Neutrino energy losses in matter

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A consistent theory for the slowing down of neutrinos by an arbitrary stable homogeneous isotropic material medium is developed. The neutrino energy losses are reduced to five (in place of the two for the case of a charged particle) properties of the medium, including the axial generalizations of its permittivities. A number of general properties of these characteristics (in particular, the relevant sum rules) permit the determination of a rigorous and universal upper bound on the neutrino energy losses. Consequently, the neutrino energy losses are less than a value on the order of the collision limit and can never be anomalously large.

1. INTRODUCTION

In passing through matter a fast particle transfers to it the energy Q per unit time. A rough estimate of such energy losses (EL) is given by the so-called "collision limit"

$$Q_0=nv\int_0^{\omega_0} d\omega\omega\frac{d\sigma}{d\omega}. \tag{1.1}$$

Here σ is the cross section for scattering of the particle by a particle of the medium at rest with energy transfer ω below the kinematic limit $\omega_0=2E^2/(2E+m)$; E and v are the energy and velocity of the particle in the rest frame of the medium, slowly changing along the particle trajectory due to the assumed weakness of its interaction with the medium; m and n are the mass and number density of the particles of the medium; here and below $\hbar=c=1$.

The true energy losses Q may differ significantly from Q_0 due to collective effects of the medium, connected with the interaction and correlations of its particles. Thus, for the EL of a charged particle these effects suppress the contribution of distant collisions and "cut off" the divergence of Q_0, due to the long-range Coulomb forces, through the screening of the particle charge by the medium (the Fermi density effect[1,2]). The description of the contribution of the collective effects constitutes the central point of the EL theory for any type of particle.

In this paper we discuss the EL of the neutrino—a particle, whose action on the medium is restricted to the weak interaction, which cause the quantity (1.1) to be small (for neutrinos of not too high energy). This leads to the main question for the theory of the EL of the neutrino, which is the question of the possible existence of a medium in which the collective effects substantially enhance the size of the EL, giving rise to the inequality

$$Q\gg Q_0. \tag{1.2}$$

Up until now the answer to this question has not been clear. The situation described above for the case of a charged particle speaks against the possibility that the inequality (1.2) could be satisfied. On the other hand, it has been repeatedly asserted in recent years theoretically and experimentally that the neutrino suffers anomalously large EL, substantially larger than (1.1) (see, e.g., Refs. 3 and 4).

The main goal of this paper is to obtain a definitive (negative) answer to this problem (see also the comment of the authors of Ref. 5). This requires the development of a systematic theory of neutrino EL, which results in a rigorous and universal upper bound on Q, coinciding in order of magnitude with (1.1). This result leaves no hope for a noticeable increase of neutrino EL due to collective effects of the medium.

The plan of the article is as follows. In Sec. 2 the general theory of EL in a medium in thermodynamic equilibrium is discussed in application to the most important case when the interaction Hamiltonian of the fast particle with the particles of the medium has the form of "current $\times$ potential" or "current $\times$ current". Section 3 contains the formulation of the theory of EL of a charged particle taking into account recoil, spin effects, and also nonzero temperature of the medium. In Sec. 4 the theory of neutrino EL is formulated. The microscopic approach to the calculation of the characteristics of the medium that enter this theory are discussed in Sec. 5, while in Sec. 7 these same characteristics are discussed from the point of view of the general theory of response functions. Section 6 contains an illustration of the general relations for the example of the simplest model of a medium, giving rise to (1.1). In Sec. 8 the expression for the upper bound on the neutrino EL is obtained. Finally, in Sec. 9 a general summary of the article is given. For simplicity we consider only nonrelativistic, homogeneous and isotropic media.

2. GENERAL RELATIONS

The expression for the EL of a particle has in general the form

$$Q=-\langle\overline{\dot{H}_0}\rangle,\ \langle H_0\rangle=\mathrm{Sp}(\rho H_0), \tag{2.1}$$

where H_0 is the particle Hamiltonian, ρ is the density matrix of the "particle + medium" system, the dot denotes differentiation with respect to time, the bar denotes a time average over a large (compared to characteristic times of the medium) interval, during which the velocity of the particle is practically constant.

In the interaction picture the quantity ρ obeys the equation

$$i\dot{\rho}=[H',\rho]$$

with the initial condition $\rho=\rho_0\rho_1$ for $t\to-\infty$, where ρ_0 is the polarization density matrix of the particle and ρ_1 is the density matrix of the medium. The Hamiltonian for the interaction of the particle with the medium is taken in the form

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$$H' = -\int d\mathbf{x} j^{\mu}(x) A_{\mu}(x), \tag{2.2}$$

where $x = (t, \mathbf{x})$, j is the particle current vector, A is a field conjugate to it, which for a charged particle coincides with the electromagnetic potential in the medium, and for the neutrino coincides with a linear combination of the vector and axial vector currents of the particles of the medium (see Sec. 4, below).

To lowest (second) order in H' the expression (2.1) can be brought to the form

$$Q = \int d^4 k k_0 \varkappa_{\mu\nu}(k) K^{\mu\nu}(k), \tag{2.3}$$

where we have introduced the correlators

$$\varkappa_{\mu\nu}(x) = \langle A_{\mu}(x) A_{\nu}(0) \rangle,$$
$$K^{\mu\nu}(x) = \langle j^{\mu}(x) j^{\nu}(0) \rangle \tag{2.4}$$

(k_{μ} is the 4-momentum transferred by the particle to the medium). Equation (2.3) can also be obtained from the expression for the imaginary part of the mass operator for the particle in the medium (see Ref. 6).

By introducing eigenfunctions of the Hamiltonian of the medium with 4-momentum p_{μ}^{n} the correlator $\varkappa$ [see (2.4)] can be brought to the form

$$\varkappa_{\mu\nu}(k) = \sum_{m,n} \rho_{1m} (A_{\mu}(0))_{mn} (A_{\nu}(0))_{nm} \delta^4 (p^m - p^n + k), \tag{2.5}$$

where the density matrix of the medium with temperature T and free energy F has the form

$$\rho_{1m} = \exp[(F - E_m)/T].$$

This correlator is connected with the retarded Green's function of the field A

$$D_{\mu\nu}(x) = i\theta(t) \langle [A_{\mu}(x), A_{\nu}(0)] \rangle \tag{2.6}$$

by the relation that follows from the fluctuation–dissipation theorem,[7]

$$\varkappa_{\mu\nu}(k) = (1 + \operatorname{cth}(k_0/2T)) \operatorname{Im} D_{\mu\nu}(k). \tag{2.7}$$

As a result of current conservation the correlator K [see (2.4)] satisfies the transversality conditions

$$k_{\mu} K^{\mu\nu} = 0, \quad K^{\mu\nu} k_{\nu} = 0. \tag{2.8}$$

If the external particle is a fermion with vector current

$$j_{\mu} = \bar{\psi} O^{\mu} \psi,$$

then the correlator takes on the form

$$K^{\mu\nu}(k) = \frac{1}{2p_0} \langle O^{\mu} S^{-}(p-k) O^{\nu} - O^{\nu} S^{+}(p+k) O^{\mu} \rangle, \tag{2.9}$$

where p_{μ} is the particle momentum 4-vector and

$$S^{-} = \langle 0 | \psi(x) \bar{\psi}(0) | 0 \rangle,$$
$$S^{+} = \langle 0 | \bar{\psi}(0) \psi(x) | 0 \rangle$$

are vacuum expectation values of free field operators.

Substitution of Eqs. (2.7) and (2.9) in the general formula (2.3) gives the relation that explicitly defines the EL of the fermion that interacts with the medium according to the law (2.2). It is convenient to extract from the expression for the EL the part that does not explicitly depend on the temperature[1]

$$q = \frac{1}{\pi} \int_0^{\infty} dk_0 k_0 \int d^3 k K^{\mu\nu} \operatorname{Im} D_{\mu\nu}. \tag{2.10}$$

The remaining temperature-dependent addition, vanishing for $T = 0$, has the form

$$Q_T = \int d^4 k (k_0 \operatorname{cth}(k_0/2T) - |k_0|) K^{\mu\nu} \operatorname{Im} D_{\mu\nu}. \tag{2.10'}$$

The absence of contributions from negative k_0 in (2.10) is connected simply with the fact that the medium in its ground state cannot emit energy.

3. LOSSES BY A CHARGED DIRAC PARTICLE

The simplest application of the general formulas of Sec. 2, of interest in its own right and simplifying the passage to the case of the neutrino, is to the problem of EL of a charged particle, when A coincides with the potential and (2.6) coincides with the Green's function of the photon in the medium. In covariant form this function looks as follows ($g_{\mu\nu}$ is the metric tensor, u_{μ} is the 4-velocity of the medium as a whole)

$$D_{\mu\nu} = -R_1 g_{\mu\nu} + R_2 u_{\mu} u_{\nu} + \ldots, \tag{3.1}$$

where the dots stand for terms proportional to k_{μ} or k_{ν} that do not contribute to (2.3) [see (2.8)]. In the rest frame of the medium ($u_0 = 1$, $\mathbf{u} = 0$) we have

$$R_1 = d_t, \quad R_2 = d_l - d_t k^2/\mathbf{k}^2, \tag{3.2}$$

where the longitudinal and transverse components of the Green's function equal

$$d_l = -4\pi/\mathbf{k}^2 \varepsilon_l, \quad d_t = 4\pi/(\mathbf{k}^2 - k_0^2 \varepsilon_t)$$

and ε_l (ε_t) is the longitudinal (transverse) dielectric permittivity.

The quantities needed for the evaluation of the correlator K have the form

$$O^{\mu} = e_0 \gamma^{\mu}, \quad \rho_0 = (\hat{p} + M)/2,$$
$$S^{\pm} = \mp\theta(\mp p_0)(\hat{p} + M)\delta(p^2 - M^2), \tag{3.3}$$

where e_0 and M is the charge and mass of the particle (unpolarized, for simplicity). Then Eq. (2.9) gives

$$K^{\mu\nu} = \frac{2\pi e_0^2}{E} [\theta(E - k_0)\delta(k^2 - 2pk) B^{\mu\nu}(k) + \theta(-E - k_0)\delta(k^2 + 2pk) B^{\mu\nu}(-k)], \tag{3.4}$$

with $E = p_0 = (p^2 + M^2)^{1/2}$ and

$$B^{\mu\nu} = \frac{1}{4} \operatorname{Sp}[(\hat{p} + M)\gamma^{\mu}(\hat{p} - \hat{k} + M)\gamma^{\nu}]$$
$$= 2p^{\mu}p^{\nu} - p^{\mu}k^{\nu} - k^{\mu}p^{\nu} + (pk) g^{\mu\nu}. \tag{3.5}$$

When Eqs. (3.1), (3.2), and (3.4) are substituted into the relations (2.10) and (2.10') it is convenient to go over to new variables $\omega = k_0$ and $t = -k^2$, the energy transfer and the negative of the square of the 4-momentum transfer. Introducing the notation

$$\Sigma = B^{\mu\nu} \operatorname{Im} D_{\mu\nu} \tag{3.6}$$

and taking into account the equality $\operatorname{Im} D_{\mu\nu}(k) = -\operatorname{Im} D_{\mu\nu}(-k)$, which follows from Eq. (2.6), we arrive at the following expressions for the "cold" part of the EL

$$q=\frac{e_0^2}{4\pi^2 Ep}\int_0^{E-M} d\omega\omega\int_{t_-}^{t_+} dt\Sigma(\omega,t) \qquad (3.7)$$

and the temperature-dependent addition[2]

$$Q_T=\frac{e_0^2}{8\pi^2 Ep}\left[\int_{-\infty}^{E-M}-\int_{E+M}^{\infty}\right]d\omega|\omega|[\mathrm{cth}(|\omega|/2T)-1]$$
$$\times\int_{t_-}^{t_+} dt\Sigma(\omega,t), \qquad (3.7')$$

where $p=(E^2-M^2)^{1/2}$ and

$$t_\pm=2[p^2-E\omega\pm p(p^2-2E\omega+\omega^2)^{1/2}]$$

[for a massless particle $t=0$, $t_+=4E(E-\omega)$ hold for $\omega<E$]. According to Eqs. (3.1) and (3.5) we have

$$\Sigma=2E^2\,\mathrm{Im}\,[d_l T_-+d_t(T_+t/(t+\omega^2)-M^2/E^2)],$$
$$T_\pm=(1-\omega/2E)^2\pm(t+\omega^2)/4E^2. \qquad (3.8)$$

For a classical particle, whose energy and momentum are large compared to ω and $t^{1/2}$, the effects of recoil, spin, etc., can be neglected.

Substitution into (3.7) of the relevant formula for Σ

$$\Sigma=2E^2\,\mathrm{Im}\,[d_l+d_t(v^2-(\mathbf{kv})^2/\mathbf{k}^2)]$$

leads to the familiar expression for the EL following from macroscopic electrodynamics.[8] Therefore for a classical particle we have

$$Q_T=0, \qquad (3.9)$$

as is directly seen from the fact that the integrand in (3.7') is odd in ω.

This is in contradiction with the recent results in Pardy,[9] who gives a temperature-dependent modification of the factor $a=v^2-1/n^2$ in the Tamm-Frank formula for Cherenkov radiation [$n(\omega)$ is the index of refraction of the medium]:

$$a=\mathrm{cth}(n\omega/2T)(v^2-1/n^2).$$

The correct (to terms linear in ω/E) modification

$$a=v^2-1/n^2-\frac{\omega}{E}\mathrm{cth}(\omega/2T)(1-1/n^2)$$

corresponds to (3.9), and for $T=0$ the condition $a\geqslant 0$ coincides with the quantum condition for Cherenkov radiation.[10]

4. NEUTRINO LOSSES

When considering the massless two-component neutrino one may assume that the momentum transfer in the interaction with the medium is small compared to the masses of the intermediate bosons. Therefore the Hamiltonian for such an interaction has the 4-fermion form and is described by Eq. (2.2) with the neutrino vector current in its standard form (Sec. 2) with

$$O^\mu=\frac{G}{2^{1/2}}\gamma^\mu\Gamma, \quad \Gamma=(1+\gamma^5)/2,$$

and the field A for the interaction of the neutrino with the medium given by

$$A_\mu=C_V I_\mu{}^V+C_A I_\mu{}^A=C_a I_\mu{}^a. \qquad (4.1)$$

Here $I_\mu V=\bar\psi\gamma_\mu\psi$ is the vector, and $I_\mu^A=\bar\psi\gamma_\mu\gamma_5\psi$ is the axial vector electron current, $C_V=\pm 1+4\sin^2\theta_W$, $C_A=\pm 1$ (upper sign for the electron neutrino, lower sign for the muon neutrino), θ_W is the Weinberg angle, G is the Fermi constant, and the letters $a, b, \ldots$ combine the V and A indices with summation over repeated indices understood. The interaction of the neutrino with the nucleon has an analogous form, as does the interaction of the antineutrino with the electron and the nucleon.

The neutrino EL is described by the general Eqs. (2.10) and (2.10'), as well as by the Eqs. (3.4), (3.6), (3.7), and (3.7') after the replacement of e_0^2 by $G^2/2$ and after setting $M=0$. In particular, in a "cold" medium, to which we confine attention from now on, the neutrino EL has the form

$$Q=q=\frac{G^2}{16\pi^2E^2}\int_0^{4E^2} dt\int_0^{\bar\omega} d\omega\omega\Sigma(\omega,t),$$
$$\bar\omega=E-t/4E. \qquad (4.2)$$

The limits of integration in this formula reflect directly the kinematics of the act of 4-momentum transfer k_μ from the neutrino to the medium.

To determine Σ [see (3.6)] it is necessary to repeat the derivation of the expression for the tensor (3.5), having replaced $\hat p+M$ by $\hat p\Gamma$ in Eqs. (3.3) that determine the density matrix and the vacuum averages. This gives

$$B^{\mu\nu}=\frac{1}{4}\mathrm{Sp}[\hat p\Gamma\gamma^\mu(\hat p-\hat k)\gamma^\nu]$$
$$=2p^\mu p^\nu-p^\mu k^\nu-k^\mu p^\nu+(pk)g^{\mu\nu}-i\varepsilon^{\mu\nu\rho\sigma}k_\rho p_\sigma. \qquad (4.3)$$

The tensor $D_{\mu\nu}$ is, according to Eq. (4.1), equal to the quantity $C_a C_b D_{\mu\nu}ab$, where

$$D_{\mu\nu}{}^{ab}=i\theta(t)\langle[I_\mu{}^a(x), I_\nu{}^b(0)]\rangle. \qquad (4.4)$$

The components diagonal in the a, b indices have the form (3.1)

$$D_{\mu\nu}{}^{aa}=-R_1{}^a g_{\mu\nu}+R_2{}^a u_\mu u_\nu+\ldots, \qquad (4.5)$$

while the off-diagonal ones are represented in the form (m is the electron mass)

$$D_{\mu\nu}{}^{VA}(k)=D_{\nu\mu}{}^{AV}(-k)=ie_{\mu\nu\rho\sigma}k^\rho u^\sigma R_3/2m. \qquad (4.6)$$

In the rest frame of the medium

$$R_1{}^a=d_t^a, \quad R_2{}^a=\frac{k^2}{\mathbf{k}^2}\left(\frac{k^2}{\mathbf{k}^2}d_l{}^a-d_t^a\right), \quad R_3=d_C. \qquad (4.7)$$

The meaning of the quantities d that enter here will be explained below.

Substitution of Eqs. (4.3)–(4.7) into (3.6) gives

$$\Sigma=\mathrm{Im}[tC_a^2R_1{}^a+2E^2C_a^2R_2{}^aT_-+2EtC_VC_AR_3T_C/m]$$
$$=\frac{2E^2t}{t+\omega^2}\mathrm{Im}\left[\frac{t}{t+\omega^2}C_a^2d_l{}^aT_-+C_a^2d_t^aT_+\right.$$
$$\left.+(t+\omega^2)C_VC_A\,d_CT_C/mE\right], \qquad (4.8)$$

where the quantities $T_\pm$ are defined by Eq. (3.8) and $T_C=1-\omega/2E$. Equations (4.2) and (4.8) together determine the neutrino EL, which is thus described by five characteristics of the medium $R^a_{1,2}$, R_3 or $d^a_{l,t}$, d_C (in contrast to the two characteristics needed for the description of the EL by a charged particle; see Sec. 3). A special microscopic calculation is needed to determine these characteristics; only two of them—$d^V_{l,t}$—expressible in terms of $d_{l,t}$ can be borrowed from standard macroscopic electrodynamics. The increase in the number of the characteristics of the medium needed to describe the neutrino EL is ultimately connected with parity nonconservation: for a charged particle it is sufficient to know the vector-like reaction of the medium to the vector interaction, while for the neutrino axial currents also enter the picture.

Of greater importance in practice is the difference between the neutrino and charged particle EL due to the fact that the former increases with energy like a power, while the latter increases only logarithmically. This fact reflects the nonrenormalizability of the 4-fermion interaction.

5. MEDIUM CHARACTERISTICS

The two alternate sets of medium characteristics—R and d, determining the EL, have different meaning: the first set plays the role of response functions (see below Sec. 7), the second is directly connected to the microscopics of the medium and can be calculated with the help of familiar methods of many-body theory.

The quantities d are connected by the relations (4.5)–(4.7) to the Green's functions (4.4). Its D^{VV} component coincides (accurate up to the factor $\eta = 1/4\pi e^2$) with the photon self-energy part, expressed in terms of the compact self-energy part (polarization operator) and the Green's function of the photon (3.1). The same applies to the remaining components D^{ab}, which may be expressed in the form (see Fig. 1)

$$D^{ab}_{\mu\nu} = \eta\left[P^{ab}_{\mu\nu} + \frac{1}{4\pi} P^{aV}_{\mu\rho} D_{\rho\sigma} P^{Vb}_{\sigma\nu}\right], \qquad (5.1)$$

where the polarization operator P is a closed loop with vector or axial vertices (e is the electron charge):

$$P^{ab}_{\mu\nu} = 4\pi i e^2 \int d^4p \, \mathrm{Sp}[\Gamma_\mu{}^a G(p+k) \gamma_\nu{}^b G(p)], \qquad (5.2)$$

Here G and Γ^a are the exact electron Green's function and vertex parts, $\gamma^V_\nu = \gamma_\nu$, $\gamma^A_\nu = \gamma_\nu \gamma_5$. It is precisely the quantity (5.2) that is calculated in one or another approximation in many-body theory.

The quantity P^{ab}, just like D^{ab}, may be parametrized by five scalars $P^a_{l,t}$, P_C with the help of relations similar to (4.5)–(4.7). The quantities d of interest to us are given in terms of these scalars and the components of the photon Green's function (Sec. 3) with the help of Eq. (5.1):

$$\begin{gathered} d_l{}^V = \eta \mathbf{k}^2(1 + \mathbf{k}^2 d_l/4\pi), \\ d_t{}^V = \eta k^2(1 + k^2 d_t/4\pi), \\ d_l{}^A = \eta P_l{}^A, \; d_t{}^A = \eta\,[P_t{}^A - k^2 d_t P_C{}^2/(16\pi m^2)], \\ d_C = -\eta k^2 d_t P_C/4\pi, \end{gathered} \qquad (5.3)$$

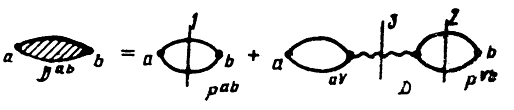

FIG. 1.

where

$$\begin{gathered} d_l = -4\pi/(\mathbf{k}^2 + P_l{}^V), \; P_l{}^V = \mathbf{k}^2(\varepsilon_l - 1), \\ d_t = -4\pi/(k^2 + P_t{}^V), \; P_t{}^V = k_0{}^2(\varepsilon_t - 1). \end{gathered} \qquad (5.4)$$

The quantities $\mathrm{Im}\, d$ that enter the EL (4.8) describe the excitation of the medium by the energy lost by the neutrino. Since the imaginary part of the Feynman diagram corresponds to processes whose diagrams are obtained by all possible cuts, the first term in (5.1) (cut 1 in Fig. 1) describes the process of "particle-hole" pair production by the weak neutrino-electron interaction. The second term in (5.1) adds to this the processes of indirect pair production through polarization of the medium induced by the weak interaction (cut 2), as well as the production of collective excitations—plasmons, excitons, phonons, etc. (cut 3). The collective effects discussed in the Introduction correspond to these last two types of processes. These processes can be described with the help of a neutrino electromagnetic form factor, reflecting the appearance of a charge and current distributions[3] induced by its weak interactions[11]

$$G\gamma^\mu \Gamma C_a P_{\mu\nu}{}^{aV}/2^{1/2} \qquad (5.5)$$

(see the dashed rectangle in Fig. 1).[3]

In the case of a nonrelativistic medium its characteristics $d^A_{l,t}$, d_C, which do not appear in ordinary macroscopic electrodynamics, may be expressed through the spin magnetization operator

$$\mathbf{M} = \frac{e}{2m} \psi^+ \boldsymbol{\sigma} \psi$$

(σ_i are Pauli matrices). Introducing the operators $\mathbf{j}^S = \nabla \times \mathbf{M}$ (spin current) and $\tau = \nabla \cdot \mathbf{M}$ and making use of (4.4) one may verify that the quantities

$$d_l{}^A e^2 k^4/4m^2 k_0{}^2, \; d_t{}^A e^2 \mathbf{k}^2/2m^2, \; -d_C e^2 \mathbf{k}^2/2m^2$$

coincide respectively with the expectation values of the retarded commutators of the pairs of operators (τ,τ), $(j_i{}^S j_i{}^S)$ and $(j_i{}^V, j_i{}^S)$, with summation over i and 1 to 3 and with $\mathbf{j}^V$ the vector current.

6. THE COLLISION LIMIT

The simplest model of a medium illustrating the content of the previous Section is provided by a degenerate weakly nonideal electron gas with homogeneous positive substrate. The Green's function and vertex parts in (5.2) may be viewed as free in such a model:

$$G^{-1}(p) = \hat{p} + \mu \hat{u} - m, \quad \mu = (p_F{}^2 + m^2)^{1/2}, \quad \Gamma_\mu{}^a = \gamma_\mu{}^a$$

(p_F is the Fermi momentum, μ is the chemical potential). Passing to the nonrelativistic limit one obtains the following expressions for the components of the polarization operator of the nonrelativistic medium in its rest frame:

$$\begin{gathered} P_l{}^V = P_t{}^A = -\frac{\mathbf{k}^2}{k^2} P_C = -2m\omega_p{}^2 \langle \Omega/L \rangle, \\ P_t{}^V = -\omega_p{}^2\left[1 + \frac{1}{2m^2} \langle [\mathbf{pk}]^2/L \rangle\right]. \\ P_l{}^A = -2\frac{\omega_p{}^2 k_0{}^2}{k^2} \langle (\mathbf{pk})/L \rangle. \end{gathered} \qquad (6.1)$$

Here $\omega_p{}^2 = 4\pi n e^2/m$ is the square of the plasma frequency,

$\Omega = [\mathbf{k}^2 + 2(\mathbf{pk})]/2m$ is the excitation energy, $L = k_0^2 - \Omega^2 + i\delta k_0$, and the angular brackets refer in this case to averaging over the momentum $\mathbf{p}$ inside the Fermi sphere. The quantities $P^V_{l,t}$ are given by the expressions of ordinary macroscopic electrodynamics.

We shall not present the unwieldy expression for the neutrino EL that results from the substitution of (6.1) into (5.3), (4.8) and (4.2) but confine ourselves to the consideration of the formal limit when the electron concentration n tends to zero. All collective effects disappear in that limit and the EL coincides with the collision limit (1.1). Indeed, as $n \to 0$, the relations (5.3) take the form $d = \eta P$ (with appropriate indices), which after going to the variables ω, t yields[4]

$$\operatorname{Im} d = \frac{\pi n}{\omega} \xi \delta(t - 2m\omega),$$

$$\xi_l^V = \xi_l^A = t + \omega^2,$$

$$\xi_t^A = 0, \quad \xi_t^V = \omega^2, \quad \xi_C = -t.$$

As a result the neutrino EL takes the form

$$Q = \frac{G^2 mn}{4\pi} \int_0^{\omega_0} d\omega\, \omega (C_V^2 \tau_- + C_A^2 \tau_+ + 2C_V C_A \tau_C), \tag{6.2}$$

where $\omega_0 = 2E^2/(2E + m)$ and

$$\tau_\pm = 1 - \frac{\omega}{E} + \frac{\omega(\omega \pm m)}{2E^2},$$

$$\tau_C = \left(1 - \frac{\omega}{2E}\right) \frac{\omega}{E}$$

Precisely the same expression is obtained on substitution in (1.1) of the familiar formula for the inelastic νe-scattering cross section,[13] which confirms the existence of the equality $Q = Q_0$ as $n \to 0$.

It is important to emphasize that this derivation of (6.2) has a purely formal meaning since at low density the electron gas is strongly nonideal: The Coulomb energy is large compared to the kinetic energy and crystallization of the system takes place (into a Wigner crystal). Correspondingly the collision limit itself serves, generally speaking, only as a formal measure of the scale of the neutrino EL.

However the quantity (1.1) acquires real meaning at relatively high neutrino energy, when the energy transferred to the medium, which is of order of ω_0, is large compared to the characteristic energy $E_0 \ll m$ of the particle of the (nonrelativistic) medium

$$E \gg (mE_0)^{1/2}, \tag{6.3}$$

which permits the neglect of the initial motion of that particle. On the other hand, when (6.3) is satisfied the momentum $|\mathbf{k}|$ transferred to the medium is of order E (on the "Bethe comb" $t \propto m\omega$; see above) and is large on the scale of internal characteristics of the medium. Consequently the neutrino scattering process is of individual, rather than collective (close collisions), character and therefore when (6.3) is satisfied the EL coincides with the collision limit (1.1):

$$Q = Q_0. \tag{6.4}$$

Consequently the collective effects of the medium manifest themselves only when the condition (6.3) is violated, i.e., in any event in the region $E \ll m$. In this region, to which we confine the discussion from now on, we have

$$Q_0 = \frac{G^2 n E^4}{6\pi m} (C_V^2 + 5C_A^2). \tag{6.5}$$

7. RESPONSE FUNCTIONS OF THE MEDIUM

The calculation of the characteristics of the medium that determine the EL in a more or less realistic model is a very difficult problem (the model of Sec. 6 is extremely idealized). A special role is therefore played by those general properties of the characteristics of the medium that are independent of a specific model.[12] It is these properties that will be considered in the present Section.

The Green's function (4.4) satisfies the condition of relativistic causality vanishing for $t \leqslant |\mathbf{x}|$ due to the presence of the θ-function and the vanishing of the commutator outside the light cone, i.e., for $t^2 \leqslant \mathbf{x}^2$. This leads to a number of general properties, possessed by any response function of the medium $R(\omega, \mathbf{k})$ that satisfies the indicated condition (see Ref. 14 and the Appendix). Namely the quantity $R(\omega, \mathbf{k} + \omega\mathbf{s})$, where $\mathbf{s}$ is an arbitrary vector with $s = 1$, is analytic for fixed $\mathbf{k}$ as a function of ω in the upper half-plane of that variable. From this we get the Leontovich dispersion relation

$$R(\omega, \mathbf{k} + \omega\mathbf{s}) = R_0 + \frac{2}{\pi} \int_0^\infty d\zeta\, \zeta \operatorname{Im} R(\zeta, \mathbf{k} + \zeta\mathbf{s})/(\zeta^2 - \omega^2 - i\delta\omega), \tag{7.1}$$

where R_0 is the limit of $R(\omega, \mathbf{k} + \omega\mathbf{s})$ as $\omega \to \infty$. If R is a scalar quantity referring to an isotropic medium then, upon setting $(\mathbf{ks}) = 0$, we have

$$R(\omega, \mathbf{k} + \omega\mathbf{s}) = R[\omega, (k^2 + \omega^2)^{1/2}].$$

Therefore in terms of the variables ω, t the quantity R is analytic in ω for fixed t, and the following relation is valid

$$R(\omega, t) = R_0 + \frac{2}{\pi} \int_0^\infty d\zeta\, \zeta \operatorname{Im} R(\zeta, t)/(\zeta^2 - \omega^2 - i\delta\omega). \tag{7.2}$$

At high frequencies the response function has the asymptotic behavior

$$R(\omega, t) = R_0 - \frac{\alpha(t)}{\omega^2} + \dots \tag{7.3}$$

Considering the corresponding limit in (7.2) we arrive at the sum rule

$$\frac{2}{\pi} \int_0^\infty d\zeta\, \zeta \operatorname{Im} R(\zeta, t) = \alpha(t). \tag{7.4}$$

As is shown in the Appendix, these properties of the response function are true not only of the tensor $D^{ab}_{\mu\nu}$ as a whole, but also of the scalars $R^a_{1,2}$, R_3 that parametrize it and which therefore obey the relations (7.2)–(7.4).

Another general property of the characteristics of the medium, relating to the sign of their imaginary parts, is formulated more conveniently in terms of the set $d^a_{l,t}$, d_C. From (2.5)–(2.7) and (4.4) follows the relation

$$[1+\operatorname{cth}(k_0/2T)]\operatorname{Im} D_{\mu\nu}^{ab} = \sum_{m,n} \rho_{1m}(I_\mu{}^a(0))_{mn}(I_\nu{}^b(0))_{nm}\delta^4(k+p_m-p_n),$$

which leads to the obvious inequality

$$\sum_j \bar f_{a,j}^{\ \mu} f_{b,j}^{\ \nu} \operatorname{Im} D_{\mu\nu}^{ab} \gtrless 0 \qquad (k_0=\omega\gtrless 0) \tag{7.5}$$

with arbitrary f. These quantities are subject below to the condition $k_\mu f^\mu_{j} = 0$ and therefore the terms omitted in (4.5) make no contribution to (7.5).

Below we obtain from (7.5) three inequalities for the quantities Im d, and correspondingly we choose three sets of coefficients f. The first two sets are different from zero for only one value of a and therefore only the diagonal tensor D^{aa} will enter (7.5). The first set corresponds to $f^0_{a,j} = 1, f^i_{a,j} = k_0 k_i/\mathbf{k}^2$ (index $j=1$), and upon substitution of (4.5) and (4.7) into (7.5) only the quantity Im d^a_l remains. The second set corresponds to $f^0_{a,j} = 0, f^i_{a,j} = \delta_{ij} - k_i k_j/\mathbf{k}^2$ ($j = 1, 2, 3$), and after the same substitutions only the quantity Im d^a_t remains. This leads to the inequalities

$$\operatorname{Im} d^a_{l,t} \gtrless 0 \qquad (\omega \gtrless 0), \tag{7.6}$$

whose meaning is that of stability conditions for the medium. The third set has the form (with ξ an arbitrary parameter)

$$f^0_{a,j}=0, \qquad f^i_{a,j}=C_a[i\xi\varepsilon_{ijk}k_k-(\delta_{ij}-k_ik_j/\mathbf{k}^2)],$$

and the substitution of (4.5)–(4.7) into (7.5) results in the inequality

$$(1+\xi^2\mathbf{k}^2)C_a^2 \operatorname{Im} d_t^a - 2\xi\frac{\mathbf{k}^2}{m}C_V C_A \operatorname{Im} d_c \gtrless 0, \qquad \omega\gtrless 0. \tag{7.7}$$

From (7.6) and (7.7) it follows, in particular, that for a cold medium

$$\Sigma > 0 \tag{7.8}$$

(such a medium is incapable of emitting energy). Indeed, the first term in the second square bracket in (4.8) in larger than zero in view of (7.6) and the positive sign of T_- in the integration region [see (3.8), (4.2)]. On the other hand the remaining terms reduce to the inequality (7.7) by the choice $\xi = (2E-\omega)^{-1}$ and add up to a positive quantity also.

8. UPPER BOUND ON THE NEUTRINO ENERGY LOSS

The general relations of the previous section lead to a number of universal results applicable to any medium of the type here considered. By these means we have previously obtained a universal formula for the EL of a charged ultrarelativistic particle.[2] On the other hand application to the neutrino leads to a determination of a universal upper bound on its EL.

We start from Eqs. (4.2) and (4.8) for the neutrino EL. It was already remarked at the end of Sec. 6 that only the region $E \ll m$ need be considered, where the quantity Q differs from Q_0. In this region the contribution of the last term (containing $C_V C_A$) in the square brackets in (4.8) is small in the parameter E/m because the quantity D^{AV} is linear in γ_5 [see (4.4)]. Omitting this term and keeping in mind that in the region of integration $T_+ \leqslant 1 + t/4E^2$ in (4.2) [see (3.8)], one readily obtains with (7.6) taken into account the following inequality:

$$\Sigma \leqslant (2E^2+t/2)C_a^2 \operatorname{Im} R_2^a. \tag{8.1}$$

Its right side is larger than zero for all ω so that when (8.1) is substituted into (4.2) we only strengthen the inequality by extending the integration over ω to infinity. The sum rule (7.4) now gives[5]

$$Q \leqslant \frac{G^2}{16\pi}\int_0^{4E^2} dt\left(1+\frac{t}{4E^2}\right)C_a^2\alpha_2^a(t). \tag{8.2}$$

It remains to determine the function α in the asymptotic behavior (7.3) of the functions R_2^a. For $a = V$ we obtain from Eqs. (4.7), (5.3), and (5.4)

$$R_2^V = \frac{t^2}{4\pi e^2(t+\omega^2)}\left[\frac{t}{t+\omega^2-\varepsilon_l\omega^2} - \frac{1}{\varepsilon_l}\right]$$

Making use of the familiar asymptotic behavior of the permittivity

$$\varepsilon_{l,t} = 1-\omega_p^2/\omega^2+\ldots, \tag{8.3}$$

we find

$$\alpha_2^V = \frac{nt^2}{[m(t+\omega_p^2)]}. \tag{8.4}$$

The denominator of this expression describes the effect of screening by the medium of the induced neutrino charge, suppresses the contribution of distant ($t < \omega_p^2$) collisions and in the case of a charged particle gives rise to the density effect.

The asymptotic form (8.3) (linearity of $\varepsilon - 1$ in the coupling constant e^2) reflects the fact that at high frequencies interaction effects in the nonrelativistic medium are small and correspondingly the polarization operator may be calculated to lowest order in the perturbation. The same applies to the axial quantity, R_2^A, whose calculation with the help of (4.7), (5.2), and (5.3) leads to similar results. However, now effects of screening (the second term in (5.1) and the last term in the expression (5.3) for d_t^A) are absent, since in the region $E \ll m$ the quantities P^{VA} and P^{AV} are small together with D^{VA} (see above). This gives

$$\alpha_2^A = nt/m. \tag{8.5}$$

Substitution of (8.4) and (8.5) into (8.2) leads to the final expression for the upper bound on the neutrino EL in the region $E \ll m$:

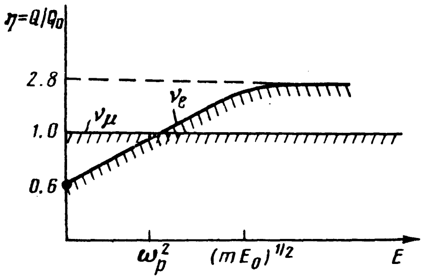

FIG. 2.

$$Q \leqslant \frac{2G^2nE^4}{3\pi m}[C_V^2\varphi(\omega_p^2/4E^2)+5C_A^2], \tag{8.6}$$

where the function

$$\varphi(x) = 6\int_0^1 dz z^2(1+z)/(z+x),$$

is equal to 5 for $x=0$ and falls monotonically to zero with increasing x. The resulting decrease in the EL is of the same nature as the density effect (see above). The dependence on the neutrino energy of the ratio $\eta = Q/Q_0$ of the neutrino energy loss to the collisional limit is shown in Fig. 2 based on Eqs. (6.3)–(6.5) and (8.6); the dashed region indicates allowed values of η.

9. CONCLUSION

The main result of this paper is the construction of a systematic quantitative theory of the EL of a neutrino moving in an arbitrary stable, homgeneous, isotrcpic medium. Just as the EL of a charged particle is expressed in terms of two characteristics of the medium (it permittivity), the EL of the neutrino is determined by five such characteristics. They are the permittivities describing the longitudinal and transverse vector responses of the medium to the vector interaction, the longitudinal and transverse axial responses to the axial interaction and the mixed axial response to the vector interaction. The calculation of these characteristics is carried out by standard methods of many-body theory.

The main qualitative conclusion of the paper reduces to a negative answer to the central question of the theory of neutrino EL formulated in the Introduction [see (1.2)]: are there stable media in which the EL exceeds substantially the collision limit (1.1), corresponding to neutrino scattering by isolaied stationary particles of the medium? According to the rigorous and universal upper bound on the neutrino EL found in this paper the corresponding ratio η (see Sec. 8) equals 1 at high neutrino energies, and at lower energies does not exceed 1 (for the muon neutrino) and a number varying between 0.6 and 2.8 (for the electron neutrino). This conclusion excludes the possibility of existence of a medium that slows down neutrinos with anomalous strength. This applies directly to "cold" media, however, without a doubt the temperature of the medium will make no qualitative difference in the situation.

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APPENDIX

If the response function $R(t,\mathbf{x}) = 0$ for $t < |\mathbf{x}|$ then the quantity

$$R(\omega, \mathbf{k}+\omega\mathbf{s}) = \int d^4x R(t,\mathbf{x})\exp[i\omega(t-\mathbf{xs})-i\mathbf{kx}], \qquad |\mathbf{s}|\leqslant 1$$

is described by an integral convergent in the upper half-plane of ω (in the region of integration $t-\mathbf{xs}\geqslant 0$) and therefore analytic in ω in that region. From this the Leontovich relation (7.1) follows by standard methods.

We prove that this relation is satisfied not only by the tensor $D_{\mu\nu}^{ab}$ itself [see (4.4)], but also by the scalars R defined by (4.5) and (4.6). To this end we restore the terms proportional to k_μ or k_ν that were omitted from (4.5):

$$D_{\mu\nu}^{aa} = -R_1^a g_{\mu\nu} + R_2^a u_\mu u_\nu + A^a k_\mu k_\nu/k^2 + B^a(k_\mu u_\nu + u_\mu k_\nu)/2(uk). \tag{A1}$$

This implies that in the rest frame of the medium the relation (7.1) is satisfied by the (0,0), (0,i) and (i,j) components of (A1) ($i,j = 1, 2, 3$):

$$-R_1^a + R_2^a + A^a k_0^2/k^2 + B^a, \tag{A2}$$

$$(A^a k_0/k^2 + B^a/2k_0)k_i, \tag{A3}$$

$$R_1^a\delta_{ij} + A^a k_i k_j/k^2. \tag{A4}$$

By writing out the relation (7.1) for (A4) and contracting it with the tensor $\delta_{ij} - s_i s_j$ and $k_i k_j$ one readily finds with $|\mathbf{s}| = 1, \mathbf{ks} = 0$ taken into account that the quantities $R_1{}^a$ and A^a satisfy the relations (7.1), (7.2). Application of the same procedure to (A3) with multiplication of the result by k_i and s_i shows that the quantity B^a also satisfies the Leontovich relations. Substitution into (A2) leads to the same conclusion for also the quantity $R_2{}^a$. Lastly, it follows from the Leontovich relation for (4.6) that what has been said above also applies to R_3.

1) The medium characteristics that enter the Green's function (2.6) depend implicitly on the temperature.

2) We note that at low temperatures the quantity Q_T is proportional not to T^2, as might be expected, but to T^4.

3) We emphasize that the form factor (5.5) describes only part of the action of the neutrino on the medium, since the first term in (5.1) is also important. The role of the latter is particularly significant for media for which the quantities ε_l and $P_l{}^V$ [see (5.3)] have a pole in the upper half-plane of the frequency:[12] only the expression (5.1) as a whole is free of this inadmissable singularity due to exact cancellation between the poles of the two terms of (5.1).

4) The δ-function reflects here the kinematics of scattering on a stationary free electron, corresponding to the so-called "Bethe comb".

5) The fact that for a charged particle this procedure gives not an upper bound but the EL itself has to do with its logarithmic dependence on the energy, as opposed to a power-law in the case of the neutrino (see end of Sec. 4). Therefore for the EL of a charged particle one may obtain upon application of the sum rule a result valid to logarithmic precision.

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