Low-temperature conductivity of one-dimensional disordered metals

Yu. V. Tarasov

Scientific-Research Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR, Kharkov

(Submitted 10 November 1989)
Zh. Eksp. Teor. Fiz. **97**, 1060–1076 (March 1990)

A theory is developed of the conductivity of one-dimensional disordered metals at temperatures low compared with the intrinsic frequency of localized electrons. An adiabatic form of perturbation theory for one-electron Green functions, and a functional representation of elastic fields in a crystal, are used to obtain an expression for the conductivity in the case of a rapidly converging series of powers of the phonon frequencies. A special technique is used for asymptotically rigorous averaging of the terms of the series over the states of impurity and phonon fields. An analysis is made of the role of phonons in the localization and delocalization of electronic states.

1. INTRODUCTION

The localization of electronic states in one-dimensional ($1D$) disordered metals, predicted in Ref. 1, has stimulated many investigations of the structure of wave functions and kinetics of electrons in such compounds. Simple physical ideas of Mott,[2,3] confirmed by rigorous calculations of Berezinskiĭ,[4] have proved very fruitful and still provide the basis for the description of many phenomena in low-dimensional conductors.

One of the most complex problems in the kinetics of electrons in $1D$ conductors is the interaction of electrons with thermal lattice vibrations. Various attempts to allow for this interaction have been made on many occasions. The first qualitative ideas were put forward by Mott[5] and they apply to the case of extremely low temperatures when phonon-assisted electron transitions between different localized states can be regarded as quite rare. According to Mott,[5] the static conductivity at a temperature T low compared with the characteristic binding energy of localized electrons T_0 should be

$$\sigma(T)\propto\exp[-(T_0/T)^{1/2}] \tag{1}$$

[a simple and clear derivation of Eq. (1) can be found in Ref. 6]. A similar dependence, though deduced using different ideas, was obtained in Refs. 7 and 8. The temperature dependence of the conductivity similar to that given by Eq. (1) is also predicted in Ref. 9 for samples of finite dimensions.

It must be however pointed out that in all these investigations the Mott law (1) is obtained using some variant of percolation theory and the characteristics of this theory in the $1D$ case make it necessary to view the results critically. In particular, the results may be affected significantly by strong fluctuations of the local electric field.[10] Subject to a number of additional assumptions, we find that instead of Eq. (1), the usual activated dependence is generally obtained.[9,11]

A more rigorous analysis of the conductivity of $1D$ metals at low temperatures obeying $T\tau_i \ll 1$ (τ_i^{-1} is the frequency of impurity scattering of electrons), for which Mott originally obtained the dependence (1), was applied in Ref. 12. A method developed in Ref. 13 was used in Ref. 12 to obtain a $\sigma(T)$ dependence close to the power law

$$\sigma(T)\propto T^4\ln^4 T\tau_i. \tag{2}$$

The ideas put forward in Ref. 12 were used in Ref. 14 to analyze the temperature and frequency dependences of the conductivity of $1D$ compounds at frequencies corresponding to the pair approximation.[15]

It should be stressed that the treatments given in Refs. 12 and 14 rely strongly on the assumption that the phonon contribution to the conductivity is due to electron hopping between distant and, therefore, poorly correlated localized states. However, it was shown in Ref. 16 that hopping over long distances can be regarded as a set of a large number of jumps between closely spaced states with strongly correlated energy levels. This gives rise to a strong correlation also of the states located far from one another, as pointed out in Ref. 14 for the case when $T=0$; moreover, this correlation follows from the very nature of calculations of the zero-phonon conductivity carried out by Berezinskiĭ.[4]

It should be pointed out also that the treatments in Refs. 12 and 14 ignore a number of important features of the electron–phonon interaction in $1D$ metals. They are, firstly, a strong Coulomb screening of the interaction of electrons with phonons in the long-wavelength part of the spectrum.[17] The usual deformation potential of such phonons vanishes, so that the main role is played by much weaker interaction mechanisms, such as the inertial (Stewart–Tolman) and cross-deformation mechanisms associated with the vibrational motion of impurities. Secondly, the frequency of the scattering of electrons by phonons was calculated in Ref. 12 allowing for the three-dimensional nature of the phonon system, whereas because of the law of conservation of momentum the $1D$ electrons can interact only with a one-dimensional phonon subsystem.[18] Finally, the role of the short-wavelength phonons in the localization of electronic states was ignored in Refs. 12 and 14. It was pointed out in Ref. 19 that, in principle, such a role is possible, but a rigorous analysis of the problem was not attempted.

Our aim will be to allow for all these factors. We shall develop an adiabatic theory which makes it possible to derive an asymptotically rigorous expression for the dissipative conductivity Re σ allowing for the dispersion in time and space at low temperatures defined by the inequality

$$T\tau_2\ll 1, \tag{3}$$

where τ_2^{-1} is the frequency of the elastic backscattering of the $1D$ electrons. The proposed method is based on a descrip-

0038-5646/90/030595-09$03.00

tion of the interaction of electrons with a classical time-dependent random Gaussian field, which is used instead of the quasiparticle picture of the electron–phonon interaction. In view of the inequality (3), the characteristic frequencies of this Gaussian field are low, compared with the "binding energy" τ_2^{-1} of the localized electronic states. This makes it possible to construct an adiabatic series of electron Green functions and a corresponding series for the conductivity σ. The dissipative part of the series consists of a "zero-phonon" term which disappears in the limit $\omega \to 0$ and a temperature correction governing the static conductivity. The temperature dependence of the latter is similar in form to Eq. (2), but it contains different powers of the temperature itself and of its logarithm.

2. FORMULATION OF THE PROBLEM AND INITIAL EQUATIONS

We shall model a $1D$ metal by a three-dimensional crystal consisting of parallel conducting atomic chains oriented along the x axis. We shall assume that the energy spectrum of electrons is rigorously one-dimensional and that the potential of defects (impurities) distributed at random over a crystal is of the short-range type.

According to the linear reaction theory, the conductivity $\sigma(k, \omega)$ can be expressed in terms of the Fourier transform of a retarded Green function of the currents[20] (k is the wave number along the chains). In the analysis of this linear reaction in the specific case of $1D$ metals it is convenient to use an approach proposed in Ref. 21. Since the kinetic processes in metals are governed by electrons originating from a small part of the Fermi surface (which in the $1D$ case consists of two planes $p_x = \pm p_0$), in the proposed approach the Schrödinger electron field operators are simplified by retaining only packets of harmonics with momenta close to $\pm p_0$:

$$\psi(x) = \frac{1}{L^{1/2}} \sum_{\alpha=\pm} e^{i\alpha p_0 x} \sum_{|k| \ll p_0} a_{\alpha p_0 + k} e^{ikx} = \begin{pmatrix} \psi_+(x) e^{ip_0x} \\ \psi_-(x) e^{-ip_0x} \end{pmatrix},$$
$$\psi^+(x) = (\psi_+^*(x) e^{-ip_0x}, \psi_-^*(x) e^{ip_0x}). \tag{4}$$

Here, L is the size of the conductor along the x axis, which we shall regard as infinite whenever this will not cause any normalization problems. The vector form of the field operators (4) allows us to define the current density operator $\hat{j}$ as follows:

$$\hat{j}(x) = ev(\psi^+(x) \hat{\sigma}_3 \psi(x)), \tag{5}$$

where v is the Fermi velocity of electrons,

$$\hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the outer brackets denote contraction of the vector indices $\alpha = \pm$.

The retarded Green function of the currents can be found conveniently using the temperature technique followed by analytic continuation of the results from the complex Bose frequencies $\omega_n = 2\pi i n T (n > 0)$ to the real frequency axis. The formalism of the generating functionals[22,23] makes it possible to represent the temperature Green function of the currents $\mathcal{G}_{jj}(x, \tau; x', \tau')$, averaged over the impurity positions, in the form of the following functional integral:

$$\mathcal{G}_{jj}(x, \tau; x', \tau') = -\frac{(ev)^2}{Z(A^+, A)} \left(\frac{\delta}{\delta A(x, \tau)} \hat{\sigma}_3 \frac{\delta}{\delta A^+(x, \tau)} \times \frac{\delta}{\delta A(x', \tau')} \hat{\sigma}_3 \frac{\delta}{\delta A^+(x', \tau')} Z(A^+, A) \right) \Bigg|_{A^+ = A = 0}. \tag{6}$$

Here,

$$Z(A^+, A) = \mathrm{const} \cdot \mathrm{Tr} \left\{ \hat{\rho}_0^{ph} \hat{T}_\tau^{ph} \int D\bar{\psi} D\bar{\psi}^+ \exp[S_\beta(\bar{\psi}^+, \bar{\psi}) + A^+ \bar{\psi} + \bar{\psi}^+ A] \right\}; \tag{7}$$

$\bar{\psi}(x,\tau)$ and $\bar{\psi}^+(x,\tau)$ are the Fermi fields corresponding to the operators of Eq. (4) considered in the temperature representation of the interaction; $A(x, \tau)$ and $A^+(x, \tau)$ are the Fermi sources (fields) which anticommute with one another and with the fields $\bar{\psi}$ and $\bar{\psi}^+$; $S_\beta \bar{\psi}^+, \bar{\psi}$ is the complete Euclidean (in the interval $[0, \beta]$) action functional for electrons ($\beta = T^{-1}$). The products of the fields in Eq. (7) are scalars in the two-dimensional space x–τ and the symbol Tr notes a trace with an unperturbed phonon density matrix $\hat{\rho}_0^{\mathrm{ph}}$ and with averaging over the realizations of the random potential of the impurities. The operator $\hat{T}_\tau^{\mathrm{ph}}$ performs "time" ordering of the τ phonon creation and annihilation operators (in the temperature representation of the interaction) which are contained in S_β.

Equations (6) and (7) allow us to go over from the quasiparticle description of the electron–phonon interaction to the interaction of electrons with a time-dependent classical random phonon field the realizations of which should be averaged. The correlation functions of this field can be determined only if we specify the form of the Hamiltonian of the electron–phonon system. It follows from the results of Refs. 17 and 24 (see also the review in Ref. 25) that the interaction of electrons in $1D$ metals is very different for short- and long-wavelength phonons. In the case of phonons with the momenta of the order of the Debye value the electron–phonon interaction can be described by the usual deformation potential Λ, whereas in the case of long-wavelength phonons the Coulomb screening causes the potential to vanish and we have to allow for weaker types of coupling such as the cross-deformation interaction associated with modulation of the static field of impurities by the lattice vibrations and the inertial interaction due to the Stewart–Tolman effect. Consequently, the interaction Hamiltonian can be represented by a sum

$$\mathcal{H}_{e\text{-}ph} = \mathcal{H}_\Lambda + \mathcal{H}_{cd} + \mathcal{H}_{in}, \tag{8}$$

where the terms differ in respect of the ranges of the phonon wave numbers and in respect of the interaction constants.

The operator structure of all the terms in the Hamiltonian (8) is the same: the terms are linear in respect of the phonon creation and annihilation operators. Therefore, averaging over the equilibrium state of phonons in Eq. (7) is carried out exactly and the result is that

$$\langle \hat{T}_\tau^{ph} \exp[S_\beta^{e,ph}(\bar{\psi}^+, \bar{\psi})] \rangle_{ph} = \exp\left(\frac{1}{2} \rho \mathcal{D} \rho \right),$$
$$\rho(x, \tau) = \bar{\psi}^+(x, \tau) \bar{\psi}(x, \tau), \tag{9}$$

where $\mathcal{D}(x - x'; \tau - \tau')$ is the correlation function of the electron–phonon interaction consisting of the three terms corresponding to three terms of the Hamiltonian (8). The explicit form of these terms will be given later.

It follows from the representation (4) that the density functional $\rho(x,\tau)$ contains only the "Fermi" components of the Fourier fields $\bar{\psi}^+$ and $\bar{\psi}$. For this reason the argument of the exponential function in the system (9) is dominated by the spatial harmonics of the correlation function $\mathscr{D}(x-x';\tau-\tau')$ with the momenta close to zero and $\pm 2p_0$. The long-wavelength harmonics describe the forward scattering of electrons, without a change in the sign of the momentum, whereas the short-wavelength harmonics correspond to the backscattering (we shall label these processes by the indices 1 and 2, respectively). Neglect of the remaining harmonics represents essentially averaging over the fast phases in the Fokker–Planck equations for the probability densities of the quantities being averaged[26] or selection of the effective diagrams in the Berezinskii technique.[4]

Since the system of equations (9) contains only the "resonant" parts of the correlation function $\mathscr{D}$ mentioned above, we can now represent the exponential function of the quadratic form of ρ as a functional integral containing the effective action and characterized by a Hamiltonian representing the motion of electrons in external time-dependent random fields, the realizations of which are averaged out:

$$\mathscr{H}^{eff}_{e-ph}=\eta(x,\tau)+\hat{a}^+\zeta(x,\tau)+\hat{a}\zeta^+(x,\tau),$$
$$\hat{a}=\begin{pmatrix}0&0\\1&0\end{pmatrix},\quad \hat{a}^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}. \tag{10}$$

Here, $\eta(x,\tau)$ are the real, whereas $\zeta(x,\tau)$ and $\zeta^+(x,\tau)$ are the complex Bose fields with a Gaussian statistics, zero average values, and the following nonzero binary correlation functions:

$$\langle\eta(x,\tau)\eta(x',\tau')\rangle=[\mathscr{D}_{cd1}(\tau-\tau')+\mathscr{D}_{in1}(\tau-\tau')]\delta(x-x'),$$
$$\langle\zeta(x,\tau)\zeta^+(x',\tau')\rangle=\mathscr{D}_{\Lambda 2}(\tau-\tau')\delta(x-x'). \tag{11}$$

The functions $\mathscr{D}_{\text{cd}1}$, $\mathscr{D}_{\text{in}1}$, and $\mathscr{D}_{\Lambda 2}$ are the amplitudes of the corresponding resonant Fourier components of the correlation function $\mathscr{D}(x-x';\tau-\tau')$ in the system (9):

$$\mathscr{D}_{cd1}(\tau-\tau')=g^2\frac{v}{\tau_{i1}}F(\tau-\tau'),$$
$$\mathscr{D}_{in1}(\tau-\tau')=(m_0vs)^2\bar{q}^{-1}F(\tau-\tau'), \tag{12}$$
$$\mathscr{D}_{\Lambda 2}(\tau-\tau')=\frac{v}{\tau_\Lambda}[\mathscr{T}_{2p_0}(\tau-\tau')+\mathscr{T}_{2p_0}(\tau'-\tau)].$$

Here $g^2\sim 1$ is the cross-deformation interaction constant;[24,27] τ^1_{i1} is the frequency of the forward scattering of electrons by impurities; m_0 is the mass of a free electron; s is the velocity of sound; $\bar{q}$ is a typical value of the momentum of long-wavelength phonons; $\tau_\Lambda^{-1}\sim\lambda\omega_{2p_0}\Delta q_0/q_D$ is the frequency governed by the strength of the deformation interaction of electrons with phonons characterized by a momentum $2p_0$; $\Delta q_0/q_D$ is the relative spectral width of a packet of such phonons (q_D is the limiting phonon momentum); $\lambda\sim(\bar{\Lambda}/\varepsilon_0)^2$ is the deformation coupling constant; ε_0 is the Fermi energy;

$$F(\tau-\tau')=\frac{1}{2NMs^2}\sum_{|q|\ll p_0}\omega_q[\mathscr{T}_q(\tau-\tau')+\mathscr{T}_q(\tau'-\tau)]; \tag{13}$$

N is the number of atoms in a one-dimensional chain; M is the mass of a unit cell. The symbol $\mathscr{T}_q(\tau-\tau')$ denotes the Matsubara Green function of phonons:

$$\mathscr{T}_q(\tau-\tau')=[N_q+\theta(\tau-\tau')]\exp[-\omega_q(\tau-\tau')],$$
$$N_q=[\exp(\beta\omega_q)-1]^{-1}, \tag{14}$$

where $\theta(\tau-\tau')$ is the θ step function.

Some comments must be made about Eqs. (10)–(13). Firstly, they are derived allowing for the fact that, because of the momentum conservation, the $1D$ electrons interact only with the subsystem of one-dimensional phonons. Consequently, the density matrix $\hat{\rho}_0^{\text{ph}}$ in Eq. (7) can be factorized and the averaging is carried out over its one-dimensional part which includes phonons characterized by $\mathbf{q}_\perp=0$.

Secondly, the δ-type nature of the correlation—with respect to the coordinate x—of the fields η, ζ, and ζ^+ in the system of equations (10) means in fact that the correlation radii of these fields are short compared with the localization length of the electronic states, which is governed by the electron backscattering time τ_2. In the case of phonons with a momentum close to $2p_0$ this condition reduces to the inequality

$$p_0l_2\sim\varepsilon_0\tau_2\gg 1,\ (l_2=v\tau_2), \tag{15}$$

and in the case of long-wavelength phonons it sets the lower limit on the temperature of a metal:

$$T\tau_2\gg s/v. \tag{16}$$

Therefore, we have passed from the operator form of the averaging of the generating functional (7) over the state of the phonon system to the function form of the averaging over the c-number Gaussian random fields with the correlation properties described by Eq. (11). In addition to these fields, which govern the effective Hamiltonian of the electron–phonon interaction (10), the complete electron Hamiltonian includes also the impurity part which is identical with Eq. (10), but does not contain the dependence on the "time" τ (Refs. 21 and 27). The correlation functions of the impurity fields considered on the assumption of weak scattering [Eq. (15)] are again δ-function-like:

$$\langle\eta_i(x)\eta_i(x')\rangle=\mathscr{D}_{i1}\delta(x-x'),$$
$$\langle\zeta_i(x)\zeta_i^+(x')\rangle=\mathscr{D}_{i2}\delta(x-x'). \tag{17}$$

We shall now transform Eq. (6) by functional differentiation allowing for the vector structure of the fields A and A^+:

$$\mathscr{G}_{jj}(x,\tau;x',\tau')$$
$$=(ev)^2\langle\operatorname{Tr}[\hat{\sigma}_3\mathscr{G}(x,\tau;x',\tau')\hat{\sigma}_3\mathscr{G}(x',\tau';x,\tau)]\rangle. \tag{18}$$

Here, $\mathscr{G}(x,\tau;x',\tau')$ is the exact Green function in the Bloch equation [in accordance with the representation given by Eq. (4), this function has the structure of a 2×2 matrix]:

$$-\left[\frac{\partial}{\partial\tau}+\mathscr{H}(x,\tau)\right]\mathscr{G}(x,\tau;x',\tau')=\delta(x-x')\delta(\tau-\tau'), \tag{19}$$

where $\mathscr{H}(x,\tau)$ is the complete Hamiltonian function of an electron, which includes the impurity and phonon fields, the symbol Tr denotes a trace of the matrix indices, and the angular brackets denote averaging over the whole set of random fields.

3. ADIABATIC EXPANSION OF GREEN FUNCTIONS

Representation of the current correlation functions of Eq. (18) in terms of the Green functions of particles moving in classical random fields is intended to facilitate the use of the averaging technique developed in Refs. 16 and 24. However, this technique can be employed only in the case of a static external field, whereas the phonon fields in the system of equations (10) cannot be regarded as such fields. We can overcome these difficulties by expressing the conductivity in terms of the Matsubara Green functions of electrons calculated in random external fields corresponding to "instantaneous" realizations of the true time-dependent fields. In the operator (along the coordinate x) form these functions are

$$\mathcal{G}_0(\zeta_m,\tau)=[\zeta_m-\hat{\mathcal{H}}(\tau)]^{-1}, \tag{20}$$

where $\zeta_m = i(2m+1)\pi/\beta$ (m is an integer) and $\hat{\mathcal{H}}(\tau)$ is the complete Hamiltonian of an electron at a moment of "time" τ. A combination of these functions makes it possible to describe the conductivity by an adiabatic series, the convergence of which is determined by the rate of change of the Hamiltonian with time.

We shall apply the formal Fourier transformation functions of Eq. (20) with respect to the variable ζ_m and thus find two types of unperturbed functions which depend on the variables τ and τ':

$$\begin{aligned}\mathcal{G}_1(\tau,\tau')&=\beta^{-1}\sum_{m=-\infty}^{\infty}\exp[-\zeta_m(\tau-\tau')]\mathcal{G}_0(\zeta_m,\tau),\\ \mathcal{G}_2(\tau,\tau')&=\beta^{-1}\sum_{m=-\infty}^{\infty}\exp[-\zeta_m(\tau-\tau')]\mathcal{G}_0(\zeta_m,\tau').\end{aligned} \tag{21}$$

These functions make it possible to transform the differential (in respect of the variable τ) equation (19) into two integral equations differing in respect of the free terms and the sequence of the factors in the integrands:

$$\mathcal{G}(\tau,\tau')=\mathcal{G}_1(\tau,\tau')+\beta^{-1}\int_0^\beta d\tau''\mathcal{G}(\tau,\tau'') \times\sum_{m=-\infty}^{\infty}\left[\frac{\partial}{\partial\tau''}\mathcal{G}_0(\zeta_m,\tau'')\right]\exp[-\zeta_m(\tau''-\tau')], \tag{22}$$

$$\mathcal{G}(\tau,\tau')=\mathcal{G}_2(\tau,\tau')-\beta^{-1}\int_0^\beta d\tau''\sum_{m=-\infty}^{\infty}\exp[-\zeta_m(\tau-\tau'')] \times\left[\frac{\partial}{\partial\tau''}\mathcal{G}_0(\zeta_m,\tau'')\right]\mathcal{G}(\tau'',\tau'). \tag{23}$$

Iteration of Eqs. (22) and (23) allows us to represent the function $\mathcal{G}(\tau,\tau')$ in the form of functional series in terms of the derivative $\partial\hat{\mathcal{H}}(\tau)/\partial\tau$. Hence, we obtain an adiabatic series for the correlation functions of the currents given by Eq. (18) and, consequently, for the conductivity $\sigma(k,\omega)$. One of the Green functions in Eq. (18) should be iterated by a series that follows from Eq. (22), whereas the other by a series that follows from Eq. (23). In this way we find the physically justified zeroth approximation for the conductivity in the limit $T\to 0$, which is identical in form with the expression obtained in Refs. 4, 16, and 28.

The problem of convergence of the adiabatic series for the conductivity can be solved in a manner similar to that adopted in traditional quantum-mechanical theory of adiabatic perturbations (see, for example, Ref. 29). Here, in fact, the relevant small parameter is the ratio of the rate of change of the Hamiltonian $\partial\hat{\mathcal{H}}/\partial\tau$ to the square of the separation between the electronic energy levels. In our case this separation can be estimated as equal to the frequency τ_2^{-1} governing the binding energy of localized states in the $1D$ system.

In the temperature range defined by Eq. (3) it is sufficient to retain only the first two terms from the whole iteration series for the conductivity. We shall consider that term in Eq. (18), which is obtained when the exact Green functions are replaced with $\mathcal{G}_1(\tau,\tau')$ and $\mathcal{G}_2(\tau,\tau')$ and which does not contain the derivative $\partial\hat{\mathcal{H}}/\partial\tau$. In the Fourier representation, this term is of the form

$$\mathcal{G}_{jj}^{(0)}(k,\omega_n)=\frac{(ev)^2}{L\beta^2}\int_0^\beta d\tau\sum_{m=-\infty}^{\infty}\langle\mathrm{Sp}[\hat\sigma_3e^{-ik\hat{x}}\mathcal{G}_0(\zeta_m+\omega_n,\tau) \times e^{ik\hat{x}}\hat\sigma_3\mathcal{G}_0(\zeta_m,\tau)]\rangle. \tag{24}$$

Here and later, Sp denotes the trace over all the matrix indices: pseudospinor and coordinates. The next term in the conductivity $\sigma^{(0)}(k,\omega)$ is an analytic continuation of the correlation function of Eq. (24) from imaginary frequencies ω_n to real ones, and it is exactly identical with the general expression obtained in Ref. 4 in the limit $T=0$. We shall not consider this term in detail, because all these features have been analyzed in detail in Refs. 16 and 28 for arbitrary values of k and ω.

We must however point out an important property of Eq. (24): it contains the Green functions of Eq. (20), which are defined by the Hamiltonian of electrons at the same moment of "time" τ. This is equivalent to a calculation of the conductivity in a static random field which represents a sum of the impurity and phonon fields frozen at this moment. The correlation functions of this combined field no longer depend on τ, and an eigenfrequency of localized electronic states τ_2^{-1} is then a sum of the impurity backscattering frequency $\tau_{i2}^{-1}=\mathcal{D}_{i2}/v$ and the phonon correction $\tau_{ph2}^{-1}=\mathcal{D}_{\Lambda 2}(0)/v$:

$$\tau_2^{-1}=\tau_{i2}^{-1}+\tau_{ph2}^{-1}. \tag{25}$$

The phonon term in Eq. (25) is related solely to the momentum dissipation of electrons by interaction with the lattice and represents in fact renormalization of the electron mass. The magnitude of this term is easily estimated if we bear in mind that the backscattering of the Fermi electrons involves a narrow packet of phonon modes, the relative number of which is

$$\Delta q_0/q_D\sim(q_Dl_2)^{-1}\ll 1.$$

It is this number that provides a measure of the randomness of the static phonon potential acting on electrons and, consequently, determines its small role in the localization of the electronic states. Using the relationships described by Eqs. (12) and (14), we find that the total elastic electron backscattering frequency allowing for the short-wavelength phonons is given by the expression

$$\tau_2^{-1}=\tau_{i2}^{-1}\left\{1-O\left[\lambda\frac{\omega_{2p_0}}{\varepsilon_0}(2N_{2p_0}+1)\right]\right\}^{-1}. \tag{26}$$

Apart from small corrections, this quantity is identical—in the adiabatic temperature limit (3)—with the frequency of electron scattering by impurities, which is consequently the main process that determines the localization length of the electronic states.

We shall now consider the second term in the correlation function of the currents proportional to the first power of the derivative $\partial\hat{\mathscr{H}}/\partial\tau$. Simple transformations can reduce it to the following form:

$$\begin{aligned}\mathscr{G}_{jj}^{(1)}(k,\omega_n)=&-\frac{(ev)^2}{L\beta^3}\iint_0^\beta d\tau\,d\tau'\sum_{l=-\infty}^{\infty}\omega_l\exp[\omega_l(\tau'-\tau)]\\&\times\sum_{s=-\infty}^{\infty}\langle\mathrm{Sp}\,[\hat{\sigma}_3e^{-ik\hat{x}}\mathscr{G}_0(\zeta_s+\omega_l,\tau)\\&\times\mathscr{G}_0(\zeta_s,\tau')\,e^{ik\hat{x}}\hat{\sigma}_3\mathscr{G}_0'(\zeta_s-\omega_n,\tau)]\\&-\mathrm{Sp}\,[\hat{\sigma}_3e^{-ik\hat{x}}\mathscr{G}_0(\zeta_s+\omega_n,\tau)\,e^{ik\hat{x}}\hat{\sigma}_3\mathscr{G}_0(\zeta_s,\tau')\,\mathscr{G}_0(\zeta_s-\omega_l,\tau)]\rangle.\end{aligned} \tag{27}$$

Analytic continuation of the function (27) to the real frequency axis yields the corresponding term in the conductivity $\sigma^{(1)}(k,\omega)$. This procedure is similar to that adopted in Ref. 30 and it is described schematically in the Appendix I.

In view of the fairly complex structure of Eq. (27), the result of its analytic continuation along the frequency axis is difficult to see (before averaging) because of a large number of terms. Since the expressions simplify somewhat as a result of averaging, we shall describe briefly the averaging procedure employed in the present study before giving the final results for $\sigma^{(1)}(k,\omega)$.

4. AVERAGING OVER REALIZATIONS OF RANDOM FIELDS

Our averaging method is based on a special representation of a one-particle retarded Green function of an electron in an arbitrary static field. In the approximation of Eq. (4) this function is a 2×2 matrix, the elements of which can be described by a combination of a finite number of functions of one spatial variable of the causal type (blocks), which are functionals of the external fields. The explicit form of the matrix Green function is obtained in Refs. 16 and 24 allowing only for the fields ζ and ζ^+ which can scatter an electron backward. We must allow also for the forward scattering associated with the field η because the Green functions in Eq. (27) depend on the various "time" arguments τ and τ'. The appropriate expressions for the matrix elements of the retarded Green function are given in the Appendix II.

One of the main features of the representation given by Eq. (II2) of the Green function in terms of $\pi_\pm^{\omega}$, $\Gamma_\pm^{\omega}$, and $\gamma_\pm^{\omega}$ is the circumstance that in the case of the δ-correlated (with respect to the spatial coordinate) random fields the averaging of the functionals of the "plus" and "minus" types is carried out independently. This follows directly from Eqs. (II3)–(II6): the (+) blocks are governed by the random fields $\eta(x')$, $\zeta(x')$, and $\zeta^+(x')$ with the coordinates $x'>x$, whereas the (−) blocks have the coordinates $x'<x$.

In the averaging of the blocks composed of functionals of the same "sign" we must allow for the fact that the functional dependences of the average quantities on the real field η and on the complex fields ζ and ζ^+ are fundamentally different. For example, it follows from Eqs. (II2) and (II5) that any blocks composed of the functionals Γ_+ and π_+ contain random fields only in the combinations

$$\begin{aligned}\xi(x,\tau)&=\zeta(x,\tau)\exp\left[-\frac{2i}{v}\int_x^\infty\eta(x',\tau)\,dx'\right],\\\xi^+(x,\tau)&=\zeta^+(x,\tau)\exp\left[\frac{2i}{v}\int_x^\infty\eta(x',\tau)\,dx'\right].\end{aligned} \tag{28}$$

A special feature of the new fields $\xi(x,\tau)$ and $\xi^+(x,\tau)$ is the vanishing of their correlation function for different "times":

$$\langle\xi(x,\tau)\xi^+(x',\tau')\rangle=\begin{cases}\mathscr{D}_2(0)\delta(x-x'), & \tau=\tau',\\ 0, & \tau\neq\tau',\end{cases} \tag{29}$$

where

$$\mathscr{D}_2(0)=\mathscr{D}_{i2}+\mathscr{D}_{\Lambda 2}(0).$$

The relationship (29) permits a change from the averaging over ζ and ζ^+ to the averaging over ξ and ξ^+ and thus allows us to perform this operation in Eq. (27) separately for the functions dependent on τ and τ'. Moreover, bearing in mind the Gaussian nature of the random fields ζ and ζ^+, we find that the conductivity $\sigma^{(1)}(k,\omega)$ is described by a relatively compact expression which contains only the terms that do not vanish as a result of the averaging:

$$\begin{aligned}\sigma^{(1)}(k,\omega)=&\frac{ie^2}{2\pi\beta^2v\omega}\int_{-\infty}^{\infty}d\varepsilon[n_F(\varepsilon-\omega)-n_F(\varepsilon+\omega)]\iint_0^\beta d\tau\,d\tau'\\&\times\sum_{l=1}^{\infty}\omega_l\exp[\omega_l(\tau'-\tau)][\Phi_1(\tau,\tau')+\Phi_2(\tau,\tau')],\end{aligned} \tag{30}$$

where

$$n_F(\varepsilon)=[\exp(\beta\varepsilon)+1]^{-1},$$

and

$$\begin{aligned}\Phi_1(\tau,\tau')=&\frac{iv^3}{L}\langle\mathrm{Sp}\,[\hat{\sigma}_3e^{-ik\hat{x}}G_0^r(\varepsilon+\omega_l,\tau)\\&\times G_0^r(\varepsilon,\tau')\,e^{ik\hat{x}}\hat{\sigma}_3G_0^a(\varepsilon-\omega,\tau)]\rangle,\end{aligned} \tag{31}$$

$$\begin{aligned}\Phi_2(\tau,\tau')=&\frac{iv^3}{L}\langle\mathrm{Sp}\,[\hat{\sigma}_3e^{-ik\hat{x}}G_0^a(\varepsilon-\omega_l,\tau)\\&\times G_0^r(\varepsilon,\tau')\,e^{ik\hat{x}}\hat{\sigma}_3G_0^r(\varepsilon-\omega,\tau)]\rangle.\end{aligned} \tag{32}$$

The operation of averaging of Eqs. (31) and (32) can be carried out in several stages. The first stage involves averaging the τ'-dependent Green functions over the fields ξ and ξ^+. Then, only the diagonal components of these functions remain finite:

$$\begin{aligned}&\langle G_{01}^r(\varepsilon,\tau'|x,y)\rangle_\xi\\&=-\frac{i}{v}\theta(x-y)\exp\left\{\left[i\frac{\varepsilon}{v}-(2l_2)^{-1}\right](x-y)\right.\\&\left.\quad-\frac{i}{v}\int_y^x\eta(x',\tau')\,dx'\right\},\end{aligned} \tag{33}$$

$$\langle G_{02}{}^{r}(\varepsilon, \tau' \,|\, x, y)\rangle_{\xi}$$

$$= -\frac{i}{v}\theta(y-x)\exp\left\{\left[i\frac{\varepsilon}{v} - (2l_2)^{-1}\right](y-x) + \frac{i}{v}\int\limits_{y}^{x}\eta(x', \tau')\,dx'\right\}. \tag{34}$$

The next stage requires averaging of the fields $\eta(x, \tau)$ and $\eta(x, \tau')$. After several transformations these fields are contained only in the phases of the exponential factors of the matrix elements of G_{i}^{0} in Eq. (II2) and then the averaging of the realizations of these fields can be carried out using an easily verifiable relationship

$$\left\langle \exp\left\{\pm\frac{i}{v}\int\limits_{y}^{x}[\eta(x', \tau') - \eta(x', \tau)]\,dx'\right\}\right\rangle = \exp\left(-\frac{\Delta\mathscr{D}_1}{2v^2}|x-y|\right), \tag{35}$$

where

$$\Delta\mathscr{D}_1 = \mathscr{D}_1(0) - \mathscr{D}_1(\tau-\tau').$$

In the final stage we have to average τ-dependent binary combinations of the Green functions in Eqs. (31) (32) of the fields ξ and ξ^{+}. This operation is carried out using recurrence relationships derived in Ref. 16. In view of the complex structure of the quantities being averaged, the averaging procedure is very cumbersome and we shall not describe it completely. The final result of the averaging is as follows.

The correlation functions (31) and (32) are represented by infinite series of auxiliary correlation functions:

$$\Phi_1 = \sum_{n=0}^{\infty}\{(R_n - R_{n+1})K_n(k) - \mathscr{M}_n[\mathscr{L}_n(k) - \mathscr{L}_{n+1}(k)] + (k\to -k)\}, \tag{36}$$

$$\Phi_2 = \sum_{n=0}^{\infty}[(\tilde{R}_n - \tilde{R}_{n+1})\tilde{K}_n(k) - (\tilde{\mathscr{M}}_n - \tilde{\mathscr{M}}_{n+1})\tilde{\mathscr{L}}_n(k) + (k\to -k)], \tag{37}$$

where the symbol $(k \to -k)$ describes the previous expression in which the sign of the wave number k is reversed. The functions of a discrete variable n in Eq. (36) satisfy a system of coupled finite-difference equations:

$$2i(\omega_l+\omega)\tau_2 R_n + n(R_{n+1} + R_{n-1} - 2R_n) = 0, \quad R_0 = 1; \tag{38}$$

$$\begin{aligned}&-(n+1)^2[K_{n+1}(k) - K_n(k)] + n^2[K_n(k) - K_{n-1}(k)]\\ &\quad -2in(\omega_l+\omega)\tau_2 K_n(k) + i[kl_2 - (\omega_l+\omega)\tau_2]K_n(k)\\ &\qquad = l_2[\mathscr{L}_n(k) - \mathscr{M}_n];\end{aligned} \tag{39}$$

$$\begin{aligned}&-n\{(n+1)[\mathscr{L}_{n+1}(k) - \mathscr{L}_n(k)] - n[\mathscr{L}_n(k) - \mathscr{L}_{n-1}(k)]\}\\ &\quad -2in(\omega_l+\omega)\tau_2\mathscr{L}_n(k)\\ &\quad +\left[1 + \frac{\Delta\mathscr{D}_1}{2\mathscr{D}_2(0)} + i(kl_2 - \omega\tau_2)\right]\mathscr{L}_n(k) = l_2R_n;\end{aligned} \tag{40}$$

$$\begin{aligned}&-(n+1)[(n+1)(\mathscr{M}_{n+1} - \mathscr{M}_n) - n(\mathscr{M}_n - \mathscr{M}_{n-1})]\\ &- 2in(\omega_l+\omega)\tau_2\mathscr{M}_n + \left[1 + \frac{\Delta\mathscr{D}_1}{2\mathscr{D}_2(0)} - i\omega_l\tau_2\right]\mathscr{M}_n = l_2R_{n+1}.\end{aligned} \tag{41}$$

The equations for $\tilde{R}_n$, $\tilde{K}_n$, $\tilde{\mathscr{L}}_n$, and $\tilde{\mathscr{M}}_n$ differ from Eqs. (38)–(41) because of the replacement $\omega \to -\omega$ and because of some modifications of the right-hand sides. One of the boundary conditions for the functions K_n, $\mathscr{L}_n$, and $\mathscr{M}_n$ is a fairly rapid fall in the limit $n \to \infty$. The following comments should be made about the values of these functions when $n = 0$: The expression for $\mathscr{L}_0$ follows from Eq. (40) and from the boundary condition for Eq. (38), whereas the corresponding condition for K_0 and $\mathscr{M}_0$ allows for the fact that these functions are finite irrespective of the values of the parameters in the equations.

With the exception of the exactly soluble equations for R_n and $\tilde{R}_n$ (see Ref. 4), the remaining equations can be solved only approximately and the asymptotic behavior of the solutions is determined by the parameter

$$\alpha_{\pm} = -2i(\omega_l \pm \omega)\tau_2, \tag{42}$$

where the plus sign corresponds to Eqs. (38)–(41) and the minus sign to the equations for the quantities with a tilde.

We shall consider the most interesting range of low frequencies $\omega\tau_2 \ll 1$ (if $\omega\tau_2 \ll 1$, the localization effects are unimportant and the conductivity can be calculated on the basis of the transport equation given in Ref. 28). In this case the dissipative conductivity is calculated bearing in mind that in the summation of l terms in Eq. (30) the most important is the range corresponding to $|\alpha_{\pm}| \lesssim 1$. We shall introduce in this range the self-similar variable $p = n\alpha_{\pm}$. Then, assuming that $|\alpha_{\pm}| \ll 1$, we can replace approximately the sums over n in Eqs. (36) and (37) with integrals and the finite-difference equations with differential equations. In the self-similar approximation the first terms in Eqs. (36) and (37) balance out mutually, and the functions $\mathscr{M}_p$, $\tilde{\mathscr{M}}_p$ as well as $\mathscr{L}_p(k)$, $\tilde{\mathscr{L}}_p(k)$ are pairwise identical. They satisfy the differential equations

$$\left[-p\frac{d}{dp}p\frac{d}{dp} + p + 1 + \frac{\Delta\mathscr{D}_1}{2\mathscr{D}_2(0)} + \begin{pmatrix}-i\omega_l\tau_2\\ ikl_2\end{pmatrix}\right]\begin{pmatrix}\mathscr{M}_p\\ \mathscr{L}_p(k)\end{pmatrix} = l_2R(p), \tag{43}$$

where $R(p) = 2p^{1/2}K_1(2p^{1/2})$ [$K_{\nu}(x)$ is the Macdonald function], which have the following solution

$$\begin{pmatrix}\mathscr{M}_p\\ \mathscr{L}_p(k)\end{pmatrix} = 4l_2\int\limits_{0}^{\infty}d\mu\,\mu\,\mathrm{cth}\frac{\pi\mu}{2}K_{i\mu}(2p^{1/2}) \times\left[\mu^2 + 1 + \frac{\Delta\mathscr{D}_1}{2\mathscr{D}_2(0)} + \begin{pmatrix}-i\omega_l\tau_2\\ ikl_2\end{pmatrix}\right]^{-1}. \tag{44}$$

Allowing for the fact that, because of the difference between the constants of the interactions of electrons with long- and short-wavelength phonons, the ratio $\Delta\mathscr{D}_1/2\mathscr{D}_2(0)$ is always small

$$\frac{\Delta\mathscr{D}_1}{2\mathscr{D}_2(0)} \ll 1, \tag{45}$$

we find that the conductivity $\sigma^{(1)}$ considered in the self-similar (SS) approximation is described by

$$\begin{aligned}&\sigma_{\mathrm{SS}}^{(1)}(k, \omega) \approx \sigma_{\mathrm{SS}}^{(1)}(0.0)/[1 + (kl_2)^2]^2,\\ &\sigma_{\mathrm{SS}}^{(1)}(0, 0) \approx \sigma_0\frac{(\overline{\omega}_q\tau_2)^3}{\varepsilon_0\tau_2}\frac{d_1}{\omega_D v}|\ln T\tau_2|.\end{aligned} \tag{46}$$

Here, $\sigma_0 \sim ne^2\tau_2/m$ is the characteristic value of the conductivity of a three-dimensional metal (allowing for the role of phonons in τ_2^{-1}), d_1 is the coefficient in front of the function $F(\tau - \tau')$ in the sum of the correlation functions (12) representing the inelastic forward electron scattering:

$$d_1 = (m_0 vs)^2/\bar{q} + g^2 \mathscr{D}_{i1}. \tag{47}$$

The coefficient d_1 is governed by the competition between the inertial and cross-deformation electron scattering mechanisms and it generally depends on temperature.

5. ANALYSIS OF RESULTS

In the preceding sections it has been shown that in the quasistatic temperature range defined by Eq. (3) the conductivity of a $1D$ metal can be represented by a rapidly converging adiabatic series. The first term in that series, which is $\sigma^{(0)}(k, \omega)$, is governed by the Green function of Eq. (24). This term manifests most clearly the effects associated with the elastic scattering of electrons by random impurity and phonon fields. It is known that such scattering in $1D$ metals results in the localization of the electronic states, i.e., the term $\sigma^{(0)}(k, \omega)$ is identical with the expression for the conductivity at $T = 0$ derived and analyzed earlier.[4,21,28,31] One difference between the results obtained in the present study and the expressions obtained earlier is the renormalization of the electron mass as a result of the scattering of electrons by short-wavelength phonons. However, it is clear from Eq. (26) that the influence of these phonons on the localization of the electronic states is weak compared with the influence of the impurity scattering. This is due to the following circumstance: although the phonon field can as a whole be regarded as a random time-dependent Gaussian potential of electrons, the localization effect is not influenced by the whole spectrum of harmonics of this field, but only by certain (resonant) packets of harmonics with the momenta $|q| \approx 2p_0$. The random nature of the field created by these harmonics is due to a slight disturbance of the translational invariance of the crystal lattice by the presence of impurities. Therefore, the degree of deviation of the phonon potential from the regular form is governed by a small parameter $(q_D l_i)^{-1} \ll 1$ and a significant influence of phonons on the localization of the electronic states is impossible in terms of the approximation described by Eq. (3).

Apart from $\sigma^{(0)}(k, \omega)$, all the other terms of the adiabatic series describing the conductivity are related to the delocalization of electrons because of the inelasticity of the electron–phonon scattering processes. In the limit (3) the dominant term among the remaining part of the series is $\sigma^{(1)}(k, \omega)$, described by Eq. (46). Its temperature dependence is governed by the relationship between T and ω_D.

We shall first consider the temperature range where $T \ll \omega_D$. Since, as demonstrated by Eq. (13), the characteristic value of the momentum of long-wavelength phonons governing the correlation functions of the forward scattering in Eq. (12) is of the same order as T/s, the following value of $\sigma^{(1)}$ is obtained from Eq. (46):

$$\sigma^{(1)}(0,0) \sim \begin{cases} \sigma_0 \left(\dfrac{s}{v}\right)^2 (T\tau_2)^2 |\ln T\tau_2|, & T\tau_{i1} < (\varepsilon_0 \tau_{i1})^2 \left(\dfrac{s}{v}\right)^3, \\ \sigma_0 \dfrac{(T\tau_2)^3 |\ln T\tau_2|}{\omega_D \tau_{i1} \varepsilon_0 \tau_2}, & T\tau_{i1} > (\varepsilon_0 \tau_{i1})^2 \left(\dfrac{s}{v}\right)^3. \end{cases} \tag{48}$$

The upper part of Eq. (48) represents the case when the inertial electron–phonon interaction mechanism predominates, whereas the lower part corresponds to the cross-deformation mechanism.

The self-similar approximation used to derive Eq. (48) in fact corresponds to the zeroth approximation in terms of the parameters of Eq. (42). Allowing for their finite nature and, consequently, for the discrete relationships (36)–(41) we obtain only small complex corrections to $\sigma^{(1)}_{\mathrm{AM}}$. Unimportant small corrections appear also on inclusion of the contribution of the nonself-similar range of summation of l terms in Eq. (30), where the absolute values of the parameters $\alpha_{\pm}$ exceed unity.

In the temperature range $T > \omega_D$ the condition of rapid convergence of the adiabatic series describing the conductivity is now no longer given by Eq. (3), but

$$\omega_D \tau_2 \ll 1. \tag{49}$$

Bearing in mind that the interaction of electrons with phonons at temperatures $T > \omega_D$ is purely of deformation nature, we find that in this case the term $\sigma^{(1)}(0, 0)$ is given by the following approximate expression:

$$\sigma^{(1)}(0,0) \sim \sigma_0 \frac{s}{v} \omega_D \tau_2 |\ln T\tau_2|. \tag{50}$$

This expression is derived on the assumption that the inequality (3) defining the temperature range does not change to the opposite but stronger inequality. However, if $T\tau_2 \gg 1$ [which is, in fact, incompatible with the condition of Eq. (49)], the situation becomes strongly nonadiabatic. In this case the correct results for the conductivity can be obtained by the diagonal method used in Ref. 31 (see also Refs. 32 and 18).

We shall conclude by noting the following circumstance. The term $\sigma^{(1)}(k, \omega)$ associated with the inelastic scattering of electrons by long-wavelength phonons, cannot be derived by a simple qualitative analysis of the kind proposed by Mott[5] and based on the concept of electron hopping between weakly correlated localized states. This is clearly related to the critical role of the interference effects in the formation of the localized states themselves[4] and in the dynamics of these states, which must be followed continuously allowing for the slow [described by the inequalities (3) and (49)] changes in the phases of the "bare" Bloch electron functions. This point of view is supported by the fact that all (without exception) terms of the adiabatic series for the conductivity can be represented by sums of the type described by Eqs. (36) and (37) and typical of interference phenomena that appear as a result of multiple scattering of the $1D$ electrons.

The author is grateful to A. A. Gogolin, D. E. Khmelnitskiĭ, S. A. Gredeskul, R. I. Shekhter, and V. A. Slyusarev for discussing this work and critical comments, and to L. V. Chebotarev for valuable discussions during this work.

APPENDIX I. ANALYTIC CONTINUATION OF THE FUNCTION $\mathscr{G}^{(1)}_{jj}(k, \omega_n)$

An analytic continuation of $\mathscr{G}^{(1)}_{jj}(k, \omega_n)$ from complex frequencies ω_n ($n > 0$) to the real axis $\omega + i0$ can be made by going over from summation of s terms in Eq. (27) to integration with respect to the complex variable ε:

$$\sum_{s=-\infty}^{\infty} F(\zeta_s)=\frac{i\beta}{2\pi}\int_C d\varepsilon\, n_F(\varepsilon)F(\varepsilon). \qquad (I1)$$

The integration contour C in Eq. (I1) is a set of circles of infinitesimally small radius covering in the positive direction the poles of the function $n_F(\varepsilon)$ Further transformations are determined by the actual form of $F(\varepsilon)$.

We shall consider, by way of example, the quantity

$$I(\omega_n,\omega_l)=\sum_{s=-\infty}^{\infty}\langle Sp[\mathcal{G}_0(\zeta_s+\omega_l)\mathcal{G}_0(\zeta_s)\mathcal{G}_0(\zeta_s-\omega_n)]\rangle, \qquad (I2)$$

which occurs in the first term of Eq. (27). The fact that all the singularities of the function $\mathcal{G}_0(\varepsilon)$ are, on the basis of its definition (20), simple poles lying along the real axis ($\varepsilon\equiv\mathbf{R}$) permits us to deform the contour C in the integral (I1) into a contour C' consisting of six straight lines located quite close to the lines $\varepsilon\equiv\mathbf{R}$, $\varepsilon\equiv\mathbf{R}+\omega_n$, and $\varepsilon\equiv\mathbf{R}-\omega_l$, as shown in Fig. 1. Substitution of variables can be used to reduce an integral over a contour C' to an integral along the real axis where, without altering the analytic properties of the integrand, we can now make the substitution $\omega_n\to\omega+i0$. Bearing in mind that the term with $\omega_l=0$ is missing from Eq. (27), we find that the required quantity $I(\omega+i0,\omega_l)$ is described by

$$I(\omega+i0,\omega_l)=\frac{i\beta}{2\pi}\int_{-\infty}^{\infty} d\varepsilon\, n_F(\varepsilon)\langle \mathrm{Sp}\{\mathcal{G}_0(\varepsilon+\omega_l)$$
$$\times[G_0^r(\varepsilon)-G_0^a(\varepsilon)]G_0^a(\varepsilon-\omega)$$
$$+\mathcal{G}_0(\varepsilon+\omega+\omega_l)G_0^r(\varepsilon+\omega)[G_0^r(\varepsilon)-G_0^a(\varepsilon)]$$
$$+[G_0^r(\varepsilon)-G_0^a(\varepsilon)]\mathcal{G}_0(\varepsilon-\omega_l)\mathcal{G}_0(\varepsilon-\omega-\omega_l)\}\rangle. \qquad (I3)$$

Here, $G_0^{r,a}(\varepsilon)=\mathcal{G}_0(\varepsilon\pm i0)$ are the retarded and advanced Green functions for a fixed instantaneous realization of the external fields.

The expression (I3) can be simplified by considering separately the cases when $l>0$ and $l<0$. This specifies the analytic properties of the functions $\mathcal{G}_0(\varepsilon\pm\omega_l)$: they are now functions of a specific (retarded or advanced) type. Averaging causes vanishing of the terms in Eqs. (I3) containing functions of just one analytic type. Bearing this point in mind, we obtain the following expressions for $I(\omega+i0,\omega_l)$:

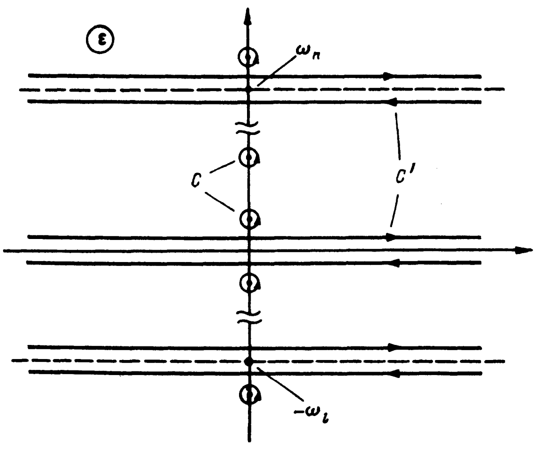

FIG. 1.

$$I(\omega+i0,\omega_l)=\frac{i\beta}{2\pi}\int_{-\infty}^{\infty} d\varepsilon[n_F(\varepsilon)-n_F(\varepsilon-\omega)]$$
$$\times\begin{cases}\langle \mathrm{Sp}[G_0^r(\varepsilon+\omega_l)G_0^r(\varepsilon)G_0^a(\varepsilon-\omega)]\rangle, & l>0;\\ -\langle \mathrm{Sp}[G_0^a(\varepsilon+\omega_l)G_0^r(\varepsilon)G_0^r(\varepsilon-\omega)]\rangle, & l<0.\end{cases} \qquad (I4)$$

An analytic continuation of the second term in Eq. (27) is obtained in a similar manner. Then, allowing for the characteristic features of the averaging procedure described in Sec. 3, we obtain Eq. (30) for $\sigma^{(1)}(k,\omega)$.

APPENDIX II. MATRIX REPRESENTATION OF THE GREEN FUNCTION

The vector form of the wave function of Eq. (4), used in the weak scattering case [Eq. (15)] leads to a matrix Green function of electrons. The factors $\psi_\pm(x)$ in Eq. (4) are continuous functions of the coordinate x and the characteristic scale of changes in these functions is determined by the localization length, which is of the order of l_2. The continuous parts of the matrix elements of the retarded Green function corresponding to these factors are found by inversion of the operator

$$\hat{T}(\varepsilon)=\varepsilon+iv\hat{\sigma}_3\partial/\partial x-\eta(x,\tau)-\zeta(x,\tau)\hat{a}^+$$
$$-\zeta^+(x,\tau)\hat{a} \qquad (II1)$$
$$(\mathrm{Im}\,\varepsilon>0)$$

in the coordinate and pseudospinor spaces. This procedure is exact for arbitrary functions η, ζ, and ζ^+ and it is described in detail in Ref. 16 for the case when $\eta=0$. Inclusion of the field η simply complicates this procedure, so that we shall give only the final results. The expression for the retarded Green function in the case of an instantaneous (corresponding to a fixed value of τ) realization of the random fields is[1]:

$$G_0^r(\varepsilon+\omega,\tau|x,y)=-\frac{1}{v}\Big\{1+\Gamma_+^\omega(y)\Gamma_-^\omega(y)$$
$$\times\exp\Big[-\frac{2i}{v}\int_{-\infty}^{\infty}\eta(x')dx'\Big]\Big\}^{-1}\begin{pmatrix}G_1^\omega & G_3^\omega\\ G_4^\omega & G_2^\omega\end{pmatrix}, \qquad (II2)$$

where

$$G_1^\omega=i\exp\Big[i\frac{\varepsilon+\omega}{v}(x-y)-\frac{i}{v}\int_y^x\eta(x')dx'\Big]\Big\{\theta(x-y)\frac{\pi_+^\omega(x)}{\pi_+^\omega(y)}$$
$$-\theta(y-x)\frac{\gamma_-^\omega(x)}{\pi_-^\omega(y)}\Gamma_+^\omega(y)\exp\Big[-2i\frac{\omega}{v}(x-y)$$
$$-\frac{2i}{v}\int_{-\infty}^{\infty}\eta(x')dx'\Big]\Big\},$$

$$G_2^\omega=i\exp\Big[-i\frac{\varepsilon+\omega}{v}(x-y)$$
$$+\frac{i}{v}\int_y^x\eta(x')dx'\Big]\Big\{\theta(y-x)\frac{\pi_-^\omega(x)}{\pi_-^\omega(y)}$$
$$-\theta(x-y)\frac{\gamma_+^\omega(x)}{\pi_+^\omega(y)}\Gamma_-^\omega(y)\exp\Big[2i\frac{\omega}{v}(x-y)$$
$$-\frac{2i}{v}\int_{-\infty}^{\infty}\eta(x')dx'\Big]\Big\},$$

$$G_3^\omega=\exp\left[i\frac{\varepsilon+\omega}{v}(x+y)-\frac{i}{v}\left(\int_{-\infty}^{x}+\int_{-\infty}^{y}\right)\eta(x')dx'\right]$$

$$\times\left[\theta(x-y)\frac{\pi_+^\omega(x)}{\pi_+^\omega(y)}\Gamma_-^\omega(y)\exp\left(-2i\frac{\omega}{v}y\right)\right.$$

$$\left.+\theta(y-x)\frac{\gamma_-^\omega(x)}{\pi_-^\omega(y)}\exp\left(-2i\frac{\omega}{v}x\right)\right],$$

$$G_4^\omega=\exp\left[-i\frac{\varepsilon+\omega}{v}(x+y)-\frac{i}{v}\left(\int_{x}^{\infty}+\int_{y}^{\infty}\right)\eta(x')dx'\right]$$

$$\times\left[\theta(x-y)\frac{\gamma_+^\omega(x)}{\pi_+^\omega(y)}\exp\left(2i\frac{\omega}{v}x\right)\right.$$

$$\left.+\theta(y-x)\frac{\pi_-^\omega(x)}{\pi_-^\omega(y)}\Gamma_+^\omega(y)\exp\left(2i\frac{\omega}{v}y\right)\right].$$

The matrix elements in Eq. (II2) are constructed from one-coordinate functions or blocks $\pi_\pm^\omega(x)$, $\Gamma_\pm^\omega(x)$ and $\gamma_\pm^\omega(x)$, which are functionals of the random fields $\eta(x', \tau)$, $\zeta(x',\tau)$ and $\zeta^+(x',\tau)$, which means that they depend parametrically also on τ. These blocks satisfy the equations

$$\frac{d\Gamma_+^\omega(x)}{dx}=-2i\frac{\omega}{v}\Gamma_+^\omega(x)-\frac{1}{v}\zeta^+(x)\exp\left[\frac{2i}{v}\int_x^\infty\eta(x')dx'\right]$$

$$+\frac{1}{v}\zeta(x)\exp\left[-\frac{2i}{v}\int_x^\infty\eta(x')dx'\right][\Gamma_+^\omega(x)]^2,$$

$$\Gamma_+^\omega(x)\to0\quad\text{for}\quad x\to\infty;\qquad\text{(II3)}$$

$$\frac{d\Gamma_-^\omega(x)}{dx}=2i\frac{\omega}{v}\Gamma_-^\omega(x)+\frac{1}{v}\zeta(x)\exp\left[\frac{2i}{v}\int_{-\infty}^x\eta(x')dx'\right]$$

$$-\frac{1}{v}\zeta^+(x)\exp\left[-\frac{2i}{v}\int_{-\infty}^x\eta(x')dx'\right][\Gamma_-^\omega(x)]^2,$$

$$\Gamma_-^\omega(x)\to0\quad\text{for}\quad x\to-\infty;\qquad\text{(II4)}$$

$$\frac{d}{dx}\frac{1}{\pi_+^\omega(x)}=\frac{\Gamma_+^\omega(x)}{\pi_+^\omega(x)}\frac{\zeta(x)}{v}\exp\left[-\frac{2i}{v}\int_x^\infty\eta(x')dx'\right],$$

$$\pi_+^\omega(x)\to1\quad\text{for}\quad x\to\infty;\qquad\text{(II5)}$$

$$\frac{d}{dx}\frac{1}{\pi_-^\omega(x)}=-\frac{\Gamma_-^\omega(x)}{\pi_-^\omega(x)}\frac{\zeta^+(x)}{v}\exp\left[-\frac{2i}{v}\int_{-\infty}^x\eta(x')dx'\right],$$

$$\pi_-^\omega(x)\to1\quad\text{for}\quad x\to-\infty;\qquad\text{(II6)}$$

$$\gamma_\pm^\omega(x)=\Gamma_\pm^\omega(x)\pi_\pm^\omega(x).\qquad\text{(II7)}$$

The most important feature of the representation given by Eq. (II2) is that, although the Green function itself is the solution of the boundary-value problem, it can be expressed in terms of the functionals $\pi_\pm^\omega$, $\Gamma_\pm^\omega$ and $\gamma_\pm^\omega$, which satisfy the conditions of the dynamic causality. The averaging of these quantities can (under some conditions) be carried out exactly, which in the final analysis makes it possible to include the interference effects associated with the multiple scattering of electrons.

[1] The time arguments of the random fields η, ζ, and ζ^+ and of the functionals of these fields are omitted from Eqs. (II2)–(II7) in order to avoid overloading these expressions.

---

[1] N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).
[2] N. F. Mott, Adv. Phys. **16**, 49 (1967).
[3] N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials*, Clarendon Press, Oxford (1971), Chap. 2.
[4] V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].
[5] N. F. Mott, Philos. Mag. **19**, 835 (1969).
[6] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors*, Springer Verlag, Berlin (1984), Chap. 9.
[7] W. Brenig, G. H. Döhler, and H. Heyszenau, Philos. Mag. **27**, 1093 (1973).
[8] V. K. S. Shante, C. M. Varma, and A. H. Bloch, Phys. Rev. B **8**, 4885 (1973).
[9] O. N. Dorokhov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 91 (1983) [JETP Lett. **38**, 105 (1983)].
[10] V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).
[11] J. Kurkijärvi, Phys. Rev. B **8**, 922 (1973).
[12] V. L. Berezinskiĭ, L. P. Gor'kov, and O. N. Dorokhov, Solid State Commun. **37**, 61 (1981).
[13] V. L. Berezinskiĭ and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **77**, 2498 (1979) [Sov. Phys. JETP **50**, 1209 (1979)].
[14] L. P. Gor'kov, O. N. Dorokhov, and F. V. Prigara, Zh. Eksp. Teor. Fiz. **85**, 1470 (1983) [Sov. Phys. JETP **58**, 852 (1983)].
[15] H. Böttger and V. V. Bryksin, Phys. Status Solidi B **78**, 9, 415 (1976).
[16] E. A. Kaner and L. V. Chebotarev, Phys. Rep. **150**, 176 (1987).
[17] E. A. Kaner and L. V. Chebotarev, Solid State Commun. **53**, 823 (1985).
[18] É. A. Kaner, Yu. V. Tarasov, and L. V. Chebotarev, Zh. Eksp. Teor. Fiz. **90**, 1392 (1986) [Sov. Phys. JETP **63**, 816 (1986)].
[19] A. A. Gogolin, S. P. Zolotukhin, V. I. Mel'nikov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 564 (1975) [JETP Lett. **22**, 278 (1975)].
[20] D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics*, Consultants Bureau, New York (1974).
[21] A. A. Abrikosov and I. A. Ryzhkin, Adv. Phys. **27**, 147 (1978).
[22] E. S. Fradkin, Tr. Fiz. Inst. Akad. Nauk SSSR **29**, 3 (1965).
[23] A. N. Vasil'ev, *Functional Methods in Quantum Field Theory and Statistics* [in Russian], Leningrad State University (1976), Chap. 5.
[24] É. A. Kaner and L. V. Chebotarev, Zh. Eksp. Teor. Fiz. **86**, 1380 (1984) [Sov. Phys. JETP **59**, 807 (1984)].
[25] E. A. Kaner and Y. V. Tarasov, Phys. Rep. **165**, 189 (1988).
[26] V. I. Klyatskin, *Embedding Method in the Theory of Wave Propagation* [in Russian], Nauka, Moscow (1986), Chap. 1.
[27] L. V. Chebotarev, Thesis for the Degree of Doctor of Physicomathematical Sciences [in Russian], Kharkov State University (1984), Chap. 5.
[28] É. A. Kaner and L. V. Chebotarev, Zh. Eksp. Teor. Fiz. **86**, 287 (1984) [Sov. Phys. JETP **59**, 164 (1984)].
[29] D. Bom, *Quantum Theory*, Prentice-Hall, Englewood Cliffs, NJ (1951).
[30] T. Holstein, Ann. Phys. (N.Y.) **29**, 410 (1964).
[31] A. A. Gogolin, V. I. Mel'nikov, and É. I. Rashba, Zh. Eksp. Teor. Fiz. **69**, 327 (1975) [Sov. Phys. JETP **42**, 168 (1975)].
[32] A. A. Gogolin, Phys. Rep. **86**, 1 (1982).

Translated by A. Tybulewicz