Interaction of two-level systems in a metal with a weak external field

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The interaction between tunnel states in a metal and an external field is considered using linear response theory. Analytic expressions for the linear response function are obtained for a number of cases covering in practice the whole range of parameters of a two-level system, including the quantity b representing the scale of the interaction of the tunneling particles with conduction electrons. For $b \ll 1$, a single solution can be obtained, valid throughout the full range of temperatures and external field frequencies. The results obtained are used to account for the anomalous temperature dependences of the velocity of sound and of the internal friction in amorphous metals and in metal–hydrogen systems. A special analysis is made of the change in the elastic properties when a metal undergoes the transition to the superconducting state. Moreover, experiments on the resonant interaction of sound with a two-well system and on inelastic neutron absorption are considered. An analysis shows that all the features of the tunnel dynamics in a metal, including the electron polaron effect and the dynamic destruction of coherent tunnel states, can be revealed directly using such experiments.

1. INTRODUCTION

The low-temperature behavior of amorphous substances belongs to a class of phenomena in which one dominant feature is below-barrier tunneling of "particles" interacting with excitations of the substance. The problem of coherent and incoherent below-barrier motion of a particle in a two-well potential with weak tunnel coupling superposed on the strong dynamic and static fluctuations of the medium is fundamental in all such phenomena including quantum diffusion of atomic particles in crystals. (The amplitude of a coherent transition between neighboring wells in a regular crystal governs the width of the resultant energy band.) It is particularly interesting to consider the tunnel motion of heavy (compared with electrons) particles in a metal when the dominant interaction is with the electron fluid. A recently developed theory has made it clear that, in spite of the existence of a small adiabatic parameter $m/M \ll 1$, below-barrier transitions are generally accompanied by the appearance of a strong electron polaron effect and a simultaneous total dynamic destruction of coherent tunnel states with increasing temperature.[1,2] In this sense the interaction with electrons is practically always strong. A strong electron polaron effect in a metal is due to the fact that an important component of a perturbed electron wave function is in the form of virtual states with low-energy electron–hole excitations, known to lead to an "orthogonality catastrophe" in the static case[3] or to an infrared divergence when transitions occur.[4–6] Kondo[6] was the first to consider the role of the orthogonality catastrophe in the diffusion of a heavy particle in a metal; his later results are summarized in review papers[7] (see also the work of Yamada *et al.*[8,9]).

When this theory is applied to the two-well problem in a metal, it yields the following decisive features: firstly, a strong renormalization of the amplitude of a tunnel transition $\tilde{\Delta}_0$ takes place, whose scale depends on the bare (unrenormalized) value of the amplitude Δ_0, on the superconducting gap Δ_s, on the well asymmetry ξ, and on the absolute temperature T; secondly, $\tilde{\Delta}_0$ vanishes exponentially at temperatures $T \gg \tilde{\Delta}_0(T)$ and only incoherent transitions accompanied by the excitation of the electron system even in the $\xi = 0$ case remain; thirdly, an intrinsic width $T_2^{-1} \sim T$ appears, which is practically independent of $\tilde{\Delta}_0$ and ξ. In many cases this width is much greater than the splitting of levels in a two-well system.

All these features imply that the task of developing a theory of propagation and absorption of sound or of a low-frequency electromagnetic field in a metal matrix containing a set of two-well systems is not trivial. In the present paper we consider only the linear response case, because it reveals all the characteristic features of the tunnel kinetics in a metal and thus makes it possible to identify the experimental manifestations of these features.

The most interesting system is undoubtedly an amorphous metal in its normal and superconducting states. Recent experiments[10–13] on the absorption and renormalization of the velocity of sound in such systems simultaneously at low and high frequencies have revealed a pattern, which—as demonstrated by the analysis of the authors themselves—in many aspects qualitatively conflicts with the model of tunnel two-well states adopted for glasses. The usual model differs from that applied to the insulating glasses[14–16] only in having a different relaxation time τ, reflecting the interaction with conduction electrons (see, for example, Refs. 16 and 17). In practice this corresponds to an allowance for the interaction with excitations in accordance with perturbation theory, which is always justified in the case of the low-temperature interaction with phonons.

Below we demonstrate that if we go beyond the framework of perturbation theory for metallic glasses and allow for the unavoidable drop in the tail of the distribution function of two-level systems in terms of the parameter $\ln \Delta_0$, we can then explain the observed anomalous temperature dependence of the acoustic properties of metallic and superconducting glasses and still retain the concept of two-well systems. The results obtained for the two-well problem are of general validity and can be used to describe the tunneling along an arbitrary collective coordinate with a heavy effective mass irrespective of the microscopic nature of two-level

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systems. These results were published briefly earlier.[2,18] The present result provides an analysis also of the possibility of using different experiments to reveal the characteristic features of the tunnel motion pattern in a metal.

2. EFFECTIVE SPIN HAMILTONIAN; COHERENT AND INCOHERENT TUNNELING

We discuss a two-level system and assume that the lower levels in the wells are separated relative to one another by ξ. We assume that the amplitude of a tunnel transition between the wells Δ_0 (or more exactly, the amplitude $\tilde{\Delta}_0$ renormalized because of the interaction with electrons), as well as the quantities ξ and T are small compared with the separation ω_0 between the levels of a particle in a well:

$$\tilde{\Delta}_0,\ \xi,\ T \ll \omega_0. \tag{2.1}$$

Under these conditions we can consider only the tunnel transitions involving the lower level.

However, as shown in Ref. 1, in an analysis of the interaction of a heavy particle with an electron liquid the fundamental point is the treatment of virtually excited states of a particle in a well. Only then can we obtain the solution of the adiabatic problem which allows for the existence of a small parameter m/M. The results obtained in Ref. 1 demonstrate that in the structure of an electron wave function perturbed by the interaction with a particle we must distinguish the "fast" virtual excitations with the energy of the electron–hole pairs within an interval $\omega_0 < \delta\varepsilon < \varepsilon_0$ and "slow" excitations within a band $\delta\varepsilon < \omega_0$ (here, ε_0 is the order of the Fermi energy ε_F or of the width of the band gap), as demonstrated in Fig. 1.

The fast excitations become adiabatically matched to a particle both during its motion in a well and in the course of below-barrier motion (the reciprocal of characteristic residence time of a particle under the barrier is usually of the same order as ω_0 and, for the sake of simplicity, we shall not distinguish these characteristics). Since $\omega_0 \ll \varepsilon_0$, this matching is equivalent to screening and leads to a corresponding renormalization of the potential relief and, to a slight extent, to renormalization of the particle mass. In a sense the particle plus the matched adiabatic excitations form a physical object which tunnels as a whole.

On the other hand, the slow excitations do not follow the particle and the corresponding part of the electron wave function is oriented toward the center of the potential well.[1] It is this nonadiabatic part of the perturbed wave function, remaining in the well during the tunneling of the particle, which is responsible for the electron polaron effect. In the course of formation of the function the band of the slow excitations becomes cut off not only at the top (ω_0), but also at the bottom (ν). This is associated with the finite lifetime τ of the particle in the well, so that an admixture of states characterized by $\delta\varepsilon < \tau^{-1}$ does not form in the available time.

If we ignore the tunneling, we find that the nonadiabatic wave function can be found as the eigenfunction $\Psi_n^{(i)}$ of a one-well Hamiltonian

$$H^{(i)} = H_p^{(i)} + H_{el} + V_1^{(i)} \tag{2.2}$$

Here, $H_p^{(i)}$ is the Hamiltonian of a particle in the renormalized ith potential well;

$$V_1^{(i)} = \sum_{kk'\sigma}{}' V_{kk'}^{(i)} a_{k\sigma}^+ a_{k'\sigma}, \qquad \nu < |\varepsilon_k - \varepsilon_{k'}| < \omega_0 \tag{2.3}$$

represents the interaction of the particle with electron–hole pairs in the energy interval given above.

In the $\Psi_n^{(i)}$ function representation the effective Hamiltonian of two-level systems can be written in the form

$$H_{eff} = {}^1/_2\xi\sigma_z + {}^1/_2\Delta_{coh}\sigma_x + {}^1/_2 V_z\sigma_z + {}^1/_2\Delta_0(\Lambda - \langle\Lambda\rangle)\sigma_x + H_{el}, \qquad \Delta_{coh} = \Delta_0\langle\Lambda\rangle. \tag{2.4}$$

Here, Λ is the polaron operator whose matrix elements $\Lambda_{nm} = \langle\Psi_n^{(i)}|\Psi_m^{(i)}\rangle$ determine the overlap integral;

$$V_z = V_2^{(1)} - V_2^{(2)}, \tag{2.5}$$

where $V_2^{(i)}$ is the interaction described by Eq. (2.3), limited to the energy interval $|\varepsilon_k - \varepsilon_{k'}| < v$. The excitations in this interval do not participate in the formation of the function $\Psi_n^{(i)}$, but it is these excitations that are responsible for scattering at low temperatures $T \sim \Delta_{coh}$ (see below).

It should be pointed out that Eq. (2.4) does not include the term associated with the influence of fluctuations of the potential barrier due to the interaction with electrons on the amplitude of a transition between the wells ("fluctuation conditioning of the barrier," a concept which first arose in an analysis[19] of the influence of the interaction with phonons on the tunnel motion). Although this effect is greatly enhanced in a metal, as demonstrated by Kondo[20] and by Zawadowski *et al.* (see, for example, Ref. 21), its role in the inelastic transitions remains—as in the case of phonons—small compared with the role of the last term in Eq. (2.4). This problem is discussed specifically in Ref. 22.

The explicit form of the polaron operator Λ was found in Ref. 22 for an arbitrary interaction of electrons with a particle. In the absence of the scattering of electrons by a particle the operator Λ assumes a particularly simple form[1,2] which will be used later:

$$\Lambda = \prod_{k>k'\sigma}{}' \exp\left\{-\frac{V_{kk'}a_{k\sigma}^+ a_{k'\sigma}}{\varepsilon_k - \varepsilon_{k'} - i\delta} + \frac{V_{kk'}a_{k'\sigma}^+ a_{k\sigma}}{\varepsilon_k - \varepsilon_{k'} + i\delta}\right\}, \tag{2.6}$$

where

$$V_{kk'} = V_{kk'}^{(1)} - V_{kk'}^{(2)}.$$

Equation (2.6) is derived subject to an implicit assumption that the lifetime of a particle in a single well satisfies $\tau \to \infty$ if we have $\nu = 0$ (the symbol for the product has a prime be-

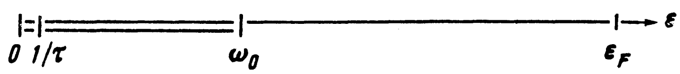

FIG. 1. Adiabatic and nonadiabatic excitations of electron–hole pairs in a Fermi liquid.

cause of the upper limit of the energy of electron–hole pairs is set by the frequency ω_0). The finite nature of the lifetime is allowed for most simply if in determination of the overlap integral we assume that the interaction acts only for a limited time and make the substitution $V \to V(t) = Ve^{-t/\tau}$. In this case the amplitude of the excitation of an electron–hole pair in Eq. (2.6) is replaced with

$$\frac{V_{kk'}}{\varepsilon_k - \varepsilon_{k'} - i/\tau}. \tag{2.7}$$

Using Eq. (2.6) subject to the substitution of Eq. (2.7), we can calculate $\Delta_{\rm coh}$ directly:

$$\Delta_{coh} = \Delta_0 \exp\left\{-{}^1/_2 \sum_{kk'\sigma}{}' \frac{|V_{kk'}|^2 n_k(1-n_{k'})}{(\varepsilon_k-\varepsilon_{k'})^2+1/\tau^2}\right\}$$

$$= \Delta_0 \exp\left\{-b\int_0^{\omega_0} dy\, y \frac{\operatorname{cth} y/2T}{y^2+1/\tau^2}\right\}. \tag{2.8}$$

Here, n_k are the electron occupancy numbers;

$$b = \rho^2(\varepsilon_F)\overline{|V_{kk'}|^2}. \tag{2.9}$$

The bar in Eq. (2.9) represents averaging over the Fermi surface.

In the limit $T \to 0$ the integral in Eq. (2.8) can be calculated by a trivial procedure and we have

$$\Delta_{coh}(T=0) \equiv \Delta_* = \Delta_0 e^{-b \ln \omega_0 \tau}. \tag{2.10}$$

In the case of a symmetric system ($\xi = 0$) the time is $\tau = 1/\Delta_*$ (here and below we have $\hbar = 1$) and it follows from Eq. (2.10) that the polaron narrowing is due to a factor $b \ln \omega_0/\Delta_*$. In future, bearing in mind Eq. (2.1), we shall assume that this factor can have an arbitrary value and in this sense the interaction with the electron subsystem is strong. However, Eq. (2.6) was derived ignoring the scattering terms, which corresponds to the condition $b \ll 1$. Throughout this paper we assume that this condition is satisfied. It should be noted that in a metal if one orbital channel of the scattering of an electron by a particle predominates, then in the most general case we have $b \leqslant 1/2$ (Refs. 8, 9, 22).

The self-consistent solution which follows from Eq. (2.10) is

$$\Delta_* = \Delta_0 \left(\frac{\Delta_0}{\omega_0}\right)^{b/(1-b)} \tag{2.11}$$

This strong nonlinear reduction in the coherent amplitude is known from the theory of tunneling with "dissipation" (see, for example, the review in Ref. 23) which appears in the "viscous" or "ohmic" regimes. In particular, when the interaction takes place with phonons in a crystal this effect occurs only in the one-dimensional case, provided moreover there is no "transport" effect (for details see Ref. 24).

The value of $\Delta_{\rm coh}$ falls with increasing temperature. Even at a relatively low temperature $T > \Delta_*$ the fall becomes exponential. It follows from Eq. (2.8) that

$$\Delta_{coh} \approx \Delta(T) e^{-{}^1/_2\Omega_T\tau}, \quad \Delta(T) = \Delta_0\left(\frac{2\pi T}{\gamma\omega_0}\right)^b, \quad \Omega_T = 2\pi bT, \tag{2.12}$$

where γ is the Euler constant. If, as before, we assume in Eq. (2.12) that $\tau = 1/\Delta_{\rm coh}$ holds, the resultant equation

$$\Delta_{coh} = \Delta_0(T)\exp(-\Omega_T/2\Delta_{coh}) \tag{2.13}$$

in the range $\Omega_T > 2\Delta(T)/e$, has only one solution $\Delta_{\rm coh} = 0$.

However, in addition to coherent tunneling there may be also noncoherent and below-barrier motion involving a change in the electron state governed by the fourth term in the Hamiltonian of Eq. (2.4). This mechanism always ensures delocalization of a particle at a finite temperature T and it therefore ensures a finite value of τ. A calculation of the probability of such a transition carried out applying perturbation theory to the transition amplitude yields

$$\tau^{-1}(\xi, T) = 2\pi \frac{\Delta_0^2}{4} \sum_{nm} \rho_m |\Lambda_{nm}|^2 \delta(E_n - E_m + \xi)$$

$$= \frac{\Delta_0^2}{4}\int_{-\infty}^{+\infty} dt \operatorname{Tr}\{\rho\Lambda^+(t)\Lambda(0)\}e^{i\xi t}. \tag{2.14}$$

Using the results of Ref. 1 for $\xi = 0$, we obtain

$$\tau^{-1} = \frac{1}{2}\frac{\Gamma^2(1+b)}{\Gamma(1+2b)}\frac{\Delta^2(T)}{\Omega_T}. \tag{2.15}$$

A comparison of Eqs. (2.12), (2.15), and (2.13) shows directly that for $\Omega_T \gg \Delta(T)$, the following inequalities are obeyed:

$$\Delta(T) \gg \tau \gg \Delta_{coh}. \tag{2.16}$$

An increase in the temperature of the system thus suppresses the coherent tunneling process so that the incoherent processes begin to play the dominant role. The second term then disappears from the Hamiltonian of Eq. (2.4) and so the spectrum of the two-level system no longer has a characteristic coherent gap and the difference between the energy levels is simply ξ. This result is related to the specific nature of the electron interaction in a metal, where the constancy of the density of states ρ near ε_F gives rise to a linear dependence $\Omega_T \propto T$ and the dimensionless parameter b need not be small. In the interaction with phonons, at least in the three- and two-dimensional cases, the coherent amplitude does not vanish and, moreover, it hardly changes at temperatures $T \gg \Delta_*$ provided only that $T \ll \Theta_D$ (for details see Ref. 24).

In general, for $\xi \neq 0$ the reciprocal of the transition time given by Eq. (2.14) is described by the following expression in the incoherent tunneling case:

$$\tau^{-1}(\xi, T) = \frac{1}{2}\frac{\Delta^2(T)\Omega_T}{\xi^2+\Omega_T^2}\frac{|\Gamma(1+b+i\xi/2\pi T)|^2}{\Gamma(1+2b)}e^{\xi/2T}. \tag{2.17}$$

This relationship, obtained using only perturbation theory with respect to Δ_0, is valid for any ratio of the parameters subject to the condition

$$\frac{\Delta^2([T,\xi]_{\max})}{\xi^2+\Omega_T^2} \ll 1. \tag{2.18}$$

The same inequality corresponds to the dominant role of the incoherent processes. In fact, for $\Omega_T > \xi$, this follows directly from the preceding analysis; in the opposite case the law of energy conservation results unavoidably in the excitation of the electron system.

The reciprocal of the time needed to establish equilibrium in a two-level system under these conditions is related to Eq. (2.17) by the simple expression

$$\gamma=(1+e^{-\xi/T})\tau^{-1}(\xi,\ T). \tag{2.19}$$

Considering the problem of a linear response of a two-level system in a metallic matrix we assume, as pointed out already, that $b\ll 1$ and that the polaron parameter $b\ln(\omega_0/\Delta_*)$ can have any value. If $b\ll 1$, we can introduce a purely formal approach shifting upward, relative to Δ_*, the lower cutoff limit ν of the nonadiabatic excitation. This results in a partial redistribution of the interaction with electron–hole pairs between the third and fourth terms in the Hamiltonian of Eq. (2.4) and alters the coherent amplitude of the transition (2.10):

$$\Delta_\nu=\Delta_0\exp\{-b\ln(\omega_0/\nu)\}. \tag{2.20}$$

If ν obeys the condition

$$b\ln(\nu/\Delta_*)\ll 1$$

or

$$\nu\ll\nu_*=\Delta_* e^{1/b}, \tag{2.21}$$

then in practice Δ_ν is equal to Δ_*.

The solution of the equation for the density matrix makes it possible to distinguish clearly two ranges. The first is characterized by the inequality

$$\Omega_T\ll\varepsilon. \tag{2.22}$$

It corresponds to coherent motion with respect to ε governed by the first two terms of the Hamiltonian (2.4) with $\Delta_{\rm coh}$ of Eq. (2.10) or (2.20) in the weak scattering case governed by the third term in Eq. (2.4). If we select the value $\nu\gg\Delta_*/\pi b$, which may be compatible with the condition (2.21) for $b\ll 1$, the interaction with excitations of energy $\delta E>\nu$ described by the fourth term of Eq. (2.4) is practically absent in the case of such motion and the strong polaron interaction is manifested only by the renormalization $\Delta_{\rm coh}$.

The second range corresponds to the inequality which is the opposite to that given by Eq. (2.22) or, in the more general case, the inequality (2.18). In this case the coherence of the system is lost [see Eq. (2.13)] and the interaction with an electron fluid cannot be regarded as weak. However, we can now use the weakness of the coherent coupling between the wells against the background of strong intrawell inelastic processes; we can then introduce the site representation and find the solution of the transport equation by means of perturbation theory in Δ_0 or, in fact, in terms of the parameter (2.18). In the range under discussion the value of the cutoff limit ν is completely unimportant, because the problem can be solved for an arbitrary value of b.

Finally, we can introduce the third range

$$\varepsilon<T<\nu<\nu_*, \tag{2.23}$$

which for $b\ll 1$ overlaps considerably both region I and region II (Fig. 2). In this region we can find the solution of the transport equation in the site representation for arbitrary values of Δ_* and Ω_T. The overlap of the regions makes it possible to obtain just one solution valid throughout the full range of variation of the parameters. We must stress that the final answer is naturally independent of ν.

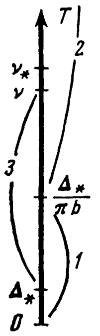

FIG. 2. Structure of various regions: 1) coherent motion; 2) incoherent tunneling; 3) high temperatures.

Figure 2 is plotted for $\xi=0$, where the concept applies also when $\xi\neq 0$. In fact, for $\xi\gg\Delta_*$, the relationships (2.18), (2.22), and (2.23) setting out the boundaries of the region and retaining the overall picture with the replacement of Δ_* and $\tilde{\varepsilon}$ result in an additional overlap of regions 1 and 2.

The proposed approach to the problem is applied below. An artificial assumption $\nu<\nu_*$ is essential to the use of perturbation theory in determination of the polaron renormalization of the spectrum [in the sense of Eq. (2.21)] when the parameter $b\ln(\omega_0/\Delta_*)$ is unrestricted.

3. INTERACTION OF AN EXTERNAL FIELD WITH A TWO-LEVEL SYSTEM IN A METAL

We assume that a particle in a two-level system experiences a weak external field. This field gives rise to an additional term in the Hamiltonian of Eq. (2.4):

$$H_{int}=\varepsilon_0\eta\sigma_z\cos\omega t, \tag{3.1}$$

where η is the characteristic energy scale and $\varepsilon_0\to 0$ is the dimensionless field amplitude. The term proportional to σ_x is small compared with Eq. (3.1) and the reduction is in the ratio Δ_0/ω_0, so that it can be ignored. When the interaction is due to an acoustic wave, then η represents the deformation potential and ε_0 is one of the components of the strain tensor. In this case the expression for the elastic moduli considered in linear response theory obviously gives

$$M_\omega=2\eta\frac{\partial}{\partial\varepsilon_0}\langle{\rm Tr}\{\rho(t)\sigma_z e^{i\omega t}\}\rangle_t, \tag{3.2}$$

where $\rho(t)$ is the density matrix of the system. We use $f(t)$ to denote the density matrix of a particle and $f(\omega)$ for its Fourier component:

$$f(\omega)=\langle{\rm Tr}_{el}\rho(t)e^{i\omega t}\rangle_t. \tag{3.3}$$

It follows from Eq. (3.2) that it suffices to know only the diagonal (in the site representation) component of the density of a particle $f_{11}=\frac{1}{2}{\rm Tr}\,\sigma_z f(\omega)$ and, bearing in mind the linearity of the problem, it suffices to calculate f_{11} in the first nonvanishing (in ε_0) order

$$M_\omega=4\eta\frac{\partial}{\partial\varepsilon_0}f_{11}(\omega). \tag{3.4}$$

[In Eq. (3.4) we used $f_{11}(\omega)+f_{22}(\omega)=0$].

The intensity matrix of the whole system in the approximation linear in ε_0 can be found from the standard equation

$$\frac{\partial\rho(t)}{\partial t}+i[H,\rho(t)]=-i[H_{int},\rho_T], \tag{3.5}$$

where ρ_T is the equilibrium density matrix (we are interested in the solution which is linear in ε_0). Substituting ρ_T in Eq. (3.5) in the form $(1/2)\varepsilon_0\eta e^{-i\omega t}\rho$ yields

$$\omega\rho-[H_0,\rho]=[H_1,\rho]+[\sigma_z,\rho_T]. \tag{3.6}$$

Depending on the range of the parameters Δ_0, ξ, and T, the separation of the Hamiltonian of Eq. (2.4) into the zeroth Hamiltonian H_0 and the interaction H_1 is best carried out in different ways using a satisfactory basis representation each time. Therefore, we shall not initially make the separation specific. Substituting the formal solution of Eq. (3.6) into the right-hand side of the same equation and going over to the density matrix of a particle in accordance with Eq. (3.3), we now obtain

$$\omega f-[H_0,f]=[\sigma_z,f_T]+\mathrm{Tr}_{el}\left[H_1,\frac{1}{\omega-L}[\sigma_z,\rho_T]\right]+J, \tag{3.7}$$

where the collision integral J is given by

$$J(\omega,\rho)=\mathrm{Tr}_{el}\left[H_1,\frac{1}{\omega-L}[H_1,\rho]\right]. \tag{3.8}$$

Here, for the sake of simplicity, we have adopted a notation (see Ref. 25) the meaning of which becomes clear if we define matrix elements in the representation of the eigenfunctions of the Hamiltonian H_0:

$$\left[\frac{1}{\omega-L}A\right]_{ik}=\frac{1}{\omega-E_{ik}}A_{ik},\qquad E_{ik}=E_i-E_k, \tag{3.9}$$

where $E_i\equiv E_a^n$ represents the eigenvalues of H_0 (a assumes two values and n is the index of the electron subsystem).

Perturbing with respect to H_1, we can ignore in Eq. (3.8) a correlation between the states of the particle and those of electrons, which is equivalent to the approximation

$$\rho=\rho_0 f, \tag{3.10}$$

where ρ_0 is the equilibrium electron density matrix. Consequently, Eq. (3.7) becomes closed relative to f. To the same accuracy we can find the equilibrium values of f_T and ρ_T from the steady-state solution of Eqs. (3.5) and (3.7) for $H_{\rm int}=0$:

$$\rho_T=\rho_0 f_0+\frac{1}{L}[\rho_0 f_0,H_1],\qquad f_T=f_0-\frac{1}{L}J(0,\rho_0,f_0), \tag{3.11}$$

where the index 0 refers to the equilibrium state for $H_1=0$.

The matrix elements of the collision integral can be represented in the form

$$J_{cd}=\sum_{ab}\Omega_{ab}{}^{cd}f_{ab}. \tag{3.12}$$

It follows from Eq. (3.8) that after direct transformation the matrix elements of the suboperator Ω are given by

$$\Omega_{ab}{}^{cd}=\sum_{nm}\rho_0(n)\left\{\delta_{bd}\sum_g\frac{(H_1)_{cg}{}^{nm}(H_1)_{ga}{}^{mn}}{\omega-E_{gb}{}^{mn}}+\delta_{ca}\sum_g\frac{(H_1)_{bg}{}^{nm}(H_1)_{gd}{}^{mn}}{\omega-E_{ag}{}^{nm}}-(H_1)_{bd}{}^{nm}(H_1)_{ca}{}^{mn}\left(\frac{1}{\omega-E_{cb}{}^{mn}}+\frac{1}{\omega-E_{ad}{}^{nm}}\right)\right\}. \tag{3.13}$$

Therefore, the problem reduces to the solution of the linear system of equations

$$\omega f_{cd}-[H_0,f]_{cd}-\sum_{ab}\Omega_{ab}{}^{cd}f_{ab}=u_{cd},$$
$$u_{cd}=[\sigma_z,f_T]_{cd}+\mathrm{Tr}_{el}\left[H_1,\frac{1}{\omega-L}[\sigma_z,\rho_T]\right]_{cd}. \tag{3.14}$$

Note that the adiabatic activation of the interaction corresponds to the substitution $\omega\to\omega+i\delta$ in Eqs. (3.13) and (3.14).

We begin with low temperatures and assume that we simultaneously have

$$\xi,\ \omega,\ T<\nu. \tag{3.15}$$

In this case the important excitations are those which do not participate in the electron polaron effect and, therefore, the interaction is governed by the third term in Eq. (2.4). However, the electron–hole pairs of energy $\delta\varepsilon>\nu$ participating in the electron polaron effect are not excited and do not interact with a tunneling particle. Therefore, the penultimate term in the Hamiltonian of Eq. (2.4) can be omitted. In accordance with the ideas put forward at the end of the preceding section, we have $\Delta_{\rm coh}=\Delta_\nu$ in the range under consideration. Our Hamiltonians H_0 and H_1 are selected to be

$$H_0=H_{el}+H_p,\ H_p={}^1/_2(\xi\sigma_z+\Delta_\nu\sigma_x),$$
$$H_1={}^1/_2V_z\sigma_z. \tag{3.16}$$

The relationship between the representation of the eigenfunctions of the Hamiltonian H_p and the site representation has the standard form ($\xi>0$):

$$|a\rangle=u|1\rangle+v|2\rangle,\ |b\rangle=-v|1\rangle+u|2\rangle,$$
$$\begin{Bmatrix}u\\v\end{Bmatrix}=\left(\frac{1\pm\xi/\varepsilon}{2}\right)^{1/2},\qquad \varepsilon^2=\Delta_\nu{}^2+\xi^2. \tag{3.17}$$

In the H_0 Hamiltonian representation of Eq. (3.16), we have

$$(H_1)_{ab}{}^{nm}=\frac{(V_z)^{nm}}{2\varepsilon}\begin{pmatrix}\xi & -\Delta_\nu\\ -\Delta_\nu & \xi\end{pmatrix}. \tag{3.18}$$

We substitute Eq. (3.18) into the definition of the superoperator Ω of Eq. (3.13). All the terms in (3.13) contain the same sum of the electron variables of the type

$$\Sigma(z)={}^1/_2\sum_{nm}{}'\rho_0(n)|V_z{}^{nm}|^2\left(\frac{1}{z+E^{nm}}+\frac{1}{z-E^{nm}}\right)$$
$$=z\sum_{kk'\sigma}\frac{|V_{kk'}|^2n_k(1-n_{k'})}{(z+\varepsilon_k-\varepsilon_{k'})(z+\varepsilon_{k'}-\varepsilon_k)}=-i\Omega(z)-2zR_\nu(z). \tag{3.19}$$

We have here

$$\Omega(z)=\pi bz\,\mathrm{Cth}(z/2T),$$
$$R_\nu(z)=\frac{1}{\pi}\int_0^\nu\frac{dy\,\Omega(y)}{y^2-z^2}=b(\ln(\nu/2\pi T)-\mathrm{Re}\,\Psi(1+iz/2\pi T)). \tag{3.20}$$

Noting that $f_{bb}=-f_{aa}$, we select f_{aa}, f_{ab}, and f_{ba} as the independent elements of the density matrix. The coefficient in front of f_{aa} in Eq. (3.12) is then $\Omega_{aa}^{cd}-\Omega_{bb}^{cd}=\tilde{\Omega}_{aa}^{cd}$. Using Eqs. (3.18) and (3.19), we obtain from Eq. (3.13)

$$\tilde\Omega_{aa}{}^{aa}=\frac{\Delta_v{}^2}{2\varepsilon^2}[\Sigma(\omega+\varepsilon)+\Sigma(\omega-\varepsilon)]; \quad \Omega_{ab}{}^{aa}=\frac{\Delta_v\xi}{2\varepsilon^2}\Sigma(\omega-\varepsilon);$$

$$\Omega_{ab}{}^{ab}=\frac{\xi^2}{\varepsilon^2}\Sigma(\omega-\varepsilon)+\frac{\Delta_v{}^2}{2\varepsilon^2}\Sigma(\omega); \quad \Omega_{ba}{}^{ab}=-\frac{\Delta_v{}^2}{2\varepsilon^2}\Sigma(\omega). \tag{3.21}$$

The Hermitian nature of the density matrix leads to the relationship $f_{cd}(\omega)=f^*_{dc}(-\omega)$, which in turn gives the result

$$\Omega_{ab}{}^{cd}(\omega)=-\Omega_{ba}{}^{dc*}(-\omega); \; u_{cd}(\omega)=-u_{dc}{}^*(-\omega). \tag{3.22}$$

The relationships in Eq. (3.22) allow us to find the remaining matrix elements of the superoperator Ω.

We now consider the first range of temperatures where the inequality of Eq. (2.22) is obeyed. Assuming $b\ll 1$, we can readily show that

$$\Sigma\ll(\varepsilon,\ \omega)_{max}. \tag{3.23}$$

In the same limit we have the usual situation in perturbation theory when the widths of the transitions and the renormalization of the spectrum are small. Therefore, the linear response can be found in the first nonvanishing approximation in terms of b.

Under these conditions the determinant of the system (3.14) is governed simply by the product of the diagonal elements

$$D=(\omega-\tilde\Omega_{aa}{}^{aa})(\omega-\varepsilon_{ab}-\Omega_{ab}{}^{ab})(\omega-\varepsilon_{ba}-\Omega_{ba}{}^{ba}). \tag{3.24}$$

The zeros of the determinant of Eq. (3.24) determine the eigenfrequencies of the system. Using Eq. (2.33), they can be represented in the form (see Ref. 25)

$$\omega^{(1)}=-i\gamma_1(\tilde\varepsilon),\ \omega^{(2)}=\mp\tilde\varepsilon-i\gamma_2(\tilde\varepsilon,\tilde\varepsilon), \tag{3.25}$$

$$\gamma_1(z)=\frac{\Delta_v^2}{\varepsilon^2}\Omega(z),\quad \gamma_2(z,\varepsilon)=\frac{\xi^2}{\varepsilon^2}\Omega(z-\varepsilon)+\frac{\Delta_v^2}{2\varepsilon^2}\Omega(z), \tag{3.26}$$

$$\tilde\varepsilon=\varepsilon-\frac{\Delta_v^2}{\varepsilon}R_v(\tilde\varepsilon). \tag{3.27}$$

In this case the energy renormalization is related to the renormalization of the tunneling amplitude. The expression (3.27) represents an expansion, linear in b ($R_v\propto b$), of the general expression for the renormalized separation between the levels:

$$\tilde\varepsilon=[\xi^2+\Delta^2(\tilde\varepsilon)]^{1/2},\quad \Delta(\tilde\varepsilon)=\Delta_v(1-R_v(\tilde\varepsilon))\approx\Delta_v e^{-R_v(\tilde\varepsilon)}. \tag{3.28}$$

The expression for $R_v(\tilde\varepsilon)$ of Eq. (3.20) in the case T, $\tilde\varepsilon\ll v$ contains a dependence on the upper limit of integration only via a term of the type $\ln v$. Bearing in mind this result and the explicit form of the expression for Δ_v given by Eq. (2.20), we obtain

$$\Delta(\tilde\varepsilon)=\Delta_0\exp(-R_{\omega_0}(\tilde\varepsilon)), \tag{3.29}$$

where the index ω_0 corresponds to the replacement of the upper limit in the integral (3.20) with ω_0. It should be noted that the dependence on the nonphysical parameter v in Eq. (3.29) disappears. In the limit $T\to 0$ in the symmetric case ($\xi=0$), we find from Eq. (3.20) that ($R_{\omega_0}=b\ln(\omega_0/\Delta_*)$, and therefore, $\Delta(\tilde\varepsilon)=\Delta_*$ [see Eq. (2.11)]. To the accuracy considered here, the quantities ε and Δ_v in Eqs. (3.25)–(3.27) can be replaced with $\tilde\varepsilon$ and $\Delta(\tilde\varepsilon)$, respectively. Unless specially mentioned, we shall assume that such a substitution is made.

In the eigenfunction representation of the Hamiltonian H_0 of Eq. (3.16) the diagonal element of the density matrix of a particle considered in the site representation f_{11}, occurring in the definition of the elastic modulus of Eq. (3.4), is given by the following expression obtained subject to the transformation (3.17):

$$f_{11}=\frac{\xi}{\varepsilon}f_{aa}-\frac{\Delta_v}{2\varepsilon}(f_{ab}+f_{ba}). \tag{3.30}$$

We can find the matrix element f_{cd} by solving the system of equations (3.14), first finding the matrix u_{cd}. This procedure is fairly tedious, but it can be carried out directly without any fundamental difficulties. It is convenient to use the relationship (3.22) and the exact relation $u_{11}=0$, which follows directly from the definition of (3.14) and which transforms to

$$u_{ab}+u_{ba}=\frac{2\xi}{\Delta_v}u_{aa}.$$

Direct calculations carried out using the expressions in Eq. (3.11) yield

$$u_{aa}=-\frac{\Delta^2(\tilde\varepsilon)\xi}{2\tilde\varepsilon^3\omega}\,\mathrm{th}(\tilde\varepsilon/2T)\{\Sigma(\omega+\tilde\varepsilon)-\Sigma(\omega-\tilde\varepsilon)+4\tilde\varepsilon R_v(\tilde\varepsilon)\},$$

$$u_{ab}-u_{ba}=-\mathrm{th}(\tilde\varepsilon/2T)\left\{\frac{2\Delta(\tilde\varepsilon)}{\tilde\varepsilon}\frac{\partial\Delta(\tilde\varepsilon)}{\partial\Delta_v}-\frac{\Delta(\tilde\varepsilon)}{\tilde\varepsilon^3}\right.$$

$$\times\left[\frac{\xi^2}{\omega}(\Sigma(\omega+\tilde\varepsilon)+\Sigma(\omega-\tilde\varepsilon)\right.$$

$$\left.\left.+2i\Omega(\tilde\varepsilon))-\frac{2\Delta^2(\tilde\varepsilon)}{\omega^2-\tilde\varepsilon^2}(\omega(\Sigma(\tilde\varepsilon)-\Sigma(\omega))+2\tilde\varepsilon(\omega-\tilde\varepsilon)R_v(\tilde\varepsilon))\right]\right\}.$$

The final expression for the elastic modulus of Eq. (3.4) or for the linear response function can be represented in the form

$$M=\frac{2\eta^2}{D}\left\{\frac{\Delta^2(\tilde\varepsilon)\xi^2}{\tilde\varepsilon^3\omega}(\omega^2-\tilde\varepsilon^2)\,\mathrm{th}(\tilde\varepsilon/2T)\right.$$

$$\times\left[\frac{i\pi b\,\mathrm{Sh}(\omega/T)}{\mathrm{Ch}(\omega/T)-\mathrm{Ch}(\tilde\varepsilon/T)}+R_v(\tilde\varepsilon+\omega)+R_v(\tilde\varepsilon-\omega)-2R_v(\tilde\varepsilon)\right]$$

$$\left.-\frac{\Delta_v}{2}(\omega-\tilde\Omega_{aa}{}^{aa})\left[u_{ab}-u_{ba}-4\frac{\Delta(\tilde\varepsilon)\xi^2}{\tilde\varepsilon^3}\mathrm{th}(\tilde\varepsilon/2T)R_v(\tilde\varepsilon)\right]\right\} \tag{3.31}$$

[D has the value given by Eq. (3.24)]. This cumbersome expression simplifies greatly in all the most interesting cases. For example, for $T\gg\tilde\varepsilon$ [if we have $b\ll 1$, this inequality must apply together with Eq. (2.22)], Eq. (3.31) becomes

$$M\approx\eta^2\frac{\Delta^2(T)}{T}\frac{\omega+i(2\Omega(\omega)-\Omega_T)}{[\omega+i\gamma_1(\omega)][(\omega+i\gamma_2(\omega,0))^2-\tilde\varepsilon^2]}. \tag{3.32}$$

Here, $\Delta(T)$ is the value of $\Delta(\tilde\varepsilon)$ of Eq. (3.28) when $T\gg\tilde\varepsilon$, which is independent of $\tilde\varepsilon$. To the same first-order in b, this expression can be simplified to

$$M\approx\eta^2\frac{\Delta^2(T)}{T}\frac{\omega+i\Omega_T}{\omega[(\omega+i\Omega_T)^2-\tilde\varepsilon^2]-i\Delta^2(T)\Omega_T}. \tag{3.33}$$

Usually in studies of the acoustic properties of metallic glasses the greatest interest lies in the second limiting case $\omega\ll T$. In this limit for an arbitrary relationship between $\tilde\varepsilon$ and T, Eq. (31) becomes

$$M\approx 2\eta^2\left\{\frac{\frac{\xi^2\beta}{2T\,\mathrm{Ch}^2(\tilde\varepsilon/2T)}i\gamma_1(\tilde\varepsilon)+\frac{\Delta^2(\tilde\varepsilon)}{\tilde\varepsilon}\varkappa\,\mathrm{th}(\tilde\varepsilon/2T)(\omega+i\gamma_1(\tilde\varepsilon))}{[\omega+i\gamma_1(\tilde\varepsilon)][(\omega+i\gamma_2(0,\tilde\varepsilon))^2-\tilde\varepsilon^2]}\right\}. \tag{3.34}$$

Here, using Eq. (3.29), we obtain

$$\varkappa=\frac{\partial\ln\Delta(\tilde\varepsilon)}{\partial\ln\Delta_v}\left(1+\frac{2\xi^2}{\tilde\varepsilon}\frac{\partial R_v}{\partial\tilde\varepsilon}-\xi^2\frac{\partial^2 R_v}{\partial\tilde\varepsilon^2}\right)\approx\frac{\tilde\varepsilon^3}{\Delta^2(\tilde\varepsilon)}\frac{\partial^2\tilde\varepsilon}{\partial\xi^2}, \tag{3.35}$$

$$\beta=\left(1-\frac{2\Delta^2(\tilde\varepsilon)}{\tilde\varepsilon}\frac{\partial R_v}{\partial\tilde\varepsilon}\right)=\frac{\tilde\varepsilon^2}{\xi^2}\left(\frac{\partial\tilde\varepsilon}{\partial\xi}\right)^2. \tag{3.36}$$

The first term in the numerator of Eq. (3.29) represents the relaxation contribution and the second is the resonance contribution. The main differences between this expression and that generally accepted (see, for example, Ref. 17) are an explicit allowance for the renormalization of the spectrum of a two-level system [Eqs. (3.27)–(3.29)] and the appearance of the factors $\varkappa$ and β (the change in the structure γ_2 away from the resonance is not so important). The quantity $\varkappa$ originates from the dependence of the polaron effect on the level splitting. It can be studied by direct calculation of the resonance contribution of two-level systems to $\mathrm{Re}\,M$ in the limit $\omega\to 0$. In fact, in this case $\mathrm{Re}\,M|_{res}$ is governed by the second derivative of the adiabatic spectrum of the system in the field of a deformation wave at fixed occupation numbers

$$\mathrm{Re}\,M|_{res}\sim-\mathrm{th}(\tilde\varepsilon/2T)\frac{\partial^2\tilde\varepsilon(t)}{\partial\varepsilon_0^2}\bigg|_{\varepsilon_0=0}, \qquad \tilde\varepsilon^2(t)=(\xi+2\varepsilon_0\cos(\omega t))^2+\Delta^2(\tilde\varepsilon(t)). \tag{3.37}$$

A comparison of the system (3.37) with Eqs. (3.34) and (3.35) demonstrates the equivalence of these expressions. In the absence of the polaron effect or when Δ is independent of the energy, the factor in question is $\varkappa=1$. In particular, this limit is realized for $T\gg\tilde\varepsilon$ or in the presence of a gap in the electron spectrum $E_{gap}>\tilde\varepsilon$ (which applies, for example, to superconductors). An allowance for the deviation of the factor β from unity from Eqs. (3.34) and (3.36) is strictly speaking invalid, because it corresponds to inclusion of terms of order b^2. However, if we find the relaxation contribution to the elastic modulus in the limit $\omega\to 0$, which is governed by a redistribution of the populations of the levels in a two-level system again in an adiabatic field of an acoustic wave [see Eq. (3.37)]

$$\mathrm{Re}\,M|_{rel}\sim-\frac{\partial}{\partial\varepsilon_0}(\mathrm{th}(\tilde\varepsilon/2T))\frac{\partial\tilde\varepsilon}{\partial\varepsilon_0}\bigg|_{\varepsilon_0=0}=\frac{\eta^2}{2T\,\mathrm{Ch}^2(\tilde\varepsilon/2T)}\left(\frac{\partial\tilde\varepsilon}{\partial\xi}\right)^2, \tag{3.38}$$

we readily obtain an expression equivalent to the first term in Eq. (3.34) with the coefficient β defined by Eq. (3.36). It should be noted that in the range $\tilde\varepsilon$, $\omega\ll T$ the expressions (3.34) and (3.33) are naturally identical.

4. SITE REPRESENTATION

We shall now consider the region labeled 3 in Fig. 2 characterized by the inequality of Eq. (2.23). In this region we can in fact ignore the quantity $\tilde\varepsilon$ in all the denominators in Eqs. (3.13) and (3.14). Retaining the Hamiltonian in the form of Eq. (3.16), we can then apply the site representation, bearing in mind the possibility of extending the definition of Eq. (3.9) to this representation:

$$\left(\frac{1}{\omega-L}A\right)_{ik}=\frac{1}{\omega-E^{nm}}A_{ik}. \tag{4.1}$$

The calculation of the matrix elements of the superoperator Ω and the supervector u simplifies greatly in the site representation because the interaction operator H_1 of Eq. (3.16) is diagonal in this representation. For example, in the matrix Ω there are only two nonzero elements:

$$\Omega_{12}{}^{12}=\Omega_{21}{}^{21}=\Sigma(\omega). \tag{4.2}$$

The nonzero components of the supervector are

$$u_{12}+u_{21}=-\frac{\Delta_v\xi}{\omega T}\left(\frac{\partial}{\partial\omega}\Sigma(\omega)+2R_v(0)\right),\qquad u_{12}-u_{21}=\frac{\Delta(T)}{\omega T}\frac{\partial\Delta(T)}{\partial\Delta_v}(\Sigma(\omega)-\Sigma(0)-\omega). \tag{4.3}$$

In this case the system of equations (3.14) has the simple form

$$\omega f_{11}+{}^1/_2\Delta_v(f_{12}-f_{21})=0;\quad(\omega-\xi-\Sigma(\omega))f_{12}+\Delta_v f_{11}=u_{12}. \tag{4.4}$$

Substituting the solution of the system into Eq. (3.4) we obtain

$$M\approx 2\eta^2\frac{\Delta^2(T)}{2T}\frac{\omega+i\Omega_T}{\omega[(\omega+i\Omega_T)^2-\tilde\varepsilon^2]-i\Delta^2(T)\Omega_T}. \tag{4.5}$$

The above expressions are identical with Eq. (3.33). This is very remarkable because the region 3 includes the range of temperatures where $\Omega(T)$ or the dynamic width of the levels are known to exceed $\tilde\varepsilon$.

We now consider region 2, which corresponds to the inequality (2.18) valid at high temperatures or for strongly asymmetric wells ($\tilde\varepsilon\approx\xi\gg\Delta_*$), when the tunneling can be considered using perturbation theory. As pointed out in Sec. 2, in the initial Hamiltonian of (2.4) we then have $\Delta_{\rm coh}\approx 0$ [see Eq. (2.12)] and the interaction is governed by the penultimate term in Eq. (2.4). Once again the site representation provides a satisfactory distribution. Substituting

$$H_1={}^1/_2\Delta_v(\Lambda-\langle\Lambda\rangle)\sigma_x$$

in Eq. (3.13), we obtain

$$\Omega_{11}{}^{11}=A_{\omega+\xi}+A_{\omega-\xi};\quad\Omega_{12}{}^{12}=\Omega_{21}{}^{21}=-\Omega_{12}{}^{21}=-\Omega_{21}{}^{12}=A_\omega; \tag{4.6}$$

$$A_z={}^1/_4\Delta_v^2\sum_{nm}\rho_0(n)|\Lambda^{nm}|^2\left(\frac{1}{z+E^{nm}}+\frac{1}{z-E^{nm}}\right). \tag{4.7}$$

The remaining matrix elements of the superoperator Ω vanish.

In calculating the function A_z it is convenient to use the transformation

$$\frac{1}{z+E^{nm}+i0}=-i\int_0^\infty e^{i(z+E^{nm})t}\,dt. \qquad (4.8)$$

Bearing in mind the explicit form of the operator Λ of Eq. (2.6), we have

$$\sum_{nm}\rho_0(n)|\Lambda^{nm}|^2 e^{iE^{nm}t}=\langle\Lambda^+(t)\Lambda\rangle=e^{-\chi(t)}, \qquad (4.9)$$

where

$$\chi(t)=\sum_{kk'\sigma}\frac{|V_{kk'}|^2}{(\varepsilon_k-\varepsilon_{k'})^2}n_k(1-n_{k'})(1-e^{-i(\varepsilon_k-\varepsilon_{k'})t}). \qquad (4.10)$$

In the expressions (4.7)–(4.10) the characteristic energy of electron–hole pairs is known to be greater than $1/\tau$ and there is no need to adopt the representation of Eq. (2.7).

We now represent $\chi(t)$ in the form

$$\chi(t)=2b\int_0^\infty\frac{du}{u}[(1-\cos(ut))\,\mathrm{Cth}(u/2T)-i\sin(ut)].$$

Simple transformations yield

$$e^{-\chi(t)}=\left(\frac{\pi T}{\gamma\omega_0}\right)^{2b}\frac{e^{-i\pi b}}{(\mathrm{Sh}\,\pi Tt)^{2b}}. \qquad (4.11)$$

Going back to Eq. (4.7) and noting that the remaining time interval can be reduced to a tabulated value, we obtain

$$A_z=\frac{\Delta_0^2(2\pi T/\gamma\omega_0)^2}{8\pi T}\frac{|\Gamma(b+iz/2\pi T)|^2}{\Gamma(2b)\sin\pi b}\,\mathrm{Sh}(z/2T-i\pi b). \qquad (4.12)$$

In this representation the equation of f_{11}, subject to Eq. (4.6), becomes decoupled to give

$$f_{11}=\frac{1}{\omega-(A_{\omega+\xi}+A_{\omega-\xi})}u_{11}. \qquad (4.13)$$

Using the general definition of Eq. (3.14) and the relationship between ρ_T and the interaction H_1 of Eq. (3.11), we find

$$u_{11}=-{}^1/{}_2\Delta_0^2\sum_{nm}|\Lambda^{nm}|^2\times\left(\frac{\rho_2^n-\rho_1^m}{(\omega+\xi-E^{nm})(E^{nm}-\xi)}+\frac{\rho_1^n-\rho_2^m}{(\omega-\xi-E^{nm})(E^{nm}+\xi)}\right).$$

Here we have written $\rho_i^n\equiv(\rho_0 f_0)_{ii}^{nn}=\rho_0(n)f_{0i}$. Using the definition (4.7), we obtain after simple transformations the expression

$$u_{11}=\frac{[(A_\xi-A_{\xi+\omega})\,\mathrm{Sh}(\xi/2T+i\pi b)+(A_{\omega-\xi}-A_{-\xi})\,\mathrm{Sh}(\xi/2T-i\pi b)]}{\omega\cos(\pi b)\,\mathrm{Ch}(\xi/2T)}. \qquad (4.14)$$

Recalling that compared with Eq. (4.13), the true density matrix contains an additional, factor $\varepsilon_0\eta/2$, we find that the modulus of Eq. (3.4) is

$$M=\frac{2\eta^2[(A_\xi-A_{\xi+\omega})\,\mathrm{Sh}(\xi/2T+i\pi b)+(A_{\omega-\xi}-A_{-\xi})\,\mathrm{Sh}(\xi/2T-i\pi b)]}{\omega\cos(\pi b)\,\mathrm{Ch}(\xi/2T)[\omega-(A_{\omega+\xi}+A_{\omega-\xi})]}. \qquad (4.15)$$

The expression (4.15) subject to (4.12) represents the solution of the linear response in the range defined by the inequality of Eq. (2.18). We note an important point that this expression is derived without postulating the parameter b to be small.

A comparison with the preceding result can be made by considering the limit $z\ll T$ ($\Omega_T\gg z$). It then follows from Eq. (4.12) that

$$A_z={}^1/{}_2\Delta^2(T)/(z+i\Omega_T); \qquad (4.16)$$

$$\Delta^2(T)=\Delta_0^2\left(\frac{2\pi T}{\gamma\omega_0}\right)^{2b}\frac{\mathrm{tg}(\pi b)\Gamma^2(b)}{2\pi\Gamma(2b)};\quad \Omega_T=2T\,\mathrm{tg}(\pi b). \qquad (4.17)$$

(We are retaining the same notation as before.) For $b\ll 1$, Eq. (4.17) is identical with Eq. (2.12).

If we assume $b\ll 1$ right from the beginning, then for any relationship between z and T, we obtain

$$A_z=\frac{\Delta^2(z)}{2(z+i\Omega(z))}. \qquad (4.18)$$

We also give the value of A_z in the limit $T\to 0$ (for an arbitrary b):

$$A_z=\frac{\Delta_0^2}{2z}\left(\frac{z}{\omega_0}\right)^{2b}\Gamma(1-2b)\cos(\pi b)e^{-i\pi b}. \qquad (4.19)$$

We now calculate the quantity of Eq. (4.15) in the high-temperature limit $T\gg\tilde\varepsilon,\omega$, using the expression given by Eq. (4.16). We then obtain

$$M=\eta^2\frac{\Delta^2(T)}{T}\frac{\omega+i\Omega_T}{\omega[(\omega+i\Omega_T)^2-\varepsilon^2]-i\Delta^2(T)\Omega_T}. \qquad (4.20)$$

The above is valid for an arbitrary value of b; $\Delta(T)$ and Ω_T have in general the values given by Eq. (4.17). We note that the denominator of Eq. (4.19) contains $\tilde\varepsilon^2=\xi^2+\Delta^2(T)$, i.e., it is identical with Eqs. (3.28) and (3.29) for $b\ll 1$ and $T\gg\tilde\varepsilon$. If right from the beginning we assume $b\ll 1$ and use Eq. (4.18), the result given by Eq. (4.20) is retained for an arbitrary value of ω (when we still have $T\gg\tilde\varepsilon$).

Comparing Eqs. (3.33), (4.5), and (4.20) we can show that for $T\gg\tilde\varepsilon$ and $b\ll 1$ there is a unique solution for the linear response in all three regions shown in Fig. 2, which covers both incoherent and purely coherent motion. It is important to note that this happens for an arbitrary scale of the electron polaron effect. The polaron effect occurs as a result of the self-consistent renormalization of the transition amplitude of Eq. (3.29), which in the $T \gg \tilde{\varepsilon}$ case has a value that depends only on temperature. The results are far from trivial if we bear in mind that it applies also to the case of the dynamic destruction of the energy structure when the width is $\Omega_T \gg \tilde{\varepsilon}$.

We now rewrite Eq. (4.20) identically, but in a somewhat different form:

$$M = \frac{\eta^2}{T} \frac{i(\xi^2+\Omega_T^2)\dfrac{\Delta^2(T)}{\tilde{\varepsilon}^2+\Omega_T^2}\Omega_T + \Delta^2(T)\left[\omega + i\dfrac{\Delta^2(T)}{\tilde{\varepsilon}^2+\Omega_T^2}\Omega_T\right]}{\omega[(\omega+i\Omega_T)^2-\tilde{\varepsilon}^2]-i\Delta^2(T)\Omega_T}. \quad (4.21)$$

We shall compare it with Eq. (3.34) derived for the region 1 subject to just the condition $\omega \ll T$ (apart from $b \ll 1$). For $T \gg \tilde{\varepsilon}$, the two expressions are identical. However, Eq. (3.34) is valid in the region 1 also for $\tilde{\varepsilon} > T$, and it covers a wide range of values of $\xi \gg \Delta(\tilde{\varepsilon})$, where the solution obtained for the noncoherent region 2 is valid independently. If we substitute Eq. (4.18) into Eq. (4.15), then for $\xi \gg \Delta(\xi)$ we obtain a complete analog of Eq. (3.34). The overlap of the solutions obtained independently for the three regions allows us to obtain a single solution valid for $\omega \ll T$ and $b \ll 1$ when the parameters have arbitrary values and the scale of the electron polaron effect is arbitrary:

$$M \approx 2\eta^2 \frac{\dfrac{i(\xi^2+\Omega_T^2)\beta}{2T\,\mathrm{Ch}^2(\tilde{\varepsilon}/2T)} \dfrac{\Delta^2(\tilde{\varepsilon})\Omega(\tilde{\varepsilon})}{\tilde{\varepsilon}^2+\Omega_T^2} + \dfrac{\Delta^2(\tilde{\varepsilon})}{\tilde{\varepsilon}}\varkappa\,\mathrm{th}(\tilde{\varepsilon}/2T)\left[\omega + i\dfrac{\Delta^2(\tilde{\varepsilon})\Omega(\tilde{\varepsilon})}{\tilde{\varepsilon}^2+\Omega_T^2}\right]}{\omega[(\omega+i\Omega(\tilde{\varepsilon}))^2-\tilde{\varepsilon}^2]-i\Delta^2(\tilde{\varepsilon})\Omega(\tilde{\varepsilon})+i(\Omega_T-\Omega(\tilde{\varepsilon}))\omega^2\Delta^2(\tilde{\varepsilon})/\tilde{\varepsilon}^2}. \quad (4.22)$$

5. DISCUSSION OF RESULTS

The results obtained in the preceding section represent the solution of the general problem of the linear response of a two-level system interacting with conduction electrons. This solution applies to a wide class of the parameters of the system. At temperatures T greater than the true separation between the levels $\tilde{\varepsilon}$ the expression given by Eq. (4.20) is valid generally for any parameters, including the parameter b of Eq. (4.20), representing the scale of the interaction of a particle with electrons. On the other hand, for $\omega \ll T$, we have a more general expression (4.22) which is valid for arbitrary values of the parameters provided that $b \ll 1$. It should also be mentioned that the description of the coherent region can naturally be made for the arbitrary value of b. Since in the incoherent regime it is sufficient to satisfy the condition $\Omega \gg \Delta_*$ or $\xi \gg \Delta_*$, we can easily see that it is necessary to satisfy the condition $b \ll 1$ only in a very narrow range of the parameters.

The dynamics of such a dissipative system can largely be established by analyzing the poles of the linear response function. At low temperatures when the condition $\Omega \ll \tilde{\varepsilon}$ is obeyed and the coherent regime applies, the poles are given by Eqs. (3.25)–(3.27). The first purely imaginary root describes relaxation of the occupation numbers (T_1^{-1}); the second and third roots form a clear spectrum of the system and represent relaxation of the phase correlations in the nondiagonal elements of the density matrix (T_2^{-1}). It is important to note that in the asymmetric case $\xi \gg \Delta(\tilde{\varepsilon})$ we have the inequality $\gamma_2 \gg \gamma_1$. We can see that in this case an isolated two-level system has its own value of T_2 which can be arbitrarily small compared with T_1. The phase relaxation process is then related to dynamic fluctuations of the positions of the energy levels of a particle in neighboring wells at zero frequency, and these fluctuations are independent of the overlap integral.

We now consider the case when $T > \tilde{\varepsilon}$. The poles M_ω can be found by solving the cubic equation

$$\omega((\omega+i\Omega_T)^2-\tilde{\varepsilon}^2)-i\Delta^2(T)\Omega_T=0. \quad (5.1)$$

In the simple case of a symmetric two-level system the solution of (5.1) becomes

$$\omega^{(1)} = -i\Omega_T; \quad \omega^{(2,3)} = {}^1/_2[-i\Omega_T \pm (4\Delta^2(T)-\Omega_T^2)^{1/2}]. \quad (5.2)$$

For $\Omega_T \ll \Delta(T)$, which may hold in the investigated range of temperatures only for $b \ll 1$, the poles of Eq. (5.2) are identical with the results given by Eq. (3.25). The situation changes greatly for $\Omega_T > 2\Delta(T)$. In fact, in this case all the roots become purely imaginary. In the limit $\Omega_T \gg \Delta(T)$ the solution (5.2) becomes

$$\omega^{(1)} = -i\Delta^2(T)/\Omega_T; \quad \omega^{(2)} = -i\Omega_T; \quad \omega^{(3)} = -i\Omega_T + i\Delta^2(T)/\Omega_T. \quad (5.3)$$

We have altered here the nomenclature of the roots, leaving the index 1 for the slowest root, which—as in the preceding case—is responsible for relaxation of the occupation numbers. Therefore, the transition from the coherent to the incoherent regime is accompanied by disappearance of the real spectrum. This result is internally related to disappearance of the coherent amplitude of the transition [see Eq. (2.13)] which occurs in a similar range of the parameters.

We now assume $\xi \neq 0$. We use the fact that for $\Omega_T \gg \Delta(T)$ or $\xi \gg \Delta(T)$ and also $\Omega_T \ll \Delta(T)$, one of the roots $\omega^{(1)}$ in Eq. (5.2) is always small compared with the others and is given approximately by[25]

$$\omega^{(1)} = -i\frac{\Delta^2(T)\Omega_T}{\tilde{\varepsilon}^2+\Omega_T^2}. \quad (5.4)$$

The remaining two roots are of the form

$$\omega^{(2,3)} = \pm\tilde{\varepsilon} - i\Omega_T\left(\xi^2/\tilde{\varepsilon}^2 + \frac{1}{2}\Delta^2(T)/\tilde{\varepsilon}^2\right), \quad \Omega_T \ll \tilde{\varepsilon}, \quad (5.5)$$

$$\omega^{(2,3)} = \pm\xi - i\Omega_T, \quad \Omega_T \gg \tilde{\varepsilon}. \quad (5.6)$$

The result of Eq. (5.6), where the real part is generally independent of Δ, again reflects the disappearance of coherent transitions in the system. When the general condition of Eq. (2.18) is obeyed, the incoherent processes predominate and we find $T_2^{-1} \gg T_1^{-1}$ which applies now also in the purely symmetric case.

Retaining the condition $T \gg \tilde{\varepsilon}$, we now consider the behavior of the linear response function of Eq. (4.20). This equation can be transformed identically to

$$M=\frac{\eta^2}{T}\left\{\frac{\omega[(\omega+i\Omega_T)^2-\xi^2]}{(\omega-\omega^{(1)})(\omega-\omega^{(2)})(\omega-\omega^{(3)})}-1\right\}. \quad (5.7)$$

In the limit $\Omega_T \ll \tilde{\varepsilon}$, when the roots $\omega^{(2,3)}$ are given by Eq. (5.5), the zeros in the numerator and denominator are different, and M_ω has a clear resonant structure. In the opposite case, $\Omega_T \gg \tilde{\varepsilon}$, the roots obey Eq. (5.6). The poles $\omega^{(2,3)}$ are cancelled by zeros of the function in the numerator of (5.7), and so the function M_ω has a relaxation structure

$$M_\omega=-\frac{\eta^2}{T}\frac{i\gamma_1}{(\omega+i\gamma_1)}. \quad (5.8)$$

This expression demonstrates the relationship between M_ω and the purely incoherent diffusion of particles. The reciprocal longitudinal relaxation time γ_1 in Eq. (5.8) is governed by the value of the root $\omega^{(1)}$.

In the incoherent approximation there are no restrictions on the value of b. This applies in particular to the expression (5.4) obtained in the case $T \gg \tilde{\varepsilon}$, provided $\Delta(\tilde{\varepsilon})$ and Ω_T are understood to be given by the expressions in Eq. (4.17). On the other hand, for $\xi > \Delta(\tilde{\varepsilon})$ in the incoherent region, we can find a slow pole using the initial expression (4.15), from which it follows directly that

$$\gamma_1=2\,\mathrm{Im}\,A_\xi. \quad (5.9)$$

Using Eq. (4.12) for A_ξ, we obtain γ_1 which is identical with that given by Eqs. (2.17) and (2.19). Again applying the concept of overlap of the regions where Eqs. (2.19), (2.17), (5.4), and (3.26) are valid, we can write down the general expression covering the full range of the parameters (see Ref. 2):

$$\gamma_1=\frac{\Delta^2(T)\Omega_T}{\varepsilon^2+\Omega_T^2}\frac{|\Gamma(1+b+i\varepsilon/2\pi T)|^2}{\Gamma(1+2b)}\mathrm{Ch}(\varepsilon/2T). \quad (5.10)$$

It should be noted that for $\tilde{\varepsilon} > T$, we should use the general expression (4.22) so that γ_1 of Eq. (5.10) determines the central part of the spectrum associated with longitudinal relaxation in the limit $\omega \to 0$.

By way of illustration, Fig. 3a shows the absorption of radiation by a two-level system, related to M_ω by

$$I_\omega \propto \mathrm{Im}\, M_\omega/(1-e^{\omega/T}). \quad (5.11)$$

We consider the symmetric case $\xi = 0$ and select $b = 0.05$. For comparison, Fig. 3b gives the function I_ω described by

$$I_\omega=\mathrm{Im}\left[\frac{1}{\omega+\Delta(T)+i\gamma_2}+\frac{1}{\omega-\Delta(T)+i\gamma_2}\right](1+e^{\omega/T})^{-1}, \quad (5.12)$$

which is derived for the symmetric case and coherent processes at low temperatures when $\gamma_2 = (1/2)\Omega(\tilde{\varepsilon})$ holds and is continued purely formally to high temperatures. We can see that a satisfactory description of the linear response predicts that an increase in T should result in a continuous evolution from the pattern corresponding to a discrete spectrum to a strong central peak whose width falls on increase in T. On the other hand, the simplest description of the resonance structure shows that an increase in T simply broadens the spectral dependence and the width increases with temperature.

We now consider how the results change when the metal goes over to the superconducting state. It is clear from the preceding discussion that the whole kinetics of a dissipative system is governed by components of the superoperator Ω, the values of which depend on two characteristics $\Omega(z)$ and $R_\nu(z)$ of Eq. (3.19). We can determine these quantities for the superconducting state using the standard u–v technique and representing the interaction with electrons [Eq. (2.3)] in the form

$$V=\sum_{kk'\sigma}V_{kk'}[(u_ku_{k'}-v_kv_{k'})b_{k\sigma}^+b_{k'\sigma}+(u_kv_{-k'}+v_ku_{-k'})\sigma(b_{k\sigma}^+b_{-k'-\sigma}^+-b_{k\sigma}b_{-k'-\sigma})], \quad (5.13)$$

where $u_k(v_k)=(1/2\pm\eta_k/2E_k)^{1/2}$, $\eta_k=v_F(k-k_F)$, $E_k=(\eta_k^2+\Delta_s^2)^{1/2}$, and $\Delta_s(T)$ is the gap in the energy spectrum of the superconductor. In this representation the expression for the function $\Omega(z)$ of Eq. (3.19) becomes[15]

$$\Omega_s(z>0)=2\pi b\,\mathrm{Cth}(z/2T)\times\left\{\int_{\Delta_s}^{\infty}dE\,g_Eg_{E+z}(E(E+z)-\Delta_s^2)(n_E-n_{E+z})+\Theta(z-2\Delta_s)\int_{\Delta_s}^{z-\Delta_s}dE\,g_Eg_{z-E}(E(z-E)+\Delta_s^2)(1/2-n_E)\right\}. \quad (5.14)$$

Here, $g_E=(E^2-\Delta_s^2)^{-1/2}$ is a function representing the density of one-electron states in the superconductor. For $z \gg 2\Delta_s$ or $T > \Delta_s$, the function $\Omega_s(z)$ retains a value typical of a normal metal. If $z \ll T$, Eq. (5.14) can be calculated trivially for any value of T (Refs. 16 and 17):

$$\Omega_s(z=0)=\frac{4\pi bT}{1+\exp(\Delta_s/T)}\equiv\Omega_s. \quad (5.15)$$

In the interval $T \ll z \leqslant 2\Delta_s$, we find that Eq. (5.14) leads to

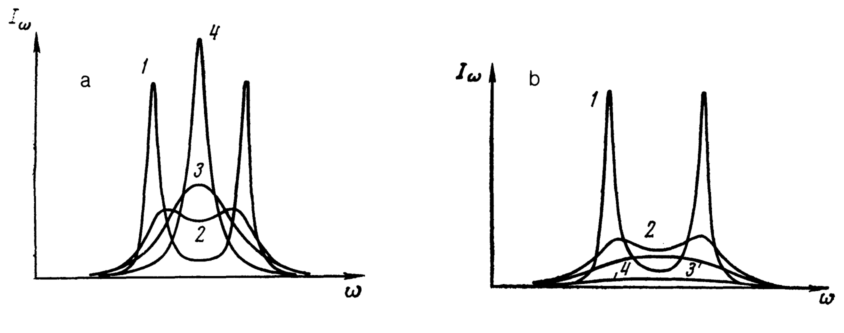

FIG. 3. Inelastic absorption of neutrons by a symmetric system ($\xi = 0$) calculated for the case when $b = 0.05$ (a) and for two Lorentzian peaks (b); $T/\Delta_* = 1$ (curve 1), 4 (curve 2), 7 (curve 3), and 20 (curve 4).

$$\Omega_s(z) \approx \frac{\pi b}{1+\exp(\Delta_s/T)} \left[\frac{\pi T z}{1+z/2\Delta_s} \right]^{1/2}. \qquad (5.16)$$

It should be noted that the function $\Omega_s(z)$ has a discontinuity at a point $z = 2\Delta_s$:

$$\Omega_s(z=2\Delta_s+0) - \Omega_s(z=2\Delta_s-0) = \pi^2 b \Delta_s \frac{\mathrm{Ch}(\Delta_s/T)}{1+\mathrm{Ch}(\Delta_s/T)}. \qquad (5.17)$$

The function $R(z)$ naturally retains its form in the range $z \gg 2\Delta_s$ or $T > \Delta_s$. If we have T, $z \ll \Delta_s$, the quantity $R(z)$ reaches a constant value:

$$R(z, T) = R_s = b \ln(2\omega_0/e\Delta_s). \qquad (5.18)$$

This result is obtained by integrating Eq. (3.20) with a natural cutoff at the lower limit $u = 2\Delta_s$.

If the renormalized value of the difference between the levels in a two-level system of a normal metal satisfies the condition $\tilde{\varepsilon} > 2\Delta_s(0)$, then the spectrum of the system and the process of longitudinal relaxation of γ_1 below the superconducting transition point T_c are in fact unchanged. The situation is different in the case of relaxation of the phase γ_2. In the case under discussion we in fact always have $\tilde{\varepsilon} \gg \Omega_s$ and we can use the relationship (3.26):

$$\gamma_2(\xi, \tilde{\varepsilon}) = (\xi/\tilde{\varepsilon})^2 \Omega_s(0) + {}^1/_2 \gamma_1(\tilde{\varepsilon}). \qquad (5.19)$$

In the superconducting states the quantity $\Omega_s(0)$ of Eq. (5.15) decays exponentially as T decreases and the classical relationship $\gamma_2 = (1/2)\gamma_1$ is rapidly attained. The decrease of γ_2 combined with constant γ_1 is particularly strong in the case of a system with a significant static shift. The linear response function of Eq. (4.22) retains its form after just one substitution $\Omega_T \to \Omega_s$.

We now assume $\tilde{\varepsilon}(T=0) \ll 2\Delta_s(0)$. According to Eq. (5.18) the cutoff of an infrared divergence then occurs on a scale of $2\Delta_s$, and determination of $\Delta(\tilde{\varepsilon})$ does not require the self-consistent solution

$$\Delta_*^s = \Delta_0 (e\Delta_s/2\omega_0)^b. \qquad (5.20)$$

It should be noted that the coherent amplitude in a superconductor is greater than the value for a normal metal at $T = 0$. The ratio of the two values is $(e\Delta_s/2\tilde{\varepsilon})^b$.

The exponential fall of Ω_s with decreasing T given by Eq. (5.15) and (5.16) is associated with a reduction in the number of normal excitations and is effectively equivalent to a weakening of the interaction. We then have

$$b^{eff} \approx b \exp(-\Delta_s/T). \qquad (5.21)$$

We thus arrive at the case of a small value of b^{eff} for a constant temperature-independent spectrum, described by Eq. (5.20). Therefore, perturbation theory operates satisfactorily and we can use Eqs. (3.26) and (5.4) to demonstrate that γ_1 and γ_2 decrease exponentially in accordance with the law $\exp(-\Delta_s/T)$.

It therefore follows that two-level systems characterized by $\tilde{\varepsilon} < 2\Delta_s(0)$ exhibit a transition from a normal metal to the superconducting state which is equivalent to the transition from the case of a strong interaction between the particle and the medium to the case of a weak interaction. The strong polaron effect is then retained, but it is now nonlinear (in Δ_0).

6. ACOUSTIC PROPERTIES OF METALLIC AND SUPERCONDUCTING GLASSES

In the preceding sections it was shown that all the dynamic characteristics of two-level systems in a metal can be revealed by an analysis of the linear response function M_ω. In this section we use examples of specific systems to study the experimental manifestations of such characteristics.

We consider the behavior of the acoustic properties of an amorphous metal in the normal and superconducting states. A typical experiment involving determination of the velocity of sound v and absorption coefficient α is carried out at frequencies $\omega \ll T$ (Refs. 10–13, 26, 27), and the realistically attainable values of the frequency are $\omega \leqslant 10^{-2}$ K; we shall analyze the situation using the general relationships of Eqs. (3.34) and (4.22). The renormalization of the sound speed is related to the real part of the elastic modulus by

$$\Delta v/v = \sum \frac{1}{2\rho v^2} \operatorname{Re} M_\omega, \qquad (6.1)$$

where ρ is the density of a metallic glass. Since a metallic glass contains a set of different two-level systems with a wide spectrum of the parameters Δ_0 and ξ, it is necessary to average Eq. (4.22) using a certain distribution function $P(\ln \Delta_0, \xi) d \ln \Delta_0 d\xi$. It is generally accepted in the theory of amorphous systems that $P = \overline{P} = \text{const}$ holds. Since P is obviously independent of ξ at low ξ, it is physically clear that when $\ln \Delta_0$ deviates from a certain value typical of a metallic glass, the function $P(\ln \Delta_0)$ should decrease.

We consider first the resonant contribution to $\Delta v/v$ associated with modulation of the energy splitting of two-level systems at fixed values of the occupation numbers and governed by the second term of the numerator of Eq. (4.22). This mechanism is effective only in the case of two-level systems with unequal populations of the levels when $T < \tilde{\varepsilon}$. In this case we have $\omega \ll \tilde{\varepsilon}$ and the denominator of Eq. (4.22) simplifies greatly. The result is

$$\Delta v/v|_{res} \approx -C \int_0^{\Delta_0^{max}} \frac{d\Delta_0}{\Delta_0} \int_0^{\xi^{max}} d\xi \, \frac{P(\ln \Delta_0)}{\overline{P}} \frac{\partial^2 \tilde{\varepsilon}}{\partial \xi^2} \operatorname{th}(\tilde{\varepsilon}/2T);$$

$$C = \frac{\overline{P}\eta^2}{\rho v^2}. \qquad (6.2)$$

In this expression the important feature is the behavior of the distribution function in the range $\Delta(\tilde{\varepsilon}) \geqslant T$, where we ignore the difference between P and $\overline{P}$. The integral of Eq. (6.2) in the case ξ^{max}, $(\Delta_*)_{max} \gg T$ [in fact $\xi^{max} \gg (\Delta_*)_{max}$] is logarithmic and, apart from an unimportant constant, can be written in the form

$$\Delta v/v|_{res} \approx -C \int_{(\Delta_* \approx T)}^{\Delta_0^{max}} \frac{d\Delta_0}{\Delta_0} \int_0^{\infty} d\xi \frac{\partial^2 \tilde{\varepsilon}}{\partial \xi^2} = -C \int_{(\Delta_* \approx T)}^{\Delta_0^{max}} \frac{d\Delta_0}{\Delta_0}. \qquad (6.3)$$

Note the use of the limiting tunneling amplitude of Eq. (2.11), which corresponds to the limit $T < \Delta_*(\Delta_0)$. We substitute the variables in accordance with $\Delta_0 \to \Delta_*$ in Eq. (6.3). It follows from Eq. (2.11) that

$$d\Delta_0/\Delta_0 = (1-b)\, d\Delta_*/\Delta_* \qquad (6.4)$$

so that the resonant contribution to the renormalization of the velocity of sound is

$$\Delta v/v|_{res}=(1-b)C\ln(\Delta_*^{max}/T). \qquad (6.5)$$

The relaxation contribution to the change in the velocity of sound is governed by the first term of Eq. (4.22) and is entirely due to the thermal two-level systems ($\tilde{\varepsilon}\leqslant T$). The main contribution to $\Delta v/v|_{rel}$ is given, to within logarithmic precision, by two-level systems with low tunnel amplitudes $\Delta(T)\ll T$ and a strong asymmetry $\Delta(T)\ll\xi$ [see Eq. (6.7) below]. Using only these inequalities we can transform the relaxation part of Eq. (4.22) to

$$M_{rel}=-2\eta^2\frac{\xi^2+\Omega_T^2}{\omega^2+\Omega_T^2}\frac{i\gamma_1}{\omega+i\gamma_1}\frac{1}{2T\,\mathrm{Ch}^2(\tilde{\varepsilon}/2T)}$$
$$=-2\eta^2\frac{i\gamma_1}{\omega+i\gamma_1}\frac{\partial\,\mathrm{th}(\xi/2T)}{\partial\xi}, \qquad (6.6)$$

where γ_1 is given by Eq. (3.26). In fact, the expression (6.6) is valid for an arbitrary value of b if γ_1 is understood to be given by Eq. (5.10). This result can be obtained directly from Eq. (4.15) in the limit $\omega\to 0(\Delta\ll\xi)$. Averaging Eq. (6.6) over $d\xi$ when $\Delta v/v|_{rel}$, we obtain

$$\Delta v/v|_{rel}\approx -C\int_{\Delta_0'}^{(\Delta_*\approx T)}\frac{d\Delta_0}{\Delta_0}\frac{P(\ln\Delta_0)}{\bar{P}}. \qquad (6.7)$$

The logarithmic divergence is cut off below for two reasons. Firstly, if we use the standard condition $\gamma_1\approx\omega$, we obtain a value Δ_0' which is given (apart from an unimportant coefficient) by

$$\Delta_0'\approx T\left(\frac{\omega}{\Omega_T}\right)^{1/2}\left(\frac{2\pi T}{\gamma\omega_0}\right)^{-b};\qquad \Delta_*'\approx T\left(\frac{\omega}{\Omega_T}\right)^{1/2(1-b)}. \qquad (6.8)$$

In the case of a normal metal for $\omega\ll$T we find $\Delta_*'\ll T$, and the relaxation contribution is always important. On the other hand, the fall of the distribution function $P(\ln\Delta_0)$ at low tunnel amplitudes introduces an effective cutoff scale of $\Delta_n^{\min}$ of the distribution ($\Delta_*^{\min}$). If we integrate with respect to $d\Delta_*$ given by Eq. (6.4) in Eq. (6.7), the result is

$$\Delta v/v|_{rel}=-(C/2)\ln(\Omega_T/\omega),\qquad \Delta_*'>\Delta_*^{min},$$
$$\Delta v/v|_{rel}=-C(1-b)\ln(T/\Delta_*^{min}),\qquad \Delta_*^{min}>\Delta_*'. \qquad (6.9)$$

Since Δ_*' falls when the frequency is reduced [see Eq. (6.8)], the first case described by Eqs. (6.9) is typical of high frequencies of sound, whereas the second is typical of low frequencies.

The overall renormalization of sound is then described by

$$\Delta v/v=-C(1/2-b)\ln(T_0/T),\qquad \Delta_*'>\Delta_*^{min}. \qquad (6.10)$$

In the opposite limit ($\Delta_*'>\Delta_*^{\min}$) the renormalization of the velocity of sound ceases to depend completely on temperature: $\Delta v/v=$ const.

It is important to note that both these results are exact; in other words, they are valid for any strength of the interaction with the electron system. This is easily demonstrated if we consider the limit $\omega\to 0$ when we can regard the interaction of a two-level system with sound as adiabatic and a complete local equilibrium in the deformation field is established in the system in the available time. Since the amplitude of sound occurs in the Hamiltonian in the combination $\xi+2\eta\varepsilon_0\cos(\omega t)$, we can use the initial expression for M of Eq. (3.4) to go over from differentiation with respect to ε_0 to differentiation with respect to ξ. Then, averaging over the distribution of the parameters of the two-level system, we find (when the lower limit of integration removes the adiabatic two-level systems) that

$$\Delta v/v=-2C\int_{(\gamma_1(\xi=T)\approx\omega)}^{\Delta_0^{max}}\frac{d\Delta_0}{\Delta_0}\frac{P(\ln\Delta_0)}{\bar{P}}\int_0^\infty d\xi\,\frac{\partial f_{11}^{(0)}}{\partial\xi}$$
$$=-C\int_{(\gamma_1(\xi=T)\approx\omega)}^{\Delta_0^{max}}\frac{d\Delta_0}{\Delta_0}\frac{P(\ln\Delta_0)}{\bar{P}}. \qquad (6.11)$$

This follows from the obvious relationships $f_{11}^{(0)}(\xi\to\infty)=1$ and $f_{11}^{(0)}(\xi=0)=1/2$. The result in question is derived without assuming weakness of the interaction (smallness of b) and reproduces rigorously the above relationships.

It should be pointed out that the result given by Eq. (6.11) demonstrates that separation of the renormalization of the velocity of sound into resonant and relaxation parts is in all cases arbitrary, as is true, for example, in strong interactions when the very concept of the spectrum of a two-level system is difficult to introduce.

We now consider the renormalization of the velocity of sound in a superconducting metallic glass. In the case of two-level systems with energies $\tilde{\varepsilon}>2\Delta_s$ there is naturally no change. If $\tilde{\varepsilon}<2\Delta_s$, in view of the linear dependence $\Delta_*\propto\Delta_0$ [see Eq. (5.20)], the factor b disappears from Eq. (6.4). At the same time Ω_T changes to the superconducting value Ω_s of Eq. (5.15). This has a significant influence on the definition of the value of Δ_0', which now becomes

$$\Delta_{0s}'\approx T\left(\frac{\omega}{\Omega_T}\right)^{1/2}\left(\frac{e\Delta_s}{2\omega_0}\right)^{-b};\qquad \Delta_{*s}'\approx T\left(\frac{\omega}{\Omega_s}\right)^{1/2}. \qquad (6.12)$$

The exponentially fast fall of Ω_s as a result of lowering of T implies that at some temperature the value of Δ_{*s}' unavoidably becomes of order T and the relaxation contribution disappears. Above this temperature both mechanisms of the renormalization of the velocity of sound are active and the total value $\Delta v/v$ is

$$\Delta v/v=-C\ln(\Delta_0^{max}/\Delta_{0s}'),\qquad \Delta_{0s}'>\Delta_0^{min}. \qquad (6.13)$$

For $\Delta_{0s}'<\Delta_0^{\min}$, the quantity Δ_{0s}' changes to $\Delta_0^{\min}$ in this expression and, as in the case of a normal metal, $\Delta v/v$ ceases to depend on temperature.

After the relaxation contribution disappears at low temperatures the value of $\Delta v/v$ is determined only by the resonant contribution, which in the superconducting state is

$$\Delta v/v=-C\ln(2\Delta_s/T)-(1-b)C\ln(\Delta_*^{max}/2\Delta_s). \qquad (6.14)$$

The continuous curve in Fig. 4 represents $\Delta v/v$ plotted separately for the case of high frequencies (Fig. 4a) and low frequencies (Fig. 4b). The dashed curves in the same figures show how $\Delta v/v$ behaves in the superconducting state. At high frequencies in the normal state the resonant contribu-

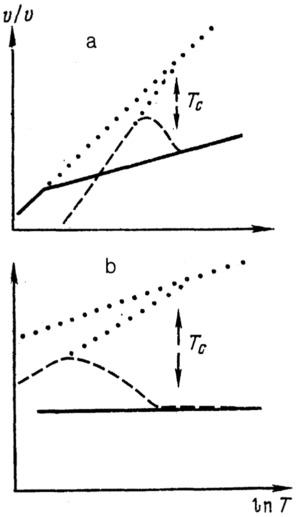

FIG. 4. Values of $\Delta v/v$ plotted as a function of T for the normal (continuous curve) and superconducting (dashed curve) phases at high (a) and low (b) frequencies of sound. The resonant contribution (represented by dotted curves) is included for illustration.

tion predominates ($b < 1/2$) and $\Delta v/v$ rises logarithmically on increase in T. The coefficient in front of $\ln T$ depends on the interaction with electrons in accordance with $(\frac{1}{2} - b)$ and the slope decreases with the strength of the interaction. At temperatures $T < T_c$ the exponential fall of Ω_s results in a steep reduction in $|\Delta v/v|_{\rm rel}$ and, consequently, rapidly increases the total value of $\Delta v/v$. This has been established in earlier investigations (see, for example, Refs. 12 and 26). However, after passing through a maximum the velocity of sound decreases because T decreases much faster than does $\Delta v/v|_{\rm res}$ in the normal phase due to a change in the coefficient in front of $\ln T$ in Eqs. (6.5) and (6.14), and the curve for the superconducting state may even intersect the curve $\Delta v/v$ for the normal metal, which is a contradiction that cannot be resolved using perturbation theory to describe the interaction of two-level systems with conduction electrons. This is precisely the pattern manifested clearly in the experiments on alloys such as $Pd_{30}Zr_{70}$ (see also Ref. 12) and $Cu_{60}Zr_{40}$ (Refs. 27 and 26) (this is less clear in the latter case because of the absence of direct measurements in the normal phase at $T < T_c$). It is interesting to note that the ratio of the slopes $\Delta v/v$ demonstrates a considerable difference in the values of b for these materials, which is correlated with the results plotted in Ref. 26.

At low frequencies in the normal phase when we have $\Delta'_* > \Delta_*^{\rm min}$, there is practically no dependence of the velocity of sound on T, as demonstrated above. In the transition to the superconducting state at low frequencies ($\omega \sim 10^3$ Hz) a "latent" interval of temperatures occurs in the vicinity of T_c as long as Δ'_{0s} of Eq. (6.12) remains less than $\Delta_0^{\rm min}$ when T is lowered. Consequently, the velocity of sound in a superconductor initially does not differ from $\Delta v/v$ for a normal metal.

The strong reduction in the temperature dependence of $\Delta v/v$ at low frequencies was first detected experimentally for $Cu_{30}Zr_{70}$ (Refs. 12 and 13). These investigations revealed clearly the existence of a latent temperature interval, followed by a rapid rise of the total value of $\Delta v/v$. The fact that $\Delta v/v$ was independent of T at low frequencies clearly manifested the role of the limit of the distribution $P(\ln \Delta_0)$ at low values of Δ_0. It should be stressed that this is a universal result and is independent of the mechanism of the interaction of a two-level system with a medium, which follows directly from the general relationship (6.11).

The coefficient α representing the absorption of sound in a normal metallic glass at frequencies $\omega \ll T$ is governed primarily by the relaxation processes. The relevant expression for α is given by the imaginary part of Eq. (6.6):

$$\alpha = -\omega \,\mathrm{Im}\, M_\omega/\rho v^3. \qquad (6.15)$$

In the relaxation regime the result is determined by thermal two-level systems ($\tilde{\varepsilon} \sim T$) satisfying the condition $\omega\gamma_1 \sim 1$, which corresponds to small amplitudes of the tunneling $\Delta_0 \sim \Delta'_0$ [see Eq. (6.8)]:

$$\alpha|_{rel} = C\frac{\pi\omega}{2v}P(\ln \Delta_0')/\bar{P}. \qquad (6.16)$$

At low frequencies ($\omega \sim 10^3$ Hz) it follows from Eq. (6.8) that the amplitude is $\Delta'_0 \sim 10^{-(3\text{-}4)}$. It is natural to assume that this value lies in the left-hand tail of the distribution of P. For a normal metal Δ'_0 decreases proportionally to $T^{1/2-b}$ as a result of cooling (see Refs. 2 and 18), which leads to a weak fall of P. In the superconducting phase the exponential fall of Ω_s has the opposite effect of rapid rise of this quantity described by $\Delta'_0 \propto T^{1/2}\exp(-\Delta_s/2T)$ [Eq. (6.12)]. Consequently, the fall of α changes to a rise, as demonstrated in Fig. 5a. When the condition $\omega/\gamma_{1\max} \approx \omega/\Omega_s \sim 1$ is reached the rise of absorption as T decreases changes to a steep fall described by $\alpha \propto \gamma_{1\,\max}$ or $\alpha \propto \gamma_{1\max}^{(\rm ph)}$ if the phonon relaxation mechanism begins to predominate.

At high sound frequencies (usually at 10^8–10^9 Hz) a typical value of Δ'_0 lies in the range where the distribution function of P changes slowly. Bearing in mind that the condition $\omega/\gamma_{1\,\max} \sim 1$ is attained relatively rapidly below T_c, it becomes clear that the rise of the absorption can only be

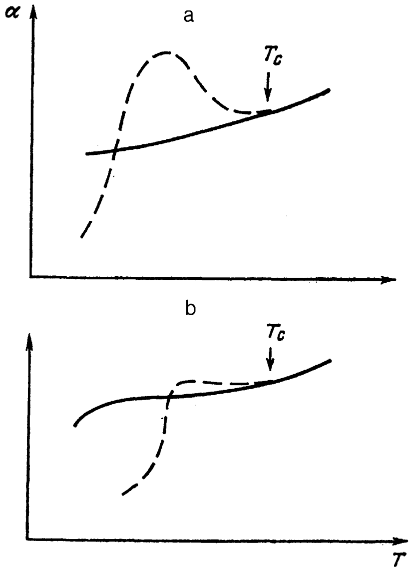

FIG. 5. Absorption of sound in metallic glasses in the normal (continuous curves) and superconducting (dashed curves) phases at low (a) and high (b) frequencies of sound.

weak and that it changes to a steep fall. This accounts for the anomalous nature of the behavior of $\alpha|_{\rm rel}$ first reported in Refs. 10–13.

The resonant absorption of high-frequency sound, which is usually absent in the case of a normal metal, becomes important when a metal undergoes the transition to the superconducting state because then the condition $\omega/\gamma_{1\,\max}\sim 1$ is readily satisfied.[26,28] The most interesting aspect of the resonant interaction of sound with two-level systems is the feasibility of checking experimentally the theoretical predictions regarding the spectrum and damping of the states.

"Hole burning"

We shall use the examples of "hole burning" (see, e.g., Refs. 28 and 29) and the acoustic echo[29,33] to show that it should be possible to observe directly the intrinsic width $\gamma_2\gg\gamma_1$ of a two-level system with a strong asymmetry unrelated to the interaction of two-level systems with one another (it is usually assumed that the intrinsic width is $\gamma_2=(1/2)\gamma_1$).

If we bear in mind the spectral analysis of the absorption near narrow resonances $\omega-\tilde{\varepsilon}\ll\omega$ of Eqs. (3.25)–(3.27) (typical of the superconducting state), we can greatly simplify the initial kinetic equation for the density matrix (3.14) and retain the diagonal terms in the matrix of the superoperator Ω of Eq. (3.12). This is always valid for $b^{\rm eff}\ll 1$. In the representation of the eigenstates of a two-level problem the equation for the density matrix is in the form of the Bloch equations:

$$\dot{f}_{aa}=-\gamma_1(f_{aa}-f_{aa}^{(0)})+(\eta\varepsilon_0\Delta_*^s/\tilde{\varepsilon})\,\mathrm{Im}\,f_{ab}e^{-i\omega t} \tag{6.17}$$

$$\dot{f}_{ab}=(i\tilde{\varepsilon}-\gamma_2)f_{ab}+{}^1/_2(i\eta\varepsilon_0\Delta_*^s/\tilde{\varepsilon})(f_{bb}-f_{aa})e^{i\omega t} \tag{6.18}$$

(we make the notation more compact by writing $\Delta\equiv\Delta_*^s$). It is well known that broadening in the case of two-level systems in glasses is not only due to the interaction with electrons or phonons, but also due to the interaction of two-level systems with one another. A characteristic value of γ_2 deduced for such interaction of the systems with one another usually lies within the range $\sim 10^{-5}$ K. In the case of insulators this value is usually the dominant one. However, in the case of metals if b is not too small and temperatures T are not too low compared with T_c, we can always ensure that the condition $\Omega_s\gg\gamma_2^{\rm TLS-TLS}$ is satisfied (here, TLS–TLS denotes the interaction of two-level systems with one another) and we can then regard the two-level systems as isolated. We assume this in the discussion below.

We assume that a saturation pulse of frequency ω_1 acts for a long time $t\gg\gamma_1^{-1}$ and a weak signal ω' acts for a time τ after saturation. The standard solution of the simple system of equations (6.17) and (6.18) then gives the following expression for the function α'_ω applicable to an isolated two-level system (see, for example, Refs. 29 and 30):

$$\Delta\alpha_{\omega'}=\alpha_{\omega'}^{(0)}-\alpha_{\omega'}=\alpha_{\omega'}^{(0)}\frac{\gamma_2^2I/I_c}{\gamma_2^2(1+I/I_c)+(\omega_1-\tilde{\varepsilon})^2}e^{-\gamma_1\tau}, \tag{6.19}$$

where $\alpha_{\omega'}^{(0)}$ is the resonant absorption in the absence of the saturation signal

$$\alpha_{\omega'}^{(0)}=\eta^2\omega'\left(\frac{\Delta}{\tilde{\varepsilon}}\right)^2\mathrm{th}(\tilde{\varepsilon}/2T)\frac{\gamma_2}{\gamma_2^2+(\omega'-\tilde{\varepsilon})^2}, \tag{6.20}$$

whereas I_c is the critical power of the pulse causing equalization of the populations of two-level systems given by[26,29,30]

$$I_c=(\gamma_1\gamma_2/\eta^2)(\tilde{\varepsilon}/\Delta)^2. \tag{6.21}$$

We have to average our results over the parameters Δ_0 and ξ. Since the time is given by $\gamma_1\sim(\Delta/\tilde{\varepsilon})^2$, the contribution of the symmetric two-level systems is suppressed particularly at high values of τ in Eq. (6.19). However, in the case of systems characterized by $\Delta<\tilde{\varepsilon}\ll T$ (but subject to the condition $T<T_c$) the quantities $\gamma_2\approx\Omega_s$ and $\gamma_1(\tilde{\varepsilon}/\Delta)^2$ are in fact independent of the ratio $\Delta/\tilde{\varepsilon}$ [see Eq. (3.26)]. For this reason the critical power of Eq. (6.21) is also independent of the asymmetry of the wells. Moreover, the last factor in Eq. (6.19) is independent of the ratio $\Delta/\tilde{\varepsilon}$. If we integrate Eq. (6.19) with respect to $d\ln\Delta_0 d\xi$, we find

$$\Delta\alpha_{\omega'}=\frac{1}{2\gamma_2\tau}\frac{\alpha_{\omega'}^{(0)}}{1+[(\omega_1-\omega')/\gamma_2(I)]^2}\frac{(1+I/I_c)^{1/2}-1}{(1+I/I_c)^{1/2}}, \tag{6.22}$$

$$\gamma_2(I)=\gamma_2((1+I/I_c)^{1/2}+1). \tag{6.23}$$

Therefore, the amplitude of a hole burnt in such a distribution should fall as $1/\tau$ without a change in its profile. If $\gamma_2=(1/2)\gamma_1$ held, then I_c and the last factor in Eq. (6.19) would depend strongly on the degree of the well asymmetry:

$$I_c(\Delta/\tilde{\varepsilon})=I_c(1)(\Delta/\tilde{\varepsilon})^2.$$

This should alter greatly the dependence of the profile of the burnt hole on time τ and on the intensity of the pump signal (the hole would become narrower with increasing τ).

Acoustic echo

We now consider the phenomenon of the acoustic echo in the classical experimental situation. We assume that an external signal of intensity $I>I_c$ acts on a resonant two-level system for a time t. If the condition $\tilde{\varepsilon}=\omega$ is satisfied and if there is no relaxation, such a signal "rotates" the diagonal and nondiagonal elements of the density matrix by an angle θ:

$$\theta=\theta_0\Delta/\omega;\qquad \theta_0=\eta\varepsilon_0 t. \tag{6.24}$$

Usually the angle θ_0 is selected near the value $^{\theta}{}_0\sim 1$ so that the main contribution to the echo signal comes from the weakly asymmetric two-level systems. However, in the case of such systems the values of γ_1 and γ_2 are of the same order of magnitude [see Eq. (3.26)] and it is difficult to carry out a self-consistent analysis of the real relationship between these quantities. The situation changes at high rotation angles when $\theta_0\gg 1$. Then comparable contributions to the echo signal come from both symmetric two-level systems and from systems with a strong asymmetry characterized by $\Delta/\omega\propto\theta_0^{-1}$. We can demonstrate this by calculating, by way of example, the average value of operator σ_y at a time t (i.e., immediately after a pulse). A trivial solution of the system of Eqs. (6.17) and (6.18) subject to the condition $\gamma_1=\gamma_2=0$

and the initial conditions $f_{ab}(0)=0$ and $f_{aa}^{(0)} = [1-\tanh(\omega/2T)]/2$ gives

$$\langle\sigma_y\rangle = 2\,\mathrm{Im}\,f_{12} = \mathrm{th}\,(\omega/2T)\,\eta\varepsilon_0\frac{\Delta}{\omega}\frac{\sin kt}{k};$$

$$k=[(\tilde{\varepsilon}-\omega)^2+(\eta\varepsilon_0\Delta/\omega)^2]^{1/2}. \qquad (6.25)$$

Averaging this expression over the parameters ξ and Δ_0, we readily find

$$\langle\sigma_y\rangle=\eta\varepsilon_0\bar{P}\,\mathrm{th}\,(\omega/2T)\int_0^1\frac{dy}{(1-y^2)^{1/2}}\int_{-\infty}^{+\infty}dx\frac{\sin[(\theta_0 y)^2+x^2]^{1/2}}{[(\theta_0 y)^2+x^2]^{1/2}}$$

$$=\eta\varepsilon_0\bar{P}\,\mathrm{th}\,(\omega/2T)\,(\pi^2/2)J_0^2(\theta_0/2). \qquad (6.26)$$

If $\theta_0 \gg 0$, the relationship given by Eq. (6.26) becomes

$$\langle\sigma_y\rangle=\bar{P}\,\mathrm{th}\,(\omega/2T)\,(\pi/t)\,(1+\sin\theta_0). \qquad (6.27)$$

Two terms in the parentheses of Eq. (6.27) represent the contributions made to $\langle\sigma_y\rangle$ by two-level systems with a strong asymmetry $\Delta/\omega \sim \theta_0^{-1}$ (first term) and of almost symmetric two-level systems $\Delta/\omega - 1 \sim \theta_0^{-1}$. The latter contribution oscillates rapidly with time t. Since it is $\langle\sigma_y\rangle$ that determines the subsequent amplitude of the echo signal in the two-pulse or three-pulse experiments, it is convenient to investigate simultaneously the dynamics of symmetric and asymmetric two-level systems. It is sufficient to separate the monotonic and rapidly oscillating parts of the echo signal. We note that the period of the oscillations of the echo amplitude can be used to find the value of the parameter $\eta = 2\pi/\varepsilon_0\delta t$ (naturally subject to the condition that the scatter of the values of η representing different two-level systems is not too large).

The expression for the amplitude of the echo signal which appears at a time $t_0 = t+\tau+t+\tau$ ($\tau \gg t$ is the free precession time of a two-level system in the intervals between the pulses) can be found from the simple solution of the system of Eqs. (6.17) and (6.18). In view of the obvious nature of the treatment [see the derivation of Eq. (6.26)] we shall give the expression for the amplitude of a two-pulse echo A_2 in the case of a single two-level system

$$A_2={}^1\!/_2\,\mathrm{th}\,(\omega/2T)\,(\theta/kt)^3\sin(kt)\,(1-\cos(kt))\,e^{-2\tau\gamma_2} \qquad (6.28)$$

and the expression for the total value of A_2 averaged over resonant two-level systems in the limit $\theta_0 \gg 1$:

$$A_2=\bar{P}\,\mathrm{th}\,(\omega/2T)\frac{\pi}{2t}\,|\,{}^3\!/_8 e^{-2\gamma_2^{(as)}\tau}$$

$$+\,(\sin\theta_0-{}^1\!/_4\sin 2\theta_0)\,e^{-2\gamma_2^{(s)}\tau}\,|. \qquad (6.29)$$

Here, $\gamma_2^{(as)}$ and $\gamma_2^{(s)}$ are the phase relaxation rates of the symmetric and asymmetric two-level systems. If we determine A_2 for different values of t, we must first be sure that the nonoscillatory part of $A_2 t$ does not change. This by itself would demonstrate that a change in the asymmetry leaves γ_2 unaffected, whereas $\gamma_1^{(as)}$ is negligible if $\theta_0 \gg 1$. When the conditions $\omega \ll T$ and $\Delta/\omega \ll 1$ are satisfied, the width $\gamma_2^{(s)}$ occurring in the oscillatory part of A_2 is related to $\gamma_1^{(as)}$ by the simple expression $\gamma_2^{(s)} = (1/2)\gamma_2^{(as)}$ [see Eq. (3.26)] which represents an independent check of the theory. It should be noted however that the oscillatory part of real systems may be smeared out because of the scatter of the values of η representing different two-level systems. Hence, it should be possible to identify and estimate the relevant distribution.

A completely analogous discussion of the amplitude of the stimulated echo A_3 which appears at the time $t_0 = t+\tau+t+\tau_0+t+\tau$ yields

$$A_3={}^1\!/_2\,\mathrm{th}\frac{\omega}{2T}(\theta/kt)^3\left[\sin^2 kt+\left(\frac{\tilde{\varepsilon}-\omega}{k}\right)^2(\cos kt-1)^2\right]$$

$$\times\sin kt\,e^{-2\gamma_2^{(as)}\tau-\gamma_1^{(as)}\tau_0}.$$

For $\gamma_2\tau_0 \gg 1$, the contribution of symmetric two-level systems to this expression can be ignored. Substituting the relationship $\gamma_1^{(as)} = (\Delta/w)^2\gamma_2^{(as)}$ and averaging over different two-level systems we find that in the case $\theta_0 \gg 1$, we obtain

$$A_3=\bar{P}\,\mathrm{th}\frac{\omega}{2T}(\pi/4t)\,e^{-2\gamma_2^{(as)}\tau}F\left(\frac{\gamma_2^{(as)}\tau_0}{2\theta_0^2}\right), \qquad (6.30)$$

$$F(z)=\int_0^\infty dx\,e^{-zx^2}\Big\{\sin^3 x\,[I_0(zx^2)-I_1(zx^2)]$$

$$+\frac{\sin x\,(\cos x-1)}{4}[I_0(zx^2)-I_2(zx^2)]\Big\}. \qquad (6.31)$$

The integral of Eq. (6.31) can be expressed in terms of the generalized hypergeometric functions ${}_2F_2$. The argument of the F function in Eq. (6.30) is simply the characteristic value of $\gamma_1^{(as)}\tau_0$ for two-level systems, which makes the dominant contribution to the echo signal. Therefore, in the three-pulse experiments we can determine directly the value of $\gamma_1^{(as)}$ and its relationship to $\gamma_2^{(as)}$.

7. ACOUSTIC PROPERTIES OF A METAL-HYDROGEN SYSTEM

An interesting example of two-level systems is a metal containing hydrogen. On one hand, the amplitude Δ_0 of a transition of a proton to a neighboring well is fairly high (see Refs. 34–44), whereas on the other hand there is practically no scatter of the values of Δ_0. This is very important because it makes it possible to avoid one of the averaging procedures characteristic of amorphous systems and thus provides an opportunity for a fuller comparison of the theory and experiment. Most detailed investigations have been carried out so far of Nb–O(N)–H(D) (Refs. 34–38 and 41–44) and Ta–O(N)–H(D) (Refs. 34, 39, 40) systems in which hydrogen exhibits tunnel transitions between energetically equivalent states near O (or N) defects. It should be mentioned that in the case of the purest metal matrices at low concentrations of gaseous defects the scatter of the levels in the neighboring wells ξ is small, which in principle limits also the second average with respect to ξ.

At realistic sound frequencies the condition $\omega \ll \tilde{\varepsilon}, T$ is usually satisfied by such systems. The linear response function of Eq. (4.22) then simplifies significantly to

$$M=-2\eta^2\left\{m\frac{\partial^2\tilde{\varepsilon}}{\partial\xi^2}\operatorname{th}(\tilde{\varepsilon}/2T)+d\left(\frac{\partial\tilde{\varepsilon}}{\partial\xi}\right)^2\frac{1}{2T\operatorname{Ch}^2(\tilde{\varepsilon}/2T)}\frac{i\gamma_1}{\omega+i\gamma_1}\right\},\quad(7.1)$$

where

$$m=\tilde{\varepsilon}^2/(\tilde{\varepsilon}^2+\Omega_T^2),\quad d=[(\xi^2+\Omega_T^2)/\xi^2][\tilde{\varepsilon}^2/(\tilde{\varepsilon}^2+\Omega_T^2)].$$

This expression differs from that generally accepted primarily because of the considerable dissipative broadening of the levels. At $T=0$ the change in the velocity of sound, related solely to the resonance mechanism, is described by the simple expression:

$$\Delta v/v=-\frac{\eta^2}{\rho v^2}\int_{-\infty}^{+\infty}P(\xi)\frac{\partial^2\tilde{\varepsilon}}{\partial\xi^2}d\xi.\quad(7.2)$$

In the case of a wide distribution over ξ, it follows from

$$\int_{-\infty}^{+\infty}d\xi\frac{\partial^2\tilde{\varepsilon}}{\partial\xi^2}=2$$

that the polaron effect and the interaction with electrons in general do not affect the values of $\Delta v/v$. We then have

$$\Delta v/v=-\frac{2\eta^2}{\rho v^2}P(0).\quad(7.3)$$

Conversely, in the case of a narrow distribution, when in fact we are interested in the value $\partial^2\tilde{\varepsilon}/\partial\xi^2$ at $\xi\approx 0$, the important feature is the intrinsic renormalization of the spectrum defined by Eq. (3.28):

$$\tilde{\varepsilon}^2=\Delta_*^2(\tilde{\varepsilon}/\Delta_*)^{2b}+\xi^2.\quad(7.4)$$

This gives the following result for a normal metal:

$$\Delta v/v=-\frac{\eta^2x}{\rho v^2}\frac{1}{\Delta_*(1-b)},\quad(7.5)$$

which depends directly on the length scale of the interaction with electrons (here, x is the density of the tunnel states of hydrogen). In the superconducting state the spectrum is $\tilde{\varepsilon}^2=(\Delta_*^s)^2+\xi^2$, where Δ_*^s is given Eq. (5.20). We then have

$$\Delta v/v=-\frac{\eta^2x}{\rho v^2}\Big/\Delta_*^s.\quad(7.6)$$

The ratio

$$(\Delta v)_n/(\Delta v)_{sc}=(e\Delta_s/2\Delta_*)^b/(1-b)\quad(7.7)$$

depends now only on the tunnel parameters and in this sense it is a very convenient quantity for the analysis of the strength of the polaron effect. The role of the polaron effect in a system of this kind was first considered in Ref. 45. The result given by Eq. (7.7) differs from that obtained in Ref. 45 by the numerical factor given in the parentheses in Eq. (7.7) and by the presence of an additional factor $(1-b)$. This is because the polaron effect is considered in Ref. 45 using perturbation theory and the self-consistency condition of Eq. (7.4), important also at low values of b, is not applied.

At high temperatures $T\gg\tilde{\varepsilon}$ (when ξ exhibits a scatter, this applies throughout the full interval) Eq. (7.1) gives the universal response which is independent of the distribution of the levels:

$$\Delta v/v=-\frac{\eta^2x}{\rho v^2}\Big/T.\quad(7.8)$$

In general, we can describe the function $P(\xi)$ by the Lorentzian profile

$$P(\xi)=\frac{x}{\pi}\frac{\xi_0}{\xi^2+\xi_0^2},\quad(7.9)$$

which is typical of a random potential created by dipole centers, with the strength of the interaction decreasing as $1/r^3$. We can find the dependence of $\Delta v/v$ on T in a wide range of temperatures if we know in fact three parameters: Δ_*, b, ξ_0. A universal combination $\eta^2x/\rho v^2$ is then established readily from the limiting relationship (7.8). In the case of a narrow distribution it is sufficient to determine experimentally the limiting values $\Delta v/v(T=0)$ for the normal and superconducting states of a metal, which makes it possible to reconstruct all the parameters using Eqs. (7.5)–(7.7). For an arbitrary distribution we need to carry out one further measurement at an intermediate temperature in order to determine ξ_0 and then the curves for $\Delta v/v$ and α considered as a function of T for a normal metal and a superconductor can be constructed unambiguously. If the values of Δ_* and Δ_*^s are found independently, for example from inelastic neutron scattering,[41,44] then in the most general case the only unknown parameter is ξ_0. In a numerical comparison of the theory and experiment we have to be cautious because in reality we may have not one but two or more types of two-level systems with similar energy parameters (this is discussed, for example, in Ref. 34). For example, the experiments on the absorption of sound in the Nb–O(N)–H(D) system (Refs. 34–36) demonstrate the existence of two types of two-level systems. In fact our experience shows the experiments on determining the sound speed in the same materials[38] demonstrate that assuming only one type of two-level system exists fails to provide a satisfactory description of the results for any values of the parameters.

8. INVESTIGATION OF SPECTRAL PROPERTIES OF TWO-WELL SYSTEMS USING ELASTIC NEUTRON SCATTERING

As pointed out already in Sec. 5, the change from coherent to noncoherent motion on increase in T results in a very characteristic evolution of the spectral density of two-level systems. If Δ_* is sufficiently large (of the order of 1 K on the temperature scale), then a satisfactory experimental method is inelastic neutron scattering with a sufficiently high energy resolution. This has been demonstrated strikingly in Refs. 41–44 using the example of the same $Nb(OH)_x$ as discussed above. These investigations revealed sharp resonances at $\omega\approx\pm\tilde{\varepsilon}$ at low temperatures and a central peak described by Eqs. (5.8) and (5.11) at high temperatures T. Unfortunately, the evolution of the spectral density at intermediate temperatures was not studied. However, Refs. 42 and 43 provide a qualitative description of the situation at such temperatures: the amplitude of the peaks decreases and they become broader at $\omega\approx\pm\tilde{\varepsilon}$ as T rises. This is accompanied by a simultaneous increase in the central peak which becomes narrower in the limit of high temperatures T, which is exactly the situation predicted in Sec. 5 (see Fig. 3). At low values of b [the system $Nb(OH_x)$ corresponds to $b=0.05$] the whole evolution at temperatures $T > \tilde{\varepsilon}$ can be studied using the simple relationship

$$I_\omega \propto -\frac{\Delta(T)}{\omega}\,\mathrm{Im}\left\{\frac{\omega+i\Omega_T}{\omega[(\omega+i\Omega_T)^2-\tilde{\varepsilon}^2]-i\Delta^2(T)\Omega_T}\right\}. \tag{8.1}$$

This expression is valid for any ω, $\tilde{\varepsilon}$, and Ω_T. Equation (8.1) describes a continuous transition from the coherent to the noncoherent regime. A relationship similar to Eq. (8.1) was obtained in Ref. 46 using the technique of functional integration in the case $b \ll 1$ and $T \gg \omega, \tilde{\varepsilon}$. In fact, as shown in Sec. 4, the range of validity of this approach is considerably wider.

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