Dynamics of high-frequency streamer

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The space-time evolution of a small-scale plasma formation at a single ionization nucleating center during rf breakdown of a gas is studied. Analysis and numerical simulation show that the quasispherical plasmoid which forms in the initial stage of the process subsequently stretches out along the direction of the field as the result of a strengthening of the field at the "poles" of the plasmoid. The polar axis runs parallel to the external electric field. The plasmoid then transforms into a rapidly growing "high-frequency streamer."

1. An important question in the theory of rf discharges in gases is in the space-time evolution of small, isolated plasma formations (plasmoids) which form as a result of the development of independent (nonoverlapping) electron avalanches around discrete ionization nucleating centers.[1] The role of these centers may be played by isolated (widely spaced) electrons, liquid and solid aerosols, small-scale inhomogeneities which result from instabilities of the discharge, seats of artificial ionization, etc.

Our purpose in this study was to derive an approximate analytic theory and to carry out a numerical simulation of the dynamics of a gas-discharge plasmoid in a linearly polarized field above the breakdown level. We assume that the dimensions of the plasmoid are smaller than the wavelength and the skin thickness. The physical factors which primarily determine the way in which the discharge propagates away from a primary breakdown region under the conditions of interest here are diffusion (free or ambipolar), electron-impact ionization of molecules, and the effect of the plasma on the rf field causing the breakdown. The initial system of equations includes an equation for the complex amplitude of the quasiharmonic electric field,

$$\mathbf{E}(\mathbf{r},t)e^{i\omega t}=-\nabla\varphi(\mathbf{r},t)e^{i\omega t},$$

in the quasistatic (irrotational) approximation and the ionization balance equation for the electron density N,

$$\operatorname{div}(\varepsilon\nabla\varphi)=0,\quad \varepsilon=1-\frac{N}{N_c}\left(1+i\frac{\nu}{\omega}\right), \tag{1}$$

$$\frac{\partial N}{\partial t}=D\Delta N+(\nu_i-\nu_a)N-\alpha N^2. \tag{2}$$

Here ε is the complex dielectric constant of the plasma, $N_c=m(\omega^2+\nu^2)/4\pi e^2$, ω is the angular frequency of the field, ν is the effective electron collision rate, D and α are the diffusion and recombination coefficients, ν_i is the rate of ionizing collisions, and ν_a is the rate at which electrons attach to molecules.

We treat the difference between the ionization and attachment rates as a given, rapidly increasing function of the field amplitude: $\nu_i-\nu_a=f(|\mathbf{E}|)$. Over a wide range of conditions, we can use a power-law approximation of this functional dependence:

$$\nu_i-\nu_a=\nu_a\left[\left(\frac{|\mathbf{E}|}{E_c}\right)^\beta-1\right] \tag{3}$$

(E_c is the so-called breakdown field). The functional relationship $\nu_i(|\mathbf{E}|)$ we are assuming here, which formally eliminates the electron temperature from consideration, holds under the conditions of localization and instantaneous heating of the electrons:

$$\delta_T\nu\tau\gg1,\quad \Lambda\gg l/\delta_T^{1/2}.$$

Here δ_T is the fraction of its energy which an electron loses in a collision with a molecule, τ and Λ are time and length scales of the amplitude variation, and l is the electron mean free path.

2. Let us analyze the solution of Eqs. (1), (2) which describe the propagation of ionization away from an initializing center in a linearly polarized external field $\mathbf{E}_0$ which is, in the absence of the plasma, uniform. We assume that the functions $N(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$ satisfy the conditions

$$N(\mathbf{r},0)=N_\Sigma\delta(\mathbf{r}), \tag{4}$$

$$\nabla\varphi(\infty,t)=-\mathbf{x}_0E_0. \tag{5}$$

Here $r=|\mathbf{r}|$ is the distance from the origin of coordinates, $\mathbf{x}_0$ is a unit vector oriented parallel to the unperturbed external field, and $\delta(\mathbf{r})$ is the three-dimensional δ-function. Under the conditions

$$N_0\equiv N_\Sigma[(\nu_i-\nu_a)/D]^{3/2}\ll N_c,\quad \alpha N_0\ll\nu_i-\nu_a,$$

which usually hold during the formation of discrete avalanches at isolated initializing electrons, the evolution of the discharge in its initial state (while the conditions $\alpha N\ll\nu_i-\nu_a$, $|\varepsilon-1|\ll1$ hold) is determined by the known spherically symmetric solution of linear equation (2) with $\alpha=0$, $\nu_i-\nu_a=\text{const}$ ($|\nabla\varphi|\equiv E_0$):

$$N(r,t)=\frac{N_\Sigma}{8(\pi Dt)^{3/2}}\exp\left[-\frac{r^2}{4Dt}+(\nu_i-\nu_a)t\right]. \tag{6}$$

Except in the brief initial time interval $t\lesssim(\nu_i-\nu_a)^{-1}$ during which the function $N(0,t)$ decreases to values $N\approx N_0$ as a result of diffusion, this solution describes a rapid growth of the electron density [it is approximately an avalanche growth for $r<2[D(\nu_i-\nu_a)]^{1/2}t$.]

The subsequent evolution of the discharge depends strongly on the predominant nonlinearity mechanism. If the density N can reach its recombination limit $N_\alpha=(\nu_i-\nu_a)/\alpha$ without perturbing the field [i.e., for $|\varepsilon(N_\alpha)-1|\ll1$], the dynamics of the plasmoid which forms will be determined by a spherically symmetric solution of Eq. (2) with constant coefficients (since $|\nabla\varphi|=E_0=\text{const}$). At large t, the discharge is a quasihomogeneous sphere in this case, with an electron density

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$N \approx N_\alpha$. It expands at the known diffusion velocity[2]

$$v_D = 2[D(\nu_i - \nu_a)]^{1/2}. \quad (7)$$

In the case $|\varepsilon(N_\alpha) - 1| \gg 1$, which is more interesting and also of more practical importance, the predominant nonlinearity mechanism in this system is not recombination but the variation in the field and thus the ionization rate ν_i in the plasma which forms. The plasmoid cannot remain spherically symmetric. Its configuration and structure depend strongly on the parameter ν/ω. We will focus on the case $\nu/\omega \gg 1$, which holds in high-pressure discharges.

In this case $\nu/\omega \gg 1$ the field perturbations become significant when the density N at the center of a plasmoid reaches values

$$N \gtrsim \omega N_c/\nu \approx \nu N_{c0}/\omega$$

($N_{c0} = m\omega^2/4\pi e^2$). Such a plasma is actually a conductor ($\varepsilon - 1 = -i\omega N/\nu N_{c0}$) in which the field amplitude decreases with increasing N, so the avalanche velocity $\nu_i - \nu_a$ decreases in the central region (with $N > \nu N_{c0}/\omega$), and the density distribution $N(r)$ becomes progressively flatter (as in the recombination case).

The avalanche velocity at the periphery, which actually determines the velocity at which the discharge propagates into the un-ionized region, depends on the local values of the amplitude, which in our case of large ν/ω (with a conducting plasma) are strengthened in the "polar" regions of the plasmoid and weakened in the "equatorial" regions (the polar axis runs parallel to the external field $\mathbf{x}_0 E_0$). As a result, the discharge propagation velocity depends on the direction, and the plasmoid stretches out along the external field.[1)] By analogy with a similar effect accompanying discharges in a static field, we call this elongating plasmoid a "high-frequency streamer."

3. Let us analyze the dynamics of a high-frequency streamer with the help of a qualitative model based on the following simplifying assumptions:

1) The plasmoid is a homogeneous ellipsoid of revolution with a sharp boundary (the width of the transition region at the boundary is much smaller than the radii of curvature of the boundary.

2) The rates (velocities) at which the major semiaxis (parallel to $\mathbf{x}_0$) and the minor semiaxis of the ellipsoid grow agree with the corresponding local values of the propagation velocity of a one-dimensional discharge with a plane boundary.

With the help of the results of Ref. 2, these velocities can be calculated approximately from (7), in which the value of v_D at the poles is determined by the amplitude of the normal field component outside the plasma, while its value at the equator is determined by the amplitude of the tangential component, which is continuous at the boundary.

The system of equations describing the evolution of a plasmoid under these assumptions (and with $\alpha = 0$) is written in the form

$$\mathbf{E}_i = \mathbf{x}_0 \frac{E_0}{1+(\varepsilon-1)n_x}, \quad \varepsilon = 1 - \frac{N}{N_c}\left(1 + i\frac{\nu}{\omega}\right), \quad (8)$$

$$\frac{\partial N}{\partial t} = [\nu_i(E_i) - \nu_a]N, \quad (9)$$

$$\frac{da}{dt} = 2\{D[\nu_i(E_n) - \nu_a]\}^{1/2}, \quad (10)$$

$$\frac{db}{dt} = 2\{D[\nu_i(E_i) - \nu_a]\}^{1/2}, \quad (11)$$

$$E_n = |\varepsilon| E_i, \quad E_i = |\mathbf{E}_i|. \quad (12)$$

Here $\mathbf{E}_i$ is the electric field in the ellipsoid; E_n is the amplitude of the external field at the ends (poles) of the ellipsoid, which is greater by a factor of $|\varepsilon|$ than the amplitude in the equatorial region, E_i; a and b are respectively the major and minor semiaxes of the ellipsoid; and n_x is the depolarization coefficient in the $\mathbf{x}_0$ direction, which depends on the ratio a/b (Ref. 5, for example). In particular, in the case $a \gg b$ we have

$$n_x = \left(\frac{b}{a}\right)^2 \ln\frac{2a}{b}. \quad (13)$$

As can be seen from (8) and (12), the behavior of the functions $E_i(t)$ and $E_n(t)$ and thus the entire nature of the evolution in which we are interested here is determined by a competition between two factors: the increase in $|\varepsilon|$ and the decrease in the depolarization coefficient n_x due to the increase in the ratio a/b. It is easy to see that if the function $\nu_i(|\mathbf{E}|)$ increases sufficiently rapidly (as it usually does in the region $|\mathbf{E}| \gtrsim E_c$) the asymptotic behavior of the solution of Eqs. (8)–(12) after a long time (with $|\varepsilon| \gg 1$ and $a \gg 1$) is such that

$$|\varepsilon| n_x \to 0, \quad E_i \to E_0 \quad (14)$$

In particular, for the power-law approximation (3) with an exponent $\beta > 1$ (for air, we should have $\beta \approx 3$–5 at $5E_c > |\mathbf{E}| > E_c$),[6] the asymptotic solution ($t \to \infty$) is

$$E_i = E_0, \quad N = N_0 e^{\gamma t}, \quad (15)$$

$$v_\parallel = \frac{da}{dt} = v_a \exp\left(\frac{1}{2}\beta\gamma t\right), \quad (16)$$

$$v_\perp = \frac{db}{dt} = v_b, \quad (17)$$

$$\frac{a}{b} \propto \frac{1}{\gamma t}\exp\left(\frac{1}{2}\beta\gamma t\right),$$

$$|\varepsilon| n_x \sim \exp[-\gamma(\beta-1)t], \quad (18)$$

where

$$v_a = 2[D\nu_i(E_0)]^{1/2}, \quad v_b = 2(D\gamma)^{1/2},$$
$$\gamma = \nu_{i0} - \nu_a, \quad \nu_{i0} = \nu_i(E_0).$$

We see that the propagation of a high-frequency streamer along the direction of $\mathbf{E}_0$ occurs with an exponentially increasing velocity $v_\parallel$ (which is determined by the field at the head of the streamer, E_n). The rapid decrease in the depolarization coefficient n_x which occurs in the process maintains the field in the plasma, E_i, at the level of the unperturbed field E_0, so the avalanche ionization within a plasmoid continues at a nondecreasing rate. This process (and the increase in the velocity $v_\parallel$ along with it) may be terminated by some effect which we have ignored here, e.g., recombination, the decrease in the exponent β at large E_n to values of less than unity, or the skin effect for the field in the plasma.

The most important condition for the existence of solution (15)–(18) is that the velocity be a sufficiently strong function of the field amplitude ($v_\parallel^2 \propto E_n^\beta, \beta > 1$). This solution thus remains the same in form if the discharge propa-

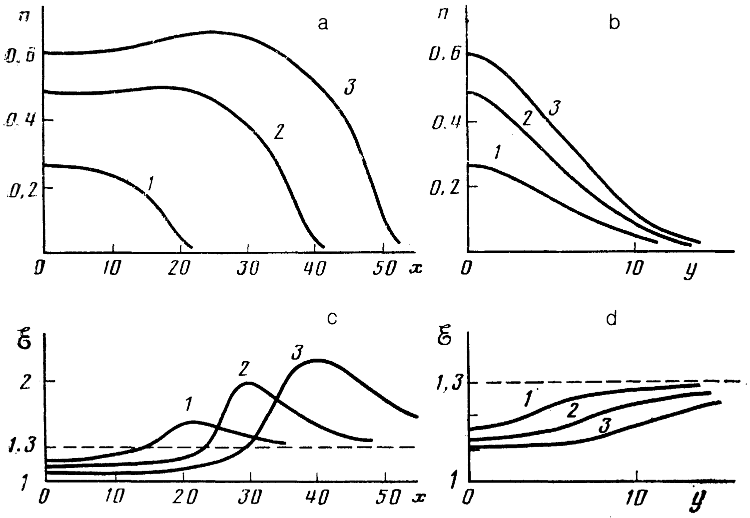

FIG. 1. Distributions of (a,b) the density and (c,d) the field amplitude in the longitudinal direction (x) and the transverse direction (y) with respect to the external field $\mathbf{E}_0$ at various dimensionless time t: 1—$t = 3.2$; 2—$t = 4.3$; 3—$t = 6.0$.

gates by some other mechanism (other than the diffusion mechanism). For example, it would remain the same in form for a regime in which the head of the streamer was a source of ionizing UV radiation; this regime occurs in strong fields and leads to fairly high velocities.[7]

At low collision rates, $\nu \ll \omega$, the evolution of the discharge becomes complex, and an analytic description of if (even at the level of qualitative models) is a separate, independent problem. Here are the three most important aspects of this case:

1) The field intensifies (more precisely, its component parallel to ∇N does) near the plasma-resonance surface $N = N_c$).

2) A general dipole (or multipole) resonance of the plasmoid can occur.[2)]

3) The maximum of the external field on the plasmoid boundary (in a certain interval of average values $N \sim N_c$) shifts away from the polar regions to the equator. This circumstance may be related to the appearance (in a certain intermediate stage of the evolution) of a tendency for the plasmoid to expand predominantly in the direction perpendicular to $\mathbf{E}_0$ (in a process accompanied by the formation of an oblate ellipsoid). In the subsequent stages, however (for $N \gg N_c$), the field maxima must nevertheless shift toward the poles, and the evolution of the discharge should evidently occur as described above (for the case $\nu/\omega \gg 1$).

4. A numerical simulation of the dynamics of a high-frequency streamer has been carried out for the two-dimensional case $\varphi = \varphi(x,y,t), N = N(x,y,t)$ under the following initial and boundary conditions:

$$N(x,y,0) = \begin{cases} N_1; & |x|, |y| < l, \\ 0; & |x|, |y| > l, \end{cases} \tag{19}$$

$$N(\pm L_1, \pm L_2, t) = 0,$$
$$\nabla\varphi(\pm L_1, \pm L_2, t) = -\mathbf{x}_0 E_0. \tag{20}$$

The boundaries $x = \pm L_1$, $y = \pm L_2$ are far from the ionization region, so the solution found must correspond closely to the asymptotic condition (5) over the entire time interval of the calculation.

The system of equations (1), (2) has been solved iteratively by Newton's method with the help of relationship (3) in finite-difference form on a rectangular mesh with a nonuniform step. We solved the system of linearized difference equations by tridiagonal inversion. The results of the numerical calculations for the parameter values $\beta = 4$, $\alpha = 0$, $N_1/N_c = 10^{-2}$, $l(\nu_a/D)^{1/2} = 1$, $\nu/\omega = 10$, and $\mathscr{E}_0 = E_0/E_c = 1.3$ are shown in Fig. 1 as plots of $n = N/N_c$ and the field amplitude $\mathscr{E} = |\mathbf{E}|/E_c$ versus the longitudinal (x) and transverse (y) coordinates at various times t. The time and length units of these figures are respectively the reciprocal of the attachment rate, ν_a^{-1}, and the attachment diffusion length $L_a = (D/\nu_a)^{1/2}$ (for convenience, we have introduced the dimensionless variables $t \to \nu_a t$, $x \to x/L_a$, $y \to y/L_a$).

It can be seen from this figure that the picture of the discharge evolution drawn by the numerical solution is close to that predicted on the basis of the qualitative model discussed above. In the initial stage of the process, while the condition $|\varepsilon - 1| \ll 1$ holds, the numerical solution is close to the known symmetric solution [similar to (6)] of the initial-value problem for a linear two-dimensional diffusion equation. For $t \gtrsim 1$, with $|\varepsilon - 1| \gtrsim 1$, the symmetry is disrupted: The field becomes significantly stronger in the polar regions, and the discharge stretches out along the external field $\mathbf{E}_0$. In particular, at the time $t \approx 6$ the ratio of the longitudinal dimension a and the transverse dimension b of the region at whose boundary the density has decreased to half its maximum value is $a/b \approx 10$. The time dependence of the longitudinal and transverse velocities $[da/dt = v_\parallel(t), db/dt = v_\perp(t)]$ found through the numerical calculation agrees satisfactorily with expression (7), in which the ionization rate ν_i should be understood as its maximum value in the corresponding part of the boundary region of the plasma.

The asymptotic regime in (15)–(18), which apparently requires an extremely large amount of computer time to reach, is not reached over the time interval of these calculations ($t \leqslant 6$). We observe some deviations from the model of a uniform ellipsoid: 1) The boundary of the ellipsoid remains blurred to a comparatively large degree, and small density maxima in which N is roughly 10% higher than the value at the center appear near the ends of the plasmoid ($y = 0$, $x = 25$) at $t \approx 6$. 2) The maxima of the field amplitude at the ends do not reach the values determined by (12),

$E_n = |\varepsilon| E_i$, and in the central region the field amplitude decreases slightly as time elapses (although for $t \geqslant 4$ this decrease, which does not exceed $E_0 - E_c$, essentially comes to a halt).

5. In conclusion, we present some numerical estimates and discuss the conditions under which these dynamic structures might be realized in the laboratory. Evidently a sufficient condition for the formation of high-frequency streamers which are isolated (i.e., which do not interact with each other) during the breakdown of a gas is that the electron diffusion length $L_e = (D\tau)^{1/2}$ be small in comparison with the average distance between the primary initializing electrons, $L_s = N_s^{-1/3}$:

$$(D\tau)^{1/2} \ll N_s^{-1/3}. \tag{21}$$

Here $\tau = (\nu_{i0} - \nu_a)^{-1} \ln (\overline{N}/N_0)$ is the time over which the electron density increases to the level $\overline{N} \sim (\nu/\omega) N_{c0}$, which corresponds (with $\nu > \omega$) to the appearance of strong field perturbations, $N_0 \propto [(\nu_i - \nu_a)/D]^{3/2}$ is the value of N found from (6) at the time $t \propto (\nu_i - \nu_a)^{-1}$; and N_s is the density of initializing electrons in the gas.

Using data in the literature[6,8] to estimate the values of the quantities involved here for various types of rf and microwave breakdown of gases, we find that condition (21) can be satisfied at source power levels which are quite feasible (which provide a sufficiently high ionization rate ν_{i0}) and at moderate initializing densities, $N_s \sim 1\ \mathrm{cm}^{-3}$.

In particular, under the experimental conditions of Ref. 1, where a discharge produced by short, intense microwave pulses was studied (the field frequency was $\omega \approx 6 \cdot 10^{10}\ \mathrm{s}^{-1}$, the field amplitude was $E_0 \approx 10^4$ V/cm, and the pulse length was $\tau_p \approx 10^{-8}$ s), for a gas (helium) pressure $P \approx 50$ torr ($\nu/\omega \gtrsim 2$), and for $N_s \sim 1\ \mathrm{cm}^{-3}$, we find

$$\nu_{i0} \approx 10^8 P \approx 5 \cdot 10^9\ \mathrm{s}^{-1}, \quad D \approx 10^7/P \approx 2 \cdot 10^5\ \mathrm{cm}^2/\mathrm{s},$$
$$\ln(\overline{N}/N_0) \approx 20, \quad L_e = (D\tau)^{1/2} \approx 0{,}03\ \mathrm{cm} \ll N_s^{-1/3}.$$

It is in this region of parameter values that a discharge consisting of a multitude of small plasmoids, stretched out along the field (with dimensions $b \sim 0.2$–0.5 cm and $a \sim 1$–2 cm), was observed in Ref. 1. For these experimental conditions, we can estimate limiting parameters of process (15)–(18), which are determined in this case by the decrease in the steepness index of the $\nu_i(|\mathbf{E}|)$ curve to values $\beta < 1$. These estimates yield

$$\nu_{i,\,max} \approx 10^9 P \approx 5 \cdot 10^{10}\ \mathrm{s}^{-1},$$
$$v_{\parallel,\,max} \approx (D\nu_{i,\,max})^{1/2} \approx 10^8\ \mathrm{cm/s},$$
$$|\varepsilon|_{max} = E_{n,\,max}/E_0 \approx 10, \quad N_{max} \approx 10\overline{N} \approx 10^{13}\ \mathrm{cm}^{-3},$$
$$a/b \approx (|\varepsilon|_{max})^{1/2} \approx 3.$$

These limiting values are reached [the asymptotic regime in (15)–(18) reaches saturation] in an extremely short time, $\Delta t \approx 2/\nu_{i0} \approx 1$ ns, which is required for the density to increase from $\overline{N}$ to N_{max}.

In the same experiments,[1] when the initializing density was deliberately increased to $N_s \sim 10^5\ \mathrm{cm}^{-3}$, and also in Refs. 9 and 10, where a discharge was produced moderately far above the breakdown threshold ($\nu_i - \nu_a \sim 10^5$–$10^6\ \mathrm{s}^{-1}$), the condition $L_e \gtrsim L_s$, which is the opposite of (21), held. In this case the primary avalanches were able to coalesce before the density reached the level $N \approx \overline{N}$, and no small-scale plasmoids were observed (at least in the early stages after the breakdown, before the onset of ionization instabilities).

The theoretical model of a high-frequency streamer in a cold gas which we have examined here for external field amplitudes above the breakdown value ($E_0/E_c > 1$) cannot be used directly to describe the corresponding effect in fields below the breakdown level (with respect to the unperturbed gas).[4,11] In the latter case, the discharge occurs because of an external agent alone, and the discharge is sustained by the heating and thermal expansion of the gas (which lower the breakdown threshold). Nevertheless, the very fact that the field amplitude and the discharge propagation velocity increase on certain regions of the curved plasma boundary could evidently play a dominant role again in this case, leading to a rapid elongation (or branching) or plasmoids which are initially produced in the discharge (as a result of, for example, an ionization-thermal instability[12,13]) and which are stretched out along the field.

Possibly it was this mechanism (an ionization-thermal instability followed by a rapid growth of plasma filaments which formed as a result of the field intensification at their ends) which was responsible for the evolution of the "branching," "streamer," and "multifilament" discharges which were described in Refs. 4, 11, and 14. A comprehensive and reliable theoretical description of the dynamics of the corresponding processes will of course require more elaborate analytic and numerical models.

[1)] That this effect might play a role in an rf discharge was also mentioned in Refs. 1 and 3 in connection with a discussion of experiments in which plasmoids elongated along the field were observed. These plasmoids were generated by breakdown at isolated electrons.[1] An externally sustained "branching" discharge was also observed in fields below the breakdown level in those experiments.[4]

[2)] For a homogeneous sphere, the resonance condition for the m-th multipole is $\varepsilon m + m + 1 = 0$ ($m = 1,2,3,\ldots$). In homogeneous formations, the resonances are strongly suppressed by the loss near the plasma-resonance surface.

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