Helical instability in an electron-hole plasma in semiconductors under exclusion conditions

V. V. Vladimirov, B. I. Kaplan, A. G. Kollyukh, and V. K. Malyutenko

Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR, Kiev
(Submitted 6 June 1989)
Zh. Eksp. Teor. Fiz. **96**, 2126–2132 (December 1989)

The first experimental and theoretical investigations have been made of a helical instability (oscillistor effect) in p-type Ge under carrier exclusion conditions when a sample is depleted of a plasma. Such an oscillistor is excited only when the surface recombination velocities on the opposite faces of a sample (parallel to the current) are significantly different. The threshold and frequency characteristics of the oscillator have been determined. Calculations have been made of the excitation criterion and of the oscillation frequency of helical waves under exclusion conditions.

1. A helical instability (i.e., an instability of the current in parallel electric **E** and magnetic **H** fields, known also as the oscillistor effect) was first observed in semiconductors by Ivanov and Ryvkin[1] who used n-type samples under conditions of injection of an electron-hole plasma. This effect was explained by a theory of helical waves put forward by Kadomtsev and Nedospasov.[2] The effect was subsequently discovered also in a homogeneous intrinsic plasma in Ge (Ref. 3): In this case a surface helical wave is excited when the surface recombination velocity s was sufficiently small that $s \ll D/d$ holds, where D is the ambipolar diffusion coefficient and d is the smallest dimension of the sample transverse to the current.

We carried out the first experimental and theoretical investigation of the oscillistor effect in p-type Ge under carrier exclusion conditions when the sample was depleted of plasma because of ambipolar drift toward the anode over a certain distance near the cathode (p^+-type contact), known as the exclusion length L_{ex} (Fig. 1). An ohmic contact played the role of the anode. An instability of the current was observed (curve 2 in Fig. 2), but only when the values of s on the wide faces of a sample parallel to the current were very different ($s_+ \gg D/d$, $s_- \ll D/d$). When the faces were subjected to the same treatment ($s_+ = s_- = s$), the effect was no longer observed (curves 1 and 3 in Fig. 2) even at low values of s and in the absence of exclusion when the role of the anode was played by a p^+-contact (homogeneous intrinsic plasma case).

We shall show later that a surface wave could not be excited in our experiments because of the strong transverse diffusion in a thin sample. Under exclusion conditions and for small values of s the oscillistor effect was not excited because of strong ambipolar motion of quasineutral helical fluctuations of the carrier density in the depletion region. At high values of s the exclusion length L_{ex} was small because of rapid surface generation and instability did not occur in the exclusion region because of strong enhancement of the longitudinal diffusion. The surface recombination processes then suppressed the oscillistor effect in the homogeneous part of the sample.

However, for $s_+ \gg s_-$, then in the exclusion region the process of surface generation created an inhomogeneous plasma with a fairly steep gradient sufficient to ensure growth of an internal helical wave.[4] The condition $L_{ex} \gg d$ should then be satisfied. Below we determine the instability criteria and the oscillation frequency for this case when $\mathbf{E} \| \mathbf{H}$.

2. Our experiments were carried out on samples of p-type Ge [$T = 300$ K, $n_i = 1.7 \times 10^{13}$ cm^{-3} is the intrinsic carrier density, $(N_A - N_D)/n_i = N = 0.05$–$0.1$ is the degree of nonneutrality of the plasma in samples of dimensions $0.05 \times 0.4 \times 0.6$ cm. The wide faces were subjected to a surface treatment which ensured $s_+ \approx 10^4$ cm/s and $s_- \approx 100$ cm/s (Fig. 1).

Figures 3 and 4 showed the frequency characteristics and the threshold dependences of the oscillistor effect [critical magnetic field $H_{cr}(U)$] in the case when $\mathbf{E} \| \mathbf{H}$. The oscillation frequency F was practically independent of the magnetic field and increased with U (Fig. 3). This type of $F(U)$ dependence is typical of strong ambipolar motion of perturbations,[3] because the phase velocity of a wave along E is governed by the average ambipolar motion velocity

$$|v_A| = |b_A E| = \frac{b_e b_h}{b_e + b_h} \frac{NE}{(\bar{f} + a)},$$

where $b_{e,h}$ are the electron and hole mobilities; $\bar{f} = \bar{n}/\bar{n}_i$; $\bar{n}$ is the average value of the carrier density; $a = b_h N/(b_e + b_h)$. The oscillistor effect was not excited in fields $H < 3$ kOe (Fig. 4) no matter how high the voltage U was. This was also due to this ambipolar motion because in weak magnetic fields characterized by $y_{e,h} = b_{e,h} H/c \ll 1$ the velocity of a Hall flux causing an instability (between static magnetic and

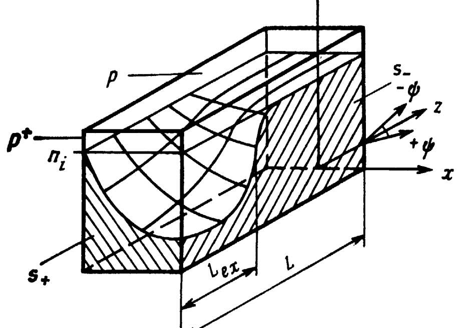

FIG. 1. Spatial distribution of carriers under exclusion conditions in a sample with p^+-type contact on the front face. The fields **E** and **H** are directed along the z axis.

 0038-5646/89/121203-04$04.00

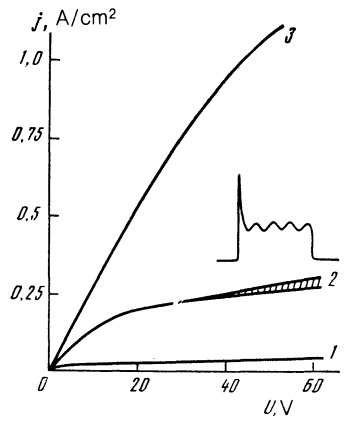

FIG. 2. Current-voltage characteristics of p-type Ge samples obtained under the following conditions: 1) $s_+ = s_- = s_{\min}$; 2) $s_+ = s_{\max}$, $s_- = s_{\min}$; 3) $s_+ = s_- = s_{\max}$. The shading of curve 2 identifies the region of existence of oscillations with an oscillogram shown as the inset.

transverse electric fields of helical perturbations) is $v_H \propto EH$ (Ref. 4) and one of the excitation conditions[3] $v_H > v_A$ is satisfied when H is high irrespective of the value of E (or U). Figure 5 shows the angular characteristics of the oscillistor frequency and amplitude (ψ is the angle between **E** and **H**). We shall discuss them later.

3. Let us estimate the main characteristics of a plasma in the exclusion region if $s_+ \gg s_-$. The lifetime of carriers in this region is governed both by the usual linear recombination time τ and by the time of ambipolar removal of carriers from the exclusion region ($\bar{\tau}_A$). If $\bar{\tau} = L_{ex}/2v_A \ll \tau$, the solution of the steady-state diffusion equation subject to the boundary conditions

$$-\frac{df}{d\xi}\bigg|_{\xi=-1} = \frac{s_- d}{D}(f-1) = 0, \qquad -\frac{df}{d\xi}\bigg|_{\xi=1} = \frac{s_+ d}{D}(f-1),$$

where $\xi = x/d$ and d is the half-thickness of the sample, is of the form

$$f = A\,\mathrm{ch}\,\beta(1+\xi),\quad A = \frac{s_+}{s_+\,\mathrm{ch}\,2\beta + (D\beta\,\mathrm{sh}\,2\beta)/d},\quad \beta = \frac{d}{(D\bar{\tau}_A)^{1/2}}.$$

The average value of the plasma density is $\bar{f} = (A/2\beta)\sinh 2\beta$.

We can estimate the exclusion length from the condition of equality of the time $\bar{\tau}_A$ and the characteristic time $2d/s_+$ taken to fill a sample with a plasma because of surface generation:

$$L_{ex} = \frac{4dv_A}{s_+},$$

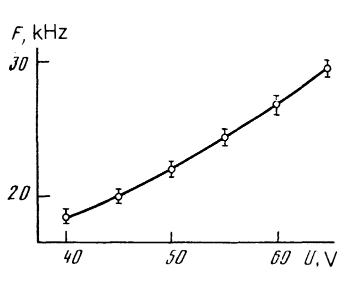

FIG. 3. Dependence of the frequency F of oscillations of the current on the voltage U obtained with $\psi = 0$ in fields $3 \leqslant H \leqslant 7$ kOe.

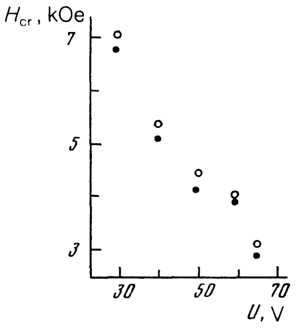

FIG. 4. Threshold characteristics of an oscillistor in the case $\psi = 0$, obtained for parallel (○) and antiparallel (●) orientations of **E** and **H**.

i.e., L_{ex} decreases on increase in s_+. Consequently, the parameter $\beta = (ds_+/2D)^{1/2}$ increases as a function of s_+ because the exclusion time $\bar{\tau}_A = 2d/s_+$ decreases. The condition $\bar{\tau}_A \ll \tau$ reduces to $s_+ \gg 2d/\tau \approx 200$ cm/s. The expression for L_{ex} can be written in terms of the current:

$$L_{ex} = \frac{4db_e b_h N\alpha}{s_+(\bar{f}+a)^2(b_e+b_h)}, \tag{1}$$

where $\alpha = j/en_i(b_e + b_h)$, $j = I/S$, I is the current, and S is the cross-section area of the sample. For $s_+ = 10^4$ cm/s, $D = 60$ cm^2/s, $b_e = 10^6$ cgs esu, and $b_h = b_e/2$, we obtain $L_{ex} \approx 0.3$ cm $\gg d$ for typical currents at which the oscillistor effect appears ($j \approx 0.25$ A/cm^2, $U = 40$ V–see curve 2 in Fig. 2). Equation (1) is valid for $L_{ex} < L$, where L is the length of the sample.

Using the condition

$$E_0(L - L_{ex}) + EL_{ex} = U,$$

where $E_0 = \alpha/(1+a)$ and $E = \alpha(\bar{f} + a)$ are, respectively, the electric fields in the intrinsic and exclusion regions, we find from the current-voltage characteristic that

$$\frac{4db_e b_h N(1-\bar{f})\alpha^2}{s_+(b_e+b_h)(\bar{f}+a)^3} + \frac{\alpha L}{1+a} = U. \tag{2}$$

We can see from Eq. (2) that at low voltages (when the currents are low and L_{ex} is short) the current-voltage characteristic is ohmic and a further increase in the voltage gives rise to a dependence $\alpha \propto U^{1/2}$. The current-voltage characteristic of Eq. (2) is in qualitative and quantitative (to within 30%) agreement with the experimental characteristic (curve 2 in Fig. 2).

In developing the oscillistor theory we shall assume $s_- d/D = 0$, $s_+ d/D \gg 1$, $L_{ex} \gg d$, and that the density profile is given by $f = \exp[\beta(\xi - 1)]$. If s_+ is too high, so that $L_{ex} \lesssim d$, the oscillistor effect is not excited because of enhancement of the longitudinal diffusion.

4. We now determine the dispersion relationship for helical waves in the case when **E**||**H** and $y_{e,h} \ll 1$. We consider quasineutral ($n' = p'$) potential perturbations ($E' = -\nabla\varphi'$) of the density and electric field of the form

$$A' = A_1(x)\exp(-i\omega t + ikz + i\varkappa y).$$

The initial continuity equations and those describing the motion of electrons (or holes), linearized in terms of small perturbations of the density N_1 and potential φ_1, can

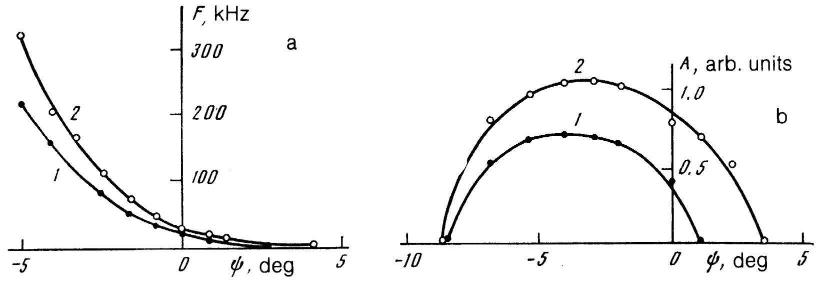

FIG. 5. Dependence of the frequency F of oscillations of the current (a) and of the amplitude A of these oscillations (b) on the angle between the fields **E** and **H**. $U = 65$ V. 1) $H = 3$ kOe; 2) 5 kOe.

be reduced to a system of two equations, the first of which is

$$N_1\left[-i\omega-ikb_eE+(\tilde{k}^2+\tilde{\varkappa}^2)\frac{D_e}{d^2}\right]-\frac{b_e}{d}\frac{d(E_x^0N_1)}{d\xi}$$
$$-\frac{D_e}{d^2}\frac{d^2N_1}{d\xi^2}$$
$$+\frac{b_ef}{d^2}\left[\frac{d^2\varphi_1}{d\xi^2}+\frac{1}{f}\frac{df}{d\xi}\frac{d\varphi_1}{d\xi}-(\tilde{k}^2+\tilde{\varkappa}^2+i\varkappa y_e)\frac{\varphi_1}{f}\frac{df}{d\xi}\right]=0, \quad (3)$$

where $N_1 = n_1/n_i$, $\tilde{k} = kd$, and $\tilde{\varkappa} = \varkappa d$, and

$$E_x^0=-\frac{1}{d}\frac{D_e-D_h}{b_e+b_h}\frac{1}{f}\frac{df}{d\xi}$$

is the ambipolar electric field.

The second equation (for holes) is analogous to Eq. (3) and is obtained by the substitutions

$$D_e\to D_h,\quad b_e\to -b_h,\quad f\to f+N.$$

The boundary conditions correspond to the equality of the electron (hole) flux along the direction of the x axis at the $\zeta = \pm 1$ faces and the surface recombination fluxes:

$$b_e\frac{d\varphi_1}{d\xi}f-D_e\frac{dN_1}{d\xi}-b_eE_x^0\,dN_1+i\tilde{\varkappa}y_e(D_eN_1-b_ef\varphi_1)$$
$$=\begin{cases} s_+dN_1, & \xi=1,\\ 0, & \xi=-1,\end{cases} \quad (4)$$

where an analogous condition for holes can be obtained by suitable substitutions.

We now simplify the derivation of the dispersion relationship (allowing for the complexity of the equations) on the basis of the following considerations. Since the instability growth rate γ_H, governed by the Hall flux in an inhomogeneous plasma, and the diffusive damping rate γ_D depend weakly on the slight nonneutrality of the plasma (in the main exclusion region $f \gg N$), in the calculations we assume $N = 0$. The growth rate γ_H is independent of diffusion (so that we shall calculate it ignoring the diffusion terms), whereas γ_D is independent of the magnetic and electric fields ($y_{e,h} \ll 1$), so that in the calculation of γ_D we shall assume that $E = H = 0$. The velocity of the ambipolar motion of perturbations in the direction of the electric field, which governs the phase velocity (and frequency) of a wave, can be calculated if we ignore the diffusion and magnetic terms but allow for nonneutrality of the plasma $N \neq 0$.

Using these approximations we find that the initial equation for calculation of the growth rate γ_H becomes

$$\frac{d^2\varphi_1}{d\xi^2}+\beta\frac{d\varphi_1}{d\xi}-\varphi_1\left[(\tilde{k}^2+\tilde{\varkappa}^2)-\frac{i\tilde{\varkappa}\tilde{k}b_ey_h\beta E}{\omega d}\right]=0 \quad (5)$$

subject to the boundary conditions

$$\varphi_1|_{\xi=1}=0,\quad \left.\frac{d\varphi_1}{d\xi}\right|_{\xi=-1}=0.$$

The eigenvalue ($-i\omega$) of this equation determines the growth rate γ_H:

$$\gamma_H=\frac{\tilde{k}\tilde{\varkappa}b_e\beta y_hE}{d(\tilde{k}^2+\tilde{\varkappa}^2+\beta^2/4)}. \quad (6)$$

It follows from Eq. (6) that for $\beta > 0$, then in the case of parallel **E** and **H** left-handed helical waves ($\tilde{k}\varkappa > 0$) are excited, whereas right-handed helical waves are excited when the orientation is antiparallel.

The equation for the determination of γ_D is

$$\frac{d^2N_1}{d\xi^2}-\left[-\frac{i\omega}{D}d^2+\tilde{k}^2+\tilde{\varkappa}^2\right]N_1=0 \quad (7)$$

subject to the boundary conditions

$$-\left.\frac{dN_1}{d\xi}\right|_{\xi=\pm1}=\begin{cases}\dfrac{s_+d}{D}N_1, & \xi=1,\\ 0, & \xi=-1.\end{cases}$$

The dispersion relationship is

$$-r_1\,\mathrm{th}\,2r_1=s_+d/D, \quad (8)$$

where

$$r_1=(-i\omega d^2/D+\tilde{k}^2+\tilde{\varkappa}^2)^{1/2}.$$

If $s_+d/D \gg 1$, we find that

$$\gamma_D=-\frac{D}{d^2}\left(\tilde{k}^2+\tilde{\varkappa}^2+\frac{\pi^2}{16}\right). \quad (9)$$

The equation for the determination of the oscillation frequency governed by the ambipolar motion is

$$(c+f)\frac{d^2\varphi_1}{d\xi^2}+\frac{df}{d\xi}\frac{d\varphi_1}{d\xi}-(c+f)(\tilde{k}^2+\tilde{\varkappa}^2)\varphi_1=0, \quad (10)$$

where

$$c=\frac{b_hN(\omega+kb_eE)}{\omega(b_e+b_h)}.$$

Integrating Eq. (10) in the interval $-1 \leqslant \xi \leqslant 1$, subject to the boundary conditions

$$\left.\frac{d\varphi_1}{d\xi}\right|_{\xi=\pm1}=0$$

we obtain the expression for c:

$$c=-\int_{-1}^{1} f\varphi_1\,d\xi \Big/ \int_{-1}^{1}\varphi_1\,d\xi. \qquad (11)$$

In the case of a smooth profile of φ_1 (or f), we find that $c = -\bar{f}$ and the wave frequency is

$$\omega=-\frac{b_e b_h \tilde{k} N E}{(b_e+b_h)\bar{f}+b_h N}\frac{1}{d}, \qquad (12)$$

i.e., the phase velocity of the wave is equal to the average polar velocity v_A and is directed in the same way as the electron drift.

At the excitation threshold we have $\gamma_H + \gamma_D = 0$ and the instability criterion becomes

$$\frac{\tilde{k}\tilde{\varkappa} b_e y_h \beta E}{\tilde{k}^2+\tilde{\varkappa}^2+\beta^2/4}>\frac{D}{d}\left(\tilde{k}^2+\tilde{\varkappa}^2+\frac{\pi^2}{16}\right). \qquad (13)$$

If we assume $\tilde{k} = \tilde{\varkappa}$, then the condition

$$\frac{d}{d\tilde{k}}(\gamma_H+\gamma_D)=0$$

can be used together with Eq. (13) to find the wave number corresponding to the lowest excitation threshold $\tilde{k}^2 = (\pi/16)\beta$. For this wave number the criterion of Eq. (13) expressed in terms of the current becomes

$$\frac{b_e y_h \beta}{D} d \frac{\alpha}{\bar{f}+a} > \frac{(2\beta+\pi)^2}{8}. \qquad (14)$$

For $s_+ = 10^4$ cm/s, $D/d = 2.5\times 10^3$ cm/s, $\beta = 1.4$, and $H = 3$ kOe, the criterion of Eq. (14) is satisfied when $j > 0.25$ A/cm^2 holds, in good agreement with the experimental results.

The minimum value of the magnetic field in which the oscillistor effect is excited is given by the condition $\gamma_H > \tilde{k} v_A/d$ or

$$y_h > N\frac{b_h(\pi/2+\beta)}{(\pi\beta)^{1/2}[(b_e+b_h)/2\beta+b_h N]}. \qquad (15)$$

For the parameters given above and $N = 0.05$, the condition (15) is satisfied if $H > 3$ kOe.

The oscillation frequency at the excitation threshold is then

$$F=\frac{\mathrm{Re}\,\omega}{2\pi}=\frac{1}{8d}(\beta/\pi)^{1/2}\frac{b_e b_h N \alpha}{(1/2\beta+a)^2(b_e+b_h)}. \qquad (16)$$

If $j = 0.25$ A/cm^2, then $N = 0.05$ and we have $F \approx 15$ kHz.

It is clear from Eq. (16) that the frequency increases as a function of the parameter β governing the degree of inhomogeneity of the plasma distribution. Variation of the angle between the electric and magnetic fields in a sample reduces the magnetoconcentration effect which either enhances the density gradient β in the exclusion region (negative values of the angle ψ in Fig. 5a) or weakens it (positive ψ). It follows from Eq. (16) that the frequency rises (negative ψ) or decreases. The oscillation amplitude decrease in the range of positive values of ψ because of a reduction in the growth rate γ_H. At high angles ψ (Fig. 5b) the oscillistor effect disappears because of a strong rise in the diffusion flux.

In the absence of exclusion (when the p^+ -type contact acts as the anode) and low values of the surface recombination velocity s, a surface helical wave in such a thin sample may be excited in much stronger magnetic fields and higher currents than those employed in our experiments. The excitation criterion[3] is

$$b_e E_0 y_h > 4\cdot 3^{1/2} D/d$$

and for $j = 0.25$ A/cm^2 ($E_0 = 18$ V/cm), this criterion is satisfied if $H > 18$ kOe holds.

The results obtained demonstrate that it is possible to develop dynamic methods for determination of the exclusion characteristics from the frequency dependences of an oscillistor.

[1]Yu. L. Ivanov and S. M. Ryvkin, Zh. Tekh. Fiz. **28**, 774 (1958) [Sov. Phys. Tech. Phys. **3**, 722 (1958)].

[2]B. B. Kadomtsev and A. V. Nedospasov, J. Nucl. Energy Part C **1**, 230 (1960).

[3]C. E. Hurwitz and A. L. McWhorter, Phys. Rev. **134**, A1033 (1964).

[4]V. V. Vladimirov, Usp. Fiz. Nauk **115**, 73 (1975) [Sov. Phys. Usp. **18**, 37 (1975)].

Translated by A. Tybulewicz