Nonlinear generation of sound in metals in the current state

N. M. Makarov, F. Pérez Rodríguez, and V. A. Yampol'skiĭ

Institute of Radio Physics and Electronics of the Academy of Sciences of the Ukrainian SSR and A. M. Gor'kiĭ State University, Khar'kov

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The nonlinear electromagnetic excitation of longitudinal sound in metals in a parallel magnetic field h_0 is studied theoretically. Sound is generated by the induction and deformation mechanisms of the electron-phonon interaction. The cases of both weak and strong nonlinearity are analyzed. It is established that under conditions of strong nonlinearity, when the sample is in the current state, the dependence of the sound field on the external field h_0 exhibits hysteresis. The dynamics of the change in the hysteresis loops of the bias field is studied as the amplitude $\mathscr{H}$ of the wave incident on the metal is increased. It is shown that under the conditions of fully developed nonlinearity the sound field consists of a series of "sharp" spikes, moving with the velocity of longitudinal sound into the interior of the metal. The spacing of the spikes is determined by the quantity $h_0/2\mathscr{H}$.

1. INTRODUCTION

In pure metals at low temperatures the most efficient mechanism of nonlinearity, determining the electromagnetic properties, is the so-called magnetodynamic mechanism. It is connected with the effect of the magnetic field of a ratio wave on the dynamics of th electrons and thereby on the kinetic coefficients of the metals (for example, the conductivity). For the type of anomalous skin effect most typical for metals, when the thickness of the skin layer δ is much smaller than the electron mean free path l and the radius of curvature R of the electron trajectories in the magnetic field of the wave, i.e.,

$$\delta \ll l \ll R = cp_F/2e\mathscr{H}, \qquad (1.1)$$

the magnetodynamic nonlinearity is characterized by the parameter

$$b = (2\mathscr{H}/\tilde{h})^{1/2}, \quad \tilde{h} = 8cp_F\delta/el^2. \qquad (1.2)$$

Here $\mathscr{H}$ is the amplitude of the radio wave ($2\mathscr{H}$ is the characteristic value of the magnetic field of the radio wave in the skin layer), p_F is the Fermi momentum, e is the absolute value of the electron charge, and c is the velocity of light. The quantity b is the ratio of the mean free path l and the range $(8R\delta)^{1/2}$ of electrons in the skin layer. For typical metals the field $\tilde{h}$ in which the curvature of the electron trajectories becomes significant and the magnetodynamic nonlinearity starts to play a role is small (0.5–5 Oe).

The magnetodynamic nonlinearity plays an important role in the process of electromagnetic generation of sound. Thus, for example, in the absence of a constant external magnetic field longitudinal sound is generated in an isotropic metal exclusively owing to nonlinearity,[1–3] and the resulting sound contains only even harmonics of the incident wave. The nonlinear excitation of sound when the anomalous skin effect (1.1) is present was first studied theoretically in the regime of weak nonlinearity ($b \ll 1$) in Ref. 1. The dependence of the amplitudes of the sound harmonics on the amplitude $\mathscr{H}$ and frequency ω of the radio wave and the electron mean free path l was calculated in Refs. 2 and 3 for a wide range of values of the nonlinearity parameter b. It was shown that the efficiency of the electromagnetic excitation of sound increases as the nonlinearity increases, and the character of this excitation is determined by the characteristics of the nonlinear anomalous skin effect.[4] The generation of the second harmonic of longitudinal sound in tungsten was first observed by Korolyuk and Khizhnyi,[5] whose experimental results are in qualitative agreement with the theoretical results of Refs. 1–3.

Nonlinear phenomena in metals become richer and more interesting when a constant external magnetic field h_0 is present. In particular, when a metal is irradiated with a radio wave with sufficiently high amplitude in the presence of an external field h_0 a rectified current and a constant magnetic field h induced by this current appear in the sample. It is interesting that hysteresis is observed in the behavior of h as a function of the external field h_0, so that the rectified current and the magnetic field h do not vanish even when h_0 is switched off. This phenomenon is termed current states in metals, and it has been well studied experimentally and theoretically (see, for example, Refs. 6 and 7). Current states are excited if the external field vector $\mathbf{h}_0$ and the magnetic field vector of the radio wave are collinear, and $|h_0| \leqslant 2\mathscr{H}$. Current states are responsible for the appearance of a member of nonlinear electrodynamic effects, e.g., nonlinear interaction of radio waves with different frequencies[8,9] and the appearance of low-frequency autowaves[10] and auto-oscillations[6,11] in metals, causing the electromagnetic response of the conduction electrons to become stochastic. Current states should also give rise to new features of the nonlinear electromagnetic generation of sound.

This paper is devoted to the theoretical analysis of nonlinear electromagnetic excitation of longitudinal sound in a metal which in a constant magnetic field h_0 is oriented parallel to its surface. The analysis is restricted to the quasistatic case when the frequency ω of the wave is much lower than electron the electron relaxation rate ν:

$$\omega \ll \nu. \qquad (1.3)$$

The inequality (1.3) permits neglecting the change in the electromagnetic field during the time of free flight of the electrons. In this paper the distribution of the sound field is found and its dependence on the amplitude $\mathscr{H}$ and frequency ω of the external signal, the quantity h_0, the electron mean free path l, and other parameters of the problem is calculated

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for both weak ($b \ll 1$) and well developed ($b \gtrsim 1$) nonlinearity. It is shown that in the current state the resulting longitudinal sound contains a collection of harmonics (ω, 2ω, 3ω, ...) of the incident wave and the sound field exhibits hysteresis as a function of the external field h_0. The dynamics of the change in the hysteresis loops as the amplitude $\mathscr{H}$ is increased is studied. It is found that under conditions of well developed nonlinearity ($b \gtrsim 1$) the field of longitudinal sound displacements consists of a series of "sharp" spikes whose relative width and height are, respectively, $\sim b^{-2}$ and $\sim b^2$. These spikes move with the velocity of longitudinal sound into the volume of the metal. In the current state neighboring spikes in the current state have opposite signs and their spacing is determined by the quantity $h_0/2\mathscr{H}$. The next successive peak is generated at the surface of the metal excited by the radio wave when the sum of the magnetic field of the radio wave on this surface and the external field h_0 vanishes. Two nonequivalent spikes appear during each period of the radio wave $2\pi/\omega$.

2. FORMULATION OF THE PROBLEM: GENERAL RELATIONS

1. Let a plane monochromatic electromagnetic wave with frequency ω and amplitude $\mathscr{H}$ be incident on the surface of a metallic half-space. We orient the x-axis along the normal to the surface into the interior of the sample ($x = 0$ at the metal–vacuum interface) and the y- and z-axes parallel to the electric and magnetic field vectors of the wave:

$$\mathbf{E}(x, t)=\{0, E(x, t), 0\};\ \mathbf{H}(x, t)=\{0, 0, H(x, t)\}. \tag{2.1}$$

We orient the constant external magnetic field $\mathbf{h}_0$ parallel to the vector $\mathbf{H}(x,t)$: $\mathbf{h}_0 \| \mathbf{H}(x,t) \| z$. We shall study the electromagnetic generation of a longitudinal sound wave in which the displacement vector is

$$\mathbf{u}(x, t)=\{u(x, t), 0, 0\}. \tag{2.2}$$

The solution of the equation of elasticity for longitudinal acoustic oscillations can be written as follows:

$$u(x, \varphi) = \frac{1}{2\rho_0 s^2}\left\{\int_0^\infty dx' [\Phi(x', \varphi-qx+qx') - \Phi(x', \varphi-qx-qx')] - \int_x^\infty dx' [\Phi(x', \varphi-qx+qx')+\Phi(x', \varphi+qx-qx')]\right\}. \tag{2.3}$$

Here $\varphi = \omega t$, ρ_0 is the density of the metal, and s and $q = \omega/s$ are the velocity and the wave number of the longitudinal sound, respectively.

The quantity $\Phi(x,\varphi)$ appearing in Eq. (2.3) is the sum of two terms:

$$\Phi(x, \varphi)=\Phi_{def}(x, \varphi)+\Phi_{ind}(x, \varphi), \tag{2.4}$$

the first of which, $\Phi_{\rm def}(x,\varphi)$, is due to the deformation mechanism while the second one, $\Phi_{\rm ind}(x,\varphi)$, is due to the induction mechanism of electron-phonon interaction.

The potential of the deformation force has the form

$$\Phi_{def}(x, \varphi)=-\int \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3} \Lambda_{xx}(\mathbf{p}) \frac{\partial f_F}{\partial \varepsilon} \chi(x, \varphi), \tag{2.5}$$

while the potential of the induction force is described by the formula

$$\Phi_{ind}(x, \varphi)=-\frac{1}{c}\int_0^x dx'' j(x'', \varphi) H_t(x'', \varphi). \tag{2.6}$$

Here $(\partial f_F/\partial \varepsilon)\chi(x,\varphi)$ is the nonequilibrium correction to the Fermi distribution function f_F, $\mathbf{p}$ is the electron momentum, $j(x,\varphi)$ is the electric current density, and $H_t(x,\varphi)$ is the total magnetic field in the metal

$$H_t(x, \varphi)=H(x, \varphi)+h_0. \tag{2.7}$$

We employ the simplest model of the quadratic and isotropic electron dispersion laws, in which the component $\Lambda_{xx}(p)$ of the deformation potential tensor is

$$\Lambda_{xx}(\mathbf{p})=-\tilde{m}(v_x^2-v_F^2/3), \tag{2.8}$$

where $\mathbf{v} = \mathbf{p}/m$ is the electron velocity, m is the effective electron mass, $\tilde{m}$ is the deformation electron mass, and v_F is the Fermi velocity of the electrons.

In deriving expression (2.3) we employed the boundary conditions that at the free boundary $x = 0$ the tangential components of the fields are continuous and the stresses vanish[12,13]:

$$\left\{\rho_0 s^2 \frac{\partial u(x, \varphi)}{\partial x} - \Phi(x, \varphi)\right\}_{x=0} = 0. \tag{2.9}$$

2. Under the conditions of the quasistatic ($\omega \ll \nu$) anomalous skin effect ($\delta \ll l$) in the region of magnetic fields where $R \gg l$ the attenuation length l_s of the sound is the largest parameter:

$$l_s \sim v_F/sq \gg 1/q,\quad l_s \sim l\nu/\omega \gg l \gg \delta. \tag{2.10}$$

It follows from this that at large distances from the surface of the metal ($x \gg l$) the first integral over x' makes the main contribution to $u(x,\varphi)$ (2.3). In addition, the field of the longitudinal displacements $u(x,\varphi)$ assumes the form of a plane wave travelling along the boundary of the sample:

$$u(x, \varphi)=u(\tilde{\varphi}),\quad \tilde{\varphi}=\varphi-qx=\omega t-qx; \tag{2.11}$$

$$u(\tilde{\varphi}) = \frac{1}{2\rho_0 s^2}\int_0^\infty dx' [\Phi(x', \tilde{\varphi}+qx') - \Phi(x', \tilde{\varphi}-qx')]. \tag{2.12}$$

According to (2.4) the sound-induced displacement $u(\tilde{\varphi})$ consists of two terms:

$$u(\tilde{\varphi})=u_{def}(\tilde{\varphi})+u_{ind}(\tilde{\varphi}). \tag{2.13}$$

The "deformation" term $u_{\rm def}(\tilde{\varphi})$ is determined by the expression (2.12), in which $\Phi(x,\varphi)$ must be replaced by $\Phi_{\rm def}(x,\varphi)$ (2.5). The "induction" term $u_{\rm ind}(\tilde{\varphi})$ is obtained by substituting into (2.12) the potential $\Phi_{\rm ind}(x,\varphi)$ from (2.6) for $\Phi(x,\varphi)$.

We note that in the linear regime Eqs. (2.3)–(2.6) and (2.11)–(2.13) transform into the corresponding expressions of Ref. 14.

3. We now analyze the induction mechanism of nonlinear excitation of longitudinal sound. It can be shown that under the conditions of the anomalous skin effect (1.1) this mechanism can complete with the deformation mechanism only in the region of small values of the parameter $q\delta$:

$$q\delta \ll 1. \tag{2.14}$$

For this reason we calculate $u_{\rm ind}(\tilde{\varphi})$ only for wavelengths q^{-1} of the sound waves that are large compared with the depth of the skin layer δ. First, we write with the help of Maxwell's equation

$$-c\partial H_t(x,\varphi)/\partial x=4\pi j(x,\varphi), \tag{2.15}$$

the potential of the induction force (2.16) in the following equivalent form:

$$\Phi_{ind}(x,\varphi)=[H_t^2(x,\varphi)-H_t^2(0,\varphi)]/8\pi. \tag{2.16}$$

The scaling length of the total magnetic field $H_t(x,\varphi)$ (2.7) along the x-axis is δ. Over this distance H_t changes from the value

$$H_t(0,\varphi)=2\mathscr{H}\cos\varphi+h_0 \tag{2.17}$$

at the surface of the metal ($x=0$) to the time(phase)-independent quantity

$$H_t(\infty,\varphi)=h+h_0, \tag{2.18}$$

in the interior of the metal (h is the induced magnetic field, which is related with the current state[7]; see introduction). It follows that the contribution of the first term in Eq. (2.16) to $u_{\rm ind}(\tilde{\varphi})$ [see Eq. (2.12)] is of order $(q\delta)^2 \ll 1$ compared with the contribution of the second term. Because of the anomalous nature of the skin effect (1.1) this contribution is also small compared with $u_{\rm def}(\tilde{\varphi})$. So, to lowest-order in $q\delta \ll 1$ the sound field $u_{\rm ind}(\tilde{\varphi})$ is determined by the second term in the formula (2.16), and according to (2.12) it is described by the expression

$$\begin{gathered} u_{ind}(\omega t-qx)=2u_1\sin(\omega t-qx)+2u_2\sin 2(\omega t-qx), \\ u_1=\mathscr{H}h_0/4\pi\rho_0\omega s, \quad u_2=\mathscr{H}^2/16\pi\rho_0\omega s. \end{gathered} \tag{2.19}$$

The formula (2.19) contains only the first and second harmonics of the longitudinal sound. The first harmonic is excited owing to the presence of the constant magnetic field h_0 and the second harmonic is a consequence of the nonlinear action of the magnetic field of the radio wave.

We note that the result obtained for the induction displacement $u_{\rm ind}(\tilde{\varphi})$ is valid for any degree of nonlinearity and anomalousness of the skin effect. The application of the formula (2.19) is determined solely by the inequality (2.14). Since this inequality gives an upper limit for the frequency ω of the external electromagnetic wave the case $q\delta \ll 1$ is the most important one in the quasistatic situation $\omega \ll \nu$.

4. The rest of this presentation will be devoted to the study of the deformation mechanism of nonlinear excitation of longitudinal sound. To calculate $\Phi_{\rm def}(x,\varphi)$ and $u_{\rm def}(\tilde{\varphi})$ it is necessary to solve the Boltzmann equation for the nonequilibrium correction $\chi(x,\varphi)$ [see Eq. (2.5)]. This equation is linearized with respect to the electric field $E(x,\varphi)$. The nonlinearity is connected with the magnetic field $H_t(x,\varphi)$ and is contained in the Lorentz force, which determines the electron trajectories. The electromagnetic fields $E(x,\varphi)$ and $H(x,\varphi)$ are found from Maxwell's equations. A procedure for solving the kinetic equation and Maxwell's equations, which takes into account the dynamics of the motion of the electrons in the external magnetic field $H_t(x,\varphi)$ (2.7), was developed in Ref. 15. We refer also to our paper[2] where a general expression was derived for the deformation force with $h_0=0$. Following Refs. 15 and 2 it is not difficult to obtain exact expressions for $\Phi_{\rm def}(x,\varphi)$. Because these expressions are complicated we shall not write them out here, especially since under the conditions of the anomalous skin effect (1.1) the exact expressions for $\Phi_{\rm def}(x,\varphi)$ can be simplified by replacing them with their asymptotic forms. Since the asymptotic expressions for $\Phi_{\rm def}(x,\varphi)$ have a significantly different form in the case of low ($b \ll 1$) and high ($b \gtrsim 1$) amplitudes of the external radio wave, in what follows we shall study these two cases separately.

3. WEAK NONLINEARITY

In the case of a weak nonlinearity and a weak external field h_0,

$$2\mathscr{H}\ll\tilde{h}, \quad h_0\ll\tilde{h}; \tag{3.1}$$

the total field $H_t(x,\varphi)$ bends the electron trajectories in the skin layer only slightly. Under such conditions electromagnetic generation of longitudinal sound is manifested in the linear $h_0\mathscr{H}$ and quadratic $\mathscr{H}^2$ approximations. The asymptotic expansion of the Fourier cosine transform

$$\Phi_{def}(k,\varphi)=2\int_0^\infty dx\cos(kx)\Phi_{def}(x,\varphi) \tag{3.2}$$

of the potential of the deformation force (2.5) has the form

$$\begin{aligned} \Phi_{def}(k,\varphi) = & -\frac{1}{2}\frac{\bar{m}}{m}\frac{\sigma_0 l}{c}\Big\{\rho h_0 k\int_0^\infty k'\,dk'\,\frac{\tilde{E}(k',\varphi)}{(k+k')^3} \\ & +\rho\frac{k}{2}\int_0^\infty k'\,dk'\,\frac{\tilde{E}(k',\varphi)}{(k+k')^2}\tilde{H}(k+k',\varphi) \\ & -(1-\rho)\Big[kE(0,\varphi)\int_0^\infty dx\frac{\sin(kx)}{x} \\ & \times\int_0^x dx''A(x'',\varphi)+\int_0^\infty dx\int_0^\infty dx'\frac{\cos(kx')}{(x'-x)^2}\Big((x'-x)A(x',\varphi) \\ & -\int_x^{x'} dx''A(x'',\varphi)\Big)\frac{\partial E(x,\varphi)}{\partial x}\Big]\Big\}. \end{aligned} \tag{3.3}$$

Here σ_0 is the static conductivity of the metal, ρ is the probability that electrons are specularly reflected from the boundary of the sample ($0 \leqslant \rho \leqslant 1$), $\tilde{H}(k,\varphi)$ is the spatial Fourier since transform of the magnetic field of the radio wave $H(x,\varphi)$

$$\tilde{H}(k,\varphi)=2\int_0^\infty dx\sin(kx)H(x,\varphi), \tag{3.4}$$

$\tilde{E}(k,\varphi)$ is the Fourier cosine transform of the electric field of the radiowave $E(x,\varphi)$, and $A(x,\varphi)$ is the vector potential

$$A(x,\varphi)=\int_0^x dx'H(x',\varphi). \tag{3.5}$$

The expression (3.3) was derived to lowest order in the parameters $h_0/\tilde{h} \ll 1$ and $b^2 \ll 1$. The electric field $\tilde{E}(k,\varphi)$ and the magnetic field $\tilde{H}(k,\varphi)$ are the solution of the linear Maxwell's equations with $h_0=0$ and contain only the first harmonic (ω) of the radio wave incident on the metal. For this reason the asymptotic expansion (3.3) contains the zeroth, first (ω), and second (2ω) harmonics. The other harmonics (3ω, 4ω, ...) are of higher order in the parameters $h_0/\tilde{h}$ and b^2. We note that the zeroth harmonic of $\Phi_{\rm def}$ vanishes in the expression (2.12) for the displacement $u_{\rm def}(\varphi)$.

To calculate the sound field $u_{\rm def}(\tilde{\varphi})$ we substitute the expression (3.3) into the (2.12) and apply the results of Ref. 16, where the distribution of the electromagnetic field in the linear regime with $h_0 = 0$ and arbitrary value of the specularity parameter ρ is found. The final expression for $u_{\rm def}(\tilde{\varphi})$ is quite involved. For this reason we present below only its asymptotic form for large and small values of the quantity $q\delta$.

1. In the limit (2.14) in which the wavelength of the sound wave is long, the deformation term in the longitudinal displacement of sound (2.13) is given by the formula

$$u_{def}(\omega t-qx)=2u_1^{(0)}\sin(\omega t-qx)+2u_1^{(+)}\sin(\omega t-qx+\pi z_0/6)$$
$$+2u_1^{(-)}\sin(\omega t-qx-\pi z_0/6)+2u_2\sin 2(\omega t-qx). \quad (3.6)$$

The first three terms in (3.6) refer to the first harmonic of the sound wave. Their amplitudes are equal to, respectively,

$$u_1^{(0)}=-\frac{1}{12\pi}\frac{\bar{m}}{m}\rho\frac{h_0\mathscr{H}}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2\frac{(q\delta_a)^2}{\sin^2(\pi z_0/2)}, \quad (3.7)$$

$$u_1^{(\pm)}=\frac{1}{12\pi}\frac{\bar{m}}{m}\rho\frac{h_0\mathscr{H}}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2\frac{(q\delta_a)^{2\mp z_0}}{\sin^2(\pi z_0/2)}g(\pm z_0), \quad (3.8)$$

where $\pi z_0 = \arccos\rho$, while $g(z)$ is given by

$$g(z)=3^{1/2}2^{-3+(1+z)/3}\frac{(1-z)(2-z)}{\sin[\pi(1+z)/3]}\{\cos(\pi z)\}^{-(1-2z)/6}$$
$$\times\left\{\frac{\cos[\pi(1-z)/3]}{\cos(\pi z/3)}\right\}^{1/4}$$
$$\times\exp\left\{\frac{\pi}{3}\sin(2\pi z)\int_0^z dx\frac{x-z}{\cos(2\pi x)-\cos(2\pi z)}\right\}. \quad (3.9)$$

The function $g(z)$ is a smooth function of its argument. When the electrons reflect specularly from the boundary of the sample ($\rho = 1$, i.e., $z_0 = 0$), $g(0) = 1/2$, while in the case of diffuse reflection ($\rho = 0$, i.e., $z_0 = 1/2$) it assumes the values

$$g(1/2)=3\cdot3^{1/4}/16\cdot2^{1/3},\quad g(-1/2)=15\cdot3^{1/4}\cdot2^{1/3}/16. \quad (3.10)$$

Under the condition (2.14) the amplitude of the second harmonic is given by the expression:

$$u_2=\frac{(1+\rho)\varkappa_0}{24\pi^4}\frac{\bar{m}}{m}\frac{\mathscr{H}^2}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2(2q\delta_a)^2\ln(1/2q\delta_a), \quad (3.11)$$

where the quantity $\varkappa_0$ is a smooth function of ρ: $\varkappa_0 = 29.46$ at $\rho = 0$ and $\varkappa_0 = 2\pi/3$ at $\rho = 1$. When the nonlinearity and the external field h_0 are weak, i.e., the conditions (3.1) hold, the depth of the skin layer δ_a is described by the linear theory:

$$\delta_a=(c^2l/3\pi^2\sigma_0\omega)^{1/3}. \quad (3.12)$$

According to the results (3.6)–(3.10) obtained above the dependence of the amplitude of the first harmonic on the parameter $q\delta_a$ is largely determined by the character of the surface scattering of the electrons. If the reflection is close to specular, i.e.,

$$\pi z_0\approx[2(1-\rho)]^{1/2}\ll\ln^{-1}(1/q\delta_a)\ll1, \quad (3.13)$$

this dependence contains a logarithmic singularity, and the first three terms in (3.6) combine into one term of the form

$$\frac{1}{3\pi^3}\frac{\bar{m}}{m}\frac{h_0\mathscr{H}}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2(q\delta_a)^2\ln^2(q\delta_a)\sin(\omega t-qx). \quad (3.14)$$

In the limit opposite to (3.13) the first harmonic is determined by a power of the parameter $q\delta_a \ll 1$. The exponent varies from 3/2 to 2, depending on the value of specularity parameter ρ.

Under the conditions (3.1) and (2.14) the deformation term (3.6) is larger than the induction term (2.19), if the wavelength of the sound wave is shorter than the electron mean free path:

$$ql\gg1. \quad (3.15)$$

2. For sound waves with short wavelengths ($q\delta_a \ll 1$) the longitudinal sound-induced displacement $u(\varphi)$ is associated primarily with the deformation mechanism of the electron-phonon interaction:

$$u(\omega t-qx)=2u_1\cos(\omega t-qx-\pi/6)+2u_2\cos[2(\omega t-qx)-\pi/6], \quad (3.16)$$

$$u_1=-\frac{\varkappa_1\rho}{3^{5/2}\pi^2}\frac{\bar{m}}{m}\frac{h_0\mathscr{H}}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2, \quad (3.17)$$

$$u_2=\frac{1}{24\pi^4}[(1-\rho)\varkappa_2-(1+\rho)\varkappa_3]\frac{\bar{m}}{m}\frac{\mathscr{H}^2}{\rho_0\omega s}\left(\frac{l}{\delta_a}\right)^2. \quad (3.18)$$

Here the quantities $\varkappa_1$, $\varkappa_2$ and $\varkappa_3$ are smooth functions of the parameter ρ. For $\rho = 0$ we have $\varkappa_2 = \pi^2 3^{1/2}/4$, and $\varkappa_3 = 3.9$; for $\rho = 1$, then we have $\varkappa_1 = 1$, and $\varkappa_3 = 3.5$.

3. So, under the conditions (3.1), when the amplitude of the radio wave $\mathscr{H}$ is small and the magnetic field h_0 is weak, the resulting longitudinal sound field $u(\tilde{\varphi})$ contains to lowest order for $x \gg l$ only two independent harmonics—the first harmonic (ω) and the second harmonic (2ω). The first harmonic (ω) is the linear (with $b = 0$) response to the constant magnetic field h_0. The second harmonic (2ω) is engendered by the nonuniformity of the magnetic field of the wave $H(x,t)$ and does not depend on the presence of the external field h_0.

4. WELL DEVELOPED NONLINEARITY; CURRENT STATES

1. In an external magnetic field with $|h_0| \leqslant 2\mathscr{H}$, when current states are excited in the metal,[6,7] the regime of well developed nonlinearity ($b \gtrsim 1$) is characterized by the trapping of some of the electrons in the potential well of the total nonuniform magnetic field $H_t(x,\varphi)$ (2.7). The trapping occurs because of the $H_t(x,\varphi)$ charges sign as a function of x and it therefore occurs in the time intervals when the sign of the total field on the surface of the metal $H_t(0,\varphi)$ (2.17) differs from that of the field $H_t(\infty,\varphi)$ (2.18) in the interior of the metal. The trapped electrons drift along the boundary of the sample along trajectories whirling near the surface, where $H_t(x,\varphi) = 0$ (Fig. 1a). At other times the electrons move in a field $H_t(x,\varphi)$ with constant sign (there are no trapped particles), and their trajectories are virtually indistinguishable from closed Larmor orbits in the magnetic field $H_t(\infty,\varphi) = h + h_0$ (Fig, 1b). Changes in the character of the electron motion occurring during each period of the

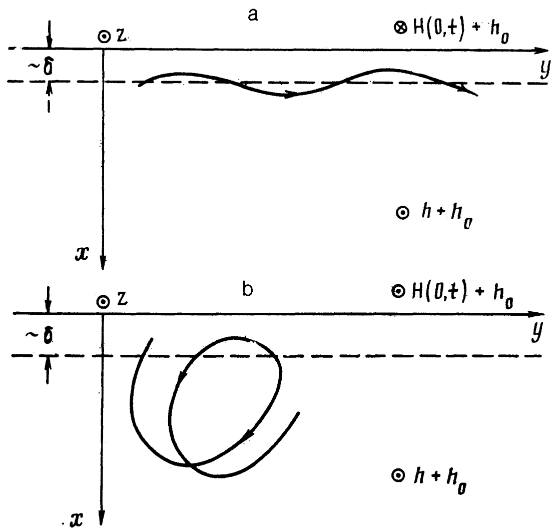

FIG. 1. The trajectories of the effective electrons in nonuniform magnetic fields (a) alternating in direction and (b) pointing in one direction only.

wave $2\pi/\omega$ cause the time-dependence of the conductivity of the metal to be jump-like, which leads to rectification of the current that induces the constant magnetic field h (the current state). It is obvious that the periodic appearance and vanishing of the trapped electrons also play an important role in the process of electromagnetic generation of sound.[2,3]

Under the conditions of well developed nonlinearity ($b \gtrsim 1$) the potential of the deformation force $\Phi_{\rm def}(x,\varphi)$ (2.5) can be represented in the following form:

$$\Phi_{def}(x,\varphi) = -\frac{4}{3\pi}\frac{\tilde{m}v_F}{e}\,\mathrm{th}\,(H_t(0,\varphi)/\tilde{\hbar})\,j(x,\varphi). \tag{4.1}$$

The formula (4.1) was derived based on asymptotically exact calculations with weak ($b \ll 1$) and strong ($b \gg 1$) nonlinearity using the method developed in Ref. 15 for constructing the asymptotic expansion of the current density $j(x,\varphi)$.

With the help of (4.1) we find the field of the longitudinal sound displacements $u_{\rm def}(\tilde{\varphi})$ in the case (2.14) when the wavelength $2\pi/q$ of the sound wave is much larger than the depth δ of the skin layer. The nonlinear current $j(x,\varphi)$ and $\Phi_{\rm def}(x,\varphi)$ decay together toward the interior of the metal over a distance $x \sim \delta$. For this reason the region $x' \sim \delta$ makes the main contribution to the integral over x' in the deformation term (2.12), and to lowest order in $qx' \sim q\delta x \ll 1$ the displacement $u_{\rm def}(\tilde{\varphi})$ can be written in the form

$$u_{def}(\tilde{\varphi}) = \frac{\omega}{\rho_0 s^3}\frac{\partial}{\partial\tilde{\varphi}}\int_0^\infty x'\,dx'\,\Phi_{def}(x',\tilde{\varphi}). \tag{4.2}$$

Substituting (4.1) into (4.2) and using Maxwell's equation (2.15) we obtain

$$u_{def}(\tilde{\varphi}) = -\frac{\tilde{m}v_F\omega c}{3\pi^2\rho_0 s^3 e}\frac{\partial}{\partial\tilde{\varphi}}\,\mathrm{th}\left(\frac{H_t(0,\tilde{\varphi})}{\tilde{\hbar}}\right)\int_0^\infty dx'[H(x',\tilde{\varphi})-h]. \tag{4.3}$$

2. We now employ an expression derived in Refs. 7 and 15 for the magnetic component $H(x,t)$ of the radio wave in the metal in the current state. We obtain finally for the longitudinal sound field

$$\begin{aligned} u_{def}(\tilde{\varphi}) = \bar{U}\sum_{r=1}^{\infty}\Big\{(\mu/r)^{1/2} & \times\Big[\frac{b^2\sin\tilde{\varphi}}{\mathrm{ch}^2[b^2(a+\cos\tilde{\varphi})]}\oint\frac{d\varphi'\cos\varphi'}{\mu S(\varphi')} \\ & \times\cos\left(r(\xi(\varphi')-\xi(\tilde{\varphi}))+\frac{\pi}{6}\right) \\ & -\frac{r\,\mathrm{th}[b^2(a+\cos\tilde{\varphi})]}{\mu S(\tilde{\varphi})}\oint\frac{d\varphi'\cos\varphi'}{\mu S(\varphi')} \\ & \times\sin\left(r(\xi(\varphi')-\xi(\tilde{\varphi}))+\frac{\pi}{6}\right)\Big]\Big\}. \end{aligned} \tag{4.4}$$

Here the following notation was introduced. The characteristic "amplitude" $\bar{U}$, determining the dependence of the displacement $u_{\rm def}(\tilde{\varphi})$ on the parameters of the problem, is

$$\bar{U} = \frac{16\cdot 3^{1/2}}{27\pi^3}\frac{\tilde{m}}{m}\frac{\mathscr{H}^2}{\rho_0\omega s}\frac{R}{\delta_a}(q\delta_a)^2. \tag{4.5}$$

The dimensionless conductivity of the metal

$$S(\varphi) = 1+\alpha\Theta\left(-\frac{2\mathscr{H}\cos\varphi+h_0}{h+h_0}\right). \tag{4.6}$$

The most distinctive property of $S(\varphi)$ is its jump-like behavior as a function of time, the nature of which was described above. $\Theta(x)$ is the Heaviside function and α is the relative change in the conductivity at the moments when groups of trapped electrons appear and disappear. In the simplest model[17] with diffuse reflection of electrons from the surface of the sample ($\rho = 0$)

$$\alpha = [\exp(1/b|\bar{\varkappa}|-1]^{-1}. \tag{4.7}$$

The ratios of the magnetic fields h_0, h, and $h + h_0$ to the doubled amplitude of the radio wave $2\mathscr{H}$ are correspondingly denoted by

$$a = h_0/2\mathscr{H},\quad \varkappa = h/2\mathscr{H},\quad \bar{\varkappa} = (h+h_0)/2\mathscr{H} = \varkappa + a. \tag{4.8}$$

The dimensionless resistance μ averaged over a period of the wave, according to (4.6), is equal to

$$\mu = \frac{1}{2\pi}\oint\frac{d\varphi}{S(\varphi)} = \frac{\pi+\alpha\beta}{\pi(1+\alpha)},\quad \beta = \arccos(-a\,\mathrm{sign}\,\bar{\varkappa}), \tag{4.9}$$

where sign x is the sign function. The function $\xi(\varphi)$ is piecewise-smooth and is defined by the following formulas:

$$\begin{aligned} \xi(\varphi) &= \frac{1}{\mu}\int_0^{\varphi}\frac{d\varphi'}{S(\varphi')} \\ &= \frac{2+\alpha}{1+\alpha}\frac{\varphi}{2\mu} + \frac{\alpha\,\mathrm{sign}\,\bar{\varkappa}}{2\mu(1+\alpha)}\begin{cases}\varphi, & -\beta\leqslant\varphi\leqslant\beta,\\ 2\beta-\varphi, & \beta\leqslant\varphi\leqslant 2\pi-\beta,\end{cases} \end{aligned} \tag{4.10}$$

$$\xi(\varphi+2\pi) = \xi(\varphi)+2\pi,\quad \beta = \arccos(-a).$$

The induced magnetic field $h = 2\mathscr{H}\varkappa$ as a function of the external field $h_0 = 2a\mathscr{H}$ undergoes hysteresis. $\varkappa$ (or h) is determined from the equation

$$\varkappa = \frac{(1-a^2)^{1/2}}{\beta+\pi\alpha^{-1}}\,\mathrm{sign}\,\bar{\varkappa}. \tag{4.11}$$

In Eqs. (4.4)–(4.11) the linear limit ($b = 0$) is obtained by making α go to zero. At $\alpha = 0$ we have $S(\varphi) = 1$,

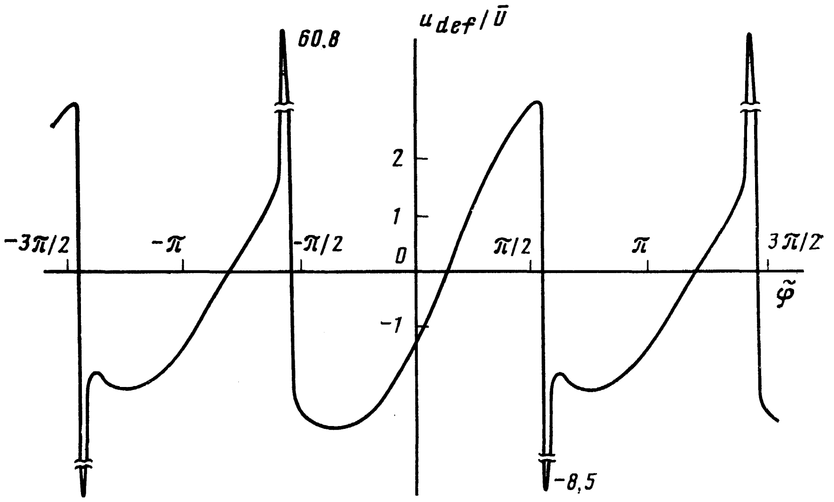

FIG. 2. The results of the calculation of the displacement $u_{\rm def}(\tilde{\varphi})$ ($\tilde{\varphi} = \omega t - qk$) for values of the nonlinearity parameter $b = 5$, the external field $h_0/2\mathscr{H} = 0.2$, and $\bar{\varkappa} > 0$.

$\mu = 1$, $\xi(\varphi) = \varphi$, $\varkappa = 0$, and only the linear term with $r = 1$ remains in the sum over r in (4.4). It vanishes for $h_0 = 0$. In the case of weak nonlinearity and small h_0 expression (4.4) is similar to the corresponding expression (3.6) of the preceding section. The limit of strong nonlinearity ($b \to \infty$) is obtained by making α go to ∞.

In the chosen model of jump-like-varying conductivity the integrals over φ' in the expression (4.4) can be computed, after which the expression can be easily analyzed.

3. Under the conditions of developed nonlinearity the sound field $u_{\rm def}(\tilde{\varphi})$ has a spiked structure. The spikes, according to the expression (4.4), are located on those fronts of the sound wave that move according to the law

$$\omega t - qx = \pm \arccos(-h_0/2\mathscr{H}) + 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots \quad (4.12)$$

The shape of the spikes is determined, generally speaking, by both terms in the expression (4.4). The second term under the condition (4.12) undergoes sharp jumps of relative width $\sim b^{-2}$ and relative magnitude $\sim \alpha^{1/3}$, while the first term contains spikes of relative height $\sim b^2$ and relative width $\sim b^{-2}$. The sound spikes are quite "sharp" even for small values of the nonlinearity parameter $b \gtrsim 3$. In the limit $b \to \infty$ they transform into δ-function peaks.

By virtue of the condition (4.12) the sound spikes are generated on the surface of the metal $x = 0$ (in the skin layer $\delta \ll q^{-1}$) when the total magnetic field on this surface $H_t(0,t)$ vanishes. For this reason, during each period of the radio wave $2\pi/\omega$ there arise two nonequivalent spikes which move with the sound velocity s into the volume of the sample. At any moment the number of peaks in the sound field in a sample with thickness $d \gg l$ is less than the integer part of the ratio qd/π. It can be shown that in the current state (with $h \neq 0$) neighboring spikes have opposite signs. The spacing of the spikes assumes successively values equal to $2\cos^{-1}(\mp h_0/2\mathscr{H})q$.

The graph of the dependence $u_{\rm def}(\tilde{\varphi})$ in Fig. 2, which was calculated for values of the external magnetic field $h_0 = 0.4\mathscr{H}$ and the nonlinearity parameter $b = 5$, illustrates the above-described spike structure of the field of longitudinal sound displacements. The plot presented corresponds to a current state with $\bar{\varkappa} > 0$. We note that with the help of this plot it is not difficult to obtain the dependence $u_{\rm def}(\tilde{\varphi})$ with $\bar{\varkappa} < 0$ and $h_0 = -0.4\mathscr{H}$. For this the following property of the field of displacements (4.4) must be used:

$$u_{def}(\tilde{\varphi}, h_0, \bar{\varkappa}) = u_{def}(\tilde{\varphi} + \pi, -h_0, -\bar{\varkappa}). \quad (4.13)$$

Spikes are present in the generated longitudinal sound owing to the jump-like behavior of the potential of the deformation field $\Phi_{\rm def}(x,\varphi)$ (4.1) as a function of the time φ. The point is that the potential engendering sound oscillations depends strongly on which electrons (trapped or Larmor) determine it at a given time and their direction of motion along the y-axis in the skin-layer. The groups of effective electrons and at the same time the direction of electron motion change as the plane on which the total magnetic field $H_t(x,\varphi)$ vanishes arrives and vanishes at the surface of the metal $x = 0$. It

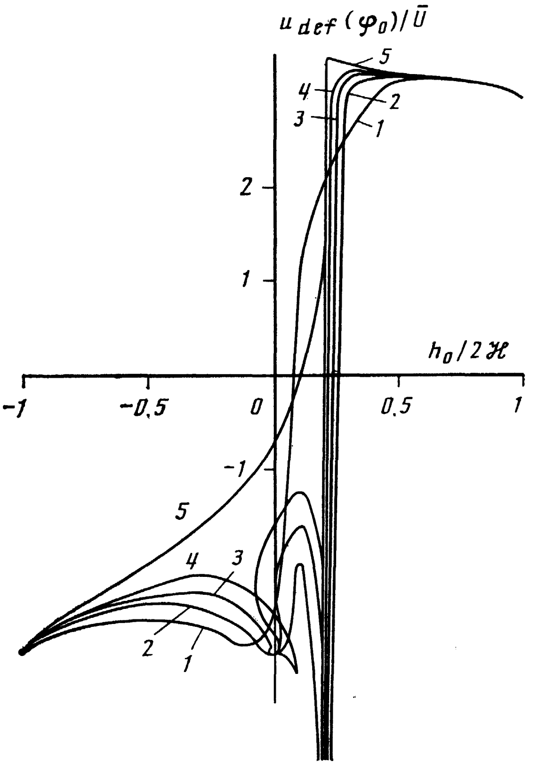

FIG. 3. Plots of the sound field $u_{\rm def}(\varphi_0)$ for $\varphi_0 = \cos^{-1}(-0.2)$ versus h_0 for different values of the parameter b [$b = 3$ (1), 5 (2), 7 (3), 10 (4), and $b \to \infty$ (5)].

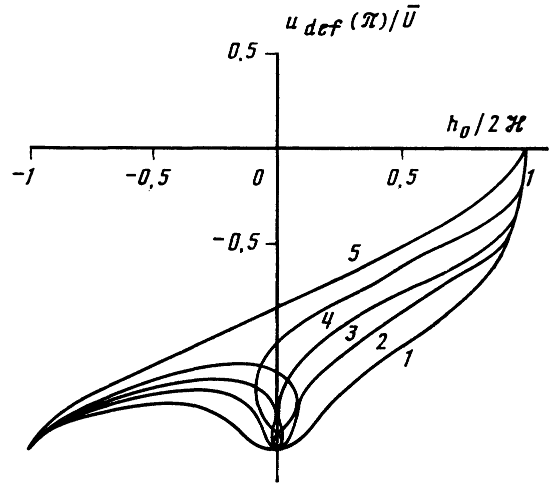

FIG. 4. Plots of $u_{\rm def}(\pi)$ versus h_0 for different values of the parameter b [$b = 3$ (1), 5 (2), 7 (3), 10 (4), and $b \to \infty$ (5)].

is precisely at these times that sound spikes are generated on the boundary of the sample.

It is well known[15,18] that the periodic appearance and disappearance of the plane on which the field $H_t(0,\varphi)$ vanishes gives rise to abrupt jumps in the electric field of the radio wave at the surface of the metal $E(0,\varphi)$ as a function of time. It is not difficult to show that the second term in the formula (4.4) for $u_{\rm def}(\tilde{\varphi})$ is proportional to $E(0,3\tilde{\varphi})$. For this reason the singularities contained in this term repeat the singularities of $E(0,\varphi)$.

4. The hysteretic dependence of the induced magnetic field $h(h_0)$ leads to hysteresis of the longitudinal sound displacement $u_{\rm def}(\tilde{\varphi})$ as a function of the external magnetic field h_0.

Figures 3 and 4 show the results of the calculation of the dependence of the sound field $u_{\rm def}(\tilde{\varphi})$ for $\varphi_0 = \cos^{-1}(-0.2)$ and $u_{\rm def}(\pi)$ on h_0 for different values of the nonlinearity parameter b (different amplitudes of the radio wave $\mathscr{H}$). It the nonlinearity b is less than the critical value $b_{\rm cr} \sim 5$, $u_{\rm def}(\varphi_0)$ and $u_{\rm def}(\pi)$ are single-valued functions of h_0. For $b = b_{\rm cr}$ points with vertical tangents appear on the curves $u_{\rm def}(h_0)$ and for $b > b_{\rm cr}$ the dependence of the sound field on h_0 is no longer single-valued. The instability of the current state on the sections between the points at which the derivative $\partial u_{\rm def}/\partial h_0$ becomes infinite leads to jumps in $u_{\rm def}(h_0)$ at these singular points. Thus in the region $b > b_{\rm cr}$ the function $u_{\rm def}(h_0)$ becomes hysteretic. Figures 5 and 6 show schematically the dynamics of the hysteresis loops of $u_{\rm def}(\varphi_0)$ and $u_{\rm def}(\pi)$ as a function of the nonlinearity parameter b. It is obvious that the quantity $b_{\rm cr}$ and the singular points on the h_0 axis for the sound wave are the same as for the induced magnetic field $h(h_0)$.

The plot of $u_{\rm def}(\varphi_0,h_0)$ contains a spike at the point $h_0 = 0.4\mathscr{H}$ (Figs. 3 and 5); this obviously agrees with the condition (4.12). Unlike the hysteresis loops, the spikes do not have a threshold with respect to the nonlinearity parameter b, so that they also exist for $b < b_{\rm cr}$. In the plot of $u_{\rm def}(\pi,h_0)$ (Figs. 4 and 6) there is no spike, since here the

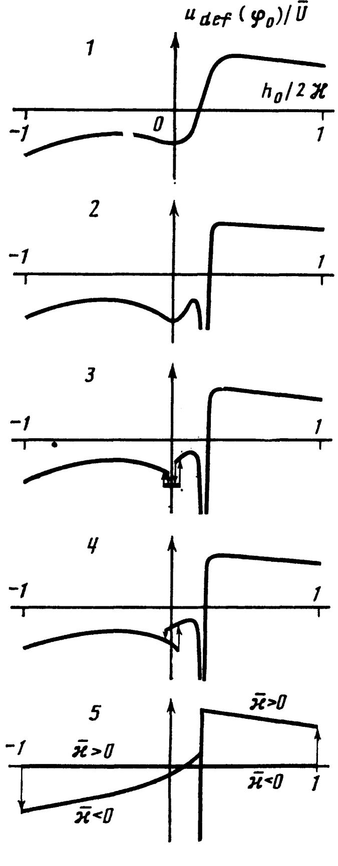

FIG. 5. Schematic diagram of the dynamics (1–5) of the change in the shape of the hysteresis loops of $u_{\rm def}(\varphi_0)$ as a function of h_0 with $\varphi_0 = \cos^{-1}(-0.2)$ as the amplitude of the incident wave $\mathscr{H}$ (the parameter b) increases (1–5, 5—$b \to \infty$).

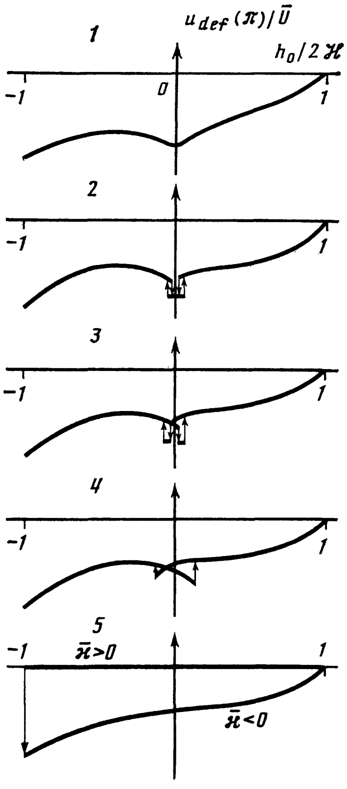

FIG. 6. Schematic diagram of the dynamics (1–5) of the change in the shape of the hysteresis loops of $u_{\rm def}(\pi)$ as a function of h_0 as the amplitude of the incident wave $\mathscr{H}$ (the parameter b) increases (1–5; 5—$b \to \infty$).

condition (4.12) can be satisfied only if $h_0 = 2\mathscr{H}$, when a point at which the sign of $H_t(0,\varphi)$ changes is not engendered on the surface of the metal. In this sense the dependence $u_{\text{def}}(\pi,h_0)$ is singular, while the curves for $u_{\text{def}}(\tilde{\varphi} \neq n\pi)$ as a function of h_0 are similar to the curve of $u_{\text{def}}(\varphi_0)$.

The hysteretic dependence of the sound wave on h_0 also gives rise to hysteresis in the amplitudes of the separate harmonics of longitudinal sound. The shape of the hysteresis loops and the dynamics of the change in shape are significantly different for the amplitudes of the first, second, and third harmonics. The multivalued nature of the amplitudes of higher-order harmonics is similar to that observed in the third-order harmonic.

5. We recall that aside from the deformation term $u_{\text{def}}(\tilde{\varphi})$ the sound field $u(\tilde{\varphi})$ (2.13) also contains an induction term $u_{\text{ind}}(\tilde{\varphi})$, which in the case (2.14) ($q\delta \ll 1$) is described by the formula (2.19). Comparing (2.19) with (4.4) shows that under conditions of strong nonlinearity ($b \gg 1$) the induction contribution to $u(\tilde{\varphi})$ must be taken into account in the region where

$$ql<b. \tag{4.14}$$

Even in this region, however, the deformation term $u_{\text{def}}(\tilde{\varphi})$ dominates, since it contains all the features of the nonlinear excitation of sound.

The results of this section were obtained for the case when the wavelength of the sound wave is large, $q\delta \ll 1$. As the parameter $q\delta$ increases (the frequency ω of the radio wave increases) the spikes observed in the sound field "spread" and in the region $q\delta \gg 1$ they vanish. In the process the hysteretic dependence of the displacement $u_{\text{def}}(\tilde{\varphi})$ and its harmonics on the magnitude of the external magnetic field h_0 remains.

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