Theory of quasielastic scattering of light by electrons in semiconductors with a nonparabolic dispersion law

V. A. Voĭtenko and I. P. Ipatova

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted 9 July 1988; resubmitted 13 June 1989)
Zh. Eksp. Teor. Fiz. **97**, 224–234 (January 1990)

It is shown that in strongly doped materials with a nonparabolic dispersion law the quasielastic-scattering spectra have the shape of Lorentzian contours. Expressions are found for the half-widths of these contours for different scattering mechanisms. It is shown that from the shape of the scattering spectrum one can determine a number of kinetic characteristics of the materials: the electron-diffusion coefficient, the thermal diffusivity, and the relaxation time of the anisotropy of the momentum distribution of the electrons. Microscopic expressions for a number of kinetic coefficients appearing in the scattering cross section are found by the Chapman-Enskog method. A general analysis, which does not use the relaxation-time approximation, makes it possible to conclude that there is collisional narrowing of the spectra of the scattering of light by fluctuations of the energy and spin of the electrons.

1. INTRODUCTION

Besides the well studied scattering of light by collective plasma oscillations in a semiconductor,[1] quasielastic scattering of light by single-particle plasma excitations is also observed.[2–6] The laws of energy and momentum conservation in the elementary act of quasielastic scattering have the form

$$\varepsilon_{\mathbf{p}+\hbar\mathbf{k}}-\varepsilon_{\mathbf{p}}=\hbar\omega, \tag{1}$$

where $\varepsilon_{\mathbf{p}}$ is the energy of an electron with quasimomentum $\mathbf{p}$ and $\hbar\mathbf{k}$ is the momentum transferred in the scattering. It can be seen from (1) that in quasielastic scattering spatial dispersion ($k\neq 0$) is important, and this makes both the analysis of the screening and the choice of the selection rules nontrivial.[7] Quasielastic scattering has been observed principally in strongly doped samples, for which the strong-screening condition

$$kr_{\text{sc}}\ll 1 \tag{2}$$

is fulfilled. A number of papers[8–10] have been devoted to exhibiting nonscreenable mechanisms of quasielastic scattering in many-valley semiconductors. The shape of the scattering spectra was practically not investigated in these papers, although it is the shape that carries important information about the kinetics of light-scattering fluctuations of electron parameters of the semiconductor.

A detailed experimental study of the shape of quasielastic-scattering spectra has been carried out by Cardona *et al.*,[3,11–13] who showed that for strongly doped samples of *n*-Si and *n*-Ge the spectra have the form of Lorentzian contours extending 100–200 cm^{-1} to the left and right of the laser excitation line. These experiments were explained in Refs. 14–16, in which it was established that the Lorentzian contours reflect diffusive motion of the carriers in a strongly doped semiconductor, when the frequent-collision condition

$$kl\ll 1, \tag{3}$$

is fulfilled, where l is the mean free path of the carriers.

In the present paper we investigate the shape of the quasielastic-scattering spectra in strongly doped semiconductors with a nonparabolic dispersion law. In these materials, three nonscreenable light-scattering mechanisms are realized: scattering by fluctuations of the energy density and momentum density of the current carriers,[8] and also scattering by spin-density fluctuations.[9] It is shown that under the condition (3) the quasielastic-scattering spectra have the shape of Lorentzian contours extending 100–200 cm^{-1} to the left and right of the frequency of the exciting laser. The widths of these Lorentzian contours turn out to be different for different scattering mechanisms. In scattering by energy fluctuations the dissipation of the fluctuations is determined by the electron thermal conductivity $\varkappa$, while the half-width Γ of the corresponding Lorentzian contour is determined by the electron thermal diffusivity $\chi(\omega)$:

$$\Gamma=k^2\chi(\omega). \tag{4}$$

In scattering by fluctuations of the momentum of the current carriers the dissipation of the fluctuations has a different, relaxational character. The shape of the scattering spectrum is, as before, Lorentzian, but the half-width of the contour is determined by that component of the electron distribution function that has the symmetry of the second spherical harmonic:

$$\Gamma=1/\tau_{p_2}. \tag{5}$$

In scattering by spin-density fluctuations, a diffusive mechanism of dissipation of the fluctuations is realized. The half-width of the corresponding Lorentzian contour is equal to

$$\Gamma=k^2D, \tag{6}$$

where D is the diffusion coefficient of the free carriers.

By the Chapman-Enskog method,[17] for the high-frequency diffusion coefficient and electron thermal conductivity $\varkappa$ we have found microscopic expressions that make it possible to analyze the shape of the quasielastic-scattering spectra in a broad range of frequencies $\omega\tau_1\lessgtr 1$, where τ_1 is the usual momentum-relaxation time. The microscopic expression found for the electron thermal conductivity makes it possible to relate the collisional narrowing of the spectral lines (the Dicke effect[18]) to the growing role of thermal diffusion upon increase of the concentration of doping impurities.

0038-5646/90/010125-06$03.00

2. SHAPE OF THE SPECTRUM FOR SCATTERING BY ENERGY-DENSITY FLUCTUATIONS

The differential cross section for electron scattering of light can be expressed in terms of the correlator of the fluctuations of the electron dielectric susceptibility χ_{ij} (Ref. 19). The cross section per unit scattering volume is equal to

$$\frac{d^2\Sigma}{d\omega d\Omega}=\frac{1}{2\pi}\frac{\omega_I^4}{c^4}e_k^I e_j^s (e_i^I e_n^s)^* \times\int dt\int d^3r\, e^{i(\mathbf{kr}-\omega t)}\langle\delta\chi_{ij}(\mathbf{r},t)\,\delta\chi_{kn}(0,0)\rangle, \qquad (7)$$

where $\mathbf{e}^I$ and $\mathbf{e}^s$ are the polarization vectors of the incident and scattered light, the angular brackets denote the correlator of the electron susceptibilities, ω_I is the frequency of the incident light, and ω is the change of frequency in the scattering. The fluctuation of the electron susceptibility for a crystal with a nonparabolic electron spectrum can be expressed phenomenologically in terms of the electron-density fluctuation δn and the energy-density fluctuation δE of the system of electrons:

$$\delta\chi_{ij}=\frac{e^2}{m\omega_I^2}(A\delta n+B\delta E)\,\delta_{ij}. \qquad (8)$$

Here m is the free-electron mass, and δE is equal to

$$\delta E=\int\frac{2d^3p}{(2\pi\hbar)^3}\varepsilon_{\mathbf{p}}\delta f_{\mathbf{p}}^{(1)}, \qquad (9)$$

where $\delta f_{\mathbf{p}}^{(1)}$ is the spherically symmetric fluctuation of the electron distribution function, and the values of the coefficients A and B for the particular case of resonance scattering are given in, e.g., Ref. 1 [formula (4.94)]. When the strong-screening condition (2) is fulfilled, the first term in (8) leads to scattering of light by plasmon-phonon modes, which is realized in the region of higher frequencies

$$\omega\sim\omega_p\gg\tau_1^{-1}$$

(ω_p is the plasma frequency), and so is not considered here. We shall be interested in the second term in (8), which gives the light scattering by fluctuations of the energy density of the current carriers. The fluctuations of the energy and particle number have the same symmetry, and are not statistically independent. The screening of such fluctuations requires special analysis, since according to (1) quasielastic scattering necessarily involves momentum transfer, i.e., the scattering cross section possesses spatial dispersion.

The shape of the scattering spectrum is determined by the law of dissipation of the light-scattering fluctuations, which, in the framework of hydrodynamics [which is valid when condition (3) is fulfilled], can be described by the diffusion and thermal-conduction equations, with thermal diffusion also taken into account. The role of the second, heavy plasma component that gives rise to the thermal diffusion is played by ionized impurities.

We write the fluctuational dissipative fluxes of the electron concentration and energy as

$$\delta\mathbf{j}=-D\nabla\delta n-\frac{D_T}{T}\nabla\delta T, \qquad (10)$$

$$\delta\mathbf{q}-\zeta\delta\mathbf{j}=\Pi\delta\mathbf{j}-\varkappa\nabla\delta T, \qquad (11)$$

where ζ is the chemical potential and Π is the Peltier coefficient. The hydrodynamic condition of neutrality has the form[19]

$$\operatorname{div}\delta\mathbf{j}=0. \qquad (12)$$

Equations (9)–(12) must be supplemented by the energy-conservation law in the form of the continuity equation

$$\frac{\partial}{\partial t}\delta E+\operatorname{div}\delta\mathbf{q}=0. \qquad (13)$$

When condition (2) is fulfilled we have

$$\frac{\partial}{\partial t}\delta E=\left(\frac{\partial E}{\partial T}\right)_n\frac{\partial}{\partial t}\delta T=c_v\frac{\partial}{\partial t}\delta T, \qquad (14)$$

where c_v is the electron specific heat at constant volume. Substituting (14) into (13), eliminating $\delta\mathbf{q}$ from (10) and (11), and taking (12) into account, we obtain an equation describing the dissipation of the temperature fluctuations that accompany the energy fluctuations:

$$\frac{\partial}{\partial t}\delta T=\chi\Delta\delta T, \qquad (15)$$

where $\chi(\omega)=\varkappa/c_v$ is the electron thermal diffusivity. After substitution of (14) into (8) and then into (7), we express the scattering cross section in terms of the spectral correlator of the temperatures

$$\langle\delta T(r,t)\,\delta T(0,0)\rangle_{\mathbf{k}\omega},$$

which, as shown in Ref. 20, satisfies Eq. (15). As the initial condition for the determination of the temperature correlator from (15) we take the value of the single-time correlator $\langle\delta T^2\rangle_{00}$ of the classical fluctuations of the temperature, which can be found, e.g., in Ref. 21 [formula (112.6)]:

$$\langle\delta T^2\rangle_{00}=T^2/c_v. \qquad (16)$$

Substituting the correlator value thus found into (7), we obtain for the cross section for light scattering by energy fluctuations the expression

$$\frac{d^2\Sigma^{(E)}}{d\omega\, d\Omega}=\left(\frac{e}{mc^2}\right)^2(\mathbf{e}^I\mathbf{e}^s)^2B^2c_vT\operatorname{Re}\left\{\frac{1}{-i\omega+k^2\chi(\omega)}\right\}. \qquad (17)$$

For $\omega\tau_1\ll1$ we can neglect the frequency dispersion of $\chi(\omega)$. Then the scattering spectrum (17) acquires the shape of a Lorentzian contour with half-width (4). At higher frequencies $\omega>1/\tau_1$ the shape of the contour deviates from Lorentzian on account of the frequency dependence of $\chi(\omega)$. If $\chi(\omega)$ decreases with increase of the concentration of doping impurities, the Lorentzian contour obtained becomes narrower with increase of the concentration. This narrowing has been observed at 300 K in the quasielastic-electron-scattering spectra of n-InP crystals in the region of electron concentrations from 3×10^{16} to 5×10^{17} cm^{-3} (Ref. 6). From the half-width of the Lorentzian contour one can establish the electron thermal diffusivity $\chi(\omega)$, i.e., the parameter that determines the thermal quasiequilibrium of the current carriers obtained by the light pumping.[22]

We have considered the case of those temperatures at which scattering of electrons by phonons is unimportant. At room temperature, for semiconductors of the InP type the condition $T\ll\hbar\omega_0$ is fulfilled, where ω_0 is the optical-phonon frequency. Therefore, $\tau_{\text{e-ph}}\gg\tau_{\text{e-e}}$, where $\tau_{\text{e-ph}}$ and $\tau_{\text{e-e}}$ are the electron-phonon and electron-electron relaxation times, respectively. The influence of the acoustic phonons on the scattering spectrum can be estimated from the continuity equation (13), if in it the transfer of energy to the phonon subsystem is taken into account by the introduction of the corresponding relaxation time $\tau_{\text{e-ph}}^{(\text{ac})}$. The expression for the half-width of the Lorentzian contour then takes the form

$$\Gamma = k^2\chi + 1/\tau_{\text{e}-\text{ph}}^{(\text{ac})}. \tag{18}$$

Finally, we note that the scattering cross section (17) is of the scalar type and is observed for parallel polarization $\mathbf{e}^I \| \mathbf{e}^s$.

3. SHAPE OF THE SPECTRUM FOR SCATTERING BY FLUCTUATIONS OF THE ANISOTROPY OF THE ELECTRON-MOMENTUM DISTRIBUTION

Nonparabolicity of the electronic band spectrum in direct-band semiconductors (GaAs, InP, etc.) leads to one more mechanism of unscreenable scattering in semiconductors—namely, to scattering by fluctuations of the average electron momentum.[8] The corresponding contribution to the fluctuation of the electron susceptibility in the case of weak nonparabolicity has the form

$$\delta\chi_{ij}^{(2)} = -\frac{e^2}{m\omega_I^2}\int \frac{2d^3p}{(2\pi\hbar)^3} B_p(\omega_I)\left(p_ip_j - \frac{1}{3}p^2\delta_{ij}\right)\delta f_{\mathbf{p}}^{(2)}. \tag{19}$$

Here $\delta f_{\mathbf{p}}^{(2)}$ is the anisotropic fluctuation of the electron distribution function, and B_p is a phenomenological coefficient that depends on the frequency ω_I. Under the condition $|E_g - \hbar\omega_I| \gg \hbar\omega$ the coefficient B_p is equal to

$$B_p(\omega_I) = \frac{2}{3}\frac{p_{cv}^4}{m^3E_g}\frac{E_g^2 + (\hbar\omega_I)^2}{[E_g^2 - (\hbar\omega_I)^2]^2}, \tag{20}$$

where p_{cv} and E_g are parameters of the Kane model.[1]

It can be seen from (19) that a contribution to the expression for $\delta\chi^{(2)}$ will be made by the part $\delta f_{\mathbf{p}}^{(2)}$ with the symmetry of the second spherical harmonic. The relaxation of a nonequilibrium distribution function of this symmetry has been considered in a description of hot-photoluminescence spectra.[23] However, the authors of Ref. 23 considered a strongly nonequilibrium situation for photo-excited carriers, when the approach to equilibrium is determined by an energy-relaxation time due to, e.g., interaction with optical phonons. In the present paper, $\delta f_{\mathbf{p}}^{(2)}$ describes small deviations from the Fermi distribution. In this case energy relaxation is unimportant, and the approach to equilibrium is determined by elastic collisions describable by the momentum relaxation time of the second spherical harmonic.

The dissipation of the light-scattering fluctuations for this scattering mechanism occurs in a relaxational manner, and not diffusively as it did in the case of light scattering by energy fluctuations. Therefore, to determine $\delta f_{\mathbf{p}}^{(2)}$ it is necessary to solve the kinetic equation with allowance for the neutrality condition, which, when (3) is fulfilled, implies the absence in the equation of a term with a self-consistent electric field.[24] The result of the calculation is

$$\frac{d^2\Sigma^{(p)}}{d\omega\, d\Omega} = \frac{3T}{10}\left(\frac{e}{mc^2}\right)^2\left(1 + |\mathbf{e}^I\mathbf{e}^s|^2 - \frac{2}{3}|\mathbf{e}^I\mathbf{e}^{s*}|^2\right) \times \int \frac{2d^3p}{(2\pi\hbar)^3}B_p^2p^4\frac{\partial f_0}{\partial\zeta}\frac{\tau_{\mathbf{p}_2}}{1+(\omega\tau_{\mathbf{p}_2})^2}. \tag{21}$$

Here f_0 is the equilibrium (Fermi) distribution function.

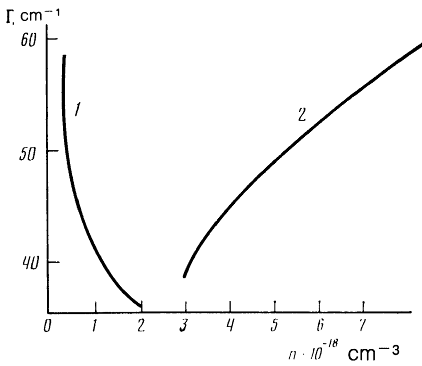

FIG. 1. Concentration dependence of the half-width of the Lorentzian contour of the quasielastic electron scattering of light for different scattering mechanisms: 1) scattering by energy-density fluctuations, with scattering cross section (17); 2) scattering by momentum-density fluctuations, with scattering cross section (21).

The scattering cross section (21) has a shape close to Lorentzian, with the half-width (5). It is nonzero both in parallel and in perpendicular polarization of the incident and scattered light. It is useful to compare the relative magnitudes of the contribution (17) from scattering by energy fluctuations and the contribution from (21) for parallel polarizations $\mathbf{e}^I$ and $\mathbf{e}^s$. In order of magnitude, the ratio of the cross sections is equal to

$$\frac{d\Sigma^{(E)}}{d\Sigma^{(P)}} \approx \left(\frac{10T}{\zeta}\right)^2. \tag{22}$$

It follows from (22) that with increase of the concentration of current carriers, i.e., with increase of ζ, the role of scattering by energy fluctuations decreases. Lowering of the temperature also leads to this effect. For example, at $T = 300$ K, exchange of the scattering mechanisms occurs at $n \sim 10^{18}$ cm^{-3} for semiconductors of the n-InP or n-GaAs type. In the figure the theoretical curve 1 describes the narrowing of the Lorentzian contour (17) with increase of the carrier concentration n, while curve 2 gives the growth (due to the decrease of $\tau_{\mathrm{p}2}$ upon increase of n) of the half-width of the Lorentzian contour (21).

4. THE SHAPE OF THE SPECTRUM OF QUASIELASTIC LIGHT SCATTERING BY SPIN-DENSITY FLUCTUATIONS

One more contribution to the quasielastic scattering involves fluctuations of the spin density of the electrons.[9] This scattering occurs via the medium of the spin-orbit interaction and is determined by the antisymmetric part of the electron-susceptibility tensor:

$$e_i^I\delta\chi_{ij}^{(\sigma)}e_j^s = B_\sigma|[\mathbf{e}^I\mathbf{e}^s]|(\delta n_\uparrow - \delta n_\downarrow). \tag{23}$$

This scattering was observed in Ref. 2, but the shape of the spectrum was not investigated. Fulfillment of the condition (3) for applicability of hydrodynamics makes it possible to describe the kinetics of the spin-density fluctuations by the continuity and diffusion equations:

$$\frac{\partial}{\partial t}(\delta n_\uparrow - \delta n_\downarrow) + \operatorname{div}(\delta\mathbf{j}_\downarrow - \delta\mathbf{j}_\uparrow) = 0,\ (\delta\mathbf{j}_\uparrow - \delta\mathbf{j}_\downarrow) = -D\nabla(\delta n_\uparrow - \delta n_\downarrow). \tag{24}$$

Here, $\delta n_\uparrow - \delta n_\downarrow$ is the difference of the populations of the spin sub-bands, defined by the spin-density operator $\hat{\sigma}_z(t)$.

In (24) there is no thermal-diffusion term, since the latter is the same for all the spin sub-bands, and purely spin relaxation has not been taken into account, since for nonmagnetic impurities the condition $\omega\tau_s \gg 1$ is fulfilled, where τ_s is the corresponding relaxation time. The frequency dependence of the scattering cross section is determined by Eqs. (24) with the following initial condition, written with allowance for the degeneracy of the electron gas [see Ref. 21, formula (115.2)]:

$$\langle(\delta n_{\downarrow}-\delta n_{\uparrow})^2\rangle'_{0,0}=T\left(\frac{\partial n}{\partial \zeta}\right)_T, \qquad (25)$$

where $n = n_{\uparrow} + n_{\downarrow}$. The scattering cross section in this case has the form

$$\frac{d^2\Sigma^{(\sigma)}}{d\omega\, d\Omega}=\left(\frac{e}{mc^2}\right)^2 [\mathbf{e}^l\mathbf{e}^s]^2 B_\sigma^2 T\left(\frac{\partial n}{\partial \zeta}\right)_T \operatorname{Re}\left\{\frac{1}{-i\omega+k^2D(\omega)}\right\}. \qquad (26)$$

The value of B_σ for the case of resonance scattering can be found in Ref. 1 [formula (4.80b)]. For $\omega\tau_1 \ll 1$ we can neglect the frequency dependence of $D(\omega)$. Then (26) acquires the shape of a Lorentzian contour with the half-width (6). The cross section (26) describes the antisymmetric resonance scattering that has been observed in n-InP (Ref. 5) in crossed polarizations of the incident and scattered light. From the half-width of the Lorentzian contour it is possible to find the diffusion coefficient of the carriers and their mobility.

The cross section (26) must be compared with the contribution (21) to the cross section in perpendicular polarizations. In order of magnitude, the ratio of the cross sections is equal to

$$\frac{d\Sigma^{(\sigma)}}{d\Sigma^{(p)}}\approx\left(\frac{\Delta}{\zeta}\right)^2\left\{\frac{\hbar\omega_I(2E_g+\Delta)\,[E_g^2-(\hbar\omega_I)^2]}{[(E_g+\Delta)^2-(\hbar\omega_I)^2][E_g^2+(\hbar\omega_I)^2]}\right\}^2. \qquad (27)$$

Here Δ is the energy of the spin-orbit splitting of the bands. For materials with $E_g \gg \Delta$ (e.g., n-GaAs), in accordance with Refs. 8 and 9, for any reasonable concentrations n the condition $\Delta \gg \zeta$ is fulfilled, and the scattering by spin-density fluctuations dominates. In semiconductors with $\Delta > E_g$ (e.g., InSb and $Cd_{0.2}Hg_{0.8}Te$), the ratio of the cross sections reduces to $(\hbar\omega_I/\zeta)^2$. Enhancement of the scattering by momentum fluctuations in narrow-band materials is due to the large degree of nonparabolicity of the bands. In addition, the Lorentzian contour (26) becomes narrower with increase of the concentration of impurities, since D decreases with increase of the concentration. On the other hand, the Lorentzian contour (21) is broadened with increase of the concentration. This broadening has been observed for n-InP in Ref. 6.

5. DERIVATION OF MICROSCOPIC EXPRESSIONS FOR THE KINETIC COEFFICIENTS

In our analysis, the kinetic coefficients are functions of ω. Picosecond studies of the scattering and energy relaxation of photo-excited carriers in semiconductors has shown that heat transfer in the electron system is realized by purely electronic thermal conduction, without the participation of phonons.[22] Therefore, it is of interest to obtain from the kinetic equation microscopic expressions for the kinetic coefficients. At $T = 300$ K the electron gas in strongly doped semiconductors, and, in particular, in films, is partially degenerate; the closeness of hetero-boundaries makes the electron collisions substantially inelastic, and so the relaxation-time approximation is inapplicable. However, it is possible to make use of the Chapman-Enskog method,[17] which, for an arbitrary degree of inelasticity, makes it possible to obtain kinetic coefficients in the form of a general expansion in a series in the eigenvalues of the collision integral.

The kinetic equation, including the term (important in the given problem) with spatial dispersion, has the form

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial x_i}(v_i f)=I_{\text{coll}}\{f\}. \qquad (28)$$

The collision integral $I_{\text{coll}}\{f\}$, with inelastic collisions taken into account, describes the approach of the nonequilibrium electron distribution function $f(r,t)$ to local equilibrium, which is defined by the Fermi function

$$\tilde{f}=\{1+\exp[(\varepsilon_p-\tilde{\zeta})/\tilde{T}]\}^{-1}, \qquad (29)$$

where the parameters $\tilde{\zeta}$ and $\tilde{T}$ vary in space and in time. The concept of local equilibrium assumes that the dissipative processes that bring the system to complete equilibrium are slow. If these processes are neglected, the electron system can be regarded as an ideal liquid, the distribution function of which has the form (29). Substitution of $\tilde{f}$ from (29) into (28) shows that the collision integral vanishes, so that $\tilde{f}$ for all values of $\tilde{\zeta}$ and $\tilde{T}$ satisfies the kinetic equation without the collision integral. If we take into account small fluctuations of the chemical potential ($\delta\zeta$) and of the temperature (δT) in such an ideal liquid, the corresponding deviation $\delta\tilde{f}$ of the distribution function f from the quasiequilibrium distribution function $\tilde{f}$ can be written in the form of a series in $\delta\zeta$ and δT:

$$\delta\tilde{f}=f-\tilde{f}=\left(\frac{\partial\tilde{f}}{\partial\zeta}\right)_T\delta\zeta+\left(\frac{\partial\tilde{f}}{\partial T}\right)_\zeta\delta T$$
$$=\left(\frac{\partial f_0}{\partial\zeta}\right)_T\delta\zeta+\left(\frac{\partial f_0}{\partial T}\right)_\zeta\delta T. \qquad (30)$$

The function $\delta\tilde{f}$ also satisfies the kinetic equation without the collision integral. Substituting into (30) the equilibrium Fermi distribution function, we obtain for $\delta\tilde{f}$

$$\delta\tilde{f}=\tilde{\theta}\left(\frac{\partial f_0}{\partial\zeta}\right)_T=\left(\delta\zeta+\frac{(\varepsilon_p-\zeta)}{T}\delta T\right)\left(\frac{\partial f_0}{\partial\zeta}\right)_T. \qquad (31)$$

To describe the relaxation of the fluctuations to complete equilibrium, it is necessary to take into account the dissipative processes that occur as a result of collisions. We introduce the distribution function δf describing the deviation from complete equilibrium, in the form

$$\delta f=f-f_0=\theta\left(\frac{\partial f_0}{\partial\zeta}\right)_T. \qquad (32)$$

The problem reduces to the determination of the function θ, the zeroth approximation for which is provided by $\tilde{\theta}$ from (31). To perform the iterations correctly, we shall make use of a variant of the Chapman-Enskog method.[17] Since $\tilde{\theta}$ and θ describe small deviations from equilibrium, corresponding to small fluctuations $\delta\zeta$ and δT, we can linearize the kinetic equation (28) in θ and take Fourier transforms with respect to $\mathbf{r}$ and t. To solve the linearized equation, we shall make use of the representation in the eigenfunctions Ψ of the linearized collision integral, which are determined by the equation

$$\sum_{\mathbf{p}'} I_{\mathbf{pp}'}\Psi_\beta(\mathbf{p}') = \nu_\beta \Psi_\beta(\mathbf{p}). \tag{33}$$

Here, the ν_β are eigenvalues of the operator $\hat{I}$. We expand $\theta(\mathbf{p})$ in a series in Ψ_β:

$$\theta(\mathbf{p}) = \sum_\beta a_\beta \Psi_\beta(\mathbf{p}) \tag{34}$$

and substitute (33) and (34) into the kinetic equation (28). We then obtain the kinetic equation in the form

$$(-i\omega+\nu_\beta)a_\beta = -\sum_{\beta'} \langle\beta|i\mathbf{kv}|\beta'\rangle a_{\beta'}. \tag{35}$$

Equation (35) can be solved by iterations in the parameter

$$(\mathbf{kv})/(-i\omega+\nu_\beta) \ll 1. \tag{36}$$

Amongst the eigenvalues ν_β are zero eigenvalues that appear as a consequence of the conservation laws satisfied in the collisions. In the absence of collisions the energy and particle-number conservation laws are fulfilled. Therefore, the zeroth eigenvalue turns out to be doubly degenerate, i.e., to it there correspond two orthonormalized eigenfunctions

$$\begin{gathered}\Psi_{01} = 1,\\ \Psi_{02} = \left[\frac{1}{Tc_v}\left(\frac{\partial n}{\partial \zeta}\right)_T\right]^{1/2}\left[\varepsilon_\mathbf{p} - \frac{3}{2}n\left(\frac{\partial \zeta}{\partial n}\right)_T\right],\end{gathered} \tag{37}$$

where the normalization factor is expressed in terms of the electronic specific heat c_v:

$$c_v = \frac{1}{T}\int \frac{2d^3p}{(2\pi\hbar)^3}\left(\frac{\partial f_0}{\partial \zeta}\right)_T\left[\varepsilon_\mathbf{p} - \frac{3}{2}n\left(\frac{\partial \zeta}{\partial n}\right)_T\right]^2. \tag{38}$$

In the zeroth approximation in the parameter (36) the function $\theta(\mathbf{p})$ reduces to the function $\tilde{\theta}(\mathbf{p})$ from (31), which makes the collision integral vanish. The expansion of $\tilde{\theta}(\mathbf{p})$ in a series in the Ψ_β is constructed in the form of an expansion in only those Ψ_β which correspond to zero eigenvalues of the collision integral $\hat{I}$, i.e., in the functions (37). There are only two nonzero expansion coefficients:

$$a_{01}^{(0)} = \delta\zeta + \left(\frac{\partial \zeta}{\partial T}\right)_n \delta T, \tag{39}$$

$$a_{02}^{(0)} = \left[\frac{c_v}{T}\left(\frac{\partial \zeta}{\partial n}\right)_T\right]^{1/2}\delta T. \tag{40}$$

In the first approximation in the parameter (36) Eq. (35) gives

$$a_\beta^{(1)} = \sum_{\alpha=1,2}\frac{\langle\beta|i\mathbf{kv}|0\alpha\rangle}{-i\omega+\nu_\beta}a_{0\alpha}^{(0)}. \tag{41}$$

Substituting (41) into (34) and (32), we obtain the nonequilibrium distribution function δf, which we can use to calculate the particle flux $\delta\mathbf{j}$ and the dissipative heat flux $(\delta\mathbf{q} - \zeta\delta\mathbf{j})$:

$$\begin{gathered}\delta\mathbf{j} = -\left(\frac{\partial n}{\partial \zeta}\right)_T \sum_\beta \langle 01|\mathbf{v}|\beta\rangle a_\beta,\\ (\delta\mathbf{q}-\zeta\delta\mathbf{j}) = T\left(\frac{\partial \zeta}{\partial T}\right)_n \delta\mathbf{j} - \left[Tc_v\left(\frac{\partial n}{\partial \zeta}\right)_T\right]^{1/2}\sum_\beta \langle 02|\mathbf{v}|\beta\rangle a_\beta.\end{gathered} \tag{42}$$

Substituting $a_{01}^{(0)}$, $a_{02}^{(0)}$, and $a_\beta^{(1)}$ from (39)–(41) into this, we obtain the macroscopic equations (10) and (11), in which the kinetic coefficients have the following forms: The tensor of the diffusion coefficients is

$$D_{lk}(\omega) = \sum_\beta \frac{\langle 01|v_l|\beta\rangle\langle\beta|v_k|01\rangle}{-i\omega+v_\beta}; \tag{43}$$

the tensor of the thermal-diffusion coefficients is

$$[D_T(\omega)]_{lk} = \left[Tc_v\left(\frac{\partial n}{\partial \zeta}\right)_T\right]^{1/2}\sum_\beta \frac{\langle 01|v_l|\beta\rangle\langle\beta|v_k|02\rangle}{-i\omega+\nu_\beta}; \tag{44}$$

and the thermoelectric coefficient is

$$e\alpha(\omega) = \left(\frac{\partial \zeta}{\partial n}\right)_T \frac{D_T(\omega)}{TD(\omega)} - \left(\frac{\partial \zeta}{\partial T}\right)_n. \tag{45}$$

In a one-component plasma the thermal-diffusion coefficient appearing in the equation

$$\delta j_i = \frac{\sigma_{ik}E_k}{e} - \frac{[D_T']_{ik}}{T}\frac{\partial}{\partial x_k}\delta T \tag{46}$$

should vanish. Equation (46) differs from (10) in the choice of variables[25]: In it, σ_{ik} is the conductivity and E_k is the fluctuating electric field. Transforming (10) to the form (46), we find

$$D_T'(\omega) = D_T(\omega) - \frac{2Tc_v}{3n}\left(\frac{\partial n}{\partial \zeta}\right)_T D(\omega). \tag{47}$$

This quantity vanishes in the limiting case $\omega\tau_1 \gg 1$. The electronic thermal conductivity $\varkappa(\omega)$ can be expressed in terms of $D_T'(\omega)$, and, in the case of an isotropic medium, has the form

$$\varkappa = \frac{1}{3}c_p\sum_{k,\beta}\frac{\langle 02'|v_k|\beta\rangle\langle\beta|v_k|02'\rangle}{-i\omega+\nu_\beta} - \left(\frac{\partial \zeta}{\partial n}\right)_T\frac{[D_T'(\omega)]^2}{TD(\omega)}, \tag{48}$$

where the wave function $\langle 02'|$ is equal to

$$\langle 02'| \equiv \Psi_{02}' = \left[\frac{1}{Tc_p}\left(\frac{\partial n}{\partial \zeta}\right)_T\right]^{1/2}(\varepsilon_\mathbf{p} - w). \tag{49}$$

Here w is the enthalpy per electron, and the normalization factor in $\langle 02'|$ is expressed in terms of the constant-pressure electronic specific heat c_p:

$$c_p = c_v + T\left(\frac{\partial p}{\partial T}\right)_V^2 \Big/ n\left(\frac{\partial p}{\partial n}\right)_T = \frac{1}{T}\int\frac{2d^3p}{(2\pi\hbar)^3}\left(\frac{\partial f_0}{\partial \zeta}\right)_T(\varepsilon_\mathbf{p} - w)^2. \tag{50}$$

The expression obtained for $\varkappa(\omega)$ contains a negative contribution that is associated with the existence of thermal diffusion and is absent in a purely electron system without impurities. With increase of the concentration of impurities, which in a semiconducting plasma play the role of the second, heavy component, the negative term in (48) grows, and $\varkappa(\omega)$ decreases. The narrowing (noted at the end of Sec. 2) of the Lorentzian light-scattering contour with increase of the concentration of impurities (the Dicke effect[18]) has the macroscopic interpretation described below.

In the case of elastic electron collisions the electron-velocity vector $\mathbf{v}$ appearing in the matrix elements of the microscopic expressions for $D(\omega)$, $D_T(\omega)$, and $\varkappa(\omega)$ is yet another eigenfunction of the collision integral, corresponding to the eigenvalue equal to the inverse transport relaxation time $\tau_{tr}^{-1} = \tau_1^{-1}$. Therefore, in the sums over β in (43), (44), and (48), there remains only one term containing τ_{tr}. The expressions for $D(\omega)$, $D_T(\omega)$, $\alpha(\omega)$, and $\varkappa(\omega)$ then go over into the known expressions[26] obtained in the relaxation-time approximation.

[1] M. V. Klein, in *Light Scattering in Solids*, ed. M. Cardona (Springer, New York, 1975), p. 147.
[2] A. Mooradian, Phys. Rev. Lett. **20**, 1102 (1968).
[3] M. Chandrasekhar, M. Cardona, and E. O. Kane, Phys. Rev. B **16**, 3579 (1977).
[4] K. T. Tsen and R. Bray, Solid State Commun. **45**, 685 (1983).
[5] B. Kh. Baĭramov, V. A. Voĭtenko, I. P. Ipatova, A. V. Subashiev, V. V. Toporov, and É. Yane, Fiz. Tverd. Tela (Leningrad) **28**, 754 (1986) [Sov. Phys. Solid State **28**, 420 (1986)].
[6] B. Kh. Baĭramov, V. A. Voĭtenko, I. P. Ipatova, *et al.*, Preprint No. 1191, A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad (1987).
[7] V. A. Voĭtenko, Fiz. Tverd. Tela (Leningrad) **26**, 1003 (1984) [Sov. Phys. Solid State **26**, 611 (1984)].
[8] P. A. Wolff, Phys. Rev. **171**, 436 (1968).
[9] D. C. Hamilton and A. L. McWorther, in *Light Scattering Spectra of Solids*, ed. G. B. Wright (Springer, New York, 1969), p. 309.
[10] P. M. Platzman, Phys. Rev. **139**, A379 (1965).
[11] G. Contreras, A. K. Sood, and M. Cardona, Phys. Rev. B **32**, 924 (1985).
[12] G. Contreras, A. K. Sood, and M. Cardona, Phys. Rev. B **32**, 930 (1985).
[13] N. Mestres and M. Cardona, Phys. Rev. Lett. **55**, 1132 (1985).
[14] I. P. Ipatova, A. V. Subashiev, and V. A. Voitenko, J. Raman Spectrosc. **10**, 221 (1981).
[15] V. A. Voĭtenko, I. P. Ipatova, and A. V. Subashiev, Izv. Akad. Nauk SSSR, Ser. Fiz. **48**, 749 (1984) [Bull. Acad. Sci. USSR, Ser. Phys. **48** (1984)].
[16] I. P. Ipatova, A. V. Subashiev, and V. A. Voitenko, Solid State Commun. **37**, 893 (1981).
[17] S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases, 2nd ed.* (Cambridge University Press, 1952).
[18] R. H. Dicke, Phys. Rev. **89**, 472 (1953).
[19] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press, Oxford, 1984.
[20] E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Pergamon Press, Oxford, 1981.
[21] L. D. Landau and E. M. Lifshitz, *Statistical Physics, 3rd ed., Vol. 1*, Pergamon Press, Oxford, 1980.
[22] J. G. Fujimoto and E. P. Ippen, in *Laser Optics of Condensed Matter*, eds. J. L. Birman, H. Z. Cummins, and A. A. Kaplyanski (Plenum Press, New York, 1988), p. 11.
[23] B. P. Zakharchenya, D. N. Mirlin, V. I. Perel', and I. I. Reshina, Usp. Fiz. Nauk **136**, 459 (1982) [Sov. Phys. Usp. **25**, 143 (1982)].
[24] V. A. Voĭtenko, Fiz. Tverd. Tela (Leningrad) **29**, 3177 (1987) [Sov. Phys. Solid State **29**, 1827 (1987)].
[25] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1959) [Russ. original, 2nd ed., Nauka, Moscow, 1986, p. 733].
[26] A. I. Anselm, *Introduction to Semiconductor Theory* (Mir, Moscow, 1981) [Russ. original, Nauka, Moscow, 1978].

Translated by P. J. Shepherd