ in this limit. Therefore, the magnitude of the first term in the right hand side of Eq. (21) is determined. Coefficients $C_1$ and $C_2$ are functions of $\beta$, $\theta$ and $\gamma^2 n_0$. Once these functions have been calculated numerically, the condition det $J = 0$ may be used to determine the eigenvalues of $\varepsilon'$. The cubic equation $f_m(\varepsilon) = 0$ must then be solved. That is an additional difficulty of the equation obtained compared with the one for Hill's determinant; this characterizes the level of difficulties of the problem considered.

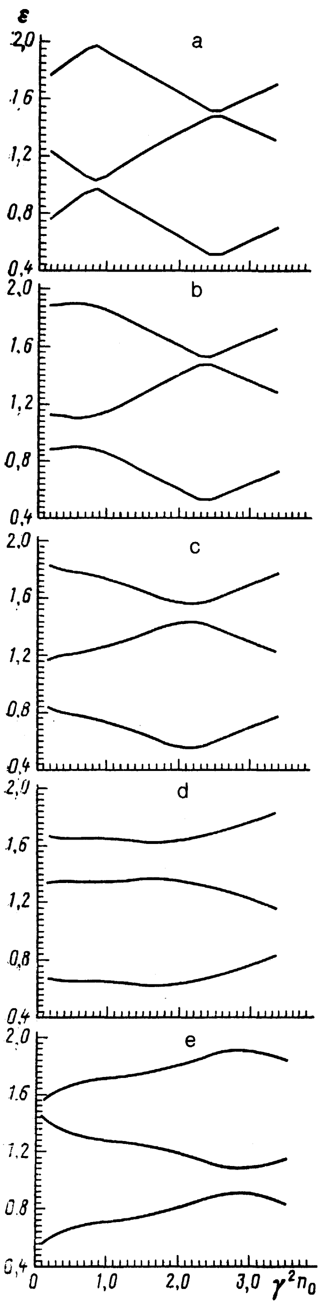


FIG. 1. Dependence of three successive quasiclassical energy levels on $\gamma_2 n_0 (\propto H_z^{-2})$. All curves are calculated for the following the InAs parameters, and for the following values of tan($\theta$):a—tan($\theta$) = 2;b—tan($\theta$) = 4;c—tan($\theta$) = 6; d—tan($\theta$) = 8; and e—tan($\theta$) = 10.

Numerical results for the $\varepsilon'$ spectrum as a function of $\gamma^2 n_0 \propto H_z^{-2}$ are shown in Figs. 1 and 2. Different traces in Fig. 1 correspond to different values of the angle $\theta$. The following relationships follow by inspection:

1) there is no level crossing in a tilted field;

2) as the difference between numbers of the approaching levels becomes greater the level splitting for small values of $\gamma\theta$ becomes smaller;

3) the region where two levels are close to each other is displaced to higher $H_z$ values as $H$ increases.

Different plots in Fig. 2 correspond to different values of the total field strength $H$. All curves exhibit the same feature: the energy level dependence on $H_z$ becomes weaker as the total field strength increases.

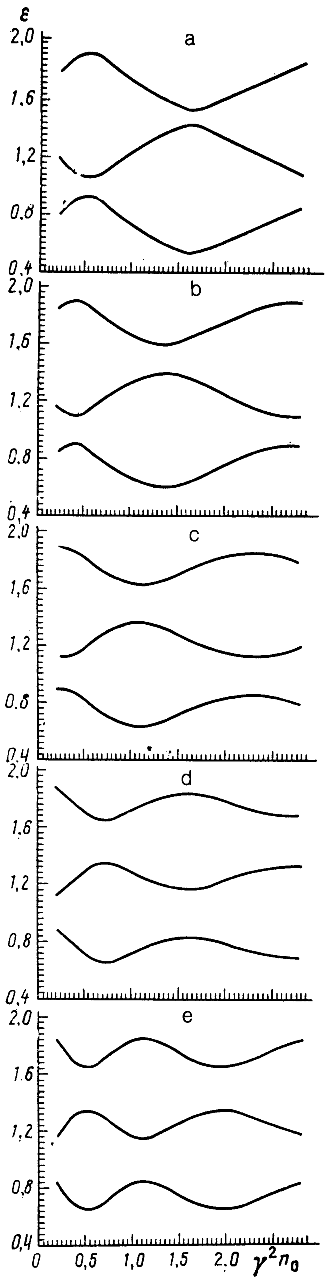

FIG. 2. Variation of the energy levels shown in Fig. 1 with $\gamma^2 n_0 \propto (H_z^{-2})$ for different values of the total magnetic field $H$ (given in units of $H_0 = m\Delta_{sp}/4m_0\mu_B$): a—$(H/H_0)^2 = 40$; c—$(H/H_0)^2 = 60$; d—$(H/H_0)^2 = 100$; and e—$(H/H_0)^2 = 150$. The same parameter values as in Fig. 1 are used.

## 4. TILTED FIELD: TUNING OF LADDERS

In Sec. 3., the properties of the energy spectrum related to the disappearance of level crossing were studied. In this section, we study the spectral properties that can lead to the appearance of beats in oscillatory phenomena. In particular, we study the case in which the energy levels of one ladder lie halfway between the levels of other ladder (tuning of ladders). All quantitative results are obtained from numerical calculations with the following parameter values: $\beta = -0.1$; $\gamma = 0.17 H_z^{-1/2}$ ($H_z$ is given in Teslas) and $N = 10^{12}$ cm$^{-2}$. These are typical values for InAs layers.

The general pattern of the spectrum, for $\gamma^2 \ll 1$, is as

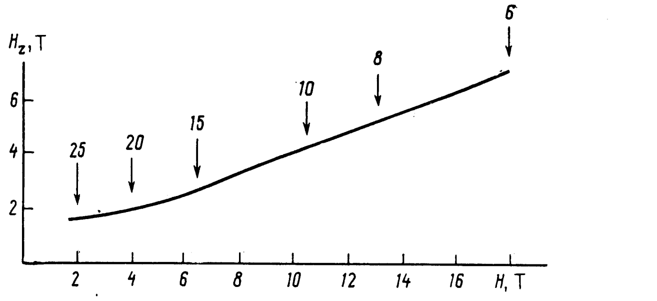

FIG. 3. Variation of the tuning field $H_z$ with total field $H$ at the Fermi energy. Numbers above arrows give the number of quantum levels below the Fermi level.

follows: in each interval of one unit of width of the spectrum there are two eigenvalues of $\varepsilon$. Therefore, the spectrum can be considered as made up of two ladders. For strong magnetic fields, when $|\beta| H/H_z > 1$ holds, i.e., when the Zeeman splitting is large, the first levels of both ladders are strongly displaced with respect to each other. Consequently, there is one level in each unit interval at the low end of the spectrum.

The maximum normal field value ($H_z = 1.72$ T) for which there is tuning of ladders near the Fermi level follows from Eqs. (15) and (16) for $s = 1$. In a tilted magnetic field, the value of the field $H_z$, which corresponds to tuning near the Fermi energy and satisfies Eq. (14), is determined as a function of $H$ from numerical calculations. Figure 3 exhibits how $H_z$ varies with the total field strength. The curve shown is smooth. The tuning field $H_z$ increases monotically as the field $H$ increases.

Figure 4 shows the distance $D$ between the nearest levels. The levels in the spectrum are numbered consecutively by an index 1; for large numbers $l \approx 2n$. The two curves in Fig. 4 correspond to nearest neighbor level distances measured from the right and the left ($D^+$ and $D^-$), respectively. The tuning of ladders corresponds to $D = 0.5$. Only the region of strong fields $H_z$ is shown in Fig. 4; for weak mag-

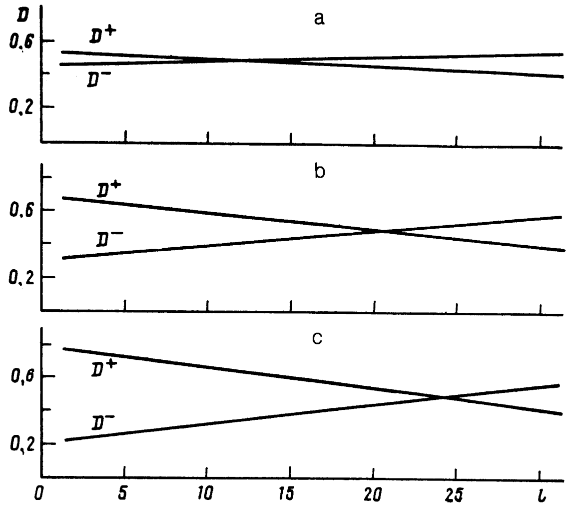

FIG. 4. Variation of distance the $D$ between nearest neighbor levels with level number $l$. For each odd value of $l$, the differences $D_l^+ = \varepsilon_{l+1} - \varepsilon_l$ and $D_l^- = \varepsilon_l - \varepsilon_{l-1}$ are shown; $l = 0$ holds for the lowest level. The same parameter values as in Fig. 1 were used: a—$H = 8$ T; b—$H = 3$ T; c—$H = H_z = 1.72$ T. All plots are for values of the tuning field $H_z$ obtained from Fig. 3.

netic fields, tuning takes place for much higher values of $l$. It follows from Fig. 4 that the level number at which tuning occurs drops as the total field $H$ increases. The fast decrease of $(D^+ - D^-)$ as the total field increases is quite significant. For $H = 18$ T tuning occurs at $l = 5$, whereas $D = 0.525$ for $l = 30$; this value can hardly be distinguished from the value of 0.5 in the scale of Fig. 4. The decrease of $(D^+ - D^-)$ for large $H$ reflects the following properties: 1) all step sizes become nearly equal within each ladder; 2) step sizes of both ladders approach each other. These properties should lead to deep effects in beat patterns of various oscillatory phenomena. For $H = 8$ T, tuning of ladders is observed in the whole range of $l$ shown in Fig. 4; as $l$ becomes larger, a phase shift develops and ladders get closer for $l \approx 13$; they separate again further on.

## 5. DISCUSSION OF EXPERIMENTAL RESULTS

A particular arrangement of levels can give rise to a series of anomalies in the magnetoresistance of 2D systems. For example, anomalies due to energy level crossings are possible.[16,17] They should particularly be observed for a range of strong fields, when 1) $\omega_c \tau \gg 1$, i.e., when the levels are well resolved, and 2) the essential levels are the ones with small numbers, i.e., they are strongly displaced with respect to each other. A different arrangement corresponds to SdH oscillations in the quasiclassical region where cosine-type oscillations, which exhibit a regular beat pattern with a large period, are observed.[5,6] They are easy to understand if one assumes that they are generated in the region $\omega_c \tau \leqslant 1$ where the transport properties are similar, in many regards, to those of the $3D$ electron gas properties and are described by elementary theory. In this interpretation, beats appear because two ladders have similar step sizes, whereas a configuration in which the ladders are tuned near the Fermi energy gives rise to beat nodes. We use such a picture below.

Studies of this type of beats in oscillatory phenomena have been proposed[12,13] to measure small SO band splitting. This method has been extensively used in the study of energy spectra of $3D$ systems.[9,18] Judging from results already obtained, this method plays no lesser role in the study of spectra of $2D$ systems. It is quite likely that specific transport properties of $2D$ systems turn out to be essential also, as has already been shown by Luo *et al.*[5b] in the interpretation of their own results. We do not pretend to interpret in detail any experimental results. Our aim is to point out those features of oscillatory phenomena which are expected to follow from the peculiarities of energy spectra established above, and to discuss experimental data briefly from that point of view.

As we noted at the beginning of this section, the nodes of oscillations should correspond to the points of ladder tuning in the framework of the quasiclassical model of SdH cosine oscillations. According to the results shown in Fig. 3, the tuning field $H_z$ increases as the total field $H$ increases. This result agrees with data of Das *et al.* (see Fig. 6 in Ref. 6) which shows that the values of the field $H_z$ where oscillation nodes occur increase noticeably as $\theta \to \pi/2$.

Well-resolved beat patterns can be observed only if particular conditions on the number of oscillations in one beat period are fulfilled. Clearly, this number should be much greater than unity. Therefore, beat patterns should become blurred as the tuning point moves into the region of small numbers. A very large beat period can also make the observation of beats difficult when, for example, there is strong damping within a period of oscillation. The sharp drop in the value of the angle between the two straight lines corresponding to $D^+$ and $D^-$ in Fig. 4 as $H$ increases should similarly lead to a sharp rise of their period; furthermore, the tuning point moves rapidly to the region of small numbers. This should give rise to damping of beats as the magnitude of the total field $H$ increases. Similar behavior has been, in fact, observed by Luo *et al.*[5b] for $H = 6$ T and it is to be expected that the scheme described above contribute significantly to this behavior.

It is also interesting to discuss the expected behavior of the SdH oscillations as $H_z$ varies in the region of high magnetic fields $H$. At large values of $H_z$, when the level width $\Gamma$ satisfies $\Gamma \ll \hbar\omega_c$, the levels are well resolved and the number of oscillations should be equal to the total number of levels crossing the Fermi level. However, as the magnitude of $H$ decreases, level resolution becomes poorer $(\Gamma \leqslant \hbar\omega_c)$, and the system goes into the cosine-type oscillation regime. Although, for high $H$, the oscillation periods of both ladders are nearly equal, beats will not be observed (since their periods are large) and the total amplitude of oscillations will be determined by the difference in the phase of oscillations of two ladders. In this case, the period of oscillations in $H_z^{-1}$ will change by a factor of two. A similar behavior has been observed by Luo *et al.*[5b] for $H > 11$ T. In a normal field, a change of the oscillation period has been observed previously for GaAs by Stormer *et al.*[19] and by Eisenstein *et al.*[20]

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