Contact phenomena in low-dimensional electron systems

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(Submitted 26 June 1989)
Zh. Eksp. Teor. Fiz. **96**, 2229–2239 (December 1989)

A theoretical investigation is reported of the problem of electrical contacts with low-dimensional systems. In contrast to the conventional contacts with macroscopic samples, the contacts under consideration represent junctions between electron systems of different dimensions. The difference between the energy spectra and the densities of states on the two sides of a contact and the specific nature of the screening of the contact electric field in a two-dimensional (or one-dimensional) electron gas give rise to qualitatively new contact phenomena. A solution is given of the problem of the distribution of the potential and carrier density in contact layers of different low-dimensional systems (such as a quantum well, a heterostructure with modulated doping, an inversion layer in a metal–insulator–semiconductor structure, or a quantum filament) both in the absence and presence of a quantizing magnetic field. A slow fall of the potential (hyperbolic in the two-dimensional case and logarithmic in the one-dimensional case) results in divergence of the capacitance of depletion layers in ideal structures. In the case of real contacts this divergence is suppressed if we allow for such factors as the finite dimensions of the contact region in a two-dimensional (one-dimensional) electron gas, the influence of a metal gate in a metal–insulator–semiconductor structure, etc. The current-voltage characteristics of such junctions are calculated allowing for quantum-mechanical reflection of electrons from the interface between electron systems of different dimensions. The specific features of formation of ohmic contacts with different low-dimensional systems are also discussed.

1. INTRODUCTION

Investigations and device applications of transport effects in low-dimensional systems require formation of electrical contacts with a two-dimensional or a one-dimensional electron gas. This can be done in a variety of ways. In the case of metal–insulator–semiconductor (MIS) structures the role of contacts is played by heavily doped source and drain regions. Metal contacts in heterostructures are usually formed by alloying or by evaporation. However, in all cases the contact region is sufficiently large that it can be considered as a bulk metal or a heavily doped semiconductor. Therefore, in contrast to a conventional contact with a macroscopic sample, a contact with a two- or one-dimensional electron gas is a junction between electron systems of different dimensions. The properties of such contacts have a number of qualitative distinguishing features which we shall consider below (some of the results are published in a preliminary form in Ref. 1).

These distinguishing features are associated, firstly, with the difference between the electron energy spectra on both sides of the contact and, secondly, with the specific nature of screening in low-dimensional systems. The first difference has the effect that the contacts in question are usually characterized by a higher resistivity and are strongly nonohmic. In fact, the Fermi level in a low-dimensional region is located higher, because of the size quantization energy E_1, than in a three-dimensional contact, even if the latter is heavily doped, so that a contact depletion layer is formed (provided the contact does not have a much lower work function than the low-dimensional material). We shall consider the properties of such a depletion layer in the next section.

2. CONTACT WITH A QUANTUM WELL

We shall consider a contact between a bulk degenerate sample and a region containing a two-dimensional electron gas (2DEG) of density n_{s0}. The analysis is simplest in the case when the 2DEG is not spatially separated from the positive charge compensating it. This situation occurs in thin films or in bulk-doped quantum wells. The distributions of the charge and potential in such a system are shown schematically in Fig. 1. These distributions can be found by solving the Laplace equation $\Delta\varphi = 0$ in the space surrounding a film and for a symmetric position of a bulk contact and identical permittivities $\varkappa$ in the regions $z<0$ and $z>0$. It is sufficient to consider only the first quandrant where $x, z>0$.

We shall begin with the case of relatively weak band bending when the contact potential eV is less than the Fermi energy $\varepsilon_F = \pi\hbar^2 n_{s0}/m^*$ of the 2DEG (m is the effective mass of an electron). If we ignore the change in the potential across the film thickness, we find that the local density n_s of the degenerate 2DEG is related linearly to the potential[1)]:

$$n_s(x)=n_{s0}[1+e\varphi(x,\ 0)/\varepsilon_F].$$

This leads to the following conditions:

$$\varphi(0,\ z)=-V, \tag{1}$$

$$\frac{\partial\varphi}{\partial z}(x,0)=\frac{2\pi e}{\varkappa}\, n_{s0}\frac{e\varphi(x,0)}{\varepsilon_F}. \tag{2}$$

The Laplace equation with such boundary conditions is readily solved and in the case of the potential in a film we obtain

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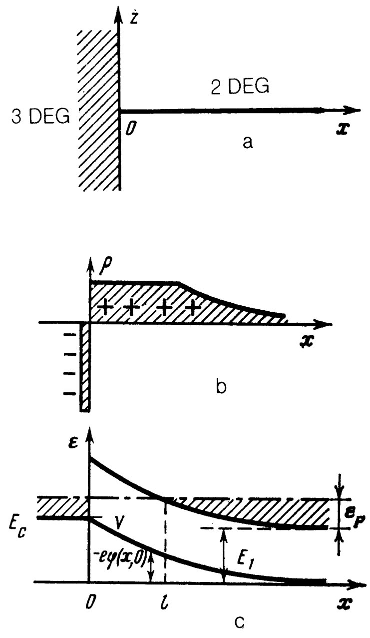

FIG. 1. Schematic representation of a contact between a two-dimensional electron gas (2DEG) and a three-dimensional gas (3DEG) (a), distribution of the surface potential (b), and qualitative behavior of the potential energy of an electron as well as bending of the energy bands in a 2DEG (c). The shaded regions are filled with electrons.

$$\varphi(x,0)=-\frac{2V}{\pi}\left[\operatorname{ci}\left(\frac{2x}{a_B}\right)\sin\left(\frac{2x}{a_B}\right)-\operatorname{si}\left(\frac{2x}{a_B}\right)\cos\left(\frac{2x}{a_B}\right)\right]. \tag{3}$$

Here, si and ci are the integral sine and cosine; $a_B = \varkappa^2/m^*e^2$ is the effective Bohr radius playing the role of the screening length in a degenerate 2DEG (Ref. 2). In spite of the presence of sines and cosines, the function (3) is monotonic and has the following asymptotic forms:

$$\varphi(x,0)\approx -V\left(1-0.54\frac{x}{a_B}+\frac{4x}{\pi a_B}\ln\frac{2x}{a_B}\right),\quad x\ll a_B, \tag{4}$$

$$\varphi(x,\ 0)\approx -Va_B/\pi x,\quad x\gg a_B. \tag{5}$$

It should be noted that a hyperbolic law similar to Eq. (5), demonstrates a fall in the charge density on a metal electrode as we move away from the 2DEG plane.

The problem becomes more complex when the band bending is strong. If we use 1 to denote the coordinate of such a point in the film at which $e\varphi(l,0) = -\varepsilon_F$, then Eq. (2) is valid only for $x > l$, whereas for $x < l$ there is total depletion and the boundary condition becomes

$$\frac{\partial\varphi}{\partial z}(x,0)=-2\pi e n_{s0}/\varkappa. \tag{6}$$

Since the boundary condition changes at the int $x = l$, the boundary condition changes it is convenient to solve the problem by the conformal transformation

$$w=u+iv=2\ln\frac{x+iz+[(x+iz)^2-l^2]^{1/2}}{l}, \tag{7}$$

which transforms the first quadrant in the xz plane to a semiaxis $u > 0$, $0 < v < \pi$ in the uv plane where the nature of the boundary conditions is now the same on both sides of the semiaxis:

$$\varphi(u,\ \pi)=-V, \tag{8}$$

$$\frac{\partial\varphi}{\partial v}(u,0)=\frac{\pi e l}{\varkappa}n_{s0}\frac{e\varphi}{\varepsilon_F}\operatorname{sh}\frac{u}{2}, \tag{9}$$

$$\frac{\partial\varphi}{\partial u}(0,v)=\frac{\pi e l}{\varkappa}n_{s0}\sin\frac{v}{2}. \tag{10}$$

It is not possible to obtain a general analytic solution. However, for $eV\gg\varepsilon_F$ (which we see later implies that $l\ll a_B$), it is quite easy to calculate the distribution of the potential in a film subject to the condition $l-x\gg a_B$, i.e., sufficiently far from the point $x = l$. For $z = 0$ and $z > l$ (i.e., for $v = 0$ and $u > 0$) we have to assume $\varphi\approx 0$. The problem is then readily solved and transformation back to the coordinates x, z gives

$$\varphi(x,0)=-\frac{2V}{\pi}\operatorname{arctg}\frac{(l^2-x^2)^{1/2}}{x}-\frac{2en_{s0}}{\varkappa}x\ln\frac{l-(l^2-x^2)^{1/2}}{l+(l^2-x^2)^{1/2}}. \tag{11}$$

The thickness of the depletion layer l has been assumed so far to be an independent parameter. In fact, it is clear that it must be expressed in terms of V and n_{s0}. This relationship is easiest to find directly from Eq. (11). We can readily see that the field in a film $\partial\varphi(x,0)/\partial x$ has no singularities and $x = l$ only if

$$l=\varkappa V/2\pi e n_{s0}. \tag{12}$$

We can see that in the 2DEG case the depletion region thickness is proportional to the contact potential and not to the root of this potential, as in the three-dimensional case.

The approach we have adopted (which is an analog of the Schottky approximation) is unsuitable for an analysis of the potential in the range $x\gtrsim l$. The solution subject to the exact boundary conditions of (8)–(10) is readily obtained for $x\gg l$ ($u\gg 1$). It is identical with Eq. (5).

The slow fall of the potential and of the charge density described by Eq. (5) is not specific to purely two-dimensional systems, but is a general law which applies (apart from a coefficient) also to films with a three-dimensional dispersion law or to a 2DEG in a quantizing magnetic field (see Sec. 5). It is related to the fact that the distribution of charges in the contact always has the geometry of a two-dimensional dipole characterized by a potential of the $1/x$ type at large distances.

In the case of a contact layer with the above distribution of the charge and potential, the specific capacitance (per unit length) is

$$C=\frac{\varkappa}{2\pi}\left(1+\frac{1}{\pi}\ln\frac{L}{l}\right), \tag{13}$$

where L is the distance under which the hyperbolic asymptote of the potential (5) is valid (see below). In the three-dimensional case the capacitance described by Eq. (13) has the following features compared with the capacitance of a depletion layer:.

1) the capacitance is practically independent of the voltage across a contact because, in accordance with Eq. (12), the thickness of the depletion layer is proportional to V;

2) the capacitances of the total depletion layer $x<l$ [first term in Eq. (13)] and of the "tail" of the potential region $x>l$ [second term in Eq. (13)] are connected in parallel and represent two regions of the same capacitor plate (the second plate is a metal contact);

3) the main charge in the contact region is concentrated not in the total depletion layer. but in the slowly decreasing tail.

We shall now consider the problem of the characteristic cutoff length L. Such complications of the above very simple model as an allowance for the finite thickness of the film or for the nonequipotential nature of the contact material do not alter the asymptotic form (5) of the potential, i.e., they do not suppress the divergence of the capacitance. The capacitance becomes finite only if we allow for the finite dimensions of the contact region or of the 2DEG itself. The smallest of these dimensions plays the role of the characteristic length L in Eq. (13).

3. CONTACT WITH AN INVERSION LAYER IN A METAL-INSULATOR-SEMICONDUCTOR STRUCTURE

We shall now consider a different semiconductor structure with a 2DEG in the form of an inversion layer of an MIS field-effect transistor. This situation differs from the case of a quantum well discussed in Sec. 2 by the following two features:

1) the energy of a quantum level E_1 and the 2DEG density n_s are mutually related;

2) the positive charge which compensates the charge of the 2DEG is concentrated in the metal electrode, which is at a distance d from the 2DEG.

The first circumstance has the effect that the contact potential V at the interface between a channel and drain or source regions is low and a total depletion layer is not formed. In fact, if a heavily doped contact is made of the same material as the region where the 2DEG is located, the contact potential is related entirely to the energy shift E_1 in the 2DEG. However, in such a contact depletion laver the value of n_s is small and, consequently, the surface field governing the quantization energy E_1 and the contact potential is also weak.

Let us estimate V in a channel of an MIS structure near a strongly degenerate three-dimensional contact characterized by the Fermi level ε_F' (Fig. 2). In our estimates we shall represent the channel by the simplest triangular well with an electric field $F=4\pi e n_s/\varkappa$. In such a well we have

$$E_1\approx 4.68(\pi^2 e^4\hbar^2 n_s^2/\varkappa^2 m^*)^{1/3}.$$

Then, the contact density $n_s(0)$ in the 2DEG is described by a self-consistent equation

$$n_s(0)=\frac{m^*}{\pi\hbar^2}\left[\varepsilon_F'-4{,}68\left(\frac{\pi^2e^4\hbar^2}{\varkappa^2 m^*}\right)^{1/3} n_s^{2/3}(0)\right]. \tag{14}$$

The solution of this equation is followed by determination of the contact potential from

$$eV=(\pi\hbar^2/m^*)\,[n_{s0}-n_s(0)]. \tag{15}$$

We can easily see that the asymptotic solutions of Eq. (14) are

$$n_s(0)\approx\begin{cases}0.1\varkappa(m^*)^{1/2}(\varepsilon_F')^{3/2}/\pi e^2\hbar, & \varepsilon_F'\ll E_B,\\ m^*\varepsilon_F'/\pi\hbar^2, & \varepsilon_F'\gg E_B,\end{cases}$$

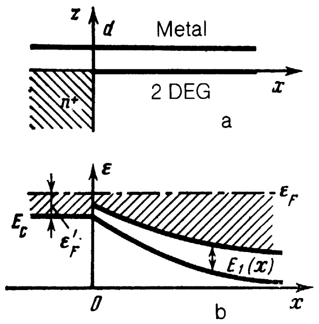

FIG. 2. Model of a contact with a 2DEG in an MIS structure (a, the heavily doped drain or source region is shown shaded) and qualitative behavior of the potential energy of an electron in the 2DEG plane allowing for an inhomogeneity of the channel (b).

where E_B is the effective Bohr energy. In any case, right up to the contact itself there are electrons in an inversion layer and the electric field of the gate does not penetrate into the bulk of the semiconductor. A calculation of the contact effects in the 2DEG can be reduced to the solution of the Laplace equation only in the insulator occupying the region $0\leqslant z\leqslant d$ and characterized by a permittivity $\tilde{\varkappa}$. If a heavily doped electrode with a potential $-V$ occupies the region defined by $x<0$ and $z<0$, and the 2DEG can be represented by the half-plane $z=0$ and $x>0$, the boundary conditions to the Laplace equation become

$$\varphi(x,0)=\begin{cases}-V, & x<0\\ -\varepsilon_F/e+{}^1/_4\tilde{a}_B\,\partial\varphi(x,0)/\partial z, & x>0,\end{cases} \tag{16}$$

$$\varphi(x,\ d)=4\pi e n_{s0}d/\tilde{\varkappa}, \tag{17}$$

($\tilde{a}_B$ is the effective Bohr radius in the case when the permittivity is $\tilde{\varkappa}$).

We shall assume $d\gg\tilde{a}_B$. Then, the potential in the 2DEG in the range $x\gg\tilde{a}_B$ is

$$\varphi(x,0)=-\frac{V\tilde{a}_Bd}{2}\sum_{n=1}^{\infty}\frac{\exp(-\pi nx/d)}{d^2+(\pi n\tilde{a}_B/4)^2}. \tag{18}$$

In the range $\tilde{a}_B\ll x\ll d$, this expression reduces to the familiar Eq. (5), whereas for $x>d$ the screening by the gate cuts off sharply the potential:

$$\varphi(x,\ 0)\approx-(V\tilde{a}_B/2d)\exp(-\pi x/d). \tag{19}$$

The specific capacitance of the contact is

$$C\approx(\tilde{\varkappa}/2\pi^2)\ln(d/\tilde{a}_B). \tag{20}$$

We can calculate it ignoring the finite nature of the dimensions of the contact, which are as a rule much greater than d (by analogy with Sec. 2), because the gate provides a much more effective screening of the contact field.

The expressions in this section can be used also to describe contact phenomena in single heterostructures with modulated doping, where the role of d is played by the total thickness of the undoped region (spacer) and of the depletion region in a wide-gap semiconductor. The only difference is that the contact regions in heterostructures (usually alloyed metallic) may have a different work function from that of a semiconductor containing a 2DEG. Depending on the value of the work function, the contact may (as in Sec. 2) or may not include a total depletion layer. As pointed out already, such a depletion layer should not occur in the MIS structure considered above. This accounts for the absence of direct experimental evidence that the contacts are nonohmic in conventional silicon MIS structures.

4. FLOW OF A CURRENT ACROSS A CONTACT

If the interface between a 2DEG and a three-dimensional region is relatively abrupt compared with the electron wavelength λ, it follows that an electron incident on the interface may suffer quantum-mechanical reflection. We shall calculate the reflection coefficient R ignoring the band bending in the 2DEG calculated in Sec. 2, which is permissible if the condition $\lambda \ll l$ is satisfied. The general formulation of the problem was discussed in Ref. 3. The results of a numerical calculation for a 2DEG bounded by a parabolic potential are also given in Ref. 3. We calculated R using a more realistic model of a rectangular quantum well of width a and with infinitely high walls located at $z = \pm a/2$ and $x > 0$.

We now consider an electron which belongs to the first level of a 2DEG and is incident from the region $x > 0$ on the interface $x = 0$. We assume that the energy of this electron E (without allowance for motion along the y axis) satisfies the condition $E_1 < E < E_2$, where $E_n = \pi^2\hbar^2 n^2/2m^* a^2$ is the energy of the nth level. Then, the complete wave function to the right of the interface (located in the 2DEG) is of the form

$$\psi^+ = \cos\left(\frac{\pi z}{a}\right)\left[\exp\left(-i\frac{[2m^*(E-E_1)]^{1/2}x}{\hbar}\right) - R\exp\left(i\frac{[2m^*(E-E_1)]^{1/2}x}{\hbar}\right)\right] + \sum_{n=1}^{\infty} A_n \cos\left[\frac{\pi(2n+1)z}{a}\right] \times \exp\left(-\frac{[2m^*(E_{2n+1}-E)]^{1/2}x}{\hbar}\right), \quad |z| \leq \frac{a}{2},$$

$$\psi^+ = 0, \quad |z| > \frac{a}{2}, \tag{21}$$

To the left of the interface the electron gas will be regarded as free and we can write down the wave function in the form

$$\psi^- = \int_0^{(2m^*E)^{1/2}/\hbar} B(k)\cos(kz) \times \exp\left[-i\left(\frac{2m^*E}{\hbar^2} - k^2\right)^{1/2} x\right] dk + \int_{(2m^*E)^{1/2}/\hbar}^{\infty} C(k)\cos(kz)\exp\left[\left(k^2 - \frac{2m^*E}{\hbar^2}\right)^{1/2} x\right] dk, \tag{22}$$

where $B(k)$ and $C(k)$ are defined in the Appendix. It is also shown in the Appendix that matching of Eqs. (21) and (22) at $x = 0$ give rise to the following system of equations for the unknown values of R and A_n:

$$(\xi-1)^{1/2}(1+R) = (1-R)(\alpha_{00}+i\beta_{00}) + \sum_{n=1}^{\infty} A_n(\alpha_{0n}+i\beta_{0n}), \tag{23}$$

$$-i[(2l+1)^2-\xi]^{1/2}A_l = (1-R)(\alpha_{l0}+\beta_{l0}) + \sum_{n=1}^{\infty} A_n(\alpha_{ln}+i\beta_{ln}),$$

$$l = 1, 2, 3, \ldots, \tag{24}$$

where $\xi = E/E_1$,

$$\alpha_{ln}(\xi) = \frac{8}{\pi^2}(-1)^{l+n}(2l+1)(2n+1) \times \int_0^{\xi^{1/2}} \frac{(\xi - x^2)^{1/2}\cos^2(\pi x/2)\,dx}{[(2l+1)^2 - x^2][(2n+1)^2 - x^2]}, \tag{25}$$

$$\beta_{ln}(\xi) = \frac{8}{\pi^2}(-1)^{l+n}(2l+1)(2n+1) \times \int_{\xi^{1/2}}^{\infty} \frac{(x^2 - \xi)^{1/2}\cos^2(\pi x/2)\,dx}{[(2l+1)^2 - x^2][(2n+1)^2 - x^2]}. \tag{26}$$

The solution of the above system provides comprehensive information on the nature of the transport of electrons across an interface between two- and three-dimensional systems. We limit ourselves only to calculation of the coefficient R at low values of the kinetic energy of an electron: $\xi - 1 \ll 1$. Under these conditions we can assume $\xi \approx 1$ throughout the system represented by Eqs. (23) and (24) (including the coefficients α and β), but with the exception of the left-hand side of the first equation. It immediately becomes clear that both A_n and $1 - R$ are proportional to $(\xi - 1)^{1/2}$, i.e., to the square root of the kinetic energy of a two-dimensional electron (this behavior of R was first pointed out in Ref. 3). A numerical calculation shows that

$$R = 1 - (2.68 - 1.36i)(\xi - 1)^{1/2}.$$

Consequently, the probability of the passage of an electron across the interface is

$$W = 1 - |R|^2 \approx 5.36[(E-E_1)/E_1]^{1/2}. \tag{27}$$

We now calculate the current-voltage characteristic of the investigated contact. In the diode approximation when $\varepsilon_F < eV \equiv e(V_0 - U)$ (V_0 is the equilibrium contact potential and U is the applied external voltage) the current density per unit length of the contact is

$$j = \frac{2.72e}{\pi\hbar^2}\left(\frac{2m^*T}{\pi E_1}\right)^{1/2}\left[\int_{eV}^{\infty}(\varepsilon - eV)^{1/2}\exp\left(\frac{\varepsilon_F - \varepsilon}{T}\right)d\varepsilon - \int_{eV_0}^{\infty}(\varepsilon - eV_0)^{1/2}\exp\left(\frac{\varepsilon_F - \varepsilon}{T}\right)d\varepsilon\right] = 5.36A^*T^2 a \times \exp\left(\frac{\varepsilon_F - eV_0}{T}\right)\left[\exp\left(\frac{eU}{T}\right) - 1\right]. \tag{28}$$

Here, $A^* = em^*/2\pi^2\hbar^3$ is the effective Richardson constant for thermionic emission. We recall that according to the above analysis, the condition for rigorous quantitative validity of Eq. (28) is ensured by the requirement $l \gg \lambda$, i.e.,

$$\frac{\varkappa\hbar}{e^2}\left(\frac{T}{m^*}\right)^{1/2}\frac{eV}{\varepsilon_F} \gg 1.$$

It is interesting to note that the current density of Eq. (28), calculated per unit area of the transverse cross section of a film, agrees (apart from a numerical factor) with the current density in a conventional three-dimensional Schottky diode. This result is far from trivial. A different density of states in a 2DEG alters the power exponent of the temperature which occurs in the preexponential factor ($T^{3/2}$ instead of T^2), but this is compensated by the factor $T^{1/2}$ related to the square-root dependence of W on the energy. In the case of a 2DEG with a smooth boundary, where $W \approx 1$, such universality of the current-voltage characteristic may not apply.

Since the contact potential (eV_0) occurring in Eq. (28) depends on the size quantization energy E_1, we can expect the resistance of the monotypic contacts (which may be determined from the frequency characteristics of a contact) with quantum wells of different width a and their current-voltage characteristics differ considerably.

5. CONTACT IN A QUANTIZING MAGNETIC FIELD

Let us now consider how a quantizing magnetic field **H**, parallel to the z axis, affects the distribution of the potential in the contact described in Sec. 2, i.e., we shall solve the problem of the screening of the contact potential in a zero-dimensional electron gas. We assume that in the interior of a 2DEG the Fermi level coincides at $T = 0$ with the Nth Landau level characterized by an occupancy ν $(0 < \nu < 1)$. For the time being we assume that the depletion-inducing contact potential eV does not exceed the separation between the Landau levels $\hbar\omega_c \equiv e\hbar H/m^*c$. Then, a film can be divided arbitrarily into two parts. In the contact part $0 < x < l_H$ (l_H will be defined later), the potential $\varphi(x,0)$ varies smoothly from V to zero. The Nth Landau level is then emptied and the film carries an uncompensated positive charge with a constant surface density $\sigma_0 = \nu e^2 H/\pi\hbar c$ (without allowance for the spin splitting). Beginning from the point $x = l_H$ the potential $\varphi(x,0)$ vanishes, but the charge in the film is still generally different from zero, because under the conditions of a purely discrete energy spectrum the relationship between the electron density and the Fermi level is multivalued.

If we bear this point in mind, we find that a calculation of the potential in a film requires solution of the Laplace equation in the first quadrant (Sec. 2) subject to the following boundary conditions at $z = 0$:

$$\partial\varphi(x, 0)/\partial z = -2\pi\sigma_0/\varkappa, \quad x < l_H,$$
$$\varphi(x, 0) = 0, \quad x \geqslant l_H. \tag{29}$$

This was precisely the problem solved in Sec. 2 in the limiting case when $eV \gg \varepsilon_F$. Therefore, the distribution of the potential in the film in the case when $x < l_H$ is described by Eq. (11) where en_{s0} is replaced with σ_0, and by analogy with Eq. (12) we have

$$l_H = \varkappa V/2\pi\sigma_0 = \hbar\varkappa cV/2e^2\nu H. \tag{30}$$

Since the quantity ν is periodic in H^{-1}, the thickness of the depletion layer l_H is an oscillatory function of the magnetic field intensity.

When we have the complete solution $\varphi(x,z)$, we can readily find also the law governing the change in the surface charge on a film $\sigma(x) = -(\varkappa/2\pi)\partial\varphi(x,0)/\partial z$ in the region where $x > l$. We can easily show that

$$\sigma(x) = \frac{4\sigma_0}{\pi}\operatorname{arctg}\left[\frac{l_H}{x + (x^2 - l_H^2)^{1/2}}\right]. \tag{31}$$

It follows from the comments in Sec. 2, that the asymptote form $\sigma(x)$ for the case when $x \gg l_H$ corresponds to the dependence (5) and that the calculations of the capacitance leading to Eq. (13) are retained only to the extent represented by the replacement of l with l_H.

The results obtained can be generalized to the case of a stronger band bending $eV > \hbar\omega_c$. The qualitative nature of the charge and potential distributions in a film are shown for this case in Fig. 3. In the limit $eV \gg \hbar\omega_c$, the distribution approaches a dependence calculated in Sec. 2 in the absence of a magnetic field.

6. CONTACT WITH A QUANTUM FILAMENT

Another interesting example of a contact with a low-dimensional system is that between a three-dimensional heavily doped semiconductor or metal ($z < 0$) and a quantized semiconductor filament, which occupies the region $z > 0$, $\rho \leqslant a$ in a cylindrical coordinate system. We assume that far from the contact the filament potential is $\varphi(\rho = a, z) = 0$. Near the contact we have $\varphi(a,z) \neq 0$ and the charge per unit length of the filament is $e\nu(z)$. In contrast to the two-dimensional case discussed in Sec. 2 and characterized by a constant density of states, we now have a relationship between $\nu(z)$ and $\varphi(a,z)$ which is no longer linear. However, we shall consider only the case of weak band bending ($eV \ll \varepsilon_F$) when the dependence $\nu[\varphi(a,z)]$ can be linearized and written in the form

$$\nu(\varphi(a, z)) \equiv \nu(z) = -\gamma\nu_0 e\varphi(a, z)/\varepsilon_F, \tag{32}$$

where ν_0 is the equilibrium linear electron density and γ is a numerical coefficient equal to 1/2 in the purely one-dimensional case when only one quantum level is filled.

In this situation the distribution of the potential around a filament is deduced from the solution of the Laplace equation subject to the boundary conditions

$$\varphi(\rho, 0) = -V, \tag{33}$$

$$\partial\varphi(a, z)/\partial\rho = r_э^{-1}\varphi(a, z), \tag{34}$$

where $\varphi_{scr} = \varkappa a\varepsilon_F/2e^2\gamma\nu_0$ is the one-dimensional screening length. The final expression for the potential is

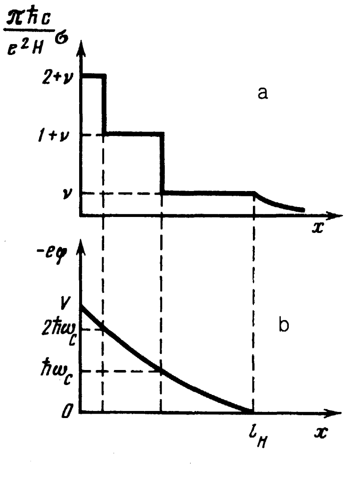

FIG. 3. Distribution of the surface charge along the coordinate in a contact subjected to a strong magnetic field (a) and qualitative behavior of the potential energy of an electron in the 2DEG plane (b).

$$\varphi(\rho,z)=-V\left[1-\frac{2}{\pi}\int_0^\infty\frac{\sin(\lambda z)}{\lambda}\frac{K_0(\lambda\rho)}{K_0(\lambda a)+\lambda r_э K_1(\lambda a)}d\lambda\right], \tag{35}$$

where $K_n(x)$ are Bessel functions with an imaginary argument.

In the one-dimensional case under discussion the contact field is screened even less than in a 2DEG and the potential along the filament falls very slowly:

$$\varphi(a,z)\approx-V\frac{r_э}{a}\left(\ln\frac{z}{a}\right)^{-1},\quad \frac{a}{r_э}\ln\frac{z}{a}\gg 1. \tag{36}$$

We can easily show that the surface charge density on the $z=0$ plane at large distances from the filament decreases as $[\rho\ln(\rho/a)]^{-1}$. Under these conditions Eq. (13) for the capacitance of an ideal contact diverges and in reality is governed by the geometric factors to an even greater extent than a contact with a 2DEG. These factors may be the length of the quantum filament and the dimensions of the metal electrode. Even when only one of these dimensions has a finite value L, we find that the contact capacitance becomes

$$C=\varkappa L/2\ln(L/a). \tag{37}$$

In contrast to Eq. (13), the quantity C is not the specific capacitance of the contact but the total.

We have thus demonstrated that the contacts with low-dimensional electron systems have a number of basically new features due to the specific nature of the energy spectrum and the processes of screening in systems of this kind (this applies also in the presence of a quantizing magnetic field). Naturally some effects and some types of low-dimensional structures are not discussed above. For example, these structures include a contact between two- and one-dimensional systems, a contact between two different two-dimensional systems (at the boundaries of which a quasione-dimensional channel may appear), etc. The effects encountered in such structures and the possibility of their experimental realization should be one of the objects of the present study.

APPENDIX

The conditions for matching the wave functions (21) and (22) at $x=0$ are

$$\int_0^{(2m^*E)^{1/2}/\hbar}B(k)\cos(kz)\,dk+\int_{(2m^*E)^{1/2}/\hbar}^\infty C(k)\cos(kz)\,dk=\begin{cases}(1-R)\cos\left(\frac{\pi z}{a}\right)+\sum_{n=1}^\infty A_n\cos\left[\frac{\pi(2n+1)z}{a}\right], & |z|\leqslant\frac{a}{2},\\ 0, & |z|>\frac{a}{2}.\end{cases} \tag{A1}$$

$$i(E-E_1)^{1/2}(1+R)\cos\left(\frac{\pi z}{a}\right)+\sum_{n=1}^\infty(E_{2n+1}-E)^{1/2}A_n\cos\left[\frac{\pi(2n+1)z}{a}\right]$$
$$=i\int_0^{(2m^*E)^{1/2}/\hbar}\left(E-\frac{\hbar^2k^2}{2m^*}\right)^{1/2}B(k)\cos(kz)\,dk-\int_{(2m^*E)^{1/2}/\hbar}^\infty\left(\frac{\hbar^2k^2}{2m^*}-E\right)^{1/2}C(k)\cos(kz)\,dk. \tag{A2}$$

Multiplying Eq. (A1) by $\cos(k'x)$ and integrating z from 0 to ∞, we have

$$2a\cos\left(\frac{ka}{2}\right)\left[\frac{1-R}{\pi^2-(ka)^2}+\sum_{n=1}^\infty A_n\frac{(-1)^n(2n+1)}{\pi^2(2n+1)^2-(ka)^2}\right]=\begin{cases}B(k), & k<(2m^*E)^{1/2}/\hbar,\\ C(k), & k>(2m^*E)^{1/2}/\hbar.\end{cases} \tag{A3}$$

The values of $B(k)$ and $C(k)$ obtained in this way are substituted in Eq. (A2). Multiplying the result successively by $\cos(\pi z/a)$, $\cos(3\pi z/a)$, etc. and integrating from z to 0 and then to ∞, we obtain a system composed of Eqs. (23) and (24).

[1)] A specific feature of a 2DEG is the fact that a similar linear relationship between n_s and φ does not require the strong inequality $|e\varphi|\ll\varepsilon_F$ and it is described rigorously right up to $|e\varphi|=\varepsilon_F$.

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