Multiple scattering of relativistic electrons in thin silicon single crystals: Theory and experiment

V. I. Kasilov, N. I. Lapin, V. I. Truten', and N. F. Shul'ga

Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR, Kharkov
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Predictions based on a model of randomly distributed chains of atoms in a crystal were tested experimentally by studying multiple scattering of relativistic electrons crossing the crystal at a small angle ψ relative to a crystallographic axis. It is shown that, in agreement with the model, in a wide range of angles $\psi \gtrsim \psi_c$ (ψ_c is the critical channeling angle) the scattering of particles in the crystal differs considerably from the same process in an amorphous medium and occurs mainly along an azimuthal angle φ. The distribution of the scattered particles with respect to φ is in satisfactory agreement with theoretical calculations.

1. INTRODUCTION

A fast charged particle moving in a crystal at a small angle relative to one of its crystallographic axes collides successively with different chains of atoms parallel to this axis. The motion of a particle in a crystal under these conditions is frequently described by a model of random chains in which collisions with different chains are assumed to occur at random (see, for example, Refs. 1–6). However, the chains of atoms with which a particle collides in a crystal are arranged periodically in the transverse plane, so that at first sight it might seem that such a structure does not exhibit any stochasticity and any motion should be regular and quasiperiodic. The reality is different: In the case under discussion the motion of a particle in a crystal can be both regular or stochastic and stochastization of the motion may be due to the action of random forces on a particle in a crystal (such as those associated with thermal vibrations of atoms and lattice defects) or due to dynamic chaos established when fast particles pass through the crystal when no random forces act on a particle.[7,8] Under chaotic motion conditions we can regard collisions of a particle with different atomic chains in a crystal as random.

The model of random chains simplifies greatly the analysis of many physical processes which accompany the passage of particles through a crystal, such as multiple scattering, emission of radiation, photogeneration of electron–positron pairs, development of electromagnetic showers, etc., so that in analyzing these processes we must know to what extent the basic assumptions of the model and its predictions are in agreement with experiments. We tackle this problem below. We report experimental angular distributions of electrons of energies $E = 158$, 190, 250, and 265 MeV transmitted by silicon single crystals which were 10 and 20 μm thick. Our experiments were carried out in order to compare quantitatively the predictions of the model of random chains with the experimental data obtained in a study of multiple scattering of electrons in a crystal.

It should be pointed out that the published experimentally determined[9] angular distributions of particles (electrons and positrons with energies of the order of 1 GeV) scattered by a crystal are in qualitative agreement with the results deduced from the random chain model. However, these data are insufficient for a quantitative comparison of the theory and experiment. This is due to the fact, according to Ref. 9, that the crystal was misoriented in a close-packed crystallographic plane, i.e., under conditions when the motion of a particle could be either regular or chaotic. Moreover, quantitative angular distributions of the particles were not obtained in Ref. 9

The experiments reported below were carried out under conditions which made it possible to compare the theoretical and experimental data.

2. EXPERIMENTS

The angular distribution of electrons scattered by a crystal was determined using the Kharkov LU-300 linear accelerator. An electron beam of 1×10^{-4} rad divergence and transverse dimensions of at most 2×2 mm was transmitted by silicon crystals 10 and 20 μm thick placed on a goniometer in a vacuum chamber. These crystals were oriented relative to the $\langle 111 \rangle$ axis on the basis of secondary electron emission.[10] Transmitted electrons were recorded using glass plates located in a cassette 4.3 m from a crystal at right-angles to the incident beam. The optimal electron dose Q was 2×10^{13} cm^{-2}. Figure 1 shows the patterns of the scattered electrons obtained for different ratios of the angle of tilt of the electron beam from the $\langle 111 \rangle$ crystallographic axis to the critical axial channeling angle ψ_c.

An analysis of the patterns was carried out conveniently using a system of polar coordinates (ρ,φ) in the plane of the glass plate and centered at the point where this plane is intersected by the axis of the crystal; the angle φ was measured from the point where the plane is intersected by the axis of the initial electron beam.

Figure 1 illustrates the fact[4] that the scattering of electrons by a crystal at low values of ψ and high E is mainly along an azimuthal angle φ forming a characteristic ring-shaped structure. This type of particle scattering is due to a coherent interaction of above-barrier particles with different atomic chains in a crystal, i.e., due to the scattering of a particle in the field of a continuous potential along the selected crystallographic axis. The scattering along the radius ρ is due to incoherent interaction with inhomogeneities of the crystal potential.

The experimental conditions were selected so that the incoherent scattering was much weaker than the coherent process. Patterns formed by the scattered electron beam were recorded photographically and analyzed with a micro-

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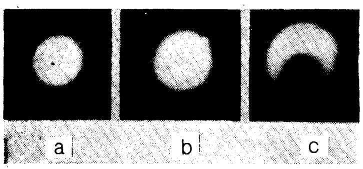


FIG. 1. Patterns formed by a beam of electrons transmitted by a crystal, obtained for different values of the angle ψ: a) $\psi=0$; b) $\psi=0.5\psi_c$; c) $\psi=\psi_c$.

photometer in directions specified by the angle φ.

The results of such photometric analysis, obtained for a given value of φ and integrated along the radius ρ, yielded the distribution $f(\varphi)$ of the particles scattered through an angle φ. They agreed with the distribution of the coherent part of the multiple scattering process. The distribution functions $f(\varphi)$ of an electron beam transmitted by a crystal obtained in this case are shown in Fig. 2.

An important quantity which determines the scattering of fast particles by a crystal is the mean-square value of the multiple scattering angle. The coherent part of this angle is given by

$$\overline{\theta^2}=4\psi^2\int_{-\pi}^{\pi} d\varphi\, f(\varphi)\sin^2\frac{\varphi}{2}, \tag{1}$$

if we know the coherent part of the particle distribution in terms of the azimuthal angle φ. Figure 3 shows the results of a determination of the orientational dependence of $\overline{(\theta^2)}^{1/2}$ on ψ/ψ_c obtained for different electron energies.

All the measurements were made under conditions which allowed us to carry out a quantitative analysis of the experimental results and to compare them with the theoretical predictions. In our experiments we used particles of lower energy and crystals of smaller thickness than in Ref. 9, so that we were able to reduce the contribution of the incoherent scattering compared with the coherent process. This was done in the range of angles $\psi\sim\psi_c$ and increased the precision of the measurements.

The crystal was misoriented well away from close-packed atoms on the crystallographic planes so that the influence of regular particle motion on angular distribution along such planes was weak.

We investigated silicon crystals oriented with the $\langle 111\rangle$ axis along the beam. In this case a theoretical analysis of the experimental data on the basis of the random chain model made it possible to ignore the asymmetry of the potential

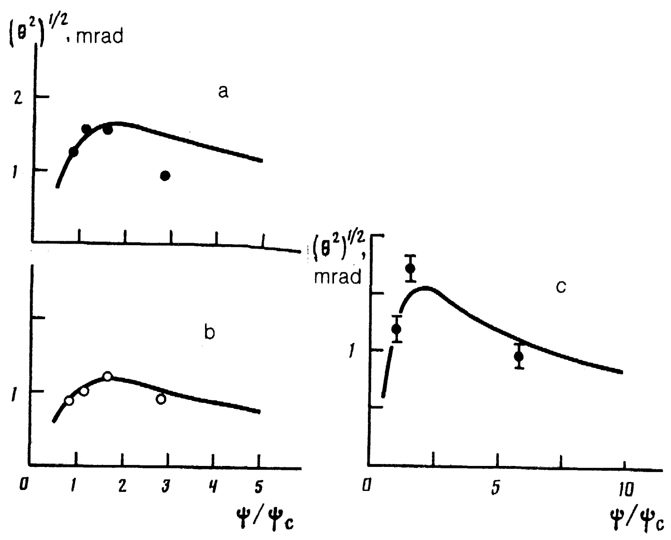


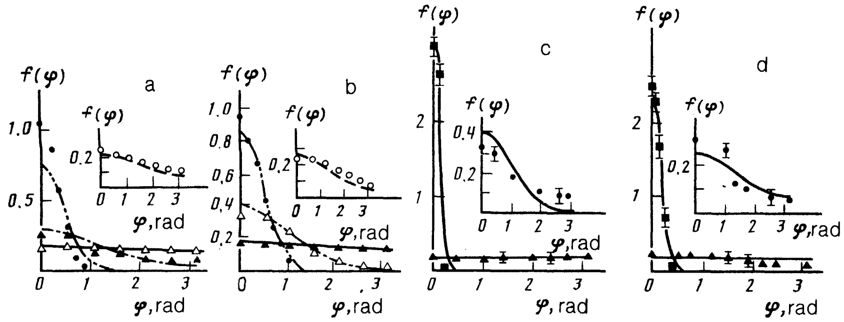
FIG. 3. Orientational dependences of the mean-square angle of multiple scattering of a beam of electrons in silicon crystals of thickness 10 μm [$E=158$ MeV (a) and 265 MeV (b)] and 20 μm [$E=250$ MeV (c)]. The experimental values are identified by points and the continuous curves represent the results of theoretical calculations.

with greater justification than in the analysis of the experimental data in Ref. 9, where the crystal orientation was relative to the $\langle 110\rangle$ axis.

The attention in our experiments was concentrated on determination of the angular distributions of the scattered particles in the range $\psi\gtrsim\psi_c$. Under these conditions and for our electron energies and crystal thicknesses we could ignore the contribution made by the particle channeling to the scattering process.

3. RANDOM CHAIN MODEL

The angular distribution of the crystal-scattered fast particles incident at a small angle ψ on one of the crystallographic axes is governed by the characteristics of the interaction of a particle with a single atomic chain and by the features of multiple scattering on different atomic chains. In the investigated case of low ψ and high E the scattering of a particle by a single atomic chain occurs mainly along the azimuthal angle φ in a plane orthogonal to the chain axis. Multiple scattering by different chains results in redistribution of the particles in the angle φ. In the random chain model the collisions of a particle with different chains are assumed to be random, whereas the potential for each chain



IG. 2. Azimuthal distributions of the coherent contribution to multiple scattering of electrons of different energies in silicon crystals: a) crystal thickness 10 μm, $E=158$ MeV, $\psi/\psi_c=2.85$ (●), 1.48 (▲), 1.14 (○), and 0.85 (△); b) same crystal, $E=265$ MeV, $\psi/\psi_c=2.83$ (●), 1.66 (△), 1.16 (○), and 0.87 (▲); c) crystal thickness 20 μm, $E=190$ MeV, $\psi/\psi_c=10$ (■), 2.2 (●), 0.7 (▲); d) same crystal, $E=$ MeV, $\psi/\psi_c=5.8$ (■), 1.5 (●), 1.0 (▲). The continuous curves are calculated theoretically.

is assumed to be cylindrically symmetric. Then, if we ignore the noncoherent particle scattering processes, the distribution of the scattered particles with respect to the angle φ at a depth z is given by the equation[2-4]

$$\frac{df(\varphi,z)}{dz}=n\,d\psi\int_{-\infty}^{\infty}db[f(\varphi+\varphi(b),z)-f(\varphi,z)],\qquad(2)$$

where n is the density of atoms in a crystal; d is the distance between the atoms in a chain; and $\varphi(b)$ is the azimuthal scattering angle for a single atomic chain when the impact parameter of a chain b is given.

The solution of this equation subject to the boundary condition $f(\varphi,0)=\delta(\varphi)$, where $\delta(\varphi)$ is the delta function, is

$$f(\varphi,z)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\cos(k\varphi)\exp\left\{-n\,dz\,\psi\int_{-\infty}^{\infty}db[1-\cos(k\varphi)]\right\}.\qquad(3)$$

Knowing $f(\varphi,z)$, we can find the square of the scattering angle of a particle by a crystal in accordance with Eq. (1):

$$\overline{\theta^2}=2\psi^2\left\{1-\exp\left[-2ndz\psi\int_{-\infty}^{\infty}db\sin^2\frac{\varphi(b)}{2}\right]\right\}.\qquad(4)$$

It follows from the above expressions that at large values of the angle ψ ($\psi\gg\psi_c$), when the argument of the exponential function in Eq. (3) is small, the angular distribution of the particles is Gaussian:

$$f(\varphi,z)=(2\pi\overline{\varphi^2})^{-1/2}\exp(-\varphi^2/2\overline{\varphi^2}),\qquad(5)$$

where $\overline{\varphi^2}$ is the mean-square value of the azimuthal angle φ.

In the angular range $\psi\lesssim\psi_c$ the argument of the exponential function in Eq. (3) is large and in this case the distribution of the particles in the angle φ becomes uniform:

$$f(\varphi,z)=1/2\pi.\qquad(6)$$

The results of calculations of the functions $f(\varphi,z)$ and $\overline{\theta^2}$ using Eqs. (3) and (4) for the values of E, z, and ψ employed in our experiments are represented by the continuous curves in Figs. 2 and 3. In these calculations we used the following approximation for the continuous potential of a chain of atoms in a silicon crystal:

$$U(\rho)=0,\quad \rho>R_{max},$$
$$U(\rho)=U_1\ln[1+cR^2/(\rho^2+\Delta^2)]-U_2,\quad \rho\leqslant R_{max},\qquad(7)$$

where $R_{\max}=a/24^{1/2}$, a is the lattice constant, R is the screening radius, $U_1=59$ eV, $U_2=3$ eV, $c=2$, $\Delta=1.4\times10^{-18}$ cm^2.

4. DISCUSSION OF RESULTS

It follows from our results that in the investigated range of particle energies and crystal thicknesses in a wide interval of misorientation angles of the crystal axes, relative to the incident beam, we can confirm the main assumptions and predictions of the random chain model. In fact, the above data considered in combination with a random chain model demonstrate the following.

1) In a wide range of angles ψ the scattering of particles occurs mainly along the azimuthal angle φ.

2) The distribution of the scattered particles with respect to φ is a smooth function of this angle, i.e., this distribution does not show clear regions corresponding to the regular motion of particles (anomalies).

3) There is a satisfactory agreement between the theoretical and experimental results.

We can thus see that the passage of fast electrons through a crystal may result in conditions under which collisions of a particle with different atomic chains can be regarded as random, in spite of the fact that in reality these chains are distributed periodically in a crystal.

We note that some investigations of the physical processes that accompany the transmission of high-energy electrons through crystals are made on the assumption that multiple scattering of particles in a crystal occurs in the same way as in an amorphous medium. In particular, this hypothesis is used in Ref. 12 in an analysis of the process of rechanneling of particles incident on a crystal at angles $\psi>\psi_c$ and also in explaining the ratios of the contributions to the radiation emitted by channeled and above-barrier particles. An analogous assumption is used frequently also in an analysis of the process of coherent emission from relativistic electrons (see, for example, Ref. 13).

It should be noted in this connection that the very first experimental data on the angular distribution of the particles scattered by a crystal[9] and the results of the present experiments show that the distribution of the particles scattered by a crystal differs considerably from those scattered by an amorphous medium, namely in a wide range of angles ψ this distribution is strongly asymmetric and the average values of the scattering angles in the azimuthal direction are much larger than the average angles in the transverse direction (the latter are of the same order of magnitude as the average angles for multiple scattering in an amorphous medium). Clearly, an analysis of these processes should be carried out on the basis of the actually observed relationships governing the scattering of particles by a crystal. Allowance for these relationships may lead to results which are very different from those obtained on the assumption that multiple scattering of particles in a crystal occurs in the same way as in an amorphous medium.

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