Phase conjugation due to stimulated Brillouin scattering in an inhomogeneous plasma

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A new mechanism of phase conjugation due to stimulated Brillouin scattering is proposed. This mechanism relies on the difference in the suppression of the scattering of the conjugated and unconjugated components because of the inhomogeneity of the plasma where the scattering takes place. The parameters of the plasma and of the incident radiation needed to ensure phase conjugation are determined.

1. INTRODUCTION

One of the important practical applications of stimulated Brillouin scattering (SBS) is phase conjugation of wavefronts.[1,2] The use of a plasma as an active medium for SBS[3-6] is promising because it makes it possible to extend phase conjugation to the long-wavelength infrared and microwave ranges. This is particularly relevant, for example, in phase conjugation of CO_2 laser radiation since SBS has been observed at the wavelength of such radiation ($\lambda \approx 10\,\mu$m) only in plasmas.[7-9]

A conventional method of phase conjugation as a result of SBS requires[1,2] that the amplitude of the density perturbation should be related locally to the amplitudes of electromagnetic waves, which is ensured by the condition $\nu l \gg 1$, where ν^{-1} is the attenuation length of sound and l is the longitudinal correlation length of the pump field.[10] This condition is readily satisfied in a low-temperature plasma, but the absorption of electromagnetic waves, which is strong because of the high frequency of elastic collisions, heats the plasma. This can reduce significantly the efficiency of SBS and phase conjugation.[11] The role of thermal nonlinearities can be minimized by increasing the plasma temperature. However, in view of the considerable difference between the electron and ion temperatures ($T_e \gg T_i$), due to the weakness of the Coulomb conditions, the Landau damping of ion sound is weak and the condition $\nu l \gg 1$ may be disobeyed. These contradictory requirements make it difficult to achieve the conventional mechanism of phase conjugation as a result of SBS in a plasma.

We shall propose a new mechanism for phase conjugation as a result of SBS in a collisionless plasma, the essence of which can be stated briefly as follows. It is known that SBS is influenced significantly by spatial variations of the plasma density and temperature, leading to failure to satisfy the phase-matching conditions needed to ensure the efficient interaction between the waves resulting in suppression of the SBS process.[12,13] In the case of stimulated scattering of radiation with a speckle structure the suppression of amplification because of the plasma inhomogeneity is stronger for components uncorrelated with the pump radiation than for the components which repeat the pump structure, so that SBS in an inhomogeneous plasma may result in phase conjugation.

We shall consider this phase conjugation mechanism in the simple case of steady-state SBS.

2. PRINCIPAL EQUATIONS

We shall assume that in a longitudinal inhomogeneous plasma with the characteristic inhomogeneity scale l we can expect scattering of the incident pump beam

$$\mathbf{E}_1 = \mathbf{e}E_0\{a_1(x,t,\mathbf{r})\exp(i\omega_1 t - ik_1 x) + \text{c.c.}\}$$

accompanied by the formation of a counterpropagating electromagnetic wave

$$\mathbf{E}_2 = \mathbf{e}E_0\left(\frac{c_1\omega_2}{c_2\omega_1}\right)^{1/2}\{a_2(x,t,\mathbf{r})\exp(i\omega_2 t + ik_2 x) + \text{c.c.}\}.$$

The scattering occurs by an ion-acoustic wave

$$\delta n = 2iN_0\left(\frac{E_0}{E_p}\right)\{a_3(x,t,\mathbf{r})\exp(i\omega_3 t - ik_3 x) + \text{c.c.}\}.$$

Here, a_i ($i = 1, 2, 3$) are the dimensionless amplitudes of the interacting waves, while $k_1 \approx k_2 \approx k$, and $k_3 = 2k$ are their wave numbers. In the vicinity of the phase-matching point ($x = 0$) the equations describing the steady-state interaction are

$$\begin{aligned} a_{1x} + \frac{i}{2k}\Delta_\perp a_1 + i\varphi_{1x}a_1 &= -\Gamma a_2 a_3, \\ -a_{2x} + \frac{i}{2k}\Delta_\perp a_2 + i\varphi_{2x}a_2 &= \Gamma a_1 a_3^*, \qquad (1) \\ \nu a_3 + a_{3x} + \frac{i}{2k_3}\Delta a_3 &= \Gamma a_1 a_2^*, \end{aligned}$$

where $\varphi_i(x)$ are the values of the phase mismatch due to variation of the plasma density; $\Gamma^{-1} = (k_0E_0N_0/E_pN_c)^{-1}$ is the characteristic nonlinear interaction length; E_0 is the amplitude of the incident wave; N_0 is the plasma concentration; $E_p = (16\pi N_0 T_e)^{1/2}$ is the characteristic plasma field; $N_c = m\omega^2/4\pi e^2$ is the critical plasma concentration; ν is the spatial damping rate of an ion-acoustic wave (damping of electromagnetic waves is ignored). It should be noted that the coefficient Γ represents a smooth (on a scale of L) function of the longitudinal coordinate $\Gamma(x)$. We shall use Γ to denote $\Gamma(x)$ at the phase-matching point $x = 0$.

We can demonstrate the existence of phase conjugation in an inhomogeneous plasma by considering the problem of stimulated scattering assuming that the pump field is constant, i.e., we assume that the amplitude a_1 satisfies the equation

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$$a_{1x}+\frac{i}{2k}\Delta_\perp a_1+i\varphi_{1x}a_1=0. \tag{2}$$

We also assume that the amplitude can be represented in the form

$$a_1=A_1(x)\Psi_1(x,\mathbf{r}), \tag{3}$$

where the function Ψ_1 has a complex spatial structure with a characteristic transverse scale ρ, which is much less than the beam width, and with a constant (over the cross section) average intensity; moreover, this function satisfies the equation

$$\Psi_{1x}+\frac{i}{2k}\Delta\Psi_1=0. \tag{4}$$

If in any cross section we know the function $\Psi(x',\mathbf{r}')$, then the solution of Eq. (4) satisfying the above boundary condition is

$$\Psi_1(x,\mathbf{r})=\int\Psi(x',\mathbf{r}')G(x,x',\mathbf{r},\mathbf{r}')d^2r',$$

where G is the Green's function of Eq. (4):

$$G(x,x',\mathbf{r},\mathbf{r}')=\frac{ik}{2\pi(x-x')}\exp\left[-\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2(x-x')}\right]\theta(x-x'). \tag{5}$$

Here, $\theta(x)$ is a unit function. Knowing the solution of Eq. (4), we can write down the amplitude of the pump wave in the form

$$a_1=\Psi_1(x,\mathbf{r})\exp[-i\varphi_1(x)]. \tag{6}$$

Without limiting the generality of the treatment we shall assume here and below that

$$\int|\Psi_1|^2d^2r=1,$$

so that the normalization constant (average intensity) is already included in the definitions of the fields and of the damping coefficient Γ.

Using the known Green's function of the equation for an ion-acoustic wave, we can rewrite the initial system in the form of an integrodifferential equation for the scattered wave amplitude:

$$-a_{2x}+\frac{i}{2k}\Delta_\perp a_2+i\varphi_{2x}a_2=\Gamma(x)a_1(x) \tag{7}$$
$$\times\int\Gamma(x')G_3^*(x,x',\mathbf{r},\mathbf{r}')a_1^*(x',\mathbf{r}')a_2(x',\mathbf{r}')dx'd^2r',$$

where

$$G_3=\frac{ik_3}{2\pi(x-x')}\exp\left[-\frac{ik_3(\mathbf{r}-\mathbf{r}')^2}{2(x-x')}-\nu(x-x')\right]\theta(x-x')$$

is the Green's function of the equation for an ion acoustic wave.

We shall obtain an equation describing amplification of a conjugate wave using a standard method[1,2] and we shall seek the solution of Eq. (7) in the form of conjugated and unconjugated waves

$$a_2=A_2(x)\exp[i\varphi_2(x)]\Psi_1^*(x,\mathbf{r})+\tilde{a}_2, \tag{8}$$

assuming that $\tilde{a}_2$ is orthogonal to Ψ_1^*, i.e., assuming that

$$\tilde{a}_2=a_2-\Psi_1\int a_2\Psi_1d^2r.$$

We substitute Eq. (8) into Eq. (7), multiply the resultant expression by $\Psi_1(x,\mathbf{r})$, and integrate over the transverse cross section of the beam. Consequently, subject to the smallness of the gain in a distance equal to the correlation length, which can be represented in the form

$$(\Gamma k\rho^2)^2\ll1, \tag{9}$$

we obtain an equation describing the conjugated wave gain:

$$-A_{2x}=\int_{-\infty}^{x}K_{\text{corr}}(x,x')A_2(x')dx', \tag{10}$$

where K_{corr} is the kernel of the equation describing the amplification of the correlated component. Its value is given by the expression

$$K_{\text{corr}}(x,x')=F(x,x')$$
$$\times\int G_3(x,x',\mathbf{r},\mathbf{r}')\Psi_1^2(x,\mathbf{r})\Psi_1^{*2}(x',\mathbf{r}')d^2r\,d^2r', \tag{11}$$

where the function F is

$$F(x,x')=\Gamma(x)\Gamma(x')\exp[-i\varphi(x)+i\varphi(x')],\quad\varphi=\varphi_1+\varphi_2.$$

We can obtain an equation describing the amplification of waves uncorrelated with the pump radiation by employing a method proposed in Ref. 10, i.e., we adopt a plane wave as a "typical representative" of an uncorrelated component and (applying the method described above) we obtain equations of the form (10) with a kernel $K=K_{\text{uncorr}}$, where

$$K_{\text{uncorr}}=F(x,x')\int G_3^*(x,x',\mathbf{r},\mathbf{r}')\Psi_1(x,\mathbf{r})\Psi_1(x',\mathbf{r}')d^2r\,d^2r'. \tag{12}$$

Equation (12) is valid subject to the same condition (9) as Eq. (10).

We can represent the kernels given by Eqs. (11) and (12) explicitly by assuming that $\Psi_1(x,\mathbf{r})$ is a Gaussian statistically homogeneous random field and by replacing integration in Eqs. (11) and (12) with respect to $\mathbf{r}'$ by statistical averaging. In the case of the equation describing a correlated component, this procedure follows:[1]

$$\int\Psi_1^2(x,\mathbf{r})\Psi_1^{*2}(x',\mathbf{r}')d^2r'=\langle\Psi_1^2(x,\mathbf{r})\Psi_1^{*2}(x',\mathbf{r}')\rangle$$
$$=2\langle\Psi_1(x,\mathbf{r})\Psi_1^*(x',\mathbf{r}')\rangle^2=2B^2(x-x',\mathbf{r}-\mathbf{r}'), \tag{13}$$

where $B(\xi,\boldsymbol{\eta})$ is the correlation function of the random field Ψ; $\xi=x-x'$, $\boldsymbol{\eta}=\mathbf{r}-\mathbf{r}'$. We note that the function $B(\xi,\boldsymbol{\eta})$ satisfies Eq. (4) (Ref. 14) and if it has the Gaussian profile with the characteristic transverse scale ρ when $\xi=0$, i.e., if $B(0,\boldsymbol{\eta})=\exp(-\eta^2/\rho^2)$, then for any value of ξ it can be represented in the form

$$B(\xi,\boldsymbol{\eta})=\left(1-\frac{2i\xi}{k\rho^2}\right)^{-1}\exp\left[-\frac{\eta^2}{\rho^2}\left(1-\frac{2i\xi}{k\rho^2}\right)^{-1}\right], \tag{14}$$

which shows that the role of the longitudinal correlation length is played by the quantity $k\rho^2 = l$ (Ref. 14). Repeating the same operations on Eq. (12), we find that

$$\int \Psi_1(x,\mathbf{r})\Psi_1^*(x',\mathbf{r}')d^2r'=B(\xi,\eta).$$

Therefore, the kernel of Eq. (10) is

$$K(x,x')=F(x,x')Z(x-x'), \tag{15}$$

where in the case of the correlated component the function $Z(\xi)$ is given by the expression

$$Z_{\text{corr}}(\xi)=2\int G_3^*(\xi,\eta)B^2(\xi,\eta)d^2\eta,$$

while for the uncorrelated component we have

$$Z_{\text{uncorr}}(\xi)=\int G_3^*(\xi,\eta)B(\xi,\eta)d^2\eta.$$

If the correlation function is Gaussian,[14] these integrals can be calculated explicitly. Consequently, if $k_3 = 2k$, we find that

$$Z_{\text{corr}}(\xi)=2e^{-\nu\xi}/(1-2i\gamma\xi),$$

$$Z_{\text{uncorr}}(\xi)=e^{-\nu\xi}/(1-i\gamma\xi), \tag{16}$$

where $\gamma^{-1} = k\rho^2 = l$ is the longitudinal correlation length.

The kernels of the integral equations for the correlated and uncorrelated components are identical in structure and differ only in the values of $x = x'$ and the characteristic longitudinal scales. We can obtain the solution of the integrodifferential equation (10) on the assumption that the scale L_{A_2} of the change in the amplitude of the scattered wave considerably exceeds the characteristic scale L_K of changes in the kernels of the integral equations:

$$L_{A_2}\gg L_K \tag{17}$$

(the limits of the validity of this expression will be explained later using the final result). Subject to this condition the function A_2 can be taken outside the integral and Eq. (10) then becomes a simple differential equation

$$-A_{2x}=g(x)A_2, \tag{18}$$

where the local growth rate $g(x)$ is given by

$$g(x) = \int_{-\infty}^{x} K(x,x')dx'. \tag{19}$$

If $\varphi(x) = 0$ we obtain the familiar special cases from Eq. (19). For example, for $\nu l \gg 1$, i.e., if the sound attenuation length is short compared with the correlation length of the pump field, we find that the local growth rates of the correlated and uncorrelated components $\operatorname{Re} g_{\text{corr}} = 2\Gamma^2/\nu$ and $\operatorname{Re} g_{\text{uncorr}} = \Gamma^2/\nu$ differ by a factor of 2 and, if the amplification length is sufficiently large, we can expect phase conjugation.[1,2] These expressions are valid if $\Gamma \ll \nu$.

In the opposite limiting case when the correlation length of the pump field is short compared with the attenuation length of sound ($\nu l \ll 1$), a calculation of the integral of Eq. (18) shows that selection of the correlated and uncorrelated (with the pump wave) structures disappears and the spatial gain is given by

$$\operatorname{Re} g_{\text{corr}} \approx \operatorname{Re} g_{\text{uncorr}} \approx \pi\Gamma^2/2\gamma, \tag{20}$$

which is valid when $\Gamma \ll \gamma$. This result is analogous to that obtained in Ref. 15 for one-dimensional fluctuations of the pump wave.

It therefore follows that in a homogeneous medium we can expect phase conjugation only if

$$\nu l\gg 1, \tag{21}$$

which expresses a local dependence of the gain on the pump wave amplitude.

3. PHASE CONJUGATION IN AN INHOMOGENEOUS PLASMA

In a hot nonisothermal plasma when the attenuation of sound is governed by the collisionless Landau damping, which should be weak at temperatures $T_e \gg T_i$, the local condition of Eq. (21) is quite likely to be disobeyed. However, we shall show that selection of the components correlated and uncorrelated with the pump wave occurs in an inhomogeneous plasma even in the absence of a local relationship between the gain and the amplitude of the pump wave, i.e., when the condition (21) is disobeyed. This effect is due to a difference in the suppression of stimulated scattering of the conjugated and unconjugated components as a result of the plasma inhomogeneity.

In calculating the effects of selection of the correlated and uncorrelated components in the case of weak attenuation of sound (i.e., when $\nu l \ll 1$), we proceed as follows. In the vicinity of the phase-matching point (we assume that there is only one such point, located at $x = 0$) we expand the difference phase

$$\varphi(x)=\int(k_1+k_2-k_3)dx$$

as a Taylor series and we retain the quadratic term:

$$\varphi(x)=\alpha x^2/2, \tag{22}$$

where $\alpha = \partial\delta k/\partial x \approx 2k_0/L_N$ (L_N is the characteristic scale of the plasma inhomogeneity) and, moreover, to simplify calculations we assume that $\Gamma(x) = \exp(-x^2/2L^2)$ (L is the layer thickness), i.e., the interaction of the waves is concentrated in a region with a characteristic length L in the vicinity of the phase-matching point. Under these assumptions the function $F(x, x')$ becomes

$$F(x,x')=\Gamma^2\exp\left(-\frac{x^2}{2L^2}-\frac{x'^2}{2L^2}-i\frac{\alpha x^2}{2}+i\frac{\alpha x'^2}{2}\right). \tag{23}$$

Now, knowing the exact form of the kernel (16), we can readily calculate the total amplification of the scattered wave over the whole interaction length, i.e., we can calculate the quantity $\exp G$, where G is given by the integral

$$\int_{-\infty}^{+\infty} g(x)dx=\int_{-\infty}^{+\infty}\int_{-\infty}^{x}K(x,x')dx\,dx'$$

$$=\Gamma^2\int_{-\infty}^{+\infty}\exp\left(-\frac{x^2}{L^2}-i\alpha x\xi+\frac{x\xi}{L^2}\right)dx$$

$$\times\int_0^{\infty}Z(\xi)\exp\left(\frac{i\alpha\xi^2}{2}-\frac{\xi^2}{2L^2}\right)d\xi. \tag{24}$$

We are interested in the value of this integral subject to the natural conditions $\alpha/\gamma^2 \ll 1$ (i.e., $l \ll \alpha^{-1}$) and $L \gg \gamma^{-1}$, which imply that many correlation lengths can be fitted within the phase mismatch length $\alpha^{-1/2}$ and within the pump localization length L. First evaluating the integral with respect to x in Eq. (24), we find that

$$G=\pi^{1/2}\Gamma^2 L\int_0^\infty Z(\xi)\exp\left\{\frac{1}{4}\left(\frac{1}{L^2}-i\alpha\right)^2\xi^2L^2+\frac{i\alpha\xi^2}{2}-\frac{\xi^2}{2L^2}\right\}d\xi,$$

and if the interaction length is sufficiently long (i.e., if $\alpha^2L^2\gg\gamma^2$), this expression becomes

$$G=(\pi\Gamma^2/\alpha)Z(0). \tag{25}$$

In the opposite limiting case when $\gamma^2\gg\alpha^2L^2$, we obtain the total logarithmic gain

$$\operatorname{Re}G\propto\Gamma^2L/\gamma,$$

which corresponds to the local growth rate of Eq. (20).

The failure of Eq. (25) to depend on L demonstrates the arbitrary nature of the approximation $\Gamma(x)$ selected solely to ensure convergence of the intermediate integrals and for convenience of calculations. It readily follows from Eq. (25) that the total gain is governed by the pump wave intensity and by the inhomogeneity parameter α, and also that—apart from the factor $Z(0)$—it is identical with the familiar expression for the gain experienced by a scattered wave in the field of a plane pump wave.[12-14] Equation (25) demonstrates that in the case of SBS in an inhomogeneous plasma there is selection between the components correlated and uncorrelated with the pump because the values of $Z(0)$ for these components differ by a factor of 2.

We now consider the spatial dependence of the local increment $g(x)$. If $|x|\ll L$, and $\alpha L^2\gg1$, this dependence becomes

$$g(x)=\Gamma^2\int_0^\infty\exp(i\alpha\xi^2/2-i\alpha\xi x)Z(\xi)\,d\xi, \tag{26}$$

where $Z(\xi)$ is given by Eq. (16).

Figure 1 shows graphically the dependence $g(y=\gamma x)$ obtained by numerical integration of Eq. (26) subject to the condition $\alpha/\gamma^2=0.1$.

Analytic expressions for the function g can be obtained for $\alpha^{1/2}|x|\gg1$:

$$g(y)=\frac{\Gamma^2}{\gamma}\begin{cases}\dfrac{2i}{\beta y}-\dfrac{2i}{\beta^2y^2}+\dfrac{4}{\beta y^2}, & x<0\\[2ex] \left(\dfrac{2\pi}{i\beta}\right)^{1/2}\dfrac{1}{y}\exp\left(-\dfrac{i\beta y^2}{2}\right)+\dfrac{2i}{\beta y}-\dfrac{2i}{\beta^2y^2}+\dfrac{4}{\beta y^2}, & \\ x>0\end{cases}$$

for the conjugated wave and

$$g(y)=\frac{\Gamma^2}{\gamma}\begin{cases}\dfrac{i}{\beta y}-\dfrac{i}{\beta^2y^2}+\dfrac{1}{\beta y^2}, & x<0\\[2ex] \left(\dfrac{2\pi}{i\beta}\right)^{1/2}\dfrac{1}{y}\exp\left(-\dfrac{i\beta y^2}{2}\right)+\dfrac{i}{\beta y}-\dfrac{i}{\beta^2y^2}+\dfrac{1}{\beta y^2}, & \\ x>0\end{cases}$$

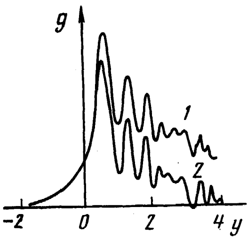

FIG. 1. Dependence of the local gain g on the dimensionless coordinate $y=x/l$ for conjugated (curve 1) and unconjugated (curve 2) waves.

for the uncorrelated wave.

It follows from the above expressions and from Fig. 1 that the amplification of the component correlated with the pump wave is concentrated in a wider region than the amplification of the uncorrelated wave.

The value of $g(0)$ for $\alpha L^2\gg1$ can also be calculated analytically. Its real part is

$$\operatorname{Re}g_{\text{corr}}(0)=\operatorname{Re}g_{\text{uncorr}}(0)=\pi\Gamma^2/4\gamma.$$

It therefore follows that the local growth rates for the correlated and uncorrelated waves are similar at low values of x: $g\sim\Gamma^2/\gamma$, and selection is due to the different characteristic amplification lengths of the components, which are of order $L_{\text{amp}}\sim\gamma/\alpha\ll L$. The condition for this approach to be valid is the smallness of the gain experienced in a distance equal to the longitudinal correlation length of Eq. (9).

It follows that in a layer of inhomogeneous transparent plasma with a single phase-matching point the process of SBS occurs under the spatial amplification conditions and the values of the gain are different for the components correlated and uncorrelated with the pump wave.

Phase conjugation in the case of weak damping in an inhomogeneous plasma is, as demonstrated by Eqs. (24) or (26), a nonlocal interference effect in no way related to the localization of the process of stimulated scattering in a distance equal to the size of one spot in a speckle structure because the phase matching is lost in a distance equal to this length.

A qualitatively different situation may be encountered in a transparent plasma with two phase-matching points (Fig. 2). If the range of the scattered or ion-acoustic wave is sufficiently long, the amplification of waves in the vicinity of each such point is no longer independent. The existence of such a feedback mechanism may have the effect that SBS occurs during lasing,[16] i.e., we can expect the existence of modes growing with time and with their growth limited only by the nonlinear effects. In view of the difference between the gains in the vicinity of the phase-matching points of the waves correlated or uncorrelated with the pump, the temporal instability growth rates are different for these waves and the wave with a transverse structure identical with the pump wave will grow fastest.

We can estimate the instability increment by assuming that the interacting waves are amplified in the vicinity of the phase-matching points and the gain is exp G, whereas in the regions between such points the waves propagate freely and are attenuated. Then, the gain experienced in one complete round trip from the interaction region and back again is

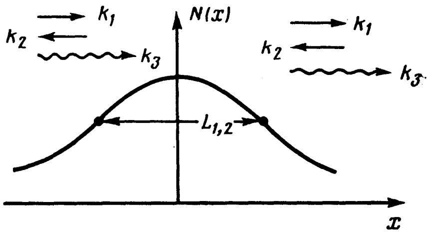

FIG. 2. Geometry of the interaction waves in a plasma with two phase-matching points.

$$\exp\,[2G-(\nu_2+\nu)\,L_{1,2}],$$

where ν_2 and ν are the spatial damping rate of the scattered and ion-acoustic waves, and $L_{1,2}$ is the distance between the phase-matching points, so that the instability growth rate is given by

$$\mu=[2G-(\nu_2+\nu)\,L_{1,2}]/\tau,$$

where $\tau = L_{1,2}\,(1/c + 1/c_s)$ is the signal phase-conjugation time in the feedback loop, and c and c_s are the group velocities of the scattered and ion-acoustic waves.

Since G is twice as large for the conjugated wave, it follows that its growth rate is greater and the quality of phase conjugation increases with time until the wave amplitudes reach the level at which they become limited by nonlinear effects.

Obviously, lasing is retained when the phase-matching points come closer together even if $L_{1,2} = 0$. This corresponds to approximation of the phase mismatch φ in the relationship (22) by an expression of the form $\varphi = \chi x^3$ type, but in contrast to Eq. (22) in this case the reduction of Eq. (10) to Eq. (18) is no longer possible, and this situation requires a separate analysis.

4. CONCLUSIONS

Phase conjugation due to the mechanism discussed here occurs in the case of strong SBS in an inhomogeneous plasma. We now summarize all the restrictions mentioned above. Selection of the growth rates requires that the scales should be $l \ll \alpha^{-1/2} \ll L$, and $\alpha l L \gg 1$, which sets the limits to L_N from above and below. If L_N is too large, the scattering is exactly the same as in an inhomogeneous layer of length L, whereas if L_N is too small, the amplification becomes much weaker. If these conditions are obeyed, the gain is given by Eq. (25).

In the case of phase conjugation by SBS in a plasma it is necessary to ensure $G \approx 15$–20, or in terms of the dimensional variables $(k_0 L_N)(E_0 N_0/E_p N_c) \approx 6$, and hence using the characteristic plasma parameters encountered in the experiments[7-9] we readily obtain an estimate of the required laser radiation power density. For example in the case of CO_2 laser radiation of wavelength $\lambda \approx 10\,\mu$m ($k \approx 6 \times 10^3$ cm^{-1}) incident on a plasma characterized by $T_e \approx 100$ eV, $L_N \approx 1$ cm, $N_0 \approx 0.3 N_c$, and $N_c \approx 10^{19}$ cm^{-3}, we should use radiation with a very reasonable value of the power density $q \approx 10^{11}$ W/cm^2.

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