Calculation of the photon structure function in QCD

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The virtual-photon structure function, measurable in deep inelastic scattering of electrons by a photon, is calculated in QCD by means of an expansion in inverse powers of the photon virtuality p^2 at intermediate values of x and intermediate $Q^2 \cong 10$ GeV^2 ($x = Q^2/2\nu$, where Q^2 is the square of the momentum transfer from the electron to the photon and ν is the energy transfer). A model is constructed which describes the resulting structure function in terms of the contributions of physical states and has the correct analytical properties in p^2. The model is used to extrapolate to the point $p^2 = 0$, i.e., the real-photon structure function is determined at intermediate values of x; good agreement with experiment is obtained without allowance for the QCD evolution. It is shown that the higher terms of order $1/p^4$ (the contribution of the gluon condensate) in the structure function of a soft longitudinal photon and in the Callan-Gross relation vanish.

1. INTRODUCTION

Since the time of Witten's paper,[1] interest in the study of photon structure functions has not weakened. As Witten showed, in the framework of perturbative QCD one can calculate the part of the structure function that dominates at large x, which is usually parametrized as follows:

$$F_2(x)=F_2^{p.t.}(x)+F_2^{HAD}(x). \qquad (1)$$

The fact that, when logarithmic corrections are taken into account, the perturbation-theory contribution $F^{p.t.}$ (see Fig. 1) contains a term $\alpha_s^{-1}(Q^2)$ has turned out to be important, and has raised hopes of a relatively "clean" determination of Λ_{QCD}. However, the uncertainty in the function F_2^{HAD}, corresponding to the nonperturbative hadronic contribution, has given rise to great difficulties on this path. To estimate the hadronic contribution, as a rule, different variants of the vector-dominance model have been used (see, e.g., the review in Ref. 2 and the recent discussion in Ref. 3), but no satisfactory algorithm for separating $F_2^{p.t.}$ and F_2^{HAD} has been proposed. Attempts made in this direction[4,5] have inevitably contained as a constituent element the introduction of an extra parameter, which has made the extraction of Λ_{QCD} from experimental data difficult. An even more artificial (from our point of view) attempt to describe experiment was made in Ref. 3, in which the logarithmic evolution began practically from Λ_{QCD} and the sensitivity to this quantity disappeared completely.

Another occasion for debates directly related to the above discussion is the question of the existence of singularities at small x that arise when one goes beyond the framework of the leading logarithmic approximation.[4–7] The degree of the divergence increases with increase of the power of α_s, and the question arises as to what is the region of the variable x in which calculations in perturbation theory remain legitimate.

To clarify these questions, knowledge of the hadronic part of the photon structure function is needed. Up to now, this part has been estimated extremely approximately.[8,9] The vector-dominance model has been employed, and it has been assumed that the vector-meson structure function can be described as a product of dependences on x that are characteristic for the region of small x (Regge behavior) and for the region of large x (quark counting). Here, the overall normalization has been fixed on the basis of the assumption that the quarks in the vector meson carry half of its momentum. With the aid of these hypotheses, the authors of Ref. 9 obtained

$$F_2^{HAD}/\alpha=0.2x^{0.4}(1-x). \qquad (2)$$

In Refs. 10 and 11 another approach to this problem was developed: The structure function of the virtual photon was calculated. In Ref. 10 it was demonstrated that in the case of a virtual photon a natural cutoff arises in the radiative corrections to the moments of the structure function, which are divergent for a real photon. In Balitskiĭ's paper,[11] which is the closest to ours, the virtual-photon structure function was calculated in the form of an expansion in inverse powers of the photon virtuality p^2, and, in particular, the contribution of the gluon condensate $\langle 0|G_{\mu\nu}^2|0\rangle$ to this expansion was taken into account. However, the method proposed by Balitskiĭ is suitable only for the calculation of the second moment of the structure function (and here, too, there are difficulties, associated with allowance for the region of small x; see below), and it is difficult to extend it to higher moments.

In the present paper, we calculate in QCD the photon structure function and, in particular, its hadronic part in the region of intermediate values of x and Q^2. The method used is conceptually close to the method proposed in Refs. 12 and 13 for the calculation of nucleon structure functions. The idea is as follows. We consider $\gamma\gamma$ scattering of two virtual photons with momenta q and p in the case when the virtuality of the first is much greater than the virtuality of the second:

$$|q^2|\gg|p^2|, \quad q^2, p^2<0.$$

Let $|p^2|$ be much greater than the characteristic hadronic

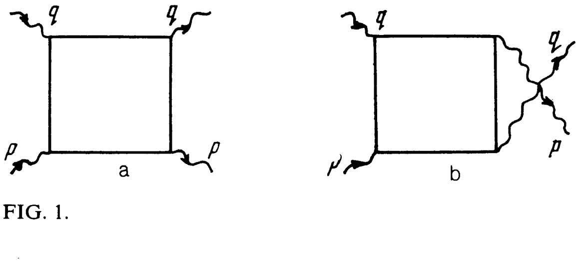

FIG. 1.

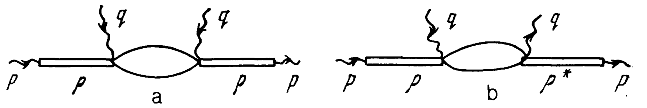

FIG. 2.

scale: $|p^2| \gg R_{\rm conf}^{-2}$, where $R_{\rm conf}$ is the confinement radius. To calculate the structure function of the virtual photon with momentum p (henceforth, we call it the soft photon) we can then use an expansion in inverse powers of p^2, i.e., an operator expansion can be performed both in q^2 and in p^2. As was shown in Ref. 13, expansion in $1/p^2$ is legitimate in regions of intermediate x, not close to the boundary values $x = 0$ and $x = 1$: As $x \to 0$ and $x \to 1$ the series in $1/p^2$ diverges.

In the expansion in powers of $1/p^2$ we take into account not only the perturbative contribution (the simplest quark loop) but also the next term due to the interaction of the quarks with the gluon condensate. We thereby find in QCD the virtual-photon structure function $F_2(x,p^2)$ with allowance for nonperturbative terms, and it becomes possible to determine the smallest values of p^2 for which the series in $1/p^2$ still converges. On the other hand, the same structure function can be represented, by means of a dispersion representation in p^2, in the form of an integral of contributions of physical states (see Fig. 2). We use the following model of the physical spectrum: the vector meson plus a continuum starting from a certain threshold p_0^2. The representation considered has the correct analytical properties in p^2 and is thereby free from the fictitious singularities in p^2 that arise in the perturbative approach. All the parameters of the hadronic contributions are fixed uniquely by comparison with results of calculations in QCD. It is shown that the resulting expression for the structure function of the soft virtual photon can be extrapolated to the point $p^2 = 0$. We thereby obtain the structure function of a real photon, with allowance for the hadronic component. Our results for the latter differ substantially from the model calculations of Refs. 8 and 9.

We note that the physical interpretation of the individual contributions to $F_2(x)$ in our approach differs from that in the usual approach. Indeed, we recall that in the language of sum rules the physical ρ meson is determined by part of a loop (that part of the dispersion integral over this loop which is dual to the ρ meson) and by the principal power corrections. Therefore, from the point of view of the sum rules, the separation of F_2 into $F_2^{\rm p.t.}$ and $F_2^{\rm HAD}$ inevitably contains double counting: It is a sum of states from two different bases—the quark-gluon basis and the hadron basis. In our approach F_2 consists from the outset of contributions of hadronic states, the principal of which is the contribution of the ρ meson. The latter is dual to part of the quark loop in Fig. 1 and contains a correction from the nonperturbative interaction. The remaining part of the quark loop can be interpreted as the contribution of the higher hadronic states.

In the present paper we confine ourselves to taking the light quarks into account, and assume them to be massless. We shall not take into account perturbative logarithmic corrections to QCD, and therefore our results will be correct in the region of not very large values of Q^2: $Q^2 \cong 10$ GeV^2.

The plan of the subsequent exposition is as follows. In Sec. 2 we describe the method of calculation of the photon structure function, based on an operator expansion of the correlator of four electromagnetic quark currents. In Sec. 3 we calculate the principal QCD diagrams determining this correlator in the kinematic region of scattering of a virtual photon by a virtual photon. In Sec. 4 we carry out a direct calculation of the structure functions of a real photon and a transverse ρ meson, and compare the results with experiment. In the Conclusion we briefly discuss the principal results of the paper and indicate possible ways of refining them.

2. METHOD OF CALCULATION OF THE PHOTON STRUCTURE FUNCTION

In its main features, the proposed method repeats the approach used in Refs. 12 and 13 to calculate the nucleon structure function, and therefore we shall confine ourselves to recalling the principal aspects. We consider the amplitude of deep inelastic forward scattering of a virtual photon (q^2) by a virtual photon (p^2) via intermediate hadronic states. The imaginary part of this amplitude is determined by short distances, if x is not too close to 0 or 1, and $|q^2|$, $|p^2| \gg R_{\rm conf}^{-2}$. This circumstance permits us to use an operator expansion—namely, to represent the indicated amplitude in the form of the sum of the very simple diagrams of Fig. 1 and power corrections to them.

Thus, we start from the four-current correlator

$$V_{\mu\nu\lambda\rho}(p,q) = -i\int dz_1\,dz_2\,dz_3 \exp\{iq(z_1-z_2)+ipz_3\}\langle 0|T\{J_\rho(z_3)J_\mu(z_1)\times J_\nu(z_2)J_\lambda(0)\}|0\rangle, \qquad (3)$$

where $J_\mu = \bar{q}\hat{Q}\gamma_\mu q$ is the electromagnetic quark current, and calculate the imaginary part Im $V_{\mu\nu\lambda\rho}$ in $s = (p+q)^2$ for fixed p^2, $q^2 < 0$ at $t = 0$. In the following, we shall be interested in a configuration in which one of the photons is strongly virtual and the other is weakly virtual, i.e., the scaling limit. Then we must retain in all the calculations the main terms of the expansion in p^2/q^2. The region $Q^2 = -q^2$ in which we shall work is assumed to be such that logarithmic QCD corrections are unimportant. All these conditions can be satisfied, in principle, if $|p^2| \cong 1$ GeV^2 and $Q^2 \cong 10$ GeV^2.

It is not difficult to verify that the arguments given in Refs. 12 and 13 for the applicability of the operator expansion are valid for Im $V_{\mu\nu\lambda\rho}$. The contribution of the unit operator to the expansion is specified by the imaginary part of the square diagram in Fig. 1. The reader familiar with the method of QCD sum rules will easily set up the next operators relevant in this problem. In the first place, one must take into account the vacuum expectation value of the operator $G_{\mu\nu}^2$, then the condensate $\bar{\psi}\psi$, etc.

We shall assume that we have been able to calculate all the principal contributions to Im $V_{\mu\nu\lambda\rho}$ (the actual calculation will be performed below, in Sec. 3). We turn to the physical interpretation of the imaginary part of the correlator (3). Multiplying it by the phonon-polarization four-vectors $e_\rho(p)$ and $e_\lambda^*(p)$ of the (in the general case, virtual) photons and summing over the polarizations, we can write for the tensor that arises the standard expansion in the structure functions of the virtual photon:

$$\sum_i e_\rho{}^i e_\lambda{}^{*i} \operatorname{Im} V_{\mu\nu\lambda\rho} = (-\delta_{\mu\nu}+q_\mu q_\nu/q^2) W_1(p^2,q^2,x) + (p_\mu - q_\mu \nu/q^2)(p_\nu - q_\nu \nu/q^2) W_2(p^2,q^2,x). \quad (4)$$

We note the fundamental advantage and simplicity of the problem under consideration in comparison with, e.g., the problem of the determination of the nucleon structure function. There is no need to distinguish the physical state of interest to us by a procedure of the borelization type. The photon is distinguished from the outset by its weak coupling with the hadronic states. Henceforth, we shall confine ourselves to the leading terms of the expansion in p^2/Q^2, i.e., to operators of twist 2 in the operator expansion of the currents interacting with the hard photon; this corresponds to the limit

$$\nu W_2 = F_2(p^2,x), \quad W_1 = F_1(p^2,x).$$

As already noted above, the method we are using is applicable in the region of intermediate x. The inapplicability of the method for small x is due to the fact that if we consider $\operatorname{Im} V(s,q^2,p^2,t)$, instead of the imaginary part $\operatorname{Im} V(s,q^2,p^2,t=0)$ of the forward scattering amplitude, then, as shown in Ref. 13, the latter has a singularity in t at

$$t = -4p^2 x/(1-x). \quad (5)$$

It can be seen from (5) that for not too small x and $|p^2| \gg R_{\rm conf}^{-2}$ the singularity in t is located far from zero, i.e., short distances play a role in the t-channel. However, as $x \to 0$ the singularity in t tends to zero, long distances come into play, and the method of operator expansion in p^2 can no longer be applied. As $x \to 1$ we find ourselves in the region of resonances in the s-channel, i.e., the operator-expansion method is again inapplicable. It follows that the method is not applicable for the calculation of moments of structure functions. Even in the approximation that we are using, when the operator $G_{\mu\nu}^2$ is taken into account a singularity arises as $x \to 0$, leading to a logarithmic divergence for the second moment. In the language of moments, this implies the need to take account of vacuum expectation values in external fields, and precisely this approach was used in Ref. 11.

3. CALCULATION OF THE FOUR-CURRENT CORRELATOR IN QCD

We turn to the direct calculation of the imaginary part of the correlator (3) on the basis of the operator expansion. The calculation of the contribution of the unit operator, i.e., of the imaginary part of the diagrams of Fig. 1, contains nothing new: These diagrams have been calculated repeatedly in the literature (see, e.g., the reviews in Refs. 14–16 and the earlier papers cited there). In the approximation of zero mass of the quarks and in leading order in p^2/Q^2 the contribution of the diagram of Fig. 1 to the structure function W_2 (i.e., the coefficient of the $p_\mu p_\nu$ structure) has the form

$$\left(\frac{3\alpha}{\pi}\sum e_q^4\right)^{-1} F_2(x,p^2) = \ln\left(\frac{2\nu}{-p^2 x}\right) x(1+2x^2-2x) + [-2x+6x^2(1-x)]. \quad (6)$$

The expression (6) arises when one sums in (4) over all polarizations of the virtual photon; i.e., in the Lorentz gauge $\Sigma_i e_\rho^i e_\lambda^{*i}$ is replaced by $\frac{1}{2}(-\delta_{\lambda\rho} + p_\lambda p_\rho/p^2)$, the density matrix of the unpolarized photon. Below, we shall also need separate expressions for the transverse-photon structure function F_2^T:

$$\sum_{i=T} e_\rho{}^i e_\lambda{}^{*i} \to {}^1/_2\left(-\delta_{\lambda\rho} + \frac{p_\lambda p_\sigma}{p^2} + \Phi_\lambda \Phi_\rho\right),$$
$$\left(\frac{3\alpha}{\pi}\sum e_q^4\right)^{-1} F_2^T(x,p^2) = \ln\left(\frac{2\nu}{-p^2 x}\right) x[x^2+(1-x)^2] - 2x + 8x^2(1-x), \quad (7)$$

and the longitudinal-photon structure function F_2^L:

$$e_\rho{}^L e_\lambda{}^{*L} \to \Phi_\lambda \Phi_\rho,$$
$$\left(\frac{3\alpha}{\pi}\sum e_q^4\right)^{-1} F_2^L(x,p^2) = 4x^2(1-x), \quad (8)$$

where

$$\Phi_\lambda = [-p^2/(\nu^2 - q^2 p^2)]^{1/2} (q_\lambda - p_\lambda \nu/p^2)$$

is the familiar expression for the polarization vector of a longitudinal photon. It is not difficult to see that $F_2^T + \frac{1}{2} F_2^L = F_2$.

Contrary to naive expectations, the longitudinal-photon structure function $F_2^L(x,p^2)$ does not vanish as $p^2 \to 0$. This anomaly in the longitudinal-photon structure function has been discussed by us previously,[17] and is connected with the zero mass of the quarks: In the case of massive quarks the expression (8) should be multiplied by $p^2/[p^2 - m_q^2/x(1-x)]$, where m_q is the quark mass. We note that the crossover from the regime with a constant (independent of p^2) function $F_2^L(x,p^2)$ to $F_2^L(x,p^2) \propto p^2$ occurs at extremely small values of p^2.

The next term of the operator expansion in p^2 for the correlator (3) is the operator $G_{\mu\nu}^2$. As always, the calculation of the coefficient functions is conveniently performed in the fixed-point gauge for the gluon field with the use of the standard expression for the propagator of a massless quark in an external gluon field. It we choose the fixed point in the upper left corner of the diagrams of Fig. 1, the diagrams of Fig. 3 are found to be nonzero. The subsequent integration over the momentum is performed in the first nonvanishing order in p^2/Q^2. For reliability, these calculations were checked by means of the analytical-computations program "REDUCE-2".

We shall give the result for the case when $\operatorname{Im} V_{\mu\nu\lambda\rho}$ is multiplied by the unpolarized-photon density matrix. Then the coefficient of the structure $p_\mu p_\mu$ in the leading twist is equal to

$$\left(\frac{3\alpha}{\pi}\sum e_q^4\right)^{-1} F_2^{\langle G^2\rangle} = -\frac{4\pi^2}{27} \frac{\langle 0|(\alpha_s/\pi) G_{\mu\nu}^2|0\rangle}{p^4 x} \quad (9)$$

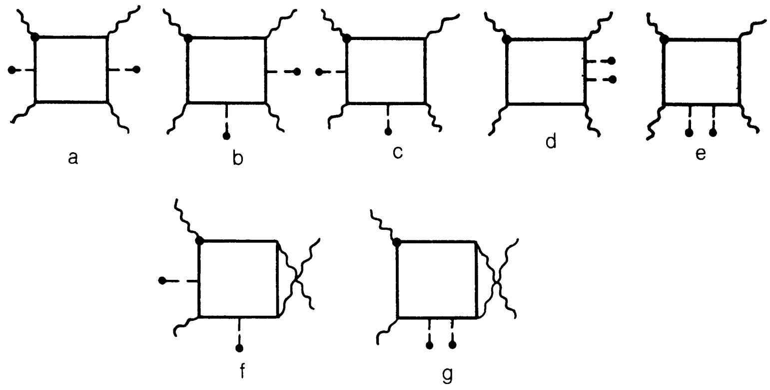

FIG. 3.

there being no contribution from the diagrams of Figs. 3f and 3g.

We return to the original expression for the contribution of the gluon condensate to Im $V_{\mu\nu\lambda\rho}$ in the case of longitudinal photons; i.e., we multiply this amplitude by $\Phi_\lambda \Phi_\rho$. After rather laborious calculations one can convince oneself that the structure function of the longitudinal photons is equal to zero in order $\langle G^2\rangle/p^4$. Thus, the gluon condensate in the leading twist gives a contribution only to the structure function of the transverse photons.

Calculations of the power correction to the structure function $F_1(x)$ and, correspondingly, to the Callan-Gross relation have been carried out separately. It was found that there is no power correction to $F_L(x) = F_2(x) - 2xF_1(x)$ from the gluon condensate.

It is possible that the absence of a power-law correction of order $\langle G^2\rangle/p^4$ in $F_2^L(x,p^2)$ has a profound cause. The fact that for massless quarks the value of $F_2^L(x,p^2)$ does not vanish as $p^2 \to 0$ has the character of an anomaly, and is due to the appearance in Im $V_{\mu\nu\lambda\rho}$ of a pole in p^2 in the diagram of Fig. 1a. In a certain sense the situation is analogous to the usual axial anomaly, in which, as is well known, nonperturbative corrections are completely absent in the calculation of the correlator

$$\int \exp\{ipx+iqy\}\langle 0|T\partial_\mu A_\mu(y), J_\alpha(x), J_\beta(0)|0\rangle,$$

where A_μ is the axial current of the quarks. If such an anomaly (based on the fact that in both cases the answer is completely determined by pole singularities) is valid, we may expect that there will be no nonperturbative corrections of any dimension to the contribution made by the loop in Fig. 1 to F_2^L.

The analytical computer calculations have made it possible to obtain the first scaling-violating power-law correction to (9). It reduces to the result that the expression (9) should be multiplied by the factor

$$1-3x^4p^2/Q^2. \qquad (10)$$

We note that the correction (10) is completely built up from a diagram of the crossing type. The analogous correction to the function F^L turns out to be equal to zero.

The expression (9) is, in essence, the principal nonperturbative contribution to the structure function of the photon in the region of intermediate values of x. In fact, the contribution of the operators $m_q\bar{\psi}_q\psi_q$ and $m_q\bar{\psi}_q\sigma G\psi_q$, which is represented in Fig. 4, is proportional to $\delta(1-x)$ or $\delta(x)$, i.e., it arises from a region in which the entire approach, generally speaking, is inapplicable. An imaginary part in s in the region of intermediate values of x will be possessed only by low-dimension radiative corrections to terms of the operator expansion, i.e., by diagrams with an extra hard-gluon exchange (of the type of the diagram in Fig. 5). Although the calculation of such diagrams is technically complicated, it can be performed subsequently to refine the results.

The region of applicability of the approximation used can be estimated from the expression for $F_2(x)$ itself. With the standard choice

$$4\pi^2\langle 0|(\alpha_s/\pi)G_{\mu\nu}^2|0\rangle = 0.45 \text{ GeV}^4$$

numerically this contribution amounts to 50% of the contribution of the simple loop for $x \cong 0.2$ and $p^2 = -0.5$ GeV^2. This implies that for smaller values of x and p^2 higher terms of the expansion, not taken into account by us, become important.

To conclude this section, we note that our result for the correction from the condensate $\langle 0|G_{\mu\nu}^2|0\rangle$ differs from the result of the calculations performed in Ref. 18 for the function F_1. In the terms proportional to $1/x$ there is a factor-of-two discrepancy, and, in addition, in Ref. 18 other terms appear that are absent in our result.

4. CALCULATION OF THE STRUCTURE FUNCTIONS OF THE PHOTON AND VECTOR MESON. COMPARISON WITH EXPERIMENT

We shall represent the expression for $F_2^{T,L}(x,p^2)$ in terms of contributions of physical states by means of disper-

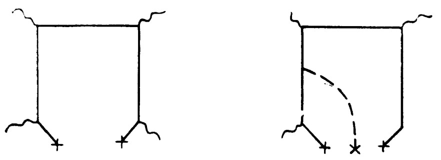

FIG. 4.

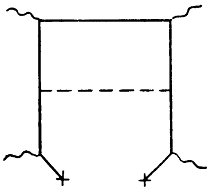

FIG. 5.

sion relations in p^2. In the general case this expression has the form

$$F_2^{T,L}(x,p^2)=\alpha(x)+\int_0^\infty \frac{\varphi^{T,L}(x,p'^2)}{p'^2-p^2}dp'^2 + \int_0^\infty dp_1'^2\,dp_2'^2\frac{\rho^{T,L}(x,p_1'^2,p_2'^2)}{(p_1'^2-p^2)(p_2'^2-p^2)}. \quad (11)$$

All the integrals in (11) converge, since the subtractive terms in the third integral in the right-hand side of (11) are included in the second integral, and the subtractive term in the second integral is included in the first integral. We take a model of the hadron spectrum in which the right-hand side of (11) is described by the contributions of the vector meson and of the continuum, which begins from the threshold p_0^2. Then $\varphi^{T,L}$ and $\rho^{T,L}$ can be written as

$$\varphi^{T,L}(x,p'^2)=\varphi_V^{T,L}(x)\delta(p'^2-m_V^2)+\gamma^{T,L}(x,p'^2)\theta(p'^2-p_0^2),$$
$$\rho^{T,L}(x,p_1'^2,p_2'^2)=C_V^{T,L}\delta(p_1'^2-m_V^2)\delta(p_2'^2-m_V^2)+\beta^{T,L}(x,p_1'^2,p_2'^2)\theta(p_1'^2-p_0^2)\theta(p_2'^2-p_0^2). \quad (12)$$

The contribution of the continuum is determined from the condition that for $|p^2|\to\infty$ (but $|p^2|\ll|q^2|$) the right-hand side of (12) coincide with the contribution [determined by the expressions (7) and (8)] of the simplest quark loop of Fig. 1. In order to find the unknown functions φ, γ, C and β, we write (7) and (8) in the form of the dispersion integrals that arise in (11). By simple transformations of the integrands in the Feynman integrals describing the contributions of the diagrams of Fig. 1 we can represent them in the form (11); namely,

$$F_2^T(x,p^2)=\frac{3\alpha}{\pi}\sum e_q^4 x\left\{-1+6x(1-x)+[x^2+(1-x)^2]\int_0^{2\nu/x}\frac{p'^2\,dp'^2}{(p'^2-p^2)^2}\right\}, \quad (13)$$

$$F_2^L(x,p^2)=\frac{12\alpha}{\pi}\sum e_q^4x^2(1-x)\left[\int_0^{2\nu/x}\frac{dp'^2}{p'^2-p^2}-\int_0^{2\nu/x}\frac{p'^2\,dp'^2}{(p'^2-p^2)^2}\right]. \quad (14)$$

After substituting (12) into (11) and comparing with (13) and (14), we obtain

$$\alpha^T(x)=-1+6x(1-x),\quad \alpha^L(x)=0,$$
$$\gamma^T(x)=0,\quad \gamma^L(x,p'^2)=4x(1-x)\theta[2\nu/x-p'^2], \quad (15)$$
$$\beta^T(x,p_1'^2,p_2'^2)=[x^2+(1-x)^2]p_1'^2\delta(p_1'^2-p_2'^2)\theta[2\nu/x-p_1'^2],$$
$$\beta^L(x,p_1'^2,p_2'^2)=-4x(1-x)p_1'^2\delta(p_1'^2-p_2'^2)\theta[2\nu/x-p_1'^2].$$

[All the quantities should be multiplied by $(3\alpha/\pi)\Sigma e_q^4$. In the language of local duality, our approach corresponds to the fact that the nonperturbative part is related to the region of small $k_\perp^2$ in the diagram of Fig. 1, i.e., to $k_\perp^2/x(1-x)<p_0^2$, where $k_\perp$ is the transverse momentum of the quark.]

To find the remaining unknown functions $\varphi_V^{T,L}(x)$ and $C_V^{T,L}(x)$, we substitute (12) and (15) into (11), compare the resulting expression with the results (7)–(9) of the QCD calculations,and require that these expressions coincide in the expansion in powers of $1/p^2$ up to terms $\sim 1/p^4$. (We restrict ourselves to terms $\sim 1/p^4$, since terms $\sim 1/p^6$ were not taken into account in the calculations in QCD.) As a result, we find

$$\varphi_V^T(x)=0,\quad \varphi_V^L(x)=\frac{12\alpha}{\pi}\sum e_q^4x^2(1-x)p_0^2, \quad (16)$$

$$C_V^T(x)=\frac{3\alpha}{2\pi}\sum e_q^4xp_0^4\left[x^2+(1-x)^2-\frac{8\pi^2}{27p_0^4x^2}\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle\right], \quad (17)$$

$$C_V^L(x)=-\frac{12\alpha}{\pi}\sum e_q^4p_0^2(p_0^2-m_V^2)x^2(1-x). \quad (18)$$

Substituting (15)–(18) into (11), we obtain the final expressions for the virtual-photon structure functions:

$$F_2^T(x,p^2)=\frac{3\alpha}{\pi}x\sum e_q^4\left\{-1+6x(1-x)+[x^2+(1-x)^2]\times\left[\ln\frac{Q^2}{x^2(p_0^2-p^2)}+\frac{p^2}{p_0^2-p^2}\right]+{}^1/_2\frac{p_0^4}{(p^2-m_V^2)^2}\times\left[x^2+(1-x)^2-\frac{8\pi^2}{27p_0^4x^2}\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle\right]\right\}, \quad (19)$$

$$F_2^L(x,p^2)=\frac{12\alpha}{\pi}\sum e_q^4x^2(1-x)\left[\frac{p_0^2}{m_V^2-p^2}+\frac{p^2}{p^2-p_0^2}-\frac{p_0^2(p_0^2-m_V^2)}{(p^2-m_V^2)^2}\right]. \quad (20)$$

The expressions (19) and (20) have the correct analytical properties in p^2, unlike the results of the perturbative calculations, in which (see Refs. 2, 9, and 14–16) the correct behavior as $p^2\to 0$ was achieved "by hand."

The function $C_V^T(x)$ determined above is connected with the structure function $f_V^T(x)$ of the transversely polarized meson by the relation

$$C_V^T(x)\Big/\sum e_q^4=\alpha(4\pi/g_V^2)m_V^4f_V^T(x), \quad (21)$$

where g_V is the photon–vector-meson transition constant for a vector meson with unit charge and consisting of quarks of the same flavor. The quantity g_V is related to the γ–ρ transition constant by

$$g_V^{-2}={}^1/_2g_\rho^{-2}, \quad (22)$$

and, experimentally,

$$g_V^2/4\pi=1.27 \quad (23)$$

(see, e.g., Ref. 19).

For a longitudinally polarized vector meson the relation (21) holds, but with the opposite sign. The reason for this is that for longitudinal photons we used the normalization condition $(e_\mu^2)_L=1$, whereas for a longitudinal vector meson we should have $(e_\mu^2)_L^V=-1$.

We now discuss the possibility of extrapolating our results (19) and (20) to the point $p^2=0$. As already noted, for $x>0.2$ in the operator expansion in p^2 we can come down to $p^2\cong 0.5$ GeV^2, at which the correction amounts to about 50%. It may be supposed that at $p^2=0.5$ GeV^2 the representation of the physical spectrum in the form of the vector-meson contribution and a continuum is a sufficiently good approximation, since we know from experiment that below the ρ meson ($m_\rho^2 = 0.6$ GeV^2) there is no appreciable contribution of states with the quantum numbers of the photon. Thus, for $x > 0.2$ we can extrapolate the structure function of the transverse photon to the point $p^2 = 0$ and obtain the real-photon structure function

$$F_2{}^T(x) = \frac{3\alpha}{\pi} x \sum_q e_q{}^4 \Big\{ -1 + 6x(1-x) + [x^2 + (1-x)^2] \ln \frac{Q^2}{x^2 p_0{}^2} + {}^1/_2 \frac{p_0{}^2}{m_\rho{}^4} \Big[x^2 + (1-x)^2 - \frac{8\pi^2}{27 p_0{}^4 x^2} \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}{}^2 | 0 \rangle \Big] \Big\}. \quad (24)$$

Upper bounds on the admissible values of x can be obtained from the following considerations. For values of x close to unity the physical states that are formed in $\gamma\gamma$ collisions are resonance states. If the initial state is also a low-lying meson (the ρ meson), then for $Q^2 \sim 5$–10 GeV^2 its contribution will be strongly suppressed by the form factor. If, however, in the initial and final states we take a sum of such resonances (ρ, ρ', ρ''), the result will be substantially greater. Thus, for $x \to 1$ our model is poor. In addition, the operator-expansion series diverges as $x \to 1$. Finally, the expression (17) for the vector-meson structure function does not lead to the behavior [$\propto (1-x)$ as $x \to 1$] that follows from quark counting. All of this points to the fact that our approach ceases to be applicable for values of x close to unity. An upper bound on the region of admissible values of x can be obtained from the requirement that for $Q^2 \sim 5$–10 GeV^2 the masses of the resonances created be significantly greater than the ρ-meson mass. It follows from this requirement that $x \leqslant 0.7$. Consequently, our approach is legitimate in the region $0.2 \leqslant x \leqslant 0.7$.

For $p^2 = 0$ the expression (24) is the complete real-photon structure function, which can be compared with experimental data. [As noted above, $F_2^L(x)$ vanishes for $|p^2| < m_q^2/x(1-x)$.] In Fig. 6 we present the values of F_2/α calculated from (19) in comparison with the results of experiment. In the calculation we took $p_0^2 = 1.5$ GeV^2—the standard value of the threshold of the continuum in the calculation of the ρ-meson mass in the sum-rule method.[20] For $0.2 \leqslant x \leqslant 0.7$ we observe reasonable agreement with experiment in a broad range of Q^2 without allowance for perturbative QCD corrections, indicating that the latter have an insignificant role for such values of Q^2. Thus, in our opinion, in this region of Q^2 it is practically impossible to determine Λ_{QCD} with sufficient accuracy. We stress once again that the agreement with experiment has been achieved without any adjustable parameters.

The hadron part of the structure function—the last term in (24)—is presented in Fig. 7 in comparison with the model expression (2) obtained in Ref. 9. It can be seen that for $x > 0.4$ our hadron part is considerably greater. By means of the expression obtained above for the vector-meson structure function one can calculate the momentum fraction carried by the quarks in the vector meson in the region of intermediate values of x. The result is

$$\int_{0,2}^{0,7} dx\, f_V{}^T(x) = 0.34, \quad (25)$$

which seems to be extremely reasonable.

In the case of the longitudinal-photon structure function we have no possibility of reliably estimating the region of values of p^2 and x in which our approach is applicable, since the correction due to the gluon condensate vanished. For $p^2 = 0$ the right-hand side of (20) is negative, i.e., extrapolation of (20) into the region of small p^2 is impossible. For $p^2 = -0.5$ GeV^2 the expression in square brackets in (20) is equal to 0.5, in comparison with the asymptotic value of 1. This gives grounds to assume that, starting from $p^2 = -0.5$ GeV^2, (20) represents the structure function of the virtual longitudinal photon. Another possibility, which was discussed above, is that all the terms of the series in $1/p^2$ vanish, so that in the region of convergence of the series only the anomalous contribution (8) remains. In this case we have a definite prediction for the longitudinal-photon structure function, namely, Eq. (8), but, of course, our model is no

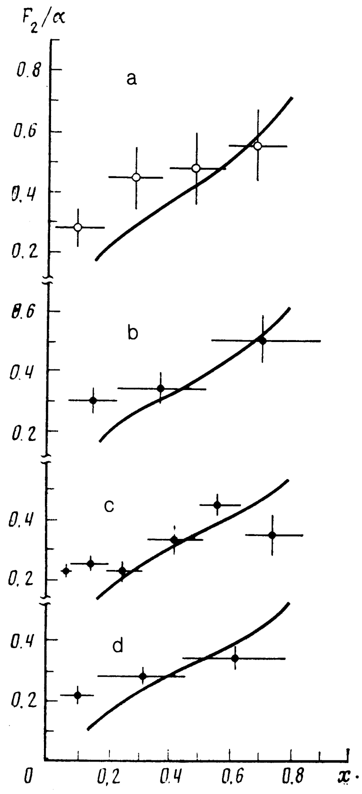

FIG. 6. Comparison of the photon structure function (24) with experimental data (● from Ref. 21, and O from Ref. 22) for different values of Q^2: a) 23 GeV^2; b) 9.2 GeV^2; c) 5.3 GeV^2; d) 4.3 GeV^2.

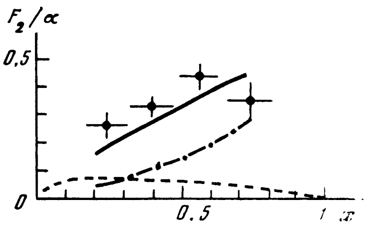

FIG. 7. The solid curve is the complete photon structure function (24); the dash-dot curve is the hadron part of the photon structure function (24); the dashed curve is the hadron part of the photon structure function (2). $Q^2 = 5$ GeV^2.

longer applicable. To clarify the question of whether (8) really does fully determine the longitudinal-photon structure function, it would be important to calculate higher power corrections in $1/p^2$.

In connection with the problem of small x, we note that, in our opinion, the hope that the $x\to 0$ singularities arising from the nonperturbative and perturbative contributions will cancel has little foundation. At least part of the singular (as $x\to 0$) contributions to the term proportional to $\langle 0|G^2_{\mu\nu}|0\rangle$ has an explicitly different nature.

5. CONCLUSION

In the present paper, on the basis of QCD and a description of the hadron spectrum that satisfies the analytical properties of the structure function with respect to the photon virtuality p^2, we have calculated the photon structure function in the region $0.2\leqslant x\leqslant 0.7$. The resulting hadron part has been found to be extremely large,and significantly larger than in previous model calculations. Allowance for this hadron part is absolutely necessary in the calculation of the QCD evolution of the photon structure function, and experimental investigation of it is evidently not a better way of determining Λ_{QCD} than the study of deep inelastic lepton-nucleon scattering. In view of the above-discussed possible absence of nonperturbative effects in the function $F_2^L(x)$, it is of great interest to study this function experimentally.

Natural refinements of the results obtained could include an analysis of the contributions of operators of higher dimensions and the contributions of higher twists, and also allowance for evolution effects.

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