Localization of two bound particles in a one-dimensional random potential

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A solvable model is proposed for two particles with attraction, placed in a one-dimensional ($1D$) disordered system. Defects induce transitions between the N internal-quantization states whose levels are lower than the total energy of the pair. The mean free paths and the localization length l_c for $N=1$ and for $1 \ll N \ll k_F l$ are calculated. It is shown that the attraction can lead to a substantial increase of l_c, up to complete delocalization at a certain value of the energy in the case of triplet pairing of electrons.

1. INTRODUCTION

The kinetic properties of a single particle (e.g., an electron) in a $1D$ random medium have been studied quite well. For any, arbitrarily weak degree of disorder localization occurs on the scale of the mean free path l. Of course, a $1D$ picture with neglect of the interaction of the particles with one another and with the thermostat is an oversimplified idealization, which, however, helps one to understand qualitative features of the behavior of more realistic systems. As a source of references one may cite the review in Ref. 1.

In the present paper we consider a very simple $1D$ model with harmonic attraction between two particles placed in weak random potential. The justification for such a system is its solvability in certain ranges of the pair energy. An exciton in a quasi-one-dimensional ($Q\,1D$) material can serve as an example of such an object. We note also that in an investigation of the electron–electron interaction in a $1D$ conductor Berezinskiĭ and Gor'kov[2] identified the repulsion (attraction) of two electrons jointly occupying the same localized state as the principal contribution. At the same time, the possibility of $1D$ delocalization due to many-particle interaction is well known.[3,4]

It would appear that two-particle attraction, leading to the formation of a bound pair, could enhance the localization induced by the disorder. However, here, in an analysis of the exact two-particle formulation, we have obtained what is in some sense a contrary result. Since the internal energy of the pair is quantized and, moreover, should not exceed the total energy, the random potential induces transitions between the N lowest levels. The calculation of the localization length l_c can be completed in two regions—for $N=1$ and for $1 \ll N \ll k_F l$. It is found that both enhancement of the localization (near the ground-state threshold) and substantial weakening of the localization are possible. Moreover, for triplet pairing a delocalized state appears at a certain energy value in the region in which $N=1$.

2. DESCRIPTION OF THE MODEL

The equation for two identical particles in the $1D$ case has the form

$$\left[-\frac{1}{2m}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)+U(x_1)+U(x_2)+V(x_1-x_2)\right] \times \Psi(x_1,x_2)=2\varepsilon\Psi(x_1,x_2). \tag{1}$$

The energy of the pair is denoted by 2ε to facilitate comparison with the same system but with the interaction switched off. We shall assess the degree of localization from the exponential decay of the transmission coefficient $T(L)$ for passage through a disordered segment of length $L\to\infty$. In these conditions the conductivity as determined by the formula of Landauer[5] is proportional to $T(L)$. Let the random potential $U(x_i)$ be nonzero for $|x_i|<L/2$:

$$U(x_i)=\sum_j U_j a\delta(x_i-aj)\theta\left(\frac{L}{2}-|x_i|\right), \tag{2}$$

$$\langle U_j\rangle=0,\ \langle U_jU_{j'}\rangle=U_0^2\delta_{jj'} \tag{3}$$

Introduction of a lattice is not essential, since we let $a\to 0$ and $aU_0\to 0$ but in such a way that the mean free path remains finite:

$$l=a\Big/\left(\frac{maU_0}{k_F}\right)^2 \gg 1/k_F, \tag{4}$$

corresponding to the $1D$ model without interaction.

We take the attraction between the particles in the very simple form

$$V(x_1-x_2)=\frac{m\omega^2}{2}\left(\frac{x_1-x_2}{2^{1/2}}\right)^2=2\varepsilon\left(\frac{x_1-x_2}{2^{1/2}}\right)^2\Big/\,\xi^2. \tag{5}$$

In the absence of the random potential we can choose the following system of separating variables for Eq. (1):

$$x=(x_1+x_2)/2^{1/2},\ y=(x_1-x_2)/2^{1/2}.$$

The quantity ξ defined by the relation (5) specifies the width of the classically allowed band

$$|y|<\xi \tag{6}$$

for all energy values. The exact solution of Eq. (1) for $U(x_i)=0$ is

$$\Psi=\sum_n v_n(y)\left[A_n\exp(ik_nx)+B_n\exp(-ik_nx)\right]/k_n^{1/2}, \tag{7}$$

where $v_n(y)$ are the normalized eigenfunctions of a linear oscillator. Traveling waves correspond to real longitudinal wave vectors

$$k_n=\{2m[2\varepsilon-\omega(1/2+n)]\}^{1/2}. \tag{8}$$

If we now include a random potential that is weak in comparison with the interaction, such that the mean free path exceeds the width of the allowed band,

$$l\gg\xi, \tag{9}$$

then the solution keeps the same form (7), except that A_n

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and B_n become functions of x that vary on the scale of l. The quasiclassical condition

$$l \gg 1/|k_n| \tag{10}$$

implies slow variation of A_n and B_n superposed on the rapid variation of the exponential factors. In addition, the quasiclassical condition (10) makes it possible to neglect the rapidly damping terms with imaginary k_n in the sum (7).

It must be said that we are considering the region

$$2\varepsilon > \omega/2. \tag{11}$$

The opposite case $2\varepsilon < \omega/2$, when strong localization arises at improbable fluctuations of the random potential, is outside the scope of this article. We note also that, according to the condition (10), we exclude from consideration the narrow energy intervals

$$\left| 2\varepsilon - \omega\left(\frac{1}{2}+n\right)\right| \lesssim \omega\left(\frac{\xi}{l}\right)\left(\frac{1}{k_F l}\right) \ll \omega,$$

in which quasiclassical behavior is violated because of the transformation of the regular propagating mode into a damped mode.

3. SCATTERING BY A DISORDERED REGION

The model we have formulated (1)–(11) permits us to apply our approach[6-8] to multichannel localization. It consists in relating the amplitudes to the left of the disordered region (A_n^L, B_n^L) and those to the right (A_u^B, B_n^R) by means of a t matrix that depends on the realization of the potential, and then writing and, if possible, solving the Fokker–Planck equation for its distribution function $W(L,\hat{t})$. A similar method was applied in Refs. 9 and 10. Without dwelling on the details, we write down the t matrix[7]

$$\begin{bmatrix} A_n^R \\ B_n^R \end{bmatrix} = \sum_{n'n''} \begin{bmatrix} u^+_{nn'} & 0 \\ 0 & u^T_{nn'} \end{bmatrix} \times \begin{bmatrix} \operatorname{ch}\Gamma_{n'} & \operatorname{sh}\Gamma_{n'} \\ \operatorname{sh}\Gamma_{n'} & \operatorname{ch}\Gamma_{n'} \end{bmatrix} \begin{bmatrix} \bar{u}_{n'n''} & 0 \\ 0 & \bar{u}^*_{n'n''} \end{bmatrix} \begin{bmatrix} A^L_{n''} \\ B^L_{n''} \end{bmatrix}. \tag{12}$$

Here, u, Γ, and $\bar{u}$ are a unitary, a diagonal real, and a unitary matrix, respectively. The relation (12) corresponds to the orthogonal ensemble that arises in the presence of time-reversal symmetry.

The derivation of the Fokker–Planck equation for the function $W(L, u, \Gamma)$ starts from the calculation of the elementary t matrix that describes the scattering by one vanishingly weak defect

$$U_j(x,y) = U_j a\left[\delta\left(\frac{x+y}{2^{1/2}} - aj\right) + \delta\left(\frac{x-y}{2^{1/2}} - aj\right)\right], \tag{13}$$

generated in Eq. (1) by the jth term of the sum (2). The corresponding increments ΔA_n, ΔB_n have, to first order in the parameter $aU_j \to 0$, the form

$$\begin{bmatrix} \Delta A_n \\ \Delta B_n \end{bmatrix} = \sum_{n'} \begin{bmatrix} -i\beta_{nn'} & -i\gamma_{nn'} \\ i\gamma^*_{nn'} & i\beta^*_{nn'} \end{bmatrix} \begin{bmatrix} A_n \\ B_n \end{bmatrix}. \tag{14}$$

The forward-scattering amplitude $\beta_{nn'}$ and backward-scattering amplitude $\gamma_{nn'}$ are determined by the Born approximation:

$$\beta_{nn'} = m\int dx\, dy \left[\frac{v_n(y)\exp(-ik_n x)}{k_n^{1/2}}\right] \times U_j(x,y)\left[\frac{v_{n'}(y)\exp(ik_{n'}x)}{k_{n'}^{1/2}}\right], \tag{15}$$

$$\gamma_{nn'} = m\int dx\, dy \left[\frac{v_n(y)\exp(-ik_n x)}{k_n^{1/2}}\right] \times U_j(x,y)\left[\frac{v_{n'}(y)\exp(-ik_{n'}x)}{k_{n'}^{1/2}}\right]. \tag{16}$$

Since the expressions (15) and (16) differ only in the sign in the combination $k_n \pm k_{n'}$, henceforth we often give only one of the relations—for $\beta_{nn'}$ or for $\gamma_{nn'}$.

The integral (16) is expressed in terms of a Laguerre polynomial:

$$\gamma_{nn'} = 2\frac{mU_j a\sqrt{2}}{(k_n k_{n'})^{1/2}} \exp[-i(k_n+k_{n'})ja\sqrt{2}]\operatorname{Re}(i^{n'-n})\frac{\phi_{nn'}(z_{nn'})}{z_{nn'}^{1/2}}, \tag{17}$$

$$z_{nn'} = (k_n+k_{n'})/(2m\omega)^{1/2}, \tag{18}$$

$$\phi_{nn'}(z) = \left(\frac{n'!}{n!}\right)^{1/2} z^{n-n'+1/2} e^{-z^2/2} L_{n'}^{n-n'}(z^2). \tag{19}$$

Thus, the function $\phi(z)$ introduced here is a normalized eigenfunction of the following Schrödinger equation:

$$\frac{1}{2}\left[-\frac{d^2}{dz^2} + z^2 + \frac{(n-n')^2 - 1/4}{z^2}\right]\phi_{nn'}(z) = (n+n'+1)\phi_{nn'}(z). \tag{20}$$

Now, when we have determined the scattering by a defect, we must turn our attention to the symmetry of the wave function $\Psi(x_1,x_2)$ for two identical particles. The relation (17) forbids transitions between states of different parity. Therefore, the sum (7) contains terms that are either only even or only odd in n. We shall discuss in more detail the first possibility—a symmetric $\Psi(x_1,x_2)$, corresponding, e.g., to a singlet pair of electrons. The analogous treatment of triplet pairing, when $\Psi(x_1,x_2)$ is antisymmetric, gives rise to no difficulties.

The relations (14) permit us to determine the respective mean free paths l_b and l_f for backward scattering

$$\frac{1}{l_b} = \frac{1}{Na\sqrt{2}} \sum_{n,n'=0}^{N-1} \langle|\gamma_{2n,2n'}|^2\rangle \tag{21}$$

and for forward scattering

$$\frac{1}{l_f} = \frac{1}{Na\sqrt{2}} \sum_{n,n'=0}^{N-1} \langle|\beta_{2n,2n'}|^2\rangle \tag{22}$$

just as was done by the author in Ref. 6. [For an antisymmetric $\Psi(x_1,x_2)$ in Eqs. (21) and (22) and below one must replace $2n$ by $2n+1$.] The number of terms in the sums (21) and (22)

$$N = \left[\frac{\varepsilon}{\omega} - \frac{1}{4}\right] + 1 \tag{23}$$

coincides with the number of channels that are open for the given energy.

4. MEAN FREE PATHS l_b AND l_f AND LOCALIZATION LENGTH l_c FOR $N=1$

The single-channel situation corresponds to the range of energies

$$^1/_2\omega<2\varepsilon<(^1/_2+2)\,\omega,\ N=1, \tag{24}$$

for which the pair moves as a rigid whole. In other words, the conditions (10) and (24) imply that scattering by weak defects is not capable of exciting the second level of the oscillator. The length l_c for such a $1D$ system coincides with l_b. Calculating the scattering amplitudes γ_{00}, and β_{00}, we find, for $N=1$, that

$$l_c=l_b=\frac{l}{2\sqrt{2}}\left(\frac{4\varepsilon-\omega}{4\varepsilon}\right)\exp\left(2\frac{4\varepsilon-\omega}{\omega}\right), \tag{25}$$

$$l_f=\frac{l}{2\sqrt{2}}\left(\frac{4\varepsilon-\omega}{4\varepsilon}\right). \tag{26}$$

As can be seen from these expressions, on the left-hand edge of the region (24), near the threshold of the ground state of the oscillator, the localization is enhanced. On the other hand, on the right-hand edge of the interval (24) the quantity l_c in a system with attraction is found to be numerically much greater than in the $1D$ model without interaction:

$$l_c\left(\varepsilon\to\frac{5\omega}{4}\right)=l\frac{\sqrt{2}}{5}e^8\sim 10^3 l. \tag{27}$$

Moreover, the forward scattering, which does not contain the exponential factor, is increased. The weakening of the localization and of the backward scattering arises because the Gaussian function of the ground state of the oscillating pair smooths the random potential.

For comparison, we give the form of the relations (23)–(26) that corresponds to an antisymmetric $\Psi(x_1,x_2)$:

$$N_a=\left[\frac{\varepsilon}{\omega}-\frac{3}{4}\right]+1, \tag{23a}$$

$$(^1/_2+1)\,\omega<2\varepsilon<(^1/_2+3)\,\omega,\ N_a=1, \tag{24a}$$

$$l_c=l_b=\frac{l}{2\sqrt{2}}\left(\frac{4\varepsilon-3\omega}{4\varepsilon}\right)\exp\left(2\frac{4\varepsilon-3\omega}{\omega}\right)\Big/\left(2\frac{4\varepsilon-3\omega}{\omega}-1\right)^2, \tag{25a}$$

$$l_f=\frac{l}{2\sqrt{2}}\left(\frac{4\varepsilon-3\omega}{4\varepsilon}\right). \tag{26a}$$

Now l_c takes its maximum value not on the right edge of the interval (24a), where the exponential factor is maximum, but at the point $2\varepsilon=7\omega/4$, where the Laguerre polynomial vanishes. Thus, for this value of the energy the pair is delocalized:

$$l_c\left(\varepsilon\to\frac{7\omega}{8}\right)=l\frac{e\omega^2}{14\sqrt{2}(8\varepsilon-7\omega)^2}\to\infty \tag{27a}$$

because of the absence of backward scattering. At the same time, however, l_f remains finite.

Since the mean free paths have turned out, generally speaking, to be of different scales, the question arises as to whether, instead of the condition (9), we can confine ourselves to the Born approximation for just the backward scattering:

$$l_b\gg\xi,$$

since it is this scattering which plays the main role in the localization. In other words, is it not possible to regard only the nondiagonal elements (16) of the matrix (14) as small? Then the expression (15) loses its applicability and the Fokker–Planck equation for $W(L,u_{nn'},\Gamma_n)$ is transformed into an integral equation in the variables $u_{nn'}$ but remains a differential equation in Γ_n. Although it seems to us that it is possible to carry out calculations of this kind, we shall remain in the framework of the restriction (9).

5. MEAN FREE PATHS l_b AND l_f FOR $N\gg 1$

As was shown above, the single-channel situation (24) is solvable, since it reduces to the $1D$ case. We now turn to another solvable region, in which the number of open channels is large:

$$N=\frac{\varepsilon}{\omega}\gg 1. \tag{28}$$

Of course, the condition (9) imposes an upper bound on N:

$$N=\frac{k_F\xi}{2\sqrt{2}}\ll\frac{k_F l}{2\sqrt{2}}. \tag{29}$$

The parameter (28) suggests that in Eq. (20) for the scattering amplitude (17) one can apply the quasiclassical approximation

$$\phi_{nn'}(z)=\frac{1}{[\pi p_{nn'}(z)]^{1/2}}2^{1/2}\cos\left(\int_{z_1}^{z}p_{nn'}(z')\,dz'+\frac{\pi}{4}\right), \tag{30}$$

$$p_{nn'}(z)=\frac{1}{z}\left[(2n+1)(2n'+1)+\frac{1}{4}-(n+n'+1-z^2)^2\right]^{1/2}. \tag{31}$$

The condition for the existence of a quasiclassical region between the turning points

$$z_{1,2}^2=n+n'+1\pm[(2n+1)(2n'+1)+^1/_4]^{1/2} \tag{32}$$

has the form

$$(2n+1)(2n'+1)+^1/_4\gg 1. \tag{33}$$

Therefore, in Eqs. (31)–(33) we must neglect the term 1/4 as an excess of accuracy. The coefficient in the function (30) is determined by the normalization integral of $\phi^2_{nn'}(z)$ over the classically accessible region.

Expression the scattering amplitudes $\gamma_{nn'}$ and $\beta_{nn'}$ in terms of the quasiclassical solution (30) makes it possible to see that the sums (21) and (22) in this case run over those n and n' for which the square root (31) is real. Bearing this comment in mind and going over from the sums (21) and (22) to integrals, we obtain

$$\frac{1}{l_b}=\frac{2\sqrt{2}}{\pi l}\int_0^1\frac{d\zeta\,d\zeta'}{(\zeta+\zeta')(1-\zeta^2-\zeta'^2)^{1/2}}, \tag{34}$$

$$\frac{1}{l_f}=\frac{2\sqrt{2}}{\pi l}\int_0^1\frac{d\zeta\,d\zeta'}{\left[(\zeta-\zeta')^2\left(1+\frac{1}{4N}-\zeta^2-\zeta'^2\right)+\left(\frac{1}{8N}\right)^2\right]^{1/2}}, \tag{35}$$

where the variables

$$\zeta=(1-n/N)^{1/2},\ \zeta'=(1-n'/N)^{1/2} \tag{36}$$

have been used. In the second integral (35) we have kept the small terms in order to make the cutoff of the logarithmic integration obvious. If we change to polar coordinates r, θ, the difference between (34) and (35) is reduced to a change of the limits of the integration over the angle:

$$\frac{1}{l_b}=\frac{1}{l}\int_{-\pi/4}^{\pi/4}\frac{d\theta}{\cos\theta}=\frac{1}{l}\ln\frac{2^{1/2}+1}{2^{1/2}-1}, \tag{37a}$$

$$\frac{1}{l_f}=\frac{1}{l}\int_{\pi/4}^{3\pi/4}\frac{d\theta}{|\cos\theta|}=\frac{1}{l}2\ln N. \tag{37b}$$

For $N\gg 1$ the results (37) are carried over without changes to the case of an antisymmetric $\Psi(x_1,x_2)$. The parameter $\ln N\gg 1$ that has appeared in the expression (37b) implies that the forward scattering is much stronger than the backward scattering. This circumstance permits us to calculate l_c.

6. LOCALIZATION LENGTH l_c FOR $N\gg 1$

In a multichannel system l_c can be determined as follows.[6,7] We rewrite the relation (12), having decoupled it into N two-component equalities:

$$\begin{bmatrix}(uA^R)_n\\(u^*B^R)_n\end{bmatrix}=\begin{bmatrix}\operatorname{ch}\Gamma_n & \operatorname{sh}\Gamma_n\\ \operatorname{sh}\Gamma_n & \operatorname{ch}\Gamma_n\end{bmatrix}\begin{bmatrix}(\bar uA^L)_n\\(\bar u^*B^L)_n\end{bmatrix}. \tag{12a}$$

The matrices $\bar u$ and u effect a transformation (at the entrance and exit) from channels that are orthogonal in the absence of defects to a new system of channels that are normal for the given realization of the random potential. The parameters Γ_n and the associated transmission coefficients

$$T_n=1/\operatorname{ch}^2\Gamma_n$$

determine the localization length l_n for the nth normal channel:

$$\frac{1}{l_n}=-\frac{d}{dL}\langle\ln T_n(L\to\infty)\rangle. \tag{38}$$

If the Γ_n are relabeled in ascending order

$$\Gamma_0<\Gamma_1<\ldots<\Gamma_{N-1},$$

the length l_c corresponding to the least-localized mode will be given by the relation

$$\frac{1}{l_c}=-\frac{d}{dL}\langle\ln T_0(L\to\infty)\rangle. \tag{39}$$

It should be noted that the asymptotic behavior of (38) and (39) is reached at

$$L\gg 1\Big/\left(\frac{1}{l_{n+1}}-\frac{1}{l_n}\right), \tag{40}$$

i.e., when the transmission coefficients in different channels differ exponentially:

$$T_0\gg T_1\gg\ldots\gg T_{N-1}. \tag{40a}$$

As a result, the expression (39) is simplified:[6]

$$\frac{1}{l_c}=\frac{1}{a\sqrt{2}}\sum_{n,n',\bar n,\bar n'}\langle u_{0n}u_{0n'}\dot{u}_{0\bar n}\dot{u}_{0\bar n'}\rangle\langle\gamma_{nn'}\dot{\gamma}_{\bar n\bar n'}\rangle. \tag{41}$$

The evolution of the matrix u also ceases to depend on the parameters Γ_n and is determined by the following Langevin equation:[6]

$$ia\frac{d}{dL}u_{nn'}=-\sum_{\bar n}u_{n\bar n}\beta_{\bar nn'}$$

$$-\sum_{s,t,\bar n}[\theta(n-\bar n)u_{ns}^*u_{\bar nt}^*\gamma_{st}^*+\theta(\bar n-n)u_{ns}u_{\bar nt}\gamma_{st}]u_{\bar nn'}. \tag{42}$$

The step function $\theta(n)$ has the value $\theta(0)=1/2$. The relations (41) and (42) with the random scattering amplitudes (15)–(17) completely determine the localization length.

To construct the Fokker–Planck equation for the distribution function $W(L,u)$ it is necessary to know the following correlators:

$$\langle\beta_{2n,2\bar n}\beta_{2n',2\bar n'}\rangle=\left(\frac{maU_0}{k_F}\right)^2\left\{\frac{4}{\zeta\zeta'}\delta_{n\bar n}\delta_{n'\bar n'}+\frac{\delta_{n\bar n'}\delta_{n'\bar n}}{\pi N\zeta\zeta'(1-\zeta^2-\zeta'^2)^{1/2}}\left|\frac{1}{\zeta-\zeta'}\right|\right\}, \tag{43}$$

$$\langle\gamma_{2n,2n'}\gamma^*_{2\bar n,2\bar n'}\rangle=\left(\frac{maU_0}{k_F}\right)^2\frac{\delta_{n\bar n}\delta_{n'\bar n'}+\delta_{n\bar n'}\delta_{n'\bar n}}{\pi N\zeta\zeta'(1-\zeta^2-\zeta'^2)^{1/2}}\left(\frac{1}{\zeta+\zeta'}\right), \tag{44}$$

where we have used the notation (36). The product of two δ symbols arises upon averaging over the rapid oscillations of (17), on account of the exponential in the longitudinal direction and on account of the quasiclassical function (30) in the transverse direction. The first term in the right-hand side of (43) corresponds to the scattering in the previous channel, and this gives an unimportant phase shift. On the contrary, the second term in (43) is very important, since it contains a logarithmically divergent contribution from the forward scattering into nearby channels. On the other hand, the correlator (44) does not have divergences. Therefore, it is necessary to take it into account only in Eq. (41) for l_c, where β is absent. In other words, the main contribution (in the parameter $\ln N\gg 1$) to the mixing over the channels is made by forward scattering. Using the stochastic equation (42) in this way, we obtain the Fokker–Planck equation for the distribution function $W(L,u_{0n},u^*_{0n'})$:

$$a\sqrt{2}\frac{\partial W}{\partial L}=\sum_{nn'}\left\{\frac{1}{2}\frac{\partial}{\partial u_{0n}}u_{0n}+\frac{1}{2}\frac{\partial}{\partial u^*_{0n}}u^*_{0n}+\frac{\partial^2}{\partial u_{0n}\partial u^*_{0n}}|u_{0n'}|^2\right\}\langle|\beta_{nn'}|^2\rangle W. \tag{45}$$

On the right, the only terms that remain are those which, after the summation over n', contain the factor $\ln N$. In the limit $L\to\infty$, when W becomes a stationary distribution function, the left-hand side of (45) can be discarded. If we now multiply the resulting equation by $|u_{0n}|^2|u_{0\bar n}|^2$ and integrate over all the random variables the following relation for the stationary correlators arises:

$$\langle|u_{0n}|^2|u_{0\bar n}|^2\rangle\left\{\sum_{n'}\langle|\beta_{nn'}|^2\rangle+\sum_{\bar n'}\langle|\beta_{\bar n\bar n'}|^2\rangle\right\}$$

$$=\sum_{n'}\langle|u_{0n'}|^2|u_{0\bar n}|^2\rangle\langle|\beta_{nn'}|^2\rangle+\sum_{\bar n'}\langle|u_{0n}|^2|u_{0\bar n'}|^2\rangle\langle|\beta_{\bar n\bar n'}|^2\rangle. \tag{46}$$

The normalized solution of the given equation is the constant

$$\langle |u_{0\,n}|^2 |u_{0\,\bar{n}}|^2 \rangle = 1/N^2. \tag{47}$$

Substituting the correlators (44) and (47) into Eq. (41) and replacing the summation over the even n by integration, we obtain the final result:

$$l_c = \frac{N}{2} l_b = \frac{\varepsilon}{2\omega} l / \ln \frac{2^{1/2}+1}{2^{1/2}-1}. \tag{48}$$

The maximum value of l_c for $N \gg 1$ is attained at the boundary of the region of applicability of (29) and is equal to

$$(l_c)_{max} \sim l(k_F l). \tag{49}$$

7. CONCLUSION

We have considered a system of two particles with interaction of the linear-oscillator type, placed in a $1D$ random potential $U(x)$. The small parameter of the problem is the strength of $U(x)$, while the interaction is taken into account exactly. We do not use approximations of the Hartree–Fock type, for which no justification can be seen here. Since the potential $U(x)$ is chosen to be the same for both particles it is natural to regard them as identical, i.e., to investigate a symmetric $\psi(x_1,x_2)$ (bosons or a singlet state of electrons) and an antisymmetric $\psi(x_1,x_2)$ (triplet pairing of electrons) separately. The generalization to the case of distinguishable particles, e.g., for an exciton, does not give rise to difficulties. In either case, the pair energy decomposes into the quantized internal energy and a remainder term associated with the longitudinal motion. Thus, only a finite number N of internal states are important. In the simplest, single-channel situation, when $N = 1$, the pair propagates as a rigid whole without excitation of the internal degrees of freedom. In this $1D$ behavior the role of the interaction reduces to a smoothing of the random potential by the ground-state wave function. Therefore, weakening of the attraction gives rise to an increase of the localization length (25) to the value (27) at which the first excited level comes into play. Moreover, for an antisymmetric $\Psi(x_1,x_2)$ the zero of the eigenfunction leads to a delocalized state [see (27a)].

The calculation of l_c can be brought to completion in the multichannel case as well ($1 \ll N \ll k_F l$). Here the answers no longer depend on the symmetry of $\Psi(x_1,x_2)$. The forward scattering (37b) becomes stronger (in the parameter $\ln N \gg 1$) than the backward scattering (37a). This intensive mixing over the channels makes it possible to arrive at the result (48) for the localization length. For $N \gg 1$ the maximum value of l_c is found to be greater by a factor of $k_F l$ than the localization length in a $1D$ system without interaction.

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