Possible formation of a toroidal current layer during a spark discharge

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The formation of a toroidal current layer due to the trapping of the magnetic field of a spark discharge by the ionized gas vortex ring produced from the material of the evaporating coating of the electrode is considered theoretically. A model of the current layer is proposed in the form of a monolayer of relativistic electrons and a sheath of positive ions. The velocity of the vortex ring layers which ensure stability of the current layer is found. The lifetimes of the current layer and vortex ring are estimated. The feasibility of physically producing the configuration is demonstrated.

The formation of current layers is possible in magnetic plasma configurations either due to a change in boundary conditions or as a result of magnetic field evolution due to external forces. The first possibility was analyzed in detail by Vainshtein.[1] One of the variants of the second possibility was considered in Ref. 2, where it was shown that a toroidal current layer can be formed as a result of the rapid decay of the external magnetic field in a toroidal chamber with rotating plasma. From the point of view of its structure this layer can be considered as a reversed or inverse θ-pinch because, if in the usual θ-pinch (Ref. 3, p. 191) plasma is rolled into a filament and a magnetic field created by an external source surrounds it and supports it, then, in contrast, in this configuration the magnetic field is rolled into a filament and plasma in the form of a toroidal current layer surrounds and maintains the magnetic field supported by a vortex ring from the external medium. This vortex ring guarantees the stability of the configuration.

In the present work, an extension of Ref. 2, we consider a scheme for creating a toroidal current layer inside the vortex ring of a spark discharge, propose a refined model of the current layer, and discuss the stability and lifetime of the configuration. This study is an attempt to establish a basis for the hypothesis that a toroidal current layer vortex (TCLV) can be produced in the laboratory.

1. TCLV STRUCTURE AND DESIGN FOR PRODUCING IT WITH A SPARK DISCHARGE

Following Ref. 2, a TCLV to a first approximation has the following structure (Fig. 1): A constant ring magnetic field created by a toroidal current layer is located inside a vortex ring. There is an almost complete vacuum in the region of space occupied by the magnetic field and, therefore, the pressure due to the vortex ring, described by the Euler equation

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\,\mathrm{grad}\,p_w, \tag{1}$$

is balanced by the magnetic pressure

$$p_m = \mu_0 H^2/2. \tag{2}$$

Here p_w and p_m are the pressures associated with the vortex ring and magnetic field, respectively, $\mathbf{v}$ is the velocity vector of the vortex ring layers, ρ is the vortex mass density, H is the magnetic field strength, and μ_0 is the magnetic permeability.

This configuration can be formed through the action of a spark discharge in the following manner (Fig. 2). At the first stage of the spark discharge[4] leaders "grow" and on the electrode, which must be covered by a liquid or porous poor conductor, strong evaporation of this covering material occurs; from this time to the end of the first stage of the spark discharge (t_0—t_1) a vortex ring is formed due to the interaction of the evaporated mass flux with edges of the funnel. At the second stage of the spark discharge, during which a potential relaxation wave advances, the discharge magnetic field is drawn into the interior of the vortex ring as the leading edge advances (t_1–t_2); when the middle part of the wave passes (t_2—t_3) the gas in the vortex ring is ionized by ultraviolet radiation of the spark discharge and an azimuthal arc discharge is initiated inside the ring; when the trailing edge of the wave passes (t_3–t_4) this arc discharge pumps out plasma from the central part of the vortex ring and changes into a toroidal current layer at the end of the spark discharge. This occurs due to the decrease of density and increase of plasma conductivity with increasing discharge current density. A ring magnetic field is formed as a result in the central part of the vortex ring in the region of almost complete vacuum which is a part of the field previously created by the spark discharge.

The TCLV configuration which is formed as a result of the above processes exists independently at the end of the spark discharge.

The process of trapping the part of the spark discharge magnetic field by the vortex ring can also be considered by representing the vortex ring in the form of a secondary winding of a toroidal transformer whose primary winding is the spark discharge passing through the center of the vortex ring. In this case the conductivity of the secondary winding grows sharply as the impulsive current passes through the

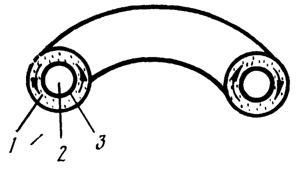

FIG. 1. TCLV structure in a first approximation: 1 is the vortex ring; 2 is the region of space occupied by the ring magnetic field; and 3 is the toroidal current layer.

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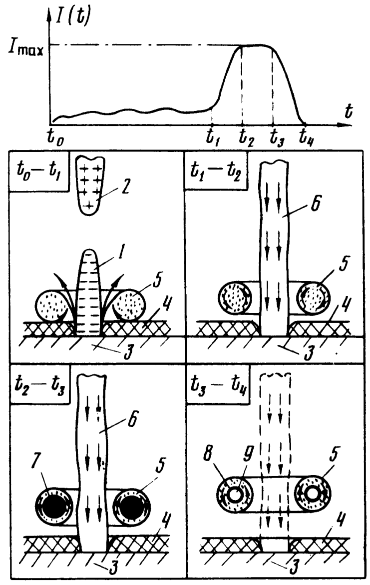

FIG. 2. Schematic of TCLV formation: 1 is the growing leader of the negative electrode; 2 is the growing leader of the positive electrode; 3 is the electrode; 4 is the friable or liquid weakly conducting cover; 5 is the vortex ring; 6 is the spark discharge at the stage of the potential relaxation wave; 7 is the azimuthal arc discharge; 8 is the toroidal current layer; 9 is the spatial region occupied by the ring magnetic field, $I(t)$ is an oscillogram of the spark discharge current; I_{max} is the current pulse value; t_0–t_1 is the period of leader growth; t_1–t_2 is the period of passage of the leading edge of the potential relaxation wave; t_2–t_3 is the period of the middle part of the wave; and t_3–t_4 is the period of passage of the trailing edge of the potential relaxation wave.

first winding, which causes the current to be maintained in the second winding after deenergization of the first winding, and consequently, the ring magnetic field to be maintained.

2. REFINED MODEL OF THE CURRENT LAYER

In order to provide the necessary strength H of the ring magnetic field the total current I_Σ in the current sheath should in view of Eq. (2) be

$$I_\Sigma=2\pi RH=2\pi R(2p_m/\mu_0)^{1/2}, \tag{3}$$

where R is the large radius of the torus. To estimate the order of magnitude of the total current we take as an example the values $R = 0.1$ m and $p_m = 1\times10^5$ Pa (atmospheric pressure). From Eq. (3) we obtain $I_\Sigma \cong 2.5\times10^{-5}$ A. It is clear that for such a current value, the toroidal current layer can exist a significant time only with very high plasma conductivity. However, the plasma is not thermally isolated, its ion temperature is low, and thus in this case only electrons with very high energy can serve as current carriers. It is supposed that they acquire this energy in the continuous acceleration regime in the process of TCLV formation. The transition of electrons to the continuous acceleration regime is possible with fulfillment if the condition

$$E[\mathrm{V/cm}]W[\mathrm{eV}]/n\sim2\cdot10^{-12}Z^2, \tag{4}$$

is satisfied (Ref. 3, p. 89), where E is the electric field strength, W is the electron energy, n is the plasma density, and Z is the ion charge. In the present case, as the azimuthal arc discharge is excited, plasma is pumped out from the central part of the vortex ring, the plasma density n decreases, and fulfillment of condition (4) becomes possible. In the process of continuous electron acceleration the arc discharge turns into a current layer and the electrons in this case are accelerated to relativistic velocities. This guarantees extremely high conductivity of the toroidal current layer.

It is natural to suppose that the relativistic electrons form a monolayer because electrons deflected toward the magnetic field are more strongly expelled by the field than the electrons of the monolayer and electrons deflected toward the vortex ring are repelled backward because of the high plasma density gradient close to the current layer.

Accepting the hypothesis of the existence of a monolayer of relativistic electrons, one must take into account neutralization of the monolayer volume charge. This neutralization is possible using a sheath of positive ions situated inside the ring magnetic field. The thickness of the sheath is determined by the depth of ion penetration into the magnetic field.

Thus the refined TCLV model (Fig. 3) in comparison with the structure of Fig. 1 contains in addition a monolayer of relativistic electrons and a sheath of positive ions. In this case the monolayer creates a ring magnetic field and the sheath located in the boundary space occupied by the ring magnetic field cancels the monolayer volume charge.

3. ELECTRON ENERGY AND CURRENT LAYER LIFETIME

To estimate the electron energy we consider two factors limiting the current layer lifetime: synchrotron radiation and electron-ion collisions.

In agreement with Ref. 5 synchrotron radiation losses of the monolayer can be described by

$$N=\frac{e^2cK}{6\pi\varepsilon_0 r_m^2}\left(\frac{W_e}{W_0}\right)^4, \tag{5}$$

where N is the power radiated by K electrons of the monolayer, r_m is the magnetic field radius, e is the electron charge, c is the speed of light, ε_0 is the dielectric permittivity, $W_0 = m_e c^2 \cong 0.51$ MeV is the electron rest energy, and W_e (MeV) is the electron kinetic energy (assumed identical for all K electrons).

Assuming that their velocity is close to the speed of light, one can find the number of electrons in the monolayer from the equation

$$K=2\pi r_m I_\Sigma/ec, \tag{6}$$

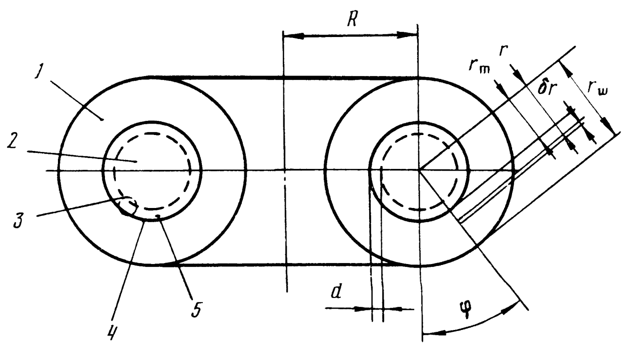

FIG. 3. Refined TCLV model: 1 is the vortex ring; 2 is the spatial region occupied by the magnetic field; 3 is the current layer; 4 is the monolayer of relativistic electrons; 5 is the positive ion sheath; R is the torus large radius; r_m is the magnetic field radius; r_w is the vortex ring radius; r is an arbitrary radius; δr is an arbitrary vortex ring layer; φ is an arbitrary angle; and d is the positive ion penetration depth ($d = d_i$).

and the energy stored by the magnetic field from the equation

$$Q_m = 2\pi^2 R r_m^2 p_m. \tag{7}$$

Then taking into account Eqs. (3) and (5)–(7) one can obtain an approximate estimate of the current layer lifetime, assuming that it is limited by synchrotron radiation:

$$\tau_s \approx \frac{Q_m}{N} = \frac{3\pi r_m^3 \varepsilon_0}{e}\left(\frac{W_0}{W_e}\right)^4 \left(\frac{p_m \mu_0}{2}\right)^{1/2}. \tag{8}$$

The losses connected with electron-ion collisions can be estimated by the electron mean free time for multiple Coulomb scattering to 90°. For deuterium the cross section for multiple Coulomb scattering is determined[6] by the approximate relation

$$\sigma_{90} = 1/nL_{90} \approx A/W_{\mathrm{keV}}^2 = B/W_e^2, \tag{9}$$

with $A = 2.6 \times 10^{-18}$ cm$^2\cdot$keV^2 and $B = 2.6 \times 10^{-28}$ m$^2\cdot$MeV^2.

We determine the plasma density in the monolayer region starting from the supposition that the number of ions equals the number of electrons, and the volume they occupy is determined by the distance d_i the positive ions penetrate into the magnetic field:

$$n = K/4\pi^2 r_m R d_i. \tag{10}$$

As given in Ref. 7 (p. 408) the ion penetration depth into the magnetic field is determined by the equation

$$d_i = m_i v_i / q_i \mu_0 H. \tag{11}$$

Here m_i, q_i, and v_i are the ion mass, charge, and velocity, respectively. Approximately setting the ion speed equal to the thermal speed at temperature T_i

$$v_i = (3kT_i/m_i)^{1/2}, \tag{12}$$

taking into account Eqs. (3), (6), and (9)–(12), we obtain for deuterium ($|q_i| = |e|$) and approximate relation for estimating the electron collision time for multiple Coulomb scattering:

$$\tau_{90} = L_{90}/c \approx W_e^2 (3kT_i m_i)^{1/2}/2p_m B. \tag{13}$$

Comparison of Eqs. (8) and (13) shows that the action of these processes on the current layer lifetime changes with the electron energy change. This provides a means, equating the right hand sides of Eqs. (8) and (13), of making an approximate estimate of the necessary electron energy:

$$W_e \sim \left[\frac{6\pi r_m^3 \varepsilon_0 B W_0^4 (p_m \mu_0)^{1/2}}{e(6kT_i m_i)^{1/2}}\right]^{1/6}. \tag{14}$$

4. TCLV STABILITY CONDITION

It is necessary to keep in mind that the above estimates for the current layer lifetime are applicable only if the configuration is stable. We note in this connection that the magnetic pressure as a function of magnetic field radius r_m and angle φ taking into account Eqs. (2) and (3),

$$p_m(r_m, \varphi) = \mu_0 I_\Sigma^2 / 8\pi^2 (R + r_m \sin\varphi)^2, \tag{15}$$

grows without bounds with a decrease of configuration size and the field in a free state is unstable. However, in a TCLV this field is located inside a vortex ring which exerts a stabilizing influence on the field. We find the configuration stability condition.

Suppose that for an arbitrary angle φ the field radius grows by a value δr_m. In this case a force δF begins to act on the vortex filament at the field boundary with area δs. In agreement with Ref. 8 the configuration is stable if the force δF is directed oppositely to the displacement δr_m:

$$\delta F = (\partial p_m/\partial r_m - \partial p_w/\partial r_m)\,\delta r_m \delta s < 0. \tag{16}$$

For solving condition (16) we find from Eq. (15) for $\varphi = -\pi/2$:

$$\partial p_m/\partial r_m = p_0/(R - r_0), \tag{17}$$

where p_0 and r_0 are the pressure and field radius, respectively, for $\varphi = -\pi/2$.

We find the second term in parentheses in expression (16) by writing the projection of Eq. (1) in the direction r (Fig. 3):

$$-\frac{v_0^2}{r_m} + \frac{d^2 r_m}{dt^2} = -\frac{1}{\rho}\frac{\partial p_w}{\partial r_m}, \tag{18}$$

from which we find

$$\partial p_w/\partial r_m = \rho v_0^2/r_0, \tag{19}$$

where v_0 is the velocity of the vortex ring layer with radius r_0. In order to obtain Eq. (19) from Eq. (18) we took $r_m = r_0$, and $d^2 r_m/dt^2 = 0$, i.e., we took as a first approximation circular motion of vortex ring layers.

Substituting Eqs. (17) and (19) into Eq. (16), we obtain the TCLV stability condition:

$$v_0 > v_c = [2r_0 p_0/\rho(R - r_0)]^{1/2}, \tag{20}$$

where v_c is the critical velocity of the vortex ring layer adjacent to the magnetic field.

5. VORTEX RING LIFETIME

When the TCLV exists independently the vortex layer velocity v_0 gradually decreases after formation of the configuration, and at the critical velocity v_c, the configuration loses stability and disappears. To estimate the lifetime we analyze the process by which the gas layers slow down. It is well known that when one gas layer moves relative to another a frictional force arises (Ref. 7, p. 199)

$$\delta F_f = -\eta \frac{\partial v_r}{\partial r}\delta s, \tag{21}$$

where η is the internal friction coefficient between layers of the vortex ring with area δs and v_r is the relative velocity of the layers. This velocity can be represented in the form

$$v_r = \frac{\partial v}{\partial r}\delta r. \tag{22}$$

Here v is the layer velocity at arbitrary radius r (Fig. 3) and δr is the layer thickness. The frictional force in the case considered is balanced by the inertial force

$$\delta F_i = -\rho \frac{\partial v}{\partial t}\delta s\,\delta r, \tag{23}$$

and by equating the right hand sides of Eqs. (21) and (23), taking into account Eq. (22), we obtain the equation of motion of the vortex ring layers

$$\frac{\partial v}{\partial t} = \frac{\eta}{\rho}\frac{\partial^2 v}{\partial r^2}. \tag{24}$$

Making the natural assumptions that the external layer of the vortex ring with radius r_w is stationary, i.e., $v(r_m,t)=0$, and the internal layer at the initial time $t=0$ moves with velocity $v(r_m,0)=v_0$ with a zero gradient $\partial v/\partial r=0$, we obtain an expression for the lifetime of the vortex ring τ_w in agreement with Ref. 9.

$$\tau_w \approx \frac{4\rho(r_w-r_m)^2}{\pi^2\eta}\ln\frac{v_0}{v_c}. \tag{25}$$

6. PHYSICAL REALIZATION OF THE TCLV CONFIGURATION

We consider an example of an actual TCLV configuration with parameters: $r_m=r_0=0.01$ m; $r_w=0.05$ m; $R=0.1$ m; $p_m=p_0=1\times10^5$ Pa; $m_i=3.3\times10^{-27}$ kg; $\rho=0.15$ kg/m^3 (deuterium); $T_i=2\times10^4$ K; $v_0/v_c=1.5$; $\eta=1\times10^5$ N s/m^2 (Ref. 10). Using these data, we obtain the following characteristics:

1) the electron energy (14) in the monolayer, $W_e\sim1.44$ MeV;

2) current layer lifetime (8), $\tau_s\approx2$ s;

3) thickness (11) of the positive ion sheath, $d_i\approx6.5\times10^{-4}$ m which is significantly less than the assumed magnetic field radius;

4) critical velocity (20), $v_c\approx385$ m/s, which is approximately 40% of the sound speed for these conditions;

5) vortex ring lifetime (25), $\tau_w\approx4$ s.

We also estimate the value of the discharge current I in the spark discharge needed to obtain electrons of the necessary energy W_e and create a ring magnetic field. From Maxwell's equations for the example considered

$$2\pi\oint E_r\,dr_m=-\int\mu_0\left(\frac{\partial \mathbf{H}}{\partial t}\right)_I ds$$

and taking into account Eq. (3), we obtain

$$E_r\approx\frac{\mu_0 r_m}{4\pi R}\frac{\partial I}{\partial t}, \tag{26}$$

where E_r is the electric field strength in the vortex ring region in the spark discharge process. The electrons passing through this field in the process of continuous acceleration, taking into account Eq. (26), acquire a momentum

$$\frac{G}{c}[W_e(W_e+2W_0)]^{1/2}=\int_{t_3}^{t_4} eE_r\,dt\approx\frac{\mu_0 r_m e}{4\pi R}\Delta I, \tag{27}$$

where ΔI is the current change of the spark discharge at the trailing edge of the potential relaxation wave, which is the one electrons pass through in the process of continuous acceleration (Fig. 2), and G is the coefficient needed to provide agreement with the electron energy units used, $G=1.6\times10^{-13}$ J/MeV.

Taking into account that the total current of the spark discharge pulse is used up not only in imparting to electrons the necessary energy (27), but also in the creation of the ring magnetic field (3), the required value of the spark discharge current is

$$I_{max}\approx\frac{4\pi RG[W_e(W_e+2W_0)]^{1/2}}{e\mu_0 r_m c}+2\pi R\left(\frac{2p_m}{\mu_0}\right)^{1/2}. \tag{28}$$

From Eq. (28) we have $I_{\max}\approx8.8\times10^5$ A for the example considered.

The estimates presented support the conclusion that the TCLV configuration, in principle, is physically realizable. Nevertheless, the great technical difficulties in producing this configuration are obvious.

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