Polarization effects in Rayleigh scattering of nonuniform electromagnetic waves

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Utilizing the solution of the kinetic equation for the polarized photon density matrix, we derive the intensity and polarization of electromagnetic radiation reflected from a randomly inhomogeneous medium. The kinetic equation makes allowance both for ordinary simple scattering and coherent reflection and refraction effects at the interface, and these give rise to inhomogeneous waves that are damped deep within the medium. In contrast to the classical transport theory result, the present solution suggests the existence of an elliptically polarized component in the reflected radiation, even with an unpolarized incident beam. We show that the effect is entirely due to the scattering of inhomogeneous waves by fluctuations in the dielectric constant of the medium. We also demonstrate that when polarization of the incident radiation is taken into consideration, the azimuthal dependence of the anomalous peak in the angular spectrum of backscattered photons is radically altered.

INTRODUCTION

The manner in which electromagnetic waves propagate within a scattering medium is governed by the ratio of the wavelength λ to the mean free path l of the radiation. If that ratio is small, i.e., $\lambda/l \ll 1$, then the electromagnetic field will be essentially homogeneous, and can be described by locally plane waves.[1,2] In other words, a photon in the interior of a medium can be viewed as a particle which, between scattering events, follows a classical trajectory. At any point along this trajectory, the photon possesses momentum $\hbar k_0 \mathbf{n}$ ($k_0 = 2\pi/\lambda$, where $\mathbf{n}$ is the unit vector tangent to the trajectory). The determinacy of the direction of the photon momentum before and after an individual scattering event makes it possible to describe the polarization state of the electromagnetic field at any point in space via its Stokes parameters.[3-5]

On the other hand, the situation is quite different near the surface of the medium. The existence of a sharp interface (compared with the mean free path l) between the medium and the vacuum results in the specular reflection and refraction of electromagnetic waves. Refraction is typically damped out deep inside the medium and is nonuniform.

Furthermore, photons near the surface do not possess a definite momentum, which makes it impossible to describe the scattering of an electromagnetic field inside the medium in terms of the Stokes parameters. The net results is that the polarization of radiation scattered in a randomly inhomogeneous medium with a sharp boundary is markedly different from the value predicted by classical transport theory.[4]

The theoretical analysis of the angular dependence of intensity and polarization of backscattered electromagnetic radiation, with due regard for refraction and specular reflection, bears an important relation to the broad-based development of methods in reflection ellipsometry as they apply to studies of surfaces and surface layers.[6]

It should be noted here that the interaction of nonuniform electromagnetic waves with an individual scattering center has previously been examined (e.g., see Refs. 7–9), with the main area of interest being the absorption and reemission of these waves by the constituent atoms of the medium.

In the present paper, we rely on the solution of the matrix kinetic equation for the polarized density matrix to obtain the angular dependence of the intensity and polarization of electromagnetic radiation reflected from a randomly inhomogeneous medium. The solution thus obtained allows both for refraction and reflection at the interface—processes that give rise to nonuniform waves—and conventional, incoherent multiple scattering. We assume that scattering introduces no change in frequency, and that the wavelength of the radiation is much larger than the relevant dimensions of the scatterer (Rayleigh scattering). The case at hand is encountered in practice when electromagnetic waves interact with atoms, molecules, and small-scale optical inhomogeneities in a medium.

THE KINETIC EQUATION FOR THE POLARIZED DENSITY MATRIX

We consider the propagation of electromagnetic radiation of frequency $\omega = k_0$ (here and in what follows we take $c = 1$) in a randomly inhomogeneous medium. For the sake of definiteness, we assume that irregularities in the refractive index are due to impurities that have typical sizes $a \ll \lambda$ and are randomly disposed within an otherwise uniform medium with a dielectric constant $\varepsilon_0 = \varepsilon_0' + i\varepsilon_0'' = 1 + \delta\varepsilon$ that differs slightly from unity (i.e., $|\varepsilon_0 - 1|^{1/2} \ll 1$).

The wave field $\mathbf{E}$ satisfies the equation

$$(\mathrm{rot}\,\mathrm{rot} - k_0{}^2\hat{\varepsilon}(\mathbf{r}))\mathbf{E}(\mathbf{r}) = 0, \qquad (1)$$

where

$$\varepsilon_{ij}(\mathbf{r}) = \delta_{ij}(1 + \delta\varepsilon\theta(\mathbf{r})) + 4\pi\alpha\delta_{ij}\sum_{a=1}^{N}\delta(\mathbf{r} - \mathbf{r}_a)$$

is the dielectric constant of the random medium, $\alpha = \alpha' + i\alpha''$ is the polarizability, the $\mathbf{r}_a$ are the coordinates of the impurities, and $\theta(\mathbf{r})$ is equal to unity inside the volume V occupied by the medium and zero outside. The fact that ε_0 and α have imaginary parts allows for photon absorption during elastic scattering.

To study energy transport by the electromagnetic field, it is necessary to construct the equation for the polarized density matrix,

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$$\rho_{ij}(\mathbf{r},\mathbf{r}')=\langle \mathbf{E}_i(\mathbf{r};\mathbf{r}_1,\ldots,\mathbf{r}_N)\mathbf{E}_j^*(\mathbf{r}';\mathbf{r}_1,\ldots,\mathbf{r}_N)\rangle.$$

The angular brackets signify averaging over the coordinates of the scatterers, i.e., integrating $\hat{\rho}$ multiplied by the coordinate-space distribution function for the impurities:

$$\rho_{ij}(\mathbf{r},\mathbf{r}')=\int d\mathbf{r}_1\ldots d\mathbf{r}_N E_i(\mathbf{r};\mathbf{r}_1,\ldots,\mathbf{r}_N)\times E_j^*(\mathbf{r}';\mathbf{r}_1,\ldots,\mathbf{r}_N)F(\mathbf{r}_1,\ldots,\mathbf{r}_N), \quad (2)$$

where $F(\mathbf{r}_1,\ldots,\mathbf{r}_N)=V^{-N}\theta(\mathbf{r}_1)\ldots\theta(\mathbf{r}_N)$.

For $\lambda/l\ll 1$, if we are not concerned with backwards reflection in the narrow angular $\Delta\theta\sim\lambda/l$,[10-14] the desired equation for $\hat{\rho}$ may be obtained by summing successive ladder diagrams, a process that has been described in detail elsewhere.[1,2,13-15]

The final result (cf. Refs. 13, 14) is

$$\rho_{ij}(\mathbf{r},\mathbf{r}')=\rho_{ij}^{coh}(\mathbf{r},\mathbf{r}')+\frac{3\nu_s}{8\pi}\int d\mathbf{r}''D_{im}(\mathbf{r},\mathbf{r}'')\rho_{ml}(\mathbf{r}'',\mathbf{r}'')D_{lj}^*(\mathbf{r}',\mathbf{r}''), \quad (3)$$

where ν_s is the elastic scattering coefficient for photons scattering from impurities (the reciprocal of ν_s, the quantity $l_s=\nu_s^{-1}$, is essentially the mean free path of photons subject to elastic scattering). Hereafter, repeated indices are summed over.

In Eq. (3), $\hat{\rho}^{\text{coh}}$ refers to a field that has not undergone incoherent scattering:

$$\rho_{ij}^{coh}=C_{ijkl}\langle E_k\rangle\langle E_l^*\rangle. \quad (4)$$

The quantities $\langle E_k\rangle$ define the mean radiation field, and are solutions of the equation

$$\left[\frac{\partial^2}{\partial x_i\partial x_l}-\delta_{il}\Delta-k_0^2\delta_{il}(1+\{\delta\varepsilon+4\pi n\alpha\}\theta(\mathbf{r}))\right]\langle E_l\rangle=0, \quad (5)$$

where $n=N/V$ is the number of inclusions (impurities) per unit volume. The boundary conditions on (5) require continuity of the mean magnetic field $\langle\mathbf{H}\rangle=\nabla\times\langle\mathbf{E}\rangle/ik_0$ and the mean tangential electric field $\langle\mathbf{E}\rangle$ at the interface. The weighting factors C_{ijkl} in Eq. (4) enables us to deal with incident radiation having arbitrary polarization.[16]

In eq. (3), $\hat{D}$ is the retarded Green's function of Eq. (5) with the appropriate boundary conditions:

$$\left[\frac{\partial^2}{\partial x_i\partial x_l}-\delta_{il}\Delta-\delta_{il}k_0^2(1+\{\delta\varepsilon+4\pi n\alpha\}\theta(\mathbf{r}))\right]D_{lj}(\mathbf{r},\mathbf{r}')=4\pi\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \quad (6)$$

Equation (3), (5), and (6) completely determine the polarized density matrix both inside and outside the medium, and enable one to calculate the intensity and polarization of scattered photons at arbitrary angles of incidence and emergence from the medium.

MEAN FIELD AND GREEN'S FUNCTION

Consider the case in which the scattering medium occupies the region of space $z>0$. Finding the mean field then reduces to the classical problem of reflection and refraction of an electromagnetic plane wave at the interface between a homogeneous medium with dielectric constant $\varepsilon=\varepsilon_0+4\pi n\alpha$ and the vacuum.[17]

Assume that a plane wave with wave vector $\mathbf{k}_0$ is incident upon the medium from the vacuum ($z=-\infty$):

$$\mathbf{E}_0(\mathbf{r})|_{z=-\infty}=\mathbf{E}_0\exp(i\mathbf{k}_0\mathbf{r}). \quad (7)$$

We resolve the field vector $\mathbf{E}_0$ into two components,

$$\mathbf{E}_0=\mathbf{E}_{0\parallel}+\mathbf{E}_{0\perp}, \quad (8)$$

where $\mathbf{E}_{0\parallel}$ is a field vector in the plane of incidence, and $\mathbf{E}_{0\perp}$, is perpendicular to it. The x axis points along $\mathbf{q}_0$, the projection of the wave vector $\mathbf{k}_0$ on the x-y plane. The mean field then takes the form

$$\langle\mathbf{E}\rangle=\mathbf{E}(z)\exp(iq_0x), \quad (9)$$

where the components of the vector $\mathbf{E}(z)$ are determined by the Fresnel equations.[17]

Making use of the explicit expressions for $\mathbf{E}(z)$,[17] as well as the fact that an arbitrarily polarized beam can be represented as a mixture of randomly and elliptically polarized components,[3-5] one can easily find the polarized density matrix (4) for arbitrarily polarized incident radiation. For example, to find $\hat{\rho}^{\text{coh}}$ inside the medium (z, $z'>0$), some simple manipulation yields

$$\hat{\rho}^{coh}(\mathbf{r},\mathbf{r}')=(\hat{F}^{(n)}+\hat{F}^{(e)})\exp\{i\varkappa_0z-i\varkappa_0^*z'+i\mathbf{q}_0(\mathbf{r}_\parallel-\mathbf{r}_\parallel')\}, \quad (10)$$

where $k_{0\perp}=k_0\mu_0=(k_0^2-q_0^2)^{1/2}$, $\varkappa_0=[k_{0\perp}^2+(\varepsilon-1)k_0^2]^{1/2}$, $\mu_0=\cos\theta_0$, θ_0 is the angle of incidence measured from the z axis, and $\mathbf{r}_\parallel$ (x,y). The matrices $\hat{F}^{(n)}$ and $\hat{F}^{(e)}$ describe the nonpolarized and elliptically polarized components, respectively:

$$\hat{F}^{(n)}=\frac{4F_0k_{0\perp}^2}{|k_{0\perp}+\varkappa_0|^2}\begin{pmatrix} |\gamma_0|^2 & 0 & -(1-\mu_0^2)^{1/2}\gamma_0 \\ 0 & 1 & 0 \\ -(1-\mu_0^2)^{1/2}\gamma_0^* & 0 & 1-\mu_0^2 \end{pmatrix}, \quad (11)$$

$$\hat{F}^{(e)}=\frac{4k_{0\perp}^2}{|k_{0\perp}+\varkappa_0|^2}\begin{pmatrix} |E_{0\parallel}|^2|\gamma_0|^2 & \gamma_0E_{0\parallel}E_{0\perp}^* & -|E_{0\parallel}|^2(1-\mu_0^2)^{1/2}\gamma_0 \\ \gamma_0^*E_{0\parallel}^*E_{0\perp} & |E_{0\perp}|^2 & -E_{0\parallel}E_{0\perp}^*(1-\mu_0^2)^{1/2} \\ -|E_{0\parallel}|^2(1-\mu_0^2)^{1/2}\gamma_0^* & -(1-\mu_0^2)^{1/2}E_{0\parallel}^*E_{0\perp} & (1-\mu_0^2)|E_{0\parallel}|^2 \end{pmatrix}, \quad (12)$$

where $\gamma_0 = \varkappa_0/k_0 = \gamma_0' + i\gamma_0''$; the quantities F_0, $\mathbf{E}_{0\parallel}$, and $\mathbf{E}_{0\perp}$ determine the intensity and polarization of the incident beam.[3] In deriving (11) and (12), we have discarded small terms of order $|\varepsilon - 1|^{1/2} \ll 1$.

Equation (6) for the Green's function $\hat{D}$ of a homogeneous, semi-infinite medium has a known solution (e.g., see Refs. 18, 19).

ANGULAR DEPENDENCE OF THE POLARIZATION AND INTENSITY OF BACKSCATTERED RADIATION

In the present context, when all photons emerging from the medium have the same frequency, the calculation of the intensity and polarization of the reflected radiation requires only that one know the photon distribution over components of the wave vector $\mathbf{k}(|\mathbf{k}| = k_0)$ parallel to the surface, $\mathbf{q} = (q_x, q_y)$, for $z \to -0$:

$$(2\pi)^2\delta(\mathbf{q}-\mathbf{q}')\hat{\rho}(\mathbf{q}; z, z)\,|_{z\to-0}$$

$$= \int d\mathbf{r}_\parallel\, d\mathbf{r}_\parallel' \exp(i\mathbf{q}\mathbf{r}_\parallel - i\mathbf{q}'\mathbf{r}_\parallel')\hat{\rho}(\mathbf{r}_\parallel, z; \mathbf{r}_\parallel', z)\,|_{z\to-0}. \qquad (13)$$

The angular distribution of intensity and polarization in the reflected radiation, $\hat{S}$, is then related to the polarized density matrix through the simple expression

$$\hat{\rho}(\mathbf{q};\ z\to-0,\ z\to-0)\,d\mathbf{q}/(2\pi)^2 = \hat{S}(\mu,\ \varphi;\ \mu_0)\,d|\mu|\,d\varphi, \qquad (14)$$

where $\mu = \cos\vartheta$, ϑ is the polar angle reckoned from the z axis, φ is the azimuthal angle in the x-y plane, and $q = k_0 \sin\vartheta$ $(q \leqslant k_0)$.

In the coordinate system defined by the triple of basis vectors

$$\mathbf{n} = \mathbf{k}/k_0 = (\cos\varphi\sin\vartheta,\ \sin\varphi\sin\vartheta,\ \cos\vartheta),$$
$$\mathbf{e}_l = \partial\mathbf{n}/\partial\vartheta = (\cos\varphi\cos\vartheta,\ \sin\varphi\cos\vartheta,\ -\sin\vartheta),$$
$$\mathbf{e}_r = [\mathbf{n},\ \mathbf{e}_l] = (-\sin\varphi,\ \cos\varphi,\ 0),$$

$\hat{S}$ is a two-dimensional matrix. Matrix elements of $\hat{S}$ in this representation are related to the Stokes parameters of the scattered photons by[3-5]

$$S_{11} = I_l, \quad S_{22} = I_r, \quad S_{12} = {}^1/_2(U - iV). \qquad (15)$$

Here I_l is the intensity of the component polarized in the scattering plane, which is formed by the z axis and the wave vector $\mathbf{k}$; I_r is the intensity of the component polarized perpendicular to that plane (i.e., parallel to the surface).

Making use of (13) to transform Eq. (3), we have

$$\hat{\rho}(\mathbf{q}; z, z) = \hat{\rho}^{coh}(\mathbf{q}; z, z)$$

$$+ \frac{3\nu_s}{8\pi}\int_0^\infty dz''\, \hat{D}(z, z''; \mathbf{q})\hat{\rho}(z'')\hat{D}^+(z, z'; \mathbf{q}), \qquad (16)$$

where $\hat{\rho}(z) \equiv \hat{\rho}(\mathbf{r},\mathbf{r})$.

From Eq. (16), we infer that in order to determine $\hat{S}$, one must first find the photon density $\hat{\rho}(z)$ in the scattering medium at $z > 0$. A closed-form solution for $\hat{\rho}(z)$ can be obtained by putting $\mathbf{r} = \mathbf{r}'$ in (3):

$$\hat{\rho}(z) = \hat{\rho}^{coh}(z) + \frac{3\nu_s}{8\pi}\int_0^\infty dz' \int \frac{d\mathbf{q}}{(2\pi)^2}\hat{D}(z, z'; \mathbf{q})\hat{\rho}(z')\hat{D}^+(z, z'; \mathbf{q}), \qquad (17)$$

where $\hat{\rho}^{coh}(z) \equiv \hat{\rho}^{coh}(\mathbf{r},\mathbf{r}')$.

In a previous investigation of the analogous equation for scalar waves,[20] it was shown that in Eq. (17), taking refraction and coherent reflection into account in the double integral has little effect on the solution. The corresponding corrections are of the same order as the ratio of the angular width $\Delta\vartheta$ of the region in which diffraction effects are significant to 4π:

$$\Delta\vartheta/4\pi \sim |\varepsilon - 1|^{1/2} \ll 1. \qquad (18)$$

Similarly, we may seek a solution of Eq. (17) by expanding in the small parameter $\xi = |\varepsilon - 1|^{1/2}$:

$$\hat{\rho}(z) = \hat{\rho}^{(0)}(z) + \xi\hat{\rho}^{(1)}(z) + \ldots \qquad (19)$$

The expression for the matrix $\hat{S}$ also takes the form of a series in ξ. Thus, to leading order (in ξ), $\hat{S}$ may be expressed solely in terms of $\hat{\rho}^{(0)}(z)$.

The equation yielding $\hat{\rho}^{(0)}$ can easily be found by discarding terms of order $|\varepsilon - 1|^{1/2} \ll 1$ in the integral over $\mathbf{q}$ on the right-hand side of Eq. (17) (i.e., by replacing $\hat{D}(z,z';\mathbf{q})$ with the Green's function $\hat{D}^{(0)}(z - z';\mathbf{q})$ for an infinite medium).

We obtain as a result

$$\hat{\rho}^{(0)}(z) = \hat{\rho}^{coh}(z)$$
$$+ \frac{3k_0^2\nu_s}{8\pi}\int_0^\infty dz' \int_0^{2\pi} d\varphi \int_0^1 \frac{\mu\, d\mu}{(2\pi)^2}\hat{D}^{(0)}(z - z'; \mathbf{q})\hat{\rho}^{(0)}(z')$$
$$\times \hat{D}^{(0)+}(z - z'; \mathbf{q}), \qquad z > 0. \qquad (20)$$

At the present level of approximation, the matrix $\hat{S}^{in}(\mu,\varphi;\mu_0)$, which describes the intensity and polarization of the incoherently reflected radiation, is given by

$$\hat{S}^{in}(\mu, \varphi; \mu_0) = \frac{3}{8\pi}\tilde{\omega}_0 \int_0^\infty d\tau'\, \hat{D}(\tau\to-0,\ \tau_1\to+0;\ \mathbf{q})\rho^{(0)}(\tau')$$
$$\times \hat{D}^+(\tau\to-0, \tau_1\to+0; \mathbf{q})\exp\left(-\frac{\tau'}{\gamma}\right), \qquad (21)$$

where $\nu = l^{-1} = (\lambda/2\pi)^{-1}(\varepsilon_0'' + 4\pi n\alpha'')$ is the photon attenuation scale factor in the medium, $\tilde{\omega} = \nu_s/\nu$ is the albedo for simple scattering (allowing for absorption in the medium), $\tau = z/l$, $\gamma = \varkappa/k_0 = \gamma' + i\gamma''$.

The reason one can substitute the Green's function $\hat{D}^{(0)}$ for an infinite medium for the Green's function $\hat{D}$ for the semi-infinite medium is related to the fact that refraction and incoherent reflection effects only affect the passage of incident and backscattered photons through the boundary of the medium, and have no bearing on multiple scattering within it. The validity of this statement is based on the assumption that $|\varepsilon - 1|^{1/2} \ll 1$.

Tensor quantities appearing in (21) will be considered in a coordinate system attached to the medium (with the z axis inwardly directed and normal to the surface, and the x axis parallel to the vector $\mathbf{q}_0$).

It can be shown that a more convenient set than the elements $\rho_{ik}^{(0)}(\tau)$ consists of the quantities $n_{\alpha\beta}(\tau)$, $n_a(\tau)$ ($\alpha,\beta = 1, 2; a = 1,\ldots,7$), which are related to the $\rho_{ik}^{(0)}(\tau)$ by the linear relations

$$\rho_{11}^{(0)}(\tau)={}^1/_2\{(F_{11}+F_{22})n_{22}(\tau)+F_{33}n_{12}(\tau)+(F_{11}-F_{22})n_1(\tau)\},$$

$$\rho_{22}^{(0)}(\tau)={}^1/_2\{(F_{11}+F_{22})n_{22}(\tau)+F_{33}n_{12}(\tau)-(F_{11}-F_{22})n_1(\tau)\},$$

$$\rho_{33}^{(0)}(\tau)=(F_{11}+F_{22})n_{21}(\tau)+F_{33}n_{11}(\tau),$$

$$\rho_{ik}^{(0)}(\tau)={}^1/_2\{(F_{ik}+F_{ki})n_a(\tau)-\operatorname{sign}(k-i)\,(F_{ik}-F_{ki})n_{a+1}(\tau)\},\tag{22}$$

where $i \neq k$; when $i = 1(2)$, $k = 2(1)$, we have $a = 2$, when $i = 1(3)$, $k = 3(1)$, we have $a = 4$, and when $i = 2(3)$, $k = 3(2)$, we have $a = 6$; also, $\hat{F} = \hat{F}^{(e)} + \hat{F}^{(n)}$.

Evaluating the elementary integral over the azimuthal angle φ on the right-hand side of Eq. (20) and making use of (22), we obtain

$$n_{\alpha\beta}(\tau)=\int\limits_0^\infty d\tau'\int\limits_0^1\frac{d\mu}{\mu}\exp\left(-\frac{|\tau-\tau'|}{\mu}\right)n_{\alpha\gamma}(\tau')\psi_{\gamma\beta}(\mu)$$
$$+\delta_{\alpha\beta}\exp\left(-\frac{\tau}{\gamma_0'}\right),\tag{23}$$

$$n_a(\tau)=\int\limits_0^\infty d\tau'\int\limits_0^1\frac{d\mu}{\mu}\exp\left(-\frac{|\tau-\tau'|}{\mu}\right)n_a(\tau')\psi_a(\mu)$$
$$+\exp\left(-\frac{\tau}{\gamma_0'}\right),\tag{24}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta, $\psi_1(\mu) = \psi_2(\mu) = \frac{3}{16}\tilde{\omega}_0(1+\mu^2)^2$, $\psi_3(\mu) = \frac{3}{4}\tilde{\omega}_0\mu_2$, $\psi_4(\mu) = \psi_6(\mu) = \frac{3}{8}\tilde{\omega}_0(1-\mu^2)(1+2\mu^2)$, $\psi_5(\mu) = \psi_7(\mu) = \frac{3}{8}\tilde{\mu}_0(1-\mu^2)$, and

$$\hat{\psi}(\mu)={}^3/_8\tilde{\omega}_0\begin{pmatrix}2(1-\mu^2)^2 & 2\mu^2(1-\mu^2)\\ \mu^2(1-\mu^2) & 1+\mu^4\end{pmatrix}.$$

The subscript a is not summed over in Eq. (24). The calculation of the integrals involving $\rho_{ik}^{(0)}(\tau)$ on the right-hand side of (21), and which are subject to constraints like (23) and (24), is a familiar problem in the classical theory of radiative transfer; the desired integrals can be expressed in terms of the appropriate Ambartsumyan-Chandrasekhar functions (see Refs. 4, 21–24 for details). Making use of the results in these references, we finally obtain for the Stokes parameters of the incoherently reflected radiation

$$I_l^{in}(\mu,\varphi;\mu_0)$$
$$=\frac{3k_0\tilde{\omega}_0}{16\pi}\frac{4k_\perp}{|k_\perp+\varkappa|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}\{|\gamma|^2[(F_{11}+F_{22})h_{22}(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+F_{33}h_{12}(\gamma',\gamma_0';\tilde{\omega}_0)]+2(1-\mu^2)[(F_{11}+F_{22})h_{21}(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+F_{33}h_{11}(\gamma',\gamma_0';\tilde{\omega}_0)]+4(1-\mu^2)^{1/2}\sin\varphi[\gamma'\operatorname{Re}F_{23}h_6(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+4(1-\mu^2)^{1/2}\cos\varphi[\gamma'\operatorname{Re}F_{13}h_4(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-\gamma''\operatorname{Im}F_{23}h_7(\gamma',\gamma_0';\tilde{\omega}_0)]$$
$$-\gamma''\operatorname{Im}F_{13}h_5(\gamma',\gamma_0';\tilde{\omega}_0)]$$
$$+2|\gamma|^2\sin 2\varphi\operatorname{Re}F_{12}h_2(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+|\gamma|^2\cos 2\varphi(F_{11}-F_{22})h_1(\gamma',\gamma_0';\tilde{\omega}_0)\},\tag{25}$$

$$I_r^{in}(\mu,\varphi;\mu_0)$$
$$=\frac{3k_0\tilde{\omega}_0}{16\pi}\frac{4k_\perp}{|k_\perp+\varkappa|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}\{(F_{11}+F_{22})h_{22}(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+F_{33}h_{12}(\gamma',\gamma_0';\tilde{\omega}_0)-2\sin 2\varphi\operatorname{Re}F_{12}h_2(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-\cos 2\varphi(F_{11}-F_{22})$$
$$\times h_1(\gamma',\gamma_0';\tilde{\omega}_0)\},\tag{26}$$

$$U^{in}(\mu,\varphi;\mu_0)$$
$$=\frac{3k_0\tilde{\omega}_0}{8\pi}\frac{4k_\perp}{|k_\perp+\varkappa|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}\{2\gamma''\operatorname{Im}F_{12}h_3(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+2(1-\mu^2)^{1/2}\sin\varphi\operatorname{Re}F_{13}h_4(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-2(1-\mu^2)^{1/2}\cos\varphi\operatorname{Re}F_{23}h_6(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+\gamma'\sin 2\varphi(F_{11}-F_{22})h_1(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-2\gamma'\cos 2\varphi\operatorname{Re}F_{12}h_2(\gamma',\gamma_0';\tilde{\omega}_0)\},\tag{27}$$

$$V^{in}(\mu,\varphi;\mu_0)$$
$$=\frac{3k_0\tilde{\omega}_0}{8\pi}\frac{4k_\perp}{|k_\perp+\varkappa|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}\{2\gamma'\operatorname{Im}F_{12}h_3(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+2(1-\mu^2)^{1/2}\sin\varphi\operatorname{Im}F_{13}h_5(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-2(1-\mu^2)^{1/2}\cos\varphi\operatorname{Im}F_{23}h_7(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$-\gamma''\sin 2\varphi(F_{11}-F_{22})h_1(\gamma',\gamma_0';\tilde{\omega}_0)$$
$$+2\gamma''\cos 2\varphi\operatorname{Im}F_{12}h_2(\gamma',\gamma_0';\tilde{\omega}_0)\},\tag{28}$$

where $h_a(x,y;\tilde{\omega}_0) = H_a(x,\tilde{\omega}_0)H_a(y,\tilde{\omega}_0)$, $h_{\alpha\beta}(x,y;\tilde{\omega}_0) = H^B_{\alpha\gamma}(y,\tilde{\omega}_0)H^B_{\gamma\beta}(x,\tilde{\omega}_0)$. Here the H-functions obey the nonlinear integral equations[4,21-24]

$$H_a(\mu,\tilde{\omega}_0)=1+\mu H_a(\mu,\tilde{\omega}_0)\int\limits_0^1\frac{H_a(\mu',\tilde{\omega}_0)\psi_a(\mu')}{\mu+\mu'}d\mu',\tag{29}$$

$$H_{\alpha\beta}{}^B(\mu,\tilde{\omega}_0)=\delta_{\alpha\beta}+\mu H_{\alpha\gamma}{}^B(\mu,\tilde{\omega}_0)\int\limits_0^1\frac{H_{\gamma\nu}(\mu',\tilde{\omega}_0)\psi_{\nu\beta}(\mu')}{\mu+\mu'}d\mu'.\tag{30}$$

The matrix $\hat{H}$ is related $\hat{H}^B$ by the transformation[22]

$$\hat{H}^B=\hat{B}\hat{H}^T\hat{B}^{-1},\quad \hat{B}={}^3/_4\begin{pmatrix}2&0\\0&1\end{pmatrix},$$

where subscript T denotes matrix transposition.

The functions $H_a(\mu,\tilde{\omega}_0)$ and $H_{\alpha\beta}(\mu,\tilde{\omega}_0)$ have been thoroughly studied, and detailed tables exist for both. The functions $H_a(\mu,\tilde{\omega}_0)$ have been tabulated for various values of the simple scattering albedo $\tilde{\omega}_0$,[4,23,24], as have the matrices $\hat{N}(\mu,\tilde{\omega}_0)$ and $\hat{H}_M(\mu,\omega_0)$, which are related to $\hat{H}(\mu,\tilde{\omega}_0)$ by

$$\hat{N}(\mu,\tilde{\omega}_0)={}^1/_2\cdot 3^{1/2}\hat{A}(\mu)\hat{H}(\mu,\tilde{\omega}_0)\hat{L},$$
$$\hat{H}_M(\mu,\tilde{\omega}_0)=\hat{L}^{-1}\hat{H}(\mu,\tilde{\omega}_0)\hat{L},\tag{31}$$

where

$$\hat{A}(\mu)=\begin{pmatrix}1-\mu^2 & \mu^2\\ 0 & 1\end{pmatrix},\quad \hat{L}=\begin{pmatrix}0 & 2^{1/2}\\ 1 & 0\end{pmatrix}.$$

Note that in the special case of conservative scattering ($\tilde{\omega}_0 = 1$), the functions $N_{\alpha\beta}(\mu,1)$ are identical, up to a constant multiplicative factor, with the well-known Chandrasekhar functions[4]

$$\hat{N}(\mu,1)={}^1/_2\cdot 3^{1/2}\begin{pmatrix}\psi(\mu) & 2^{1/2}\phi(\mu)\\ \chi(\mu) & 2^{1/2}\zeta(\mu)\end{pmatrix}. \tag{32}$$

With Eq. (32) in hand, it is easily shown that when $|\mu|$ and μ_0 are both much greater than $|\varepsilon-1|^{1/2}$ ($\tilde{\omega}_0=1$), so that coherent effects can be neglected, the solution (25)–(28) goes over into the classical transport theory result.[4] Equations (25)–(28) provide a complete solution for the intensity and polarization of incoherent electromagnetic radiation reflected from a random medium, allowing both for coherent reflection and refraction of the incident and backscattered photons, and for ordinary multiple scattering.

DISCUSSION

A. Anomalous reflection. It was shown in a previous paper[20] that for scattering of a scalar field at grazing incidence upon a medium that is optically less dense than the vacuum ($\mathrm{Re}\,\varepsilon<1$), a nonspecular peak appears in the angular spectrum of the reflected radiation near the critical angle $\vartheta_c=\arccos\mu_c$, where $\mu_c=(1-\mathrm{Re}\,\varepsilon)^{1/2}$. For small scatterers (i.e., those that are much smaller than the wavelengths), the shape and height of the anomalous peak are independent of the azimuthal angle φ. A related problem of some importance is to analyze the effect of polarization of the incident radiation on the angular dependence of the backscattered flux density

$$J^{in}(\mu,\varphi;\mu_0)=(|\mu|/2\pi)(I_l^{in}(\mu,\varphi;\mu_0)+I_r^{in}(\mu,\varphi;\mu_0))$$

for $\mathrm{Re}\,\varepsilon<1$.

For the sake of definiteness, we consider an incident wave polarized perpendicular to the plane of incidence: $\hat{F}^{(n)}=0$, $\hat{F}^{(e)}\neq 0$, $E_{0\parallel}=0$. It can be shown that at grazing incidence, with $|\mu|$ and μ_0 both of order $|\varepsilon-1|^{1/2}\ll 1$, the main contribution to the reflected angular spectrum comes from simple scattering, and

$$J^{in}(\mu,\varphi;\mu_0)\approx\frac{3\tilde{\omega}_0E_{0\perp}^2}{64\pi^2}\frac{4k_\perp^2}{|k_\perp+\varkappa|^2}\frac{4k_{0\perp}^2}{|k_\perp+\varkappa_0|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}(\cos^2\varphi+|\gamma|^2). \tag{33}$$

Equation (33) makes it clear that in the narrow range of angles $\Delta\varphi\lesssim|\varepsilon-1|^{1/2}$ near the plane $\varphi=\pm\pi/2$, the scattered intensity is a factor $|\gamma^2|\ll 1$ less than in the plane of incidence $\varphi=0$. Furthermore, there is no anomalous peak in that range of angles (Fig. 1).

This effect arises because when the polarized radiation interacts with a small particle, the differential scattering cross section $d^2\sigma/d|\mu|d\varphi$ is proportional to the squared modulus of the vector product of the incident field $\mathbf{E}_{0\perp}$ and the wave vector of the scattered field: $d^2\sigma/d|\mu|d\varphi\sim|[\mathbf{E}_{0\perp},\mathbf{k}]|^2$.[17] For $|\mu|\sim|\varepsilon-1|^{1/2}\ll 1$, the vectors $\mathbf{E}_{0\perp}$ and $\mathbf{k}$ in the azimuthal plane $\varphi=\pm\pi/2$ are almost collinear, and the intensity of the radiation scattered in the direction ($\mu,\varphi=\pm\pi/2$) turns out to be much lower than the intensity scattered in the direction ($\mu,\varphi=0$).

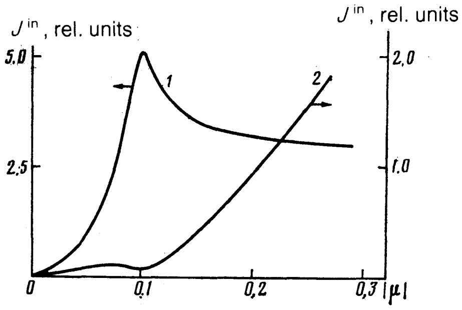

FIG. 1. Plots of the flux density $J^{in}(\mu,\varphi;\mu_0)$ of backscattered photons as a function of $|\mu|$ at $\varphi=0$ (curve 1) and $\varphi=\pi/2$ (curve 2). Parameters of the medium are $\mu_c=(1-\mathrm{Re}\,\varepsilon)^{1/2}=0.1$, $\mathrm{Im}\,\varepsilon=0.001$ ($\mu_0=0.9\mu_c$).

We see, then, that taking the polarization of the incident electromagnetic waves into consideration radically alters the azimuthal dependence of the reflected angular spectrum.

B. Unpolarized incident radiation: elliptically polarized resultant. The least trivial polarization effect due to scattering of refracted nonuniform electromagnetic waves is the appearance of an elliptically polarized component in the reflected spectrum upon grazing incidence ($\mu_0\lesssim|\varepsilon-1|^{1/2}$) of an unpolarized beam ($\hat{F}^{(e)}=0$). In fact, substituting Eq. (11) for $\hat{F}^{(n)}$ into (28), which defines the fourth Stokes parameter V^{in}, we find $V^{in}\neq 0$ over the entire range of photon exit angles $|\mu|\leqslant 1$:

$$\begin{aligned}V^{in}(\mu,\varphi;\mu_0)&=\frac{3k_0\tilde{\omega}_0F_0}{8\pi}\frac{4k_\perp}{|k_\perp+\varkappa|^2}\frac{4k_{0\perp}^2}{|k_{0\perp}+\varkappa_0|^2}\frac{\gamma'\gamma_0'}{\gamma'+\gamma_0'}\{-2\sin\varphi(1-\mu^2)^{1/2}\\&\times(1-\mu_0^2)^{1/2}h_5(\gamma',\gamma_0';\tilde{\omega}_0)\gamma_0''\\&+\gamma''\sin 2\varphi(1-|\gamma_0|^2)h_1(\gamma',\gamma_0';\tilde{\omega}_0)\}.\end{aligned} \tag{34}$$

An established result from classical radiation transport theory[4] is that $V^{in}=0$ for an unpolarized incident beam.[1)] An example shows physically why this is so. Let a linearly polarized wave be incident upon a medium. The corresponding refracted nonuniform wave is a superposition of elliptically polarized plane waves. Therefore, upon grazing incidence of an unpolarized beam, the corresponding radiation inside the medium is in general elliptically polarized. On the other hand, it is well known[4,5] that waves resulting from the scattering of elliptically polarized radiation by an isotropic scattering center are also elliptically polarized. Thus, the presence of an elliptically polarized component in the reflected radiation is entirely due to the fact that nonuniform waves are being scattered by inhomogeneities. There is no such effect in the classical radiation transport theory[4] where

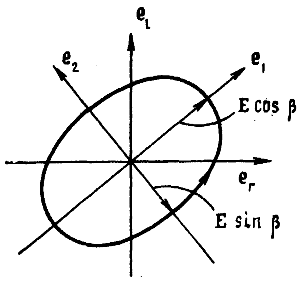

FIG. 2. The polarization ellipse for the polarized component.

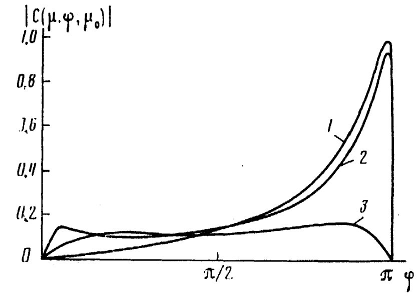

FIG. 4. Plots of ellipticity $|C(\mu,\varphi;\mu_0)|$ vs azimuthal angle φ for $|\mu| = 0.5\mu_c$ (curve 1), $|\mu| = 10\mu_c$ (curve 2), and $|\mu| = 50\mu_c$ (curve 3). Parameters of the medium: $\mu_c = (1 - \mathrm{Re}\ \varepsilon)^{1/2} = 0.1$; $\mathrm{Im}\ \varepsilon = 0.01$ $(\mu_0 = 0.9\mu_c)$.

nonuniform waves are not taken into consideration.

By virtue of the symmetry of the problem, scattered elliptically polarized waves in the plane of incidence $(\varphi = 0,\pi)$ cancel one another, and we have $V^{in} = 0$. For radiation not in the plane of incidence, this mutual cancellation does not take place, and the emergent radiation is either left-hand $(0 < \varphi < \pi)$ or right-hand $(\pi < \varphi < 2\pi)$ elliptically polarized).

Clearly, the more plane waves there are in the expansion of a nonuniform wave, the greater the value of V^{in}, i.e., the greatest effect obtains for a nonuniform wave when the direction of the $\varkappa$ component of the incident (scattered) wave vector normal to the surface is completely indeterminate: $\gamma''/\gamma' \gtrsim 1$. This inequality is satisfied for grazing angles $|\mu|$, $\mu_0 \lesssim |\varepsilon - 1|^{1/2}$ both for media that are optically lense dense than vacuum $(\mathrm{Re}\,\varepsilon < 1)$, and for strongly scattering (or absorbing) media $(\mathrm{Im}\,\varepsilon \gtrsim |1 - \mathrm{Re}\,\varepsilon|)$.

In general, an elliptically polarized component can come into being no matter what the relationship between the wavelength λ and the size a of an individual scatterer.

By way of example, we calculate the ellipticity $C(\mu,\varphi;\mu_0) = \sin 2\beta$ (Fig. 2) of the polarized component of incoherently reflected radiation described by the Stokes parameters

$$([(Q^{in})^2+(U^{in})^2+(V^{in})^2]^{1/2}, Q^{in}, U^{in}, V^{in}),$$

$$\begin{aligned}
C(\mu, \varphi; \mu_0) &= V^{in}[(Q^{in})^2+(U^{in})^2+(V^{in})^2]^{-1/2} \\
&= -4\sin\varphi[(1-\mu_0^2)^{1/2}(1-\mu^2)^{1/2}\gamma_0'' \\
&\quad -\gamma''\cos\varphi(1-|\gamma_0|^2)]\{[(1+|\gamma_0|^2) \\
&\quad \times(|\gamma|^2-1)+2(1-\mu^2)(1-\mu_0^2) \\
&\quad -4\cos\varphi(1-\mu^2)^{1/2}(1-\mu_0^2)^{1/2}(\gamma'\gamma_0'-\gamma''\gamma_0'') \\
&\quad -\cos 2\varphi(1-|\gamma_0|^2)(1+|\gamma|^2)]^2 \\
&\quad +16\sin^2\varphi[((1-\mu^2)^{1/2}(1-\mu_0^2)^{1/2}\gamma_0'' \\
&\quad -\gamma''\cos\varphi(1-|\gamma_0|^2))^2+((1-\mu^2)^{1/2}(1-\mu_0^2)^{1/2}\gamma_0' \\
&\quad +\gamma'\cos\varphi(1-|\gamma_0|^2))^2]\}^{-1/2}.
\end{aligned} \tag{35}$$

Equation (35) was derived by assuming simple (single) scattering. In Figs. 3 and 4, we have plotted $C(\mu,\varphi;\mu_0)$ as a function of azimuthal angle φ for various values of μ, corresponding both to media optically less dense than the vacuum $(\mathrm{Re}\,\varepsilon < 1, |1 - \mathrm{Re}\,\varepsilon| \gg \mathrm{Im}\,\varepsilon)$ and to strongly scattering (absorbing) media $(\mathrm{Im}\,\varepsilon \gtrsim |1 - \mathrm{Re}\,\varepsilon|)$.

Clearly, for small $|\mu| \sim |\varepsilon - 1|^{/12}$, the maximum value of $C(\mu,\varphi;\mu_0)$ is of order unity—i.e., the radiation scattered in that direction is almost circularly polarized.

Near grazing incidence $(|\mu| \lesssim |\varepsilon - 1|^{1/2})$, the forward-backward asymmetry in $C(\mu,\varphi;\mu_0)$ apparent in Figs. 3, 4 (in other words, asymmetry under the operation $\varphi \to \pi - \varphi$) is related to the fact that for $|\mu|, \mu_0 \sim |\varepsilon - 1|^{1/2}$, both the incident and scattered waves in the medium are nonuniform. In Eq. (34) for $V^{in}(\mu,\varphi;\mu_0)$, this leads to an additional interference term proportional to γ''. When the incoming radi-

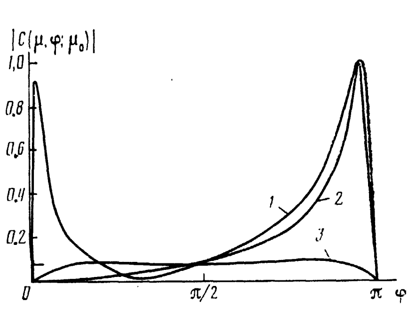

FIG. 3. Plots of ellipticity $|C(\mu,\varphi;\mu_0)|$ vs azimuthal angle φ for $|\mu| = 0.5\mu_c$ (curve 1), $|\mu| = \mu_c$ (curve 2), and $|\mu| = 5\mu_c$, (curve 3). Parameters of the medium: $\mu_c = (1 - \mathrm{Re}\ \varepsilon)^{1/2} = 0.1$; $\mathrm{Im}\ \varepsilon = 0.001$ $(\mu_0 = 0.9\mu_c)$.

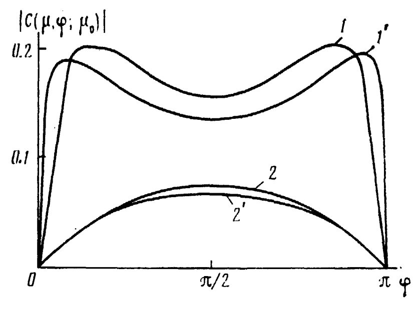

FIG. 5. Plots of ellipticity $|C(\mu,\varphi;\mu_0)|$ vs azimuthal angle φ for $|\mu| = 0.4$ (curves 1, 1′) and $|\mu| = 0.9$ (curves 2, 2′). Curves 1 and 2 have been calculated for the simple-scattering approximation, while 1′ and 2′ are based on the exact relations (25)–(28). Parameters of the medium: $\mu_c = (1 - \mathrm{Re}\,\varepsilon)^{1/2} = 0.1$; $\mathrm{Im}\,\varepsilon = 0.001$; $\tilde{\omega}_0 = 1$ $(\mu_0 = 0.5\mu_c)$.

ation is no longer at grazing incidence ($|\mu| \gg |\varepsilon - 1|^{1/2}$), both this term and its associated asymmetry disappear (Figs. 3, 4).

As the cosine of the exit angle $|\mu|$ increases, the simple-scattering approximation loses its applicability, and it becomes necessary to use the exact relations (25)–(28) to find the Stokes parameters of both the polarized and unpolarized components.

Making allowance for multiple scattering of photons in the medium results in the depolarization of the reflected radiation, and the contribution of the polarized component relative to the total intensity turns out to be smaller than in the simple-scattering approximation. This result is qualitatively the same as the analogous result in classical radiative transfer theory.[4]

The most important effect of multiple scattering is on the magnitude of the ellipticity $C(\mu,\varphi;\mu_0)$ of the reflected radiation. It can be shown that for intermediate exist angles, $|\mu| \approx 0.3$–0.6, multiple scattering changes $|C(\mu,\varphi;\mu_0)|$ markedly. In particular, the contribution made by multiply scattered waves has a strong influence on the polarization characteristics of the radiation scattered near the forward and backward directions: the ellipticity in the range of angles $|\varphi|$, $|\pi - \varphi| \lesssim \pi/6$, as calculated with the exact expressions (25)–(28), can be several times greater than the value of $|C(\mu,\varphi;\mu_0)|$ in the simple-scattering approximation.

As for the simple-scattering approximation, a comparison of numerical results for $C(\mu,\varphi;\mu_0)$ based on (25)–(28) and (35) demonstrates that it is valid in the limiting cases of grazing egress ($|\mu| \ll 1$) and near-normal egress ($1 - |\mu| \ll 1$).

To illustrate the foregoing discussion, Fig. 5 displays plots of the azimuthal dependence of $|C(\mu,\varphi;\mu_0)|$ for various μ, calculated both in the simple-scattering approximation and using the exact expressions (25)–(28).

CONCLUSION

In this paper, we have obtained a solution of the kinetic equation describing multiple scattering of polarized electromagnetic radiation in a disordered medium, making allowance for coherent reflection and refraction of waves at the boundary of the medium; this leads to the appearance of nonuniform waves. We have, for the case of Rayleigh scattering from an isolated center, calculated the intensity and polarization of the reflected radiation for arbitrary angles of incidence and egress from the medium. In contrast to the result obtained from classical radiation transport theory,[4] our solution suggests that there will be an elliptically polarized component in the reflected radiation, even when the incident beam is unpolarized. Our analysis implies that this effect is most easily observable in media that are either optically less dense than the vacuum ($\mathrm{Re}\,\varepsilon < 1$) or strongly scattering (absorbing) ($|1 - \mathrm{Re}\,\varepsilon| \lesssim \mathrm{Im}\,\varepsilon$). Finally, we have shown that allowing for the polarization of the incident radiation has a significant effect on the azimuthal dependence of the anomalous peak in the angular spectrum of the scattered photons.

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[1)] Note also that for an unpolarized beam at grazing incidence, there is no elliptically polarized component in the specularly reflected radiation.[17]

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