"Supersolitons" in periodically inhomogeneous long Josephson junctions

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An investigation is made of the dynamics of fluxon chains in long Josephson junctions with a periodic lattice of local inhomogeneities. In the commensurate case a chain as a whole is in a pinned state as long as the density of the bias current density is below a certain critical value. It is shown that defects in the form of an excess fluxon or a "hole" may propagate in a pinned chain. The long-wavelength approximation is used to deduce the evolution equation of a local deformation of a chain: the result is an "elliptic sine-Gordon equation" which has exact soliton solutions ("supersolitons") describing such defects. The current-voltage characteristics are found for the motion of a supersoliton in the presence of dissipation and a bias current (when the density of this current is less than the critical value). Supersoliton excitations are then predicted on the basis of a direct numerical solution of a perturbed sine-Gordon equation describing a periodically inhomogeneous junction. The soliton solutions of the elliptic sine-Gordon equation are also obtained numerically. Although the latter equation is in all probability nonintegrable, a numerical investigation shows in particular that a collision of two solitons of opposite polarities is in practice absolutely elastic. Both models are used to calculate the current-voltage characteristics of a ring-shaped inhomogeneous junction. An experimental study is reported of a linear Josephson junction containing a regular lattice of deliberately formed inhomogeneities. Steps on the current-voltage characteristics of such a junction are found to occur at a voltage that depends strongly on the applied magnetic field. These features are attributed to the motion of supersolitons in a junction.

1. INTRODUCTION

Topological soliton excitations in the form of magnetic flux quanta (fluxons) play an important role in the dynamics of long Josephson junctions.[1,2] It is particularly illuminating to investigate the dynamics of periodic chains of fluxons in long Josephson junctions containing a periodic lattice of deliberately formed inhomogeneities.[3–7] In particular, the dependence of the effective force (bias current density) which breaks a fluxon chain pinned to an inhomogeneity lattice on the chain density (proportional to the magnetic field at the edges of the junction), predicted theoretically and confirmed experimentally,[6] has sharp maxima at points corresponding to the situations when the chain becomes commensurate with the inhomogeneity lattice. If the bias current density is less than this critical value, the chain as a whole is at rest. However, since a fluxon chain has a finite rigidity, it follows that deformation waves may travel along a pinned chain. If a chain is almost commensurate with the lattice, i.e., if a sufficiently long segment of the chain corresponds to one extra or one missing fluxon, the corresponding defect may travel along a chain as a deformation wave under the influence of a bias current of density below the critical value. This effect was first found in a numerical experiment.[7] Our aim was to carry out a more detailed theoretical and experimental investigation of this effect.

In the second section of the present paper we shall consider the propagation of long-wavelength deformations in a pinned chain of fluxons on the basis of a familiar model[8] describing an inhomogeneous long Josephson junction in the presence of dissipative losses and a bias current. An approach analogous to the Whitham method[9] will be used to derive an effective evolution equation for long-wavelength deformations, which is the "elliptic sine-Gordon equation." Although this equation is not exactly integrable, in contrast to the conventional sine-Gordon equation, it has an exact solution in the form of a topological soliton which we shall call a "supersoliton." Depending on its polarity, a supersoliton describes either an excess or a missing fluxon in an infinite chain. In the presence of an arbitrary bias current and dissipation a supersoliton moves at a constant velocity which is uniquely defined.

In the third section we shall give the results of a numerical-investigation of the initial model of an inhomogeneous long Josephson junction, based on the usual perturbed sine-Gordon equation (the results were given first in Ref. 7), and of the new elliptic sine-Gordon equation. We shall show that the profile and velocity of a supersoliton found using both models are in good agreement and they also agree with analytic expressions obtained in the second section. We shall quote the results of a numerical investigation (based on the elliptic sine-Gordon equation) of collisions between two solitons of the opposite polarity. Since this equation is nonintegrable, a collision should be accompanied by radiative losses (emission of small-amplitude waves). Our numerical results showed that the losses are extremely weak, so that a collision looks almost absolutely elastic in a wide range of parameters. The reason for this is not quite clear.

In the fourth section we shall give the results of an experimental investigation of the current-voltage characteristics of linear long Josephson junctions with a lattice of local inhomogeneities of the microresistance type (inhomogeneous long Josephson junctions of this type had been studied earlier[4]). The experimental current-voltage characteristics can be interpreted unambiguously as the result of motion of supersoliton excitations in a chain of fluxons pinned to inhomogeneities.

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2. THEORETICAL MODEL

Evolution of a local phase shift $\varphi(x, t)$ of the wave function of superconducting electrons (proportional also to the magnetic flux in a junction) in an inhomogeneous long Josephson junction can be described by the following perturbed sine-Gordon equation[8]

$$\varphi_{tt}-\varphi_{xx}+\sin\varphi=-\alpha\varphi_t-\gamma+\varepsilon\sum_{n=-\infty}^{+\infty}\delta(x-an)\sin\varphi, \tag{1}$$

where α is the dissipative constant; γ is the bias current density; a is the period of a lattice of local inhomogeneities; the parameter ε represents the influence of a single inhomogeneity. The $\varepsilon>0$ and $\varepsilon<0$ cases correspond to what are known as a microresistance and a microshort, i.e., to local regions in a junction where the critical Josephson current density is reduced or enhanced, respectively.

We shall analyze the model described by Eq. (1) using perturbation theory (for a review see Ref. 10) and assuming that the parameters α, γ, and are small[2)]; on the other hand, the lattice period a can be arbitrary. In the zeroth approximation ($\alpha=\gamma=\varepsilon=0$) a chain of fluxons at rest is described by the following exact solution of the sine-Gordon equation:

$$\varphi(x)=\pi-2\,\mathrm{am}[(x-\xi)/k], \tag{2}$$

where am is the elliptic Jacobi amplitude and the elliptic modulus k ($0<k<1$) is an arbitrary parameter governing the chain period

$$L=2kK(k) \tag{3}$$

[here $K(k)$ is a complete elliptic integral of the first type], and ξ is an arbitrary constant (chain coordinate). We shall assume that the following condition of commensurability of the lattice and chain is satisfied:

$$a=pL, \tag{4}$$

where p is an arbitrary integer.

The total Hamiltonian of the model (1) (when $\alpha=0$) is equal to the sum of the Hamiltonian of the unperturbed system

$$H_0=\int_{-\infty}^{+\infty}\left[\frac{1}{2}\varphi_t^2+\frac{1}{2}\varphi_x^2+(1-\cos\varphi)\right]dx, \tag{5}$$

the Hamiltonian of the interaction with the lattice

$$H_\varepsilon=-\varepsilon\sum_{n=-\infty}^{+\infty}[1-\cos\varphi(x=an)], \tag{6}$$

and a term representing the bias current

$$H_\gamma=\gamma\int_{-\infty}^{+\infty}\varphi(x)\,dx. \tag{7}$$

We shall now assume that a chain of fluxons is deformed over a large distance $\lambda\gg L$. It is then sufficient to assume that the quantity ξ in Eq. (2) is a slowly varying function $\xi(x,t)$ ($\xi_x\sim L/\lambda$), whereas the elliptic modulus k remains constant. Then, substituting Eq. (2) in the terms (5)–(7), we can readily calculate the total Hamiltonian of the system $H\equiv H_0+H_\varepsilon+H_\gamma$, expressed in terms of $\xi(x,t)$:

$$H=\int_{-\infty}^{+\infty}dx\left[\frac{1}{2}\rho(\xi_t^2+\xi_x^2)-\frac{2\varepsilon}{a}\mathrm{cn}^2\left(\frac{\xi}{k}\right)-G\xi\right], \tag{8}$$

where cn is the elliptic cosine, $\rho\equiv 4k^{-2}E(k)/K(k)$ is the density of the chain mass [$E(k)$ is a complete elliptic integral of the second kind], and $G\equiv\pi\gamma k/K(k)$.

The Hamiltonian (8) gives rise to the following equation of motion:

$$\xi_{tt}-\xi_{xx}+4\varepsilon(a\rho k)^{-1}\mathrm{sn}\left(\frac{\xi}{k}\right)\mathrm{cn}\left(\frac{\xi}{k}\right)\mathrm{dn}\left(\frac{\xi}{k}\right)=\rho^{-1}G-\alpha\xi_t, \tag{9}$$

where the last (dissipative) term can be deduced from the energy balance; sn and dn are, respectively, the elliptic sine and the "delta amplitudes." The maximum velocity corresponding to the left-hand side of Eq. (9) is the same as the maximum velocity for Eq. (1) (which in our notation is equal to unity), i.e., it is equal to the velocity of the Swihart waves. Equation (9) (with $G=\alpha=0$) can obviously be reduced to the dimensionless form

$$\Xi_{TT}-\Xi_{XX}+2\,\mathrm{sgn}(\varepsilon)\,\mathrm{sn}\left(\frac{\Xi}{2}\right)\mathrm{cn}\left(\frac{\Xi}{2}\right)\mathrm{dn}\left(\frac{\Xi}{2}\right)=0, \tag{10}$$

where Ξ, T, and X can be expressed in a self-evident manner in terms of ξ, t, and x. It is natural to call Eq. (10) the elliptic sine-Gordon equation. It depends on a continuous parameter k and on the sign parameter $\mathrm{sgn}(\varepsilon)$, (i.e., in fact there are two different elliptic sine-Gordon equations corresponding to $\varepsilon>0$ and $\varepsilon<0$). If k is low, Eq. (10) is close to the conventional sine-Gordon equation:

$$\Xi_{TT}-\Xi_{XX}+\sin\Xi=-{}^5/_8k^2\sin(2\Xi)+O(k^4). \tag{11}$$

It is well known that the sine-Gordon equation (11) with a small perturbation $\sim\sin(2\Xi)$ is nonintegrable (see, for example, Ref. 10). It follows that Eq. (10) should also be nonintegrable.

Finally, we note that the spectrum $\omega(q)$ of weak excitations described by the linearized equation (10) is of the optical (gap) type, exactly as in the case of the conventional sine-Gordon equation. If $\varepsilon>0$, the ground state is $\Xi_0=4Kn$ ($n=0, 1, 2, \ldots$) and the spectrum can be of the form $\omega^2(q)=1+q^2$, where ω is the frequency and q is the wave number. If $\varepsilon<0$, we have $\Xi_0=2K(2n+1)$ and against the background of the ground state we obtain $\omega^2=k^2+q^2$.

We shall now turn back to Eq. (9). If $G=0$, it has the following exact solution describing a supersoliton at rest:

$$\xi(x)=kF\{\arcsin[(1-k^2)\mathrm{ch}^2(2xk^{-1}(\varepsilon/a\rho)^{1/2})+k^2]^{-1/2}\},$$
$$-\infty<x<0, \tag{12a}$$

$$\xi(x)=2kK(k)-\xi(-x),\qquad 0<x<+\infty \tag{12b}$$

if $\varepsilon>0$, and

$$\xi(x)=-kF\{\arccos[(\mathrm{ch}^2(2xk^{-1}[(1-k^2)|\varepsilon|/a\rho]^{1/2})-k^2)^{-1/2}\times(1-k^2)^{1/2}]\},\qquad -\infty<x<0, \tag{13a}$$

$$\xi(x)=-\xi(-x),\qquad 0<x<+\infty \tag{13b}$$

if $\varepsilon<0$. Here, $F(z)$ is an incomplete elliptic integral of the first kind.

The supersolitons described by Eqs. (12) and (13) are isolated by the boundary condition

$$\xi(x=+\infty)-\xi(x=-\infty)=2kK(k)\equiv L \tag{14}$$

[we recall that L is the period (3) of a fluxon chain]. Therefore, a supersoliton described by Eq. (12) or (13) can indeed be regarded as a "hole" in a fluxon chain. A supersoliton of the opposite polarity, i.e., the solution (12) or (13) with the sign reversed, describes fully analogously an excess fluxon in a chain. The solution for a moving supersoliton (in the case when $G = \alpha = 0$) can be obtained from Eq. (12) or (13) in an obvious manner by the Lorentz transformation.

In connection with nonintegrability of Eq. (10) we recall that nonintegrable equations can have exact one-soliton solutions, but collisions of solitons should be elastic because of emission of radiation.[10,13] In spite of the quite obvious nonintegrability of Eq. (10), discussed above, a numerical investigation of collisions of two supersolitons of opposite polarities, reported below in Sec. 3, demonstrates that such a collision appears in practice as absolutely elastic in a wide range of parameters.

We shall now consider the case when G and α are both nonzero. In the range defined by

$$\gamma^2<\gamma_c^2=\frac{4}{27}\left(\frac{\varepsilon}{\pi pk^3}\right)^2[(5k^2-2k^4-2)(1+k^2)+2(1+k^4-k^2)^{3/2}] \tag{15}$$

a fluxon chain remains pinned as a whole by a lattice of inhomogeneities.[6] On the other hand, a chain defect in the form of a hole or an excess fluxon, described by the supersoliton solution, moves at an equilibrium velocity v which can be found from the energy balance:

$$\frac{v^2}{1-v^2}=\frac{\pi^2pk^3K^2(k)\gamma^2}{8E(k)\alpha^2\varepsilon}\left(\ln\frac{1+k}{1-k}\right)^{-2} \tag{16}$$

if $\varepsilon > 0$ and

$$\frac{v^2}{1-v^2}=\frac{\pi^2pk^3K^2(k)\gamma^2}{32E(k)\alpha^2|\varepsilon|}(\arcsin k)^{-2} \tag{17}$$

if $\varepsilon < 0$ (this should be compared with the familiar expression

$$\frac{v^2}{1-v^2}=\frac{\pi^2\gamma^2}{16\alpha^2} \tag{18}$$

for the equilibrium velocity of a solitary fluxon in a homogeneous junction[8]).

The quantity $\langle\varphi_t\rangle$ averaged over the coordinate is known to be proportional to the voltage across a Josephson junction.[1,2] Using Eqs. (2) and (14), we obtain the following result:

$$\langle\varphi_t\rangle=2\pi v/l, \tag{19}$$

where l is the total length of the junction. It therefore follows that a supersoliton creates the same voltage as an ordinary solitary fluxon moving at the velocity v. On the other hand, the current-voltage characteristics, i.e., the dependence $\langle\varphi_t\rangle(\gamma)$ is different for a supersoliton and an ordinary fluxon because of the different dependences $v(\gamma)$ [compare Eqs. (16), (17), and (18)].

We shall now consider the limits of validity of our theory. It follows from Eqs. (12) and (13) that the characteristic size λ of a supersoliton can be estimated as follows:

$$\lambda^2\sim pk\{|\varepsilon|\ln[(1-k^2)^{-1}]\}^{-1}. \tag{20}$$

Equation (9) can be regarded as the Whitham type equation[9] for envelopes. The condition of its validity $L\ll\lambda$ can be reduced, subject to Eqs. (3) and (20), to the form

$$L^3\ll a/|\varepsilon|. \tag{21}$$

We can therefore see that the above description applies to fluxon chains with a moderately low density (but sufficiently rigid).

Finally, we note that if we consider a model of a periodically inhomogeneous long Josephson junction with a harmonic modulation function[14] described by the equation [see Eq. (1)]

$$\varphi_{tt}-\varphi_{xx}+\sin\varphi=-\alpha\varphi_t-\gamma+\varepsilon a^{-1}\cos(2\pi x/a)\sin\varphi,$$

we obtain not Eq. (9) for $\xi(x, t)$ but the conventional sine-Gordon equation [with terms of the same type on the right hand side as in Eq. (9)]. However, harmonic modulation can hardly be realized experimentally.

3. NUMERICAL MODELING

In this section we shall describe the results of our numerical solution of the perturbed sine-Gordon equation for a periodically modulated Josephson junction of Eq. (1), and of the elliptic sine-Gordon equation (9) derived in the preceding section. In both cases we shall use a finite-difference stabilized explicit method,[15] valued for its simplicity and effectiveness when applied to the nonlinear Klein–Gordon equations.

We shall first describe briefly the results important to our case and relating to Eq. (1) (a more detailed account can be found in Ref. 7). Integration of Eq. (1) was carried out using periodic boundary conditions for a Josephson junction closed to form a ring:

$$\varphi(l, t)=\varphi(0, t)+2\pi n_{fl}, \tag{22}$$

$$\varphi_x(l, t)=\varphi_x(0, t), \tag{23}$$

where n_{fl} is the number of unipolar fluxons in a ring junction (n_{fl} = const). The parameters of this junction assumed in our numerical modeling were as follows: $1 = 8.0$; $\alpha = 0.05$; $n_{fl} = 4$–7. The last term in Eq. (1) was approximated as follows:[3,4]

$$\varepsilon\sum_{n=0}^{N-1}\delta(x-an-x_0)\rightarrow\operatorname{sgn}(\varepsilon)\sum_{n=0}^{N-1}\left[1-\operatorname{th}^2\frac{2(x-an-x_0)}{\varepsilon}\right], \tag{24}$$

where the following numerical values were used: $\varepsilon = 0.2$, $a = 1.6$, and $x_0 = 0.8$.

The initial conditions for a chain of vortices at the first point of the current-voltage characteristic were selected in the form of a function $\varphi^0(x, t)$, which is linear in x and which satisfies the boundary conditions (22) and (23). The initial velocity of a chain selected in accordance with the energy balance relationship is given by Eq. (18). In the calculation of the other points of the current-voltage characteristics, we assumed that the initial conditions $\varphi^i(x, t)$ were in the form of the phase distributions corresponding to steady-state oscillations of φ_t in the preceding adjacent point on the current-voltage characteristic. The step in the difference scheme along the spatial and time coordinates was the same: $\Delta x = \Delta t = 0.025$. The relative precision of the deteamination of the average voltage φ_t (of the current-voltage characteristic) across the junction was $\approx 1\%$. In calculations of the selected parts of the current-voltage characteristics the distributed current γ was varied first in one direction and then in the opposite direction in order to find all the points on the characteristic. For convenience, we introduced a reduced voltage $u = \varphi_t l / 2\pi$, so that the motion in a ring chain of n_{fl} fluxons at the maximum velocity $v = 1$ corresponded to an average voltage $\langle u \rangle = n_{fl}$.

Figure 1a shows the current-voltage characteristic calculated numerically for a commensurate configuration of a chain of $n_{fl} = 5$ fluxons in a ring junction containing a lattice of $N = 5$ inhomogeneities. The lowest-energy state of this system corresponds to the case when each fluxon is pinned by an inhomogeneity. In this case the force pinning the fluxon chain is strong, which corresponds to a high critical value of the current γ_c. Since the chain is fairly dense, the reduced magnetic-field in the junction $\varphi_x(x)$ is quite high and on the average constant: it is only weakly modulated near the individual fluxons (Fig. 2a). In the course of motion of a chain of fluxons it is found that an increase in the external current γ increases the voltage monotonically: this increase is first nearly linear and then an asymptotic value $\langle u \rangle \sim n_{fl}$ is reached (this is the main Swihart peak).

The results of our calculation of the current-voltage characteristic for two incommensurate configurations characterized by $n_{fl} \neq N$ are presented in Figs. 1b and 1c. In this case the number of inhomogeneities is still the same ($N = 5$), but only the number of fluxons n_{fl} is varied. If $n_{fl} = 4$ and $n_{fl} = 6$, then in addition to the Swihart peak, the current-voltage characteristic shows a clear step at $\langle u \rangle \approx 1.0$. This is accompanied by a major redistribution of the field $\varphi_x(x)$ in the junction corresponding to a strong deformation of the fluxon chains. If $n_{fl} = 4$, a region with a lower value of the field φ_x (Fig. 2b) appears in the junction, whereas for $n_{fl} = 6$ there is a local region with a higher value of the field φ_x (Fig. 2c). According to the ideas put forward in Sec. 2, these singularities represent nonlinear collective

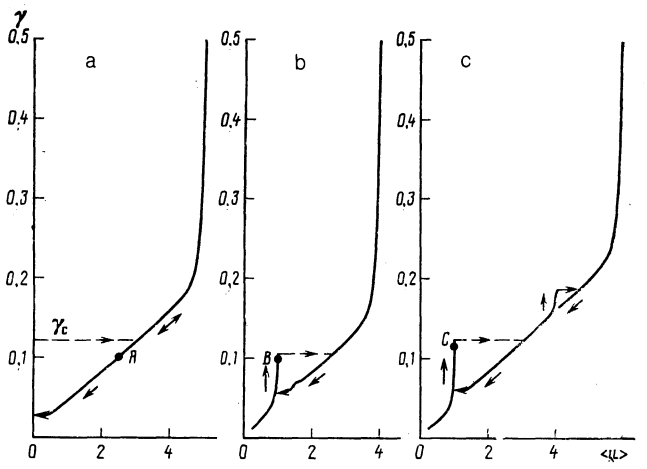

FIG. 1. Numerically calculated current-voltage characteristics of a long ring junction containing a lattice of N local inhomogeneities in a chain of n_{fl} unipolar fluxons: a) $n_{fl} = N = 5$; b) $n_{fl} = 4$, $N = 5$; c) $n_{fl} = 6$, N = 6.

excitations in a fluxon chain, i.e., they are supersolitons. If $u \approx 1.0$, the chain as a whole is at rest, but a supersoliton travels along this chain at a velocity $v \approx 1$ and supersolitons of opposite polarities move in opposite directions.

Figure 1c ($n_{fl} = 6$) shows not only the main supersoliton step at $\langle u \rangle \approx 1.0$, but also a weak singularity at $\langle u \rangle \approx 4.0$. Its origin becomes obvious if we allow for the fact that, in addition to the simplest commensurate configuration $a = L$ [$p = 1$ in Eq. (4)], there are also other configurations, particularly $a = 2L$ ($p = 2$). The step at $\langle u \rangle \approx 4.0$ can be interpreted as the appearance, in a chain of $n_{fl} = 6$ fluxons, of four supersoliton excitations ($n_{ss} = -4$) corresponding to the second ($p = 2$) commensurate configuration. The correctness of this explanation is supported by the fact that if $n_{fl} = 7$, then the calculated current-voltage characteristic exhibits steps at $\langle u \rangle \approx 2.0$ ($n_{ss} = 2$ for $p = 1$) and at $\langle u \rangle \sim 3.0$ ($n_{ss} = -3$ for $p = 2$).

An additional numerical calculation shows that for the same-values of n_{fl} and $N = 5$ as before, the asymptotic values of the voltage steps $\langle u \rangle$ remain unchanged in a wide range of junction lengths $6.0 < 1 < 12.0$. This demonstrates

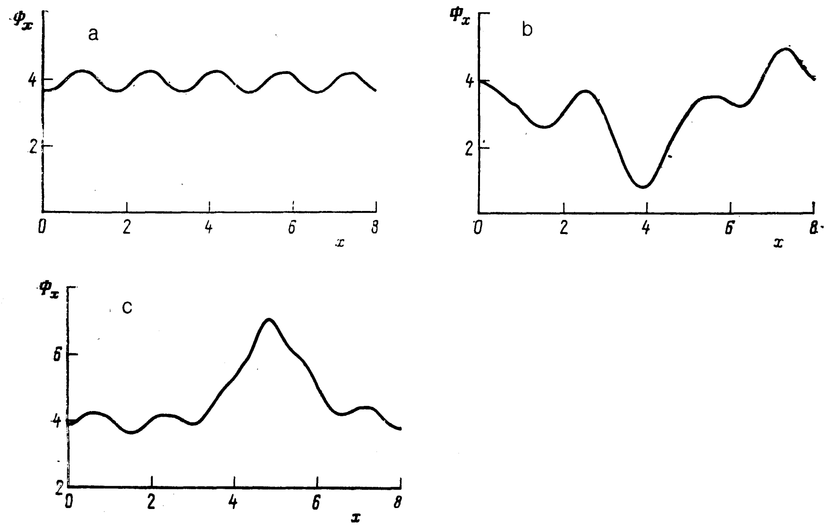

FIG. 2. Distribution of the local magnetic field Φ_x in a long ring junction corresponding to the individual points on the current-voltage characteristic a) at the point A (Fig. 1a); b) at the point B (Fig. 1b); c) at the point C (Fig. 1c).

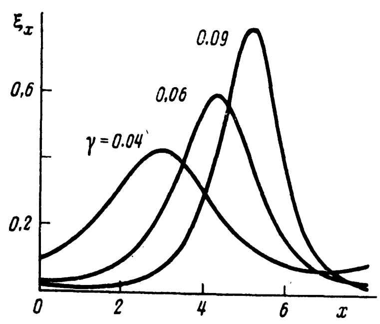

FIG. 3. Numerically calculated steady-state supersoliton profile described by the elliptic sine-Gordon equation (9).

that the steps on the current-voltage characteristic observed in this case are unrelated to a resonant interaction between a chain of fluxons and its radiation emitted at inhomogeneities,[5] but can be explained only if we assume the presence of supersolitons in the chain.

We shall now consider the results obtained from the elliptic sine-Gordon equation (9). We investigated numerically a system with the following parameters: $k = 0.478$; $L = a = 2kK(k) = 1.6$; $1 = aN = 8.00$; $\varepsilon = 0.2$–1.0; $\alpha = 0.05$. The step in the difference scheme was again the same along the spatial and time coordinates: $\Delta x = \Delta t = 0.025$. The boundary conditions for Eq. (9) were assumed to be

$$\xi(l,t) = \xi(0,t) + n_{ss}L, \tag{25}$$

$$\xi_x(l,t) = \xi_x(0,t), \tag{26}$$

where n_{ss} is the number of supersolitons, and $p = 1$ ($n_{ss} > 0$ corresponds to excess fluxons and $n_{ss} < 0$ to holes). Our initial conditions were in the form of the exact solution represented by Eqs. (12a) and (12b) for the unperturbed equation (9), as well as the normalized exact solution of the conventional sine-Gordon equation (1). In both cases the characteristic times needed to find the solution did not exceed 100–150 time units. Figure 3 shows the stationary (steady-state) solutions for three values of the current γ.

The relationship (16) was checked by a numerical calculation of the current-voltage characteristic on the assumption that $p = 1$ for three values of ε: 0.2, 0.5, and 1.0. The results are plotted in Fig. 4a. In the $\varepsilon = 0.2$ case the current-voltage characteristic is in qualitative agreement with the results of calculations carried out using the initial model represented by Eq. (1) and corresponds to a step on the characteristic at $\langle u \rangle \approx 1.0$ when $n_{fl} = 6$ (Fig. 1c). A comparison of the results of these calculations with Eq. (16) was made by converting the calculated current-voltage characteristics to shared reduced coordinates θ and γ, where θ is defined by

$$\theta = \frac{4\alpha\varepsilon^{1/2}}{\pi\gamma(1/\langle v\rangle^2 - 1)^{1/2}}. \tag{27}$$

The characteristics converted in this way are plotted in Fig. 4b. It follows from the theory that the dependence of the quantity given by Eq. (27) on γ disappears and the points obtained for all three current-voltage characteristics fit well the theoretical value

$$\theta = \frac{2k^{1/2}K(k)}{[2E(k)]^{1/2}\ln[(1+k)/1-k)]}, \tag{28}$$

which is equal to 0.618 if $k = 0.478$.

Finally, bearing in mind that in all probability Eq. (9) is nonintegrable, it would be of interest to investigate a collision of two supersolitons of the opposite polarity using the unperturbed model corresponding to Eq. (9) and assuming that $\alpha = \gamma = 0$. This was done for a ring system (junction) of length $l = 16.00$ on the assumption that $k = 0.478$ and $\varepsilon = 0.5$. The dissipation represented by $\alpha = 0.05$ and an external current $\gamma = 0.1$, governing the initial velocities $\pm v_0$ of supersolitons ($v_0 \approx 0.75$), were included right from the beginning of these calculations. After establishment of steady-state motion in a ring (Fig. 5a), which usually required 300–400 time units, it was assumed that the dissipation in the current disappeared: $\alpha = \gamma = 0$. Next, the time intervals between consecutive collisions were found over a long period (3000–5000 time units). The results indicated that in a wide range of initial velocities $0.6 < v_0 < 1.0$ a collision was practically absolutely elastic and the supersoliton velocity remained constant to within 0.5% after 400–600 collisions (Fig. 5b). Figure 5c shows the distribution of the quantity $\xi_x(x)$ in such a system after 5000 time units (about 450 collisions) from the moment corresponding to the situation illustrated in Fig. 5a. The "ripples" between supersoli-

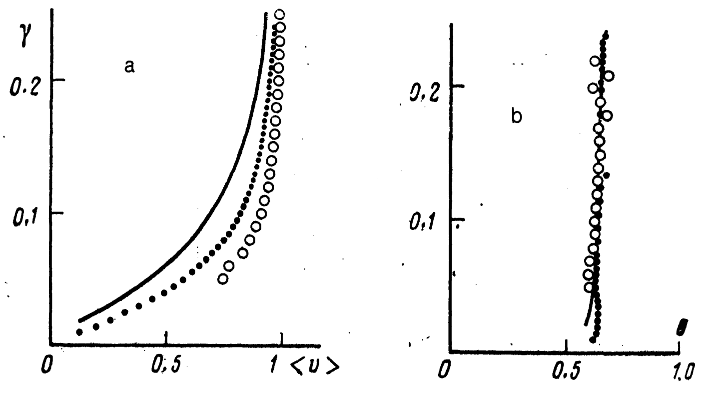

FIG. 4. a) Current-voltage characteristic $\langle v \rangle$ (γ) corresponding to the motion of one ($n_{ss} = 1$) supersoliton, found numerically using Eq. (9). b) Same characteristic converted in accordance with Eq. (27). The continuous curves correspond to $\varepsilon = 1.0$; ●) $\varepsilon = 0.5$; O) $\varepsilon = 0.2$.

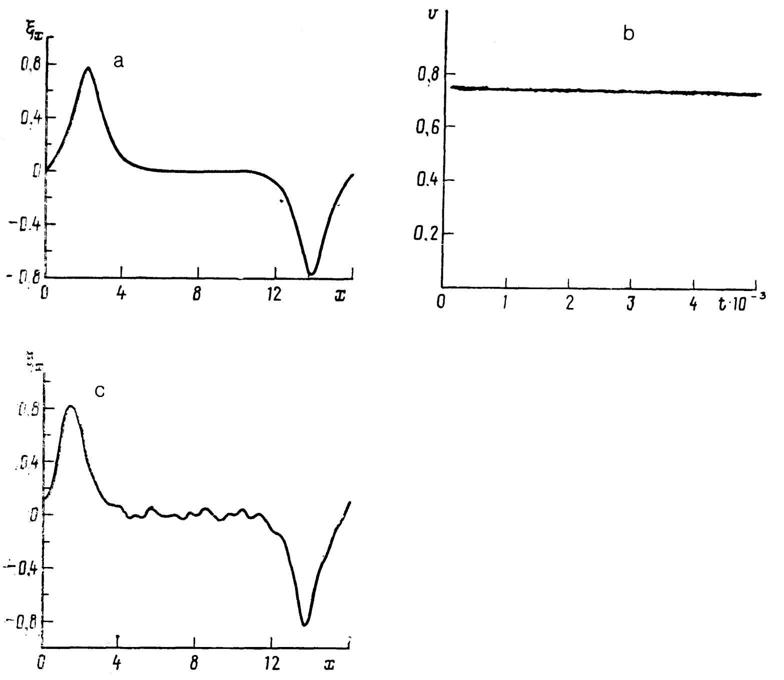

FIG. 5. Collisions of two supersolitons of the opposite polarity traveling at velocities $\pm v$ in the model of Eq. (9) characterized by $\alpha = \gamma = 0$: a) initial configuration;, b) time dependence of the velocity v; c) wave field configuration at $t = 5000$ (after 470 collisions).

tons clearly represent a very weak radiation field emitted by supersolitons as a result of their collisions.

4. EXPERIMENTS

Distributed Nb/NbO_x/Pb Josephson junctions were fabricated by a conventional method employing magnetron sputtering of niobium, followed by photolithography, plasma oxidation of niobium, and thermal evaporation of lead. Periodic modulation of the critical current density was created by silicon monoxide strips ("inhomogeneities") evaporated thermally and shaped by photolithography on the surface of a niobium film before oxidation and deposition of lead. These strips were perpendicular to the long dimension of the junction and their width along the junction was 12 μm; the thickness of the strips was ≈ 1500 Å. The results reported below were obtained for a junction of 498×20 μm size with $N = 9$ strips representing inhomogeneities. The spatial modulation period of the critical current density was then $a\lambda_J \approx 50$ μm, where λ_J is the Josephson penetration depth. Details of the procedure used in our measurements had been described earlier.[4] These measurements were carried out at $T = 4.2$ K and it was estimated that at this temperature λ_J was ≈ 30 μm.

Figure 6 shows the dependence of the critical current I_c of the investigated junction (at which a voltage appeared across the junction) on the magnetic field H directed in the plane of the junction and at right-angles to its long dimension. The dependence $I_c(H)$ exhibited two sharp maxima at fields $H_1 = 1.49$ Oe and $H_2 = 3.01$ Oe. These maxima corresponded to the exact commensurability of the spatial periods of the chain and of the inhomogeneity lattice. The first maximum at $H = H_1$ corresponded to $p = 1$ in Eq. (4), whereas the second at $H = H_2$ corresponded to $p = 2$. We investigated experimentally the current-voltage characteristics of this junction in magnetic fields close to H_1 and H_2.

In the range of fields $H_1 < H < H_2$ we observed an unusual singularity in the current-voltage characteristic: it was a step labeled with an asterisk (*) in Fig. 7a. In addition to this step, we also observed (Fig. 7) a set of Fiske steps[16,17] of two types. The first type was observed at low voltages and represented conventional Fiske steps for a long junction[18] with a characteristic separation of ≈ 20 μV on the voltage scale. These steps represented resonances occurring throughout the long junction between a fluxon oscillation mode and the radiation which appeared near the edge of the junction as a result of collisions of fluxons with this edge. We also observed two sharp steps (second type of the Fiske step) separated bv voltages of ≈ 200 μV. These were typical of junctions with regular inhomogeneities under "superradiance" conditions[5,18] when a fluxon mode was in resonance with the

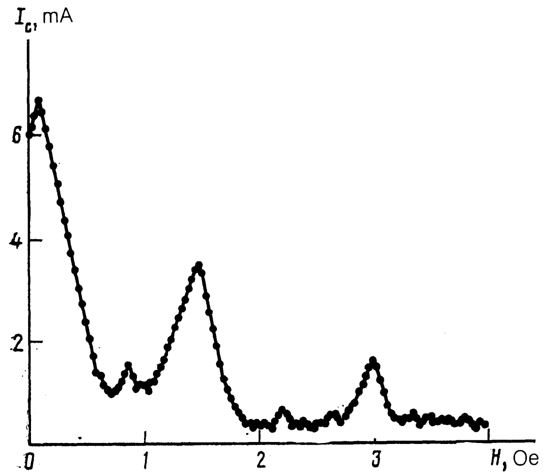

FIG. 6. Dependence of the critical current I_c flowing through an Nb/NbO_x/Pb junction with a lattice of inhomogeneities on the applied magnetic field H.

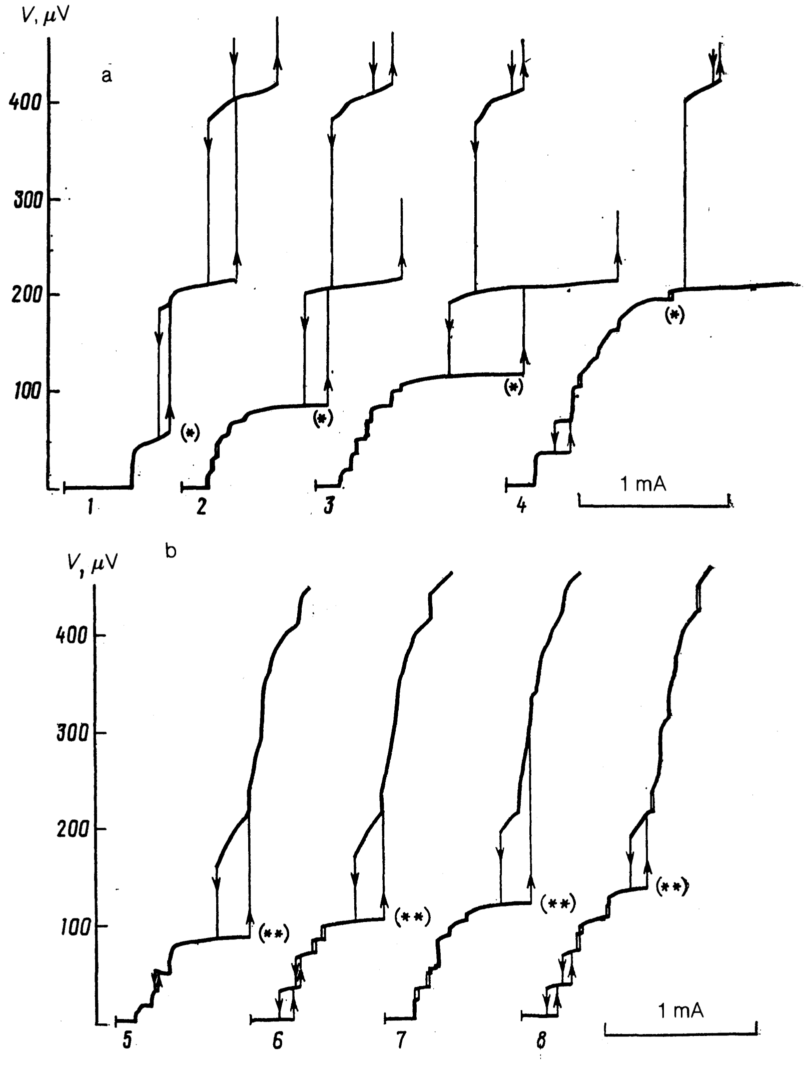

FIG. 7. Part of the current-voltage characteristic recorded at low voltages for an Nb/NbO_x/Pb junction with an inhomogeneity lattice: a) in the vicinity of the first peak of $I_c(H)$ in fields $H \geqslant H_2$, where $H = 1.83$ Oe (1), 1.98 Oe (2), 2.14 Oe (3), and 2.36 Oe (4); b) in the vicinity of the second peak of $I_c(H)$ in the range $H \geqslant H_2$, where $H = 3.41$ Oe (5), 3.47 Oe (6), 3.53 Oe (7), and 3.63 Oe (8).

radiation concentrated in short sections of the junction between the adjacent inhomogeneities. The motion of a fluxon chain as a whole ensured synchronization of the Fiske modes of the individual segments of the junction. The voltage at the Fiske steps of both types was practically unaffected by variation of the magnetic field H and only their width on the current scale changed. On the other hand, the voltage position of the step labeled (*) in Fig. 7a depended strongly on the field H. A similar singularity of the current-voltage characteristic, but observed in a narrower range of magnetic fields, appeared also when $H > H_2$. The latter step was labeled (**) in Fig. 7b.

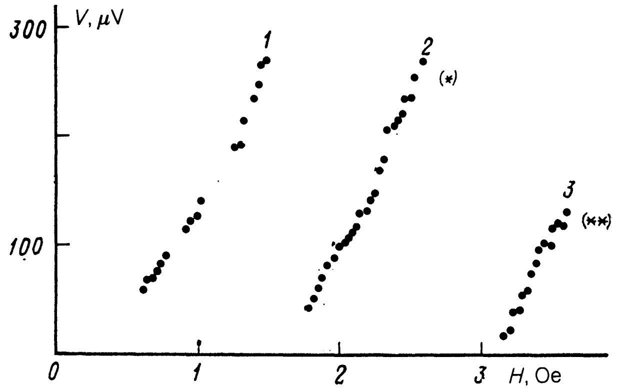

FIG. 8. Dependence of the maximum voltage at the steps in the current-voltage characteristics on the magnetic field H: 1) main Swihart step; 2) step (*) (see Fig. 7a); 3) step (**) (see Fig. 7b).

We investigated the behavior of the (*) and (**) singularities in the full range of fields in which they were observed. The results are plotted in Fig. 8. Moreover, in the range of weak fields we observed the main (Swihart) peak on the current-voltage characteristic due to the motion of the whole chain of fluxons at its maximum velocity in the junction. Since in a magnetic field the fluxon chain period was

$$L\lambda_J \approx \Phi_0/\Lambda H$$

(here Φ_0 is a magnetic flux quantum, $\Lambda = \lambda_{Nb} + \lambda_{Pb} + d$ is the depth of penetration of the magnetic field into a superconductor in the region of a Josephson spacer, and d is the effective thickness of an insulating spacer), the voltage v_{fl} corresponding to the main peak should be proportional to H:

$$V_{fl} \approx \frac{n_{fl}\Phi_0\bar{c}}{l\lambda_J} = \bar{c}\Lambda H \tag{29}$$

(we ignored here the contribution of inhomogeneities to the cross-sectional area of the junction at right-angles to the magnetic field; $\bar{c}$ is the Swihart velocity). The commensurability of a fluxon chain with the inhomogeneity lattice occurred when

$$H \approx H_p = \Phi_0 p / \Lambda a \lambda_J. \tag{30}$$

In spite of the fact that the number of fluxons n_{fl} was conserved in a linear junction (but this was not true of a ring junction, see Sec. 3) when the external magnetic field H was sufficiently high, we could speak of a constant average number of fluxons proportional to H. We could similarly assume that when the fluxon chain and the inhomogeneity lattice were incommensurate, there was a certain average number of supersolitons in the junctions and it could be estimated from

$$n_{ss} = l\left[\frac{1}{L} - \frac{p}{a}\right] = \frac{l\lambda_J\Lambda}{\Phi_0}(H - H_p). \tag{31}$$

Like fluxons, supersolitons were created at one of the edges of the junction, crossed the junction, and disappeared at the opposite edge. Therefore, the number of supersolitons n_{ss} [and, consequently, the voltage step on a current-voltage characteristic due to the motion of supersolitons–see Eq. (29)] was found to be proportional to the deviation of the magnetic field H from the commensurability field H_p. This was fully confirmed by the experimental results for the singularities (*) and (**), as demonstrated clearly in Fig. 8. The slope dV/dH of the supersoliton singularities was very nearly equal to the slope of the main peak [see Eq. (29)], as expected on the basis of physical considerations (because the topological charge Φ_0 and the maximum velocity $\bar{c}$ were the same for fluxons and supersolitons).

In the range of fields $H < H_p$ we should have $n_{ss} < 0$, which follows from Eq. (31). This corresponds to supersolitons of the opposite polarity (when instead of an excess fluxon we have a hole in a fluxon chain). However, clear steps of this type were not observed experimentally. In this range of fields we found only anomalously strong and nonmonotonic (with respect to the fields) changes in the amplitude of the Fiske steps.

Therefore, the singularities of the current-voltage characteristic of a long Josephson junction with a built-in inhomogeneity lattice observed in the present study can be interpreted unambiguously as the result of motion of supersolitons in a fluxon chain. This can be regarded as an experimental confirmation of the existence of a new type of nonlinear excitations in a nonlinear sine-Gordon system.

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[2)]A detailed investigation of the static properties of one fluxon interacting with a solitary microresistance were investigated in detail in Ref. 11. The treatment given in Ref. 11 dealt analytically with the case when the parameter ε was not small, i.e., the usual perturbation theory of Ref. 10 was inapplicable. The results of Ref. 11 had been confirmed by experiments reported in Ref. 12.

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