Quasi-one-dimensional electron systems in semiconductors

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The equilibrium and simplest dynamical properties of a system of electrons localized in a parabolic channel are described systematically for a channel which is substantially filled, i.e., for $l^* \gg 1$, where l^* is the maximum number of levels in the channel located below the Fermi level. Expressions are derived for the equilibrium characteristics of the channel produced by a parabolic confining potential $V(x)$. The possibility is discussed of retaining a parabolic shape with renormalized constants for the effective potential $\tilde{V}(x)$ (which quantizes one-electron motion in the channel). The sensitivity of various components of the quasi-one-dimensional channel conductivity to the explicit form of the potential $V(x)$ is investigated. Some features of Shubnikov–de Haas oscillations for a parabolic channel in a magnetic field perpendicular to the channel plane are described. The spectrum of transverse plasma oscillations in a parabolic channel is determined. The results of the calculations are used to interpret the various observed properties of quasi-one-dimensional electron channels.

One of the latest achievements in the area of creation of low-dimensional conducting systems is the preparation of periodic quasi-one-dimensional channels with a controllable set of parameters. A schematic view of such a construction is depicted in Fig. 1. A regular heterostructure, for example the GaAs type, is "covered" by a periodically modulated metallic gate (in Fig. 1 the gate consists of a periodic system of metallic strips). Moreover, using an external potential V_g between the gate and the $2d$ electron layer corresponding to extraction of electrons from the potential well, it is possible to achieve modulation of the electron density in this system in the x-direction within within very wide limits, up to a disruption of the continuity of the $2d$ electron density and the appearance of a system of quasi-one-dimensional electron channels conducting only in one direction (the y-direction). A series of experiments carried out recently on similar systems[1–4] have demonstrated the high quality of the proposed structures and the nontrivial behavior of the basic dynamical characteristics of $1d$ channels in constant and varying electric fields. Some of the questions that arise, dealing with the details of interpretation of the corresponding data, up to now do not have a definitive answer.

From the point of view of theory, the above quasi-one-dimensional channels, artificially generated by external restoring forces, are an ideal object for demonstrating the substantial role of electron-electron interactions in the formation of the properties of similar channels. This was first clearly shown by Laux *et al.*[5], who considered, using the model of Fig. 1, the equilibrium characteristics of a single channel as a function of the geometry of the problem and the gate potential V_g. These mainly numerical calculations show that the electron density $n(x)$ in a channel is nonuniform in the x-direction, the width $2a$ of a channel is essentially a function of the potential V_g, and the quantization spectrum for single-electron motion in the x-direction is very sensitive to the total number of electrons N_L in the channel per unit length.

The goal of the present work is a self-consistent description of the simplest equilibrium and dynamical properties of a single quasi-one-dimensional channel, allowing for full Coulomb interaction between electrons collected in the channel. As one of the initial prerequisites, we assume that the confining potential has the parabolic form

$$V(x) = V_0 + {}^1/_2 kx^2 \qquad (1)$$

with constants V_0 and k taken as parameters of the theory. The reasonable nature of such an approximation for $V(x)$ in the limit $a \ll L$, where L is the distance between neighboring metallic plates of the gate, follows from general considerations on the structure of the electrostatic fields far from the

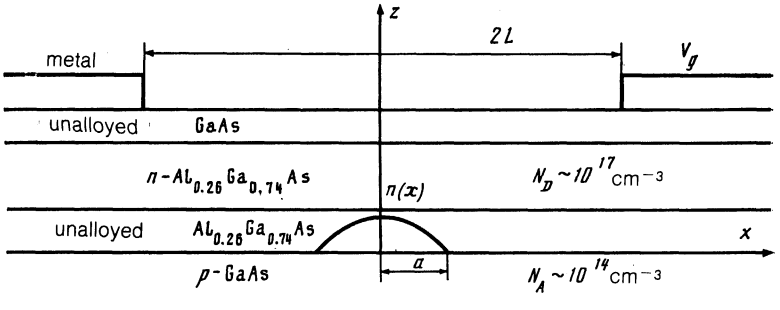

FIG. 1. A design for a device that permits creation of quasi-one-dimensional electron channels with a controllable parameter set. Electrons occupy the channel along the strip $-a \leqslant x \leqslant a$. The potential difference V_g is applied between the metallic layers and the channel, with a width of $2a$.

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sources of these fields, and is also confirmed by numerical calculations.[5] To this accuracy (that is, to lowest order in the parameter $a/L \ll 1$) the electron-electron interaction in the channel can be considered unscreened.[1)] The influence of neighboring channels will be taken into account in perturbation theory after elucidation of the fundamental properties of a single channel. A magnetic field H, which substantially enlarges the possibilities of experiment, is directed along the z-axis.

The results of this study are explained in the following. First we discuss the equilibrium properties of a channel in a confining potential $V(x)$ described by Eq. 1 (a parabolic channel), partially reproducing the numerical results of Ref. 5 under conditions when the channel contains many electronic subbands beneath the Fermi level. The possibility, in this limiting case, of obtaining analytical expressions for the basic characteristics of the channel is very advantageous for further progress in describing the dynamical properties of a parabolic channel. The second section is devoted to a discussion of the details of dc conductivity of a channel in the ballistic regime. This problem, studied in detail in Refs. 6–8, is very sensitive to the real form of the confining potential; as will be seen below, arguments arise in favor of the parabolicity of a real channel, the properties of which were studied in Refs. 6 and 7. Finally, the third section of this work contains information on plasma oscillations in a parabolic channel. Results obtained here are very useful in interpreting the experimental data on the excitation of plasma oscillations in quasi-one-dimensional periodic systems.[1–4]

1. EQUILIBRIUM PROPERTIES OF A PARABOLIC CHANNEL

A. We will first study the classical variant of the problem of the equilibrium of a system of electrons with a total density N_L in a parabolic channel $V(x)$ (Eq. 1) at zero temperature. As is well known, in this case the electrostatic problem requires solution of the integral equation relating to the equilibrium density $n(x)$ of electrons. The equation itself arises from the condition that the total chemical potential be constant along a channel "loaded" with electrons. In explicit form this condition looks like:

$$V(x)+e\varphi(x)=\mu, \tag{2}$$

where μ is the position of the Fermi level, or

$$V_0+\frac{1}{2}kx^2+\frac{e^2}{\varkappa}\int_{-a}^{a} n(s)\ln\frac{L}{|x-s|}\,ds=\mu, \quad \int_{-a}^{a} n(s)\,ds=N_L. \tag{2a}$$

Here $\varphi(x)$ is the potential due to Coulomb interaction of the electrons; $\varkappa$ is the dielectric constant of the semiconductor in which the electronic system is "embedded"; $2a$ is the width of the channel; the parameters V_0 and k, as noted above, are considered to be specified functions of V_g; and the length L entering into the argument of the logarithm takes account of the screening action of the metallic electrodes.

The solution of Eq. (2a) for $n(x)$ looks like (see, for example, Ref. 9):

$$V_0+\frac{e^2N_L}{2\varkappa}\ln\frac{2L}{a}=\mu, \tag{3}$$

$$n(x)=\frac{2N_L}{\pi a}\left(1-\frac{x^2}{a^2}\right)^{1/2}, \quad a^2=\frac{2e^2N_L}{k\varkappa}. \tag{4}$$

The relation (3) demonstrates that the density N_L depends weakly on the real form of the potential $V(x)$ (this dependence is contained only in the argument of the logarithm through the channel width a). A similar property holds also for channels arising in a longitudinal confining potential $V(x)$. In this case relations (3) and (4) have the form

$$\frac{1}{\pi}\int_{-a}^{a}\frac{V(s)\,ds}{(a^2-s^2)^{1/2}}+\frac{e^2N_L}{2\varkappa}\ln\frac{2L}{a}=\mu, \tag{3a}$$

$$\frac{1}{\pi}\int_{-a}^{a}\frac{dV}{ds}\,s\,(a^2-s^2)^{-1/2}\,ds=N_L. \tag{4a}$$

B. The simplest way to evaluate the quantum corrections to the definitions (3) and (4) is to use the Thomas-Fermi approximation, which gives in place of the equilibrium condition (2) a generalization accounting for the energy of the zero-point oscillations of the electrons in the channel[10]:

$$V(x)+e\varphi(x)+\frac{\pi\hbar^2}{2m^*}n(x)=\mu, \quad \int_{-a}^{a} n(s)\,ds=N_L. \tag{5}$$

Here m^* is the electronic effective mass.

An analytical solution of Eq. (5) has not yet been found. In this connection it is reasonable to analyze the possibility of solving it approximately in the quasiclassical regime. Looking at the definition (4) of $n(x)$, it is not hard to see that the integral term $e\varphi(x)$ in (5) is of order $(e^2N_L/\varkappa)\ln(L/a)$. As for the term with the energy of the zero-point oscillations, an estimate gives $\hbar^2(m^*)^{-1}n(x)\sim\hbar^2N_L/m^*a$. Evidently the Coulomb energy of the electron system exceeds the energy of their zero-point oscillations if the inequality

$$a\gg a_B, \quad a_B=\varkappa\hbar^2/m^*e^2, \tag{6}$$

is fulfilled; here a_B is the Bohr radius.

In the region given by (6) the energy of the zero-point oscillations can be considered in perturbation theory (an approximation sufficient for the discussion below). The effective potential $\tilde{V}(x)$ quantizing the electron motion in the x-direction looks like

$$\tilde{V}(x)\equiv V(x)+e\varphi(x)=\mu-(\pi\hbar^2/2m^*)n(x). \tag{7}$$

The total number of levels l^* in the quasiclassical well $\tilde{V}(x)$ of (7) is equal to

$$l^*=\frac{(m^*)^{1/2}}{2^{1/2}\pi\hbar}\int_0^a[\mu-\tilde{V}(x)]^{1/2}\,dx=\frac{\Gamma(5/4)}{\Gamma(7/4)}(N_La)^{1/2}, \tag{8}$$

where $\Gamma(x)$ is the gamma function and $l^*\gg 1$.

C. A more systematic way to evaluate quantum corrections to the definitions (3) and (4) has the starting point of assuming classical motion of electrons in the field of $\tilde{V}(x)$. The electronic spectrum $\varepsilon(\mathbf{p},x)$ is given by the expression

$$\varepsilon(\mathbf{p},x)=(1/2m^*)(p_x^2+p_y^2)+\tilde{V}(x), \tag{9}$$

where

$$\tilde{V}(x)=V_0+V_c^0+{}^1/_2\tilde{k}x^2, \quad \tilde{k}=k-2N_Le^2/\varkappa a^2,$$
$$V_c^0=(e^2N_L/2\varkappa)\ln(2L/a), \quad -a\leqslant x\leqslant a. \tag{9a}$$

The potential $\tilde{V}(x)$ has the form (9a) if the density $n(x)$ is determined by expression (4), with the parameter a remaining free.

Using the spectrum $\varepsilon(\mathbf{p},x)$ of (9) and the general definition of the total number of electrons N_L localized in the channel, we have

$$N_L=g\int_{-\infty}^{\infty}\frac{d\mathbf{p}}{(2\pi\hbar)^2}\int_{-a}^{a}dx\left\{\exp\left[\frac{\mathbf{p}^2/2m^*+\tilde{V}(x)-\mu}{\Gamma}\right]+1\right\}^{-1}, \tag{10}$$

where g is the g-factor. Integration over $d\mathbf{p}$ gives ($g=2$)

$$N_L=\frac{2m^*}{\pi\hbar^2}\left\{\int_{-a}^{a}[\mu-\tilde{V}(x)]dx+T\int_{-a}^{a}\ln\left[1+\exp\left(\frac{\tilde{V}(x)-\mu}{T}\right)\right]dx\right\}, \tag{11}$$

$$\tilde{V}(a)-\mu=0. \tag{11a}$$

In (10) and (11) the natural limits of integration over x are used: $-a\leqslant x\leqslant a$.

The temperature-dependent integral in (11) can be evaluated approximately, considering that in the region of integration over x we have $\tilde{V}(x)-\mu\leqslant 0$. As a result,

$$\frac{1}{a}\tilde{k}a^2=\delta\mu,\quad \delta\mu=\mu-V_0-\frac{e^2N_L}{2\varkappa}\ln\frac{2L}{a}, \tag{12}$$

$$N_L=\frac{2m^*}{\pi\hbar^2}\left[\frac{11}{12}a\delta\mu+T^2\left(\frac{2}{\tilde{k}\delta\mu}\right)^{1/2}f\left(\frac{\delta\mu}{T}\right)\right], \tag{12a}$$

where

$$f(x)=\int_0^x\frac{dse^{-s}}{(1-s/x)^{1/2}}=\begin{cases}1, & x\gg 1,\\ 2x, & x\ll 1.\end{cases}$$

Here the relations (12) are identical to (11a).

In the case $\delta\mu/T\gg 1$ of practical interest the system (12), (12a) reduces to the following equations for a and N_L:

$$\left(k-\frac{2e^2N_L}{\varkappa a^2}\right)a^3=\frac{6}{11}\frac{\pi\hbar^2}{m^*}N_L, \tag{13}$$

$$\frac{6\pi\hbar^2}{11m^*}N_L=a\delta\mu\equiv a\left(\mu-V_0-\frac{e^2N_L}{2\varkappa}\ln\frac{2L}{a}\right). \tag{13a}$$

It is apparent that in the limit $\hbar\to 0$ or $m^*\to\infty$ the definitions of a and N_L tend to the classical expressions for a (Eq. 4) and N_L (Eq. 3). As for the evaluation of l^*, in the given case

$$l^*\approx\delta\mu/\hbar\tilde{\omega}\approx(N_La)^{1/2},\quad \tilde{\omega}^2=\tilde{k}/m^*, \tag{14}$$

which coincides with the expression for l^* in Eq. 8.

The formalism of paragraph C, section 1 can be called the self-consistent Thomas-Fermi approximation. On this basis, as well as (5), rests the assumption of the absence of quantization of the electronic spectrum (9). All information on the quantization process appears in the final expression only through the Fermi statistics for electrons; that is, the use of the definition (10) for N_L. The principal feature of this form of the theory, compared with the canonical approximation of Eq. 5 and Eq. 7, is the demonstration that it is possible to keep parabolic approximation to the potential $\tilde{V}(x)$ in Eq. 9a with effective constants $\tilde{V}_0$ and $\tilde{k}$ which account for Coulomb renormalization [as opposed to the definition of $\tilde{V}(x)$ of Eq. 7, where a deviation from parabolicity arises immediately]. To account for anharmonicity in the self-consistent Thomas-Fermi approximation we extend the definition of $n(x)$:

$$n(x)=n_0(1-x^2/a^2)^{1/2}[U_0(x)+n_2U_2(x)+\ldots] \tag{15}$$

[$U_1(x)$ is the appropriate Chebyshev polynomial], and define an additional self-consistency condition which together with (11a) would allow calculation of the coefficients n_1 in the expansion (15). This program has not yet been carried out.

2. CONDUCTIVITY OF A CHANNEL IN THE BALLISTIC REGIME

In approaching the problem of the conducting properties of a one-dimensional channel it is necessary to recall that a complete theory of this phenomenon for long channels with a finite mean free path for one-electron excitations must incorporate the body of knowledge in this area formulated by different authors[11-13] in the 1970s. The collisionless regime of electron motion along the channel, discussed below, is trivial from the viewpoint of this theoretical approach, but it is optimally suited to the manifestation of the Coulomb features of the problem, not arising from dissipation.

We note that the problem of the conductivity σ of a quasi-one-dimensional channel in the ballistic approximation is applicable to experiments[6,7] already discussed in detail in a series of interesting studies,[6,8,14] using, in part, a square-well form of the confining potential $V(x)$. But, in accordance with the results of the preceeding section, in the square-well model, difficulties arise in determining the connection between the width $2a$ of the channel and the quantity N_L. In fact, for a right-angle channel the derivative dV/dx has the form of a δ-function at the ends of the interval $x=\pm a$. In this case the general relation (4a) giving the desired connection between a and N_L is divergent. Any continuous potential $V(x)$, including the simplest parabolic approximation (1), avoids the question of this divergence, but at the same time eliminates the perturbation theory explicitly used in Refs. 8 and 14. Thus, the choice arises: the square-well potential $V(x)$, already recommended in the description of certain features in the behavior of the conductivity σ, or the smooth (specifically, parabolic) potential $V(x)$, allowing one to naturally tie the channel characteristics with the geometry of the problem and the potential V_g (see Refs. 5 and 15) and also to analyze the properties of the plasma oscillations (see section 3). Obviously, it is necessary to evaluate the degree of sensitivity of the properties of σ to the details of the dependence of $V(x)$.

A. The most interesting observed feature of σ is its discontinuous growth with increasing V_g. The value of a single jump $\Delta\sigma=e^2/h$ does not depend on the number of steps.[6,7] Calculation of σ in the ballistic regime for an electronic spectrum with an arbitrary discrete part ε_l shows that

$$\sigma=\frac{e^2}{h}\sum_l\left[1+\exp\left(\frac{\varepsilon_l-\mu}{T}\right)\right]^{-1},\quad \Delta\sigma=\frac{e^2}{h}. \tag{16}$$

For a square-well potential $V(x)$ the determination of σ in the form (16) is carried out in Refs. 6 and 8. Thus, the discontinuous behavior of σ and the universality of the jump $\Delta\sigma=e^2/h$ are quite general properties of σ in the ballistic regime, independent of the explicit form of $V(x)$.

Number of point	V_g, V	$V_g - V_g^m$, V, $V_g^m = -1.54$ V	N_L, 10^6 cm^{-1}	a, 10^{-6} cm	k/k_0	N_L/N_L^0
0	−1.46	0.08	0.75	3.5	1.0	1.0
1	−1.41	0.13	1.2	4.7	0.92	1.6
2	−1.35	0.19	1.75	5.9	0.85	2.3
3	−1.3	0.24	2.2	6.6	0.83	2.9

B. The fact that the number of steps in the $\sigma(V_g)$ dependence grows approximately linearly as a function of $V_g - V_g^m$, where V_g^m is the maximum value of V_g for which the channel still displays conducting properties,[6] permits a constructive discussion in the framework of the parabolic model.[2)] This number of steps can be established in correspondence with the maximum number l^* of electronic subbands in the channel occupied by electrons. Thus, it follows from Ref. 6 that

$$l^*_{\text{exp}} \propto V_g - V_g^m. \tag{17}$$

The theoretical value of l^* in the parabolic approximation is given by Eqs. (8) or (14). The behavior of $l^*(V_g)$ amounts to a dependence of the values of N_L and a on $V_g - V_g^m$. If we note that the classical channel parameters $n(x)$ and a from (4) qualitatively reproduce the numerical calculations[5] [in both cases the density $n(x)$ goes to zero for $x = \pm a$, is inhomogeneous over the interval $|x| < a$, and has no appreciable plateau in the central part of the distribution; the width a of the channel grows with N_L] then we can assume that in the initial stages of filling the channel with electrons (up to numbers $l \lesssim 10$) the potential $V(x)$ in Ref. 5 is basically parabolic. In this connection it is natural to use the relationship between N_L and $V_g - V_g^m$, found numerically in Ref. 5: $N_L \propto V_g - V_g^m$. Besides, working with the data of Figs. 3 and 4 from Ref. 5, with the aid of the "parabolic" formulas (4), we find at several points along V_g the curvature k (see Table I). The number of points in this table is not large, but it is obvious that the relative change in N_L as a function of V_g is noticeably larger than the corresponding change in k. Thus, the dependence of $l^*(V_g)$ that interests us is basically determined by the behavior of $N_L(V_g)$. Taking account of the definition of l^* in (8), the width $2a$ in (4) and the linear connection between N_L and $V_g - V_g^m$, we have

$$l^*_{\text{theor}} \propto (V_g - V_g^m)^\lambda, \quad \lambda \geqslant 3/4. \tag{18}$$

The "greater than" sign in the evaluation of λ takes into account the contribution to the $l^*(V_g)$ dependence of the change of the channel curvature k with increasing $V_g - V_g^m$.

The result (18) is close to the experimental behavior of l^* in (17), although it does not coincide. To eliminate the difference between (18) and (17) we might possibly consider that the real behavior of $N_L(V_g)$ from Ref. 5 slightly exceeds linearity. Besides, it is completely probable that there is an influence on the result (17) due to inhomogeneity of the channel along the current direction, which takes place in experiments.[6,7]

C. A few words on the conductivity in a magnetic field H perpendicular to the plane of the channel follow. First of all, in a parabolic channel with an effective curvature $\tilde{k}$ from (9a), all electronic states are nondegenerate and delocalized in an $H \neq 0$. The corresponding spectrum

$$\begin{gathered}\varepsilon_l(p_y) = V_0 + V_c^0 + \hbar\tilde{\omega}_c(l+1/2) + p_y^2/2M, \\ \tilde{\omega}_c^2 = (k_c+\tilde{k})/m^*, \quad k_c = m^*\omega_c^2, \quad \omega_c = eH/m^*c, \\ M^{-1} = (m^*)^{-1}[1 - k_c/(k_c+\tilde{k})]\end{gathered} \tag{19}$$

resembles the spectrum $\varepsilon_l(p_y) = \varepsilon_l + p_y^2/2m^*$ of an electron in a channel without a magnetic field, with the substitution $m^* \to M$. This allows us to use, in obtaining the conductivity of a magnetized parabolic channel, the kinetic language, as, for example, is done in Ref. 16. As a result it turns out that the oscillating part $\delta\sigma(H)$ of the conductivity has the form

$$\begin{gathered}\delta\sigma \propto \exp(2\pi i \delta\mu_H/\hbar\tilde{\omega}_c), \quad \delta\mu_H = \mu - V_0 - V_c^0 - 1/2\hbar\tilde{\omega}_c, \\ \delta\mu_H = [3\pi\hbar^2\tilde{\omega}_c N_L/(2M)]^{2/3}.\end{gathered} \tag{19a}$$

The presence in the argument of the oscillatory exponent for $\delta\sigma$ of the combination frequency $\tilde{\omega}_c$ allows us to justify the phenomenological calculations of oscillations proposed in Ref. 17. However, the dependence on magnetic field is also contained in the factor $\delta\mu_H$—a fact not taken into account up to now in studying Shubnikov–de Haas oscillations in quasi-one-dimensional channels.

Thus, the parabolic approximation is acceptable for solving practically all the qualitative questions arising in the discussion of the dc conductivity properties of one-dimensional channels with a controllable parameter set.

3. PLASMA OSCILLATIONS

The spectrum of the plasma oscillations in two-dimensional finite systems without dynamical screening has hardly been studied up to the present. The standard evaluation of the eigenfrequencies

$$\omega_1^2 = 2\pi e^2 n_s/\varkappa m^* L, \tag{20}$$

where n_s is the average electron density in the channel, and L is its width, makes sense only as an estimate for $2d$ systems with a sharp profile and stops being valid even qualitatively in situations when the electron density profile is inhomogeneous in the whole width of the channel, as happens, for example, in a parabolic channel [see the definition of $n(x)$ in Eq. 4].

Besides the question of the real value of the eigenfrequencies of the plasma oscillations in a channel with an arbitrary distribution $n(x)$, a problem arises in the applicability of perturbation theory in calculating these frequencies. The density oscillations in the electronic channel with mobile boundaries should give rise to a displacement of these boundaries; that is, should be accompanied by effects in which the density variations δn are comparable with the equilibrium

density $n(x)$ of electrons at the boundary. A quantitative description of such perturbations is possible only in a nonlinear theory, so that a general theory of plasma oscillations in $2d$ channels with mobile boundaries should be nonlinear, and has not been constructed up to now.

The compromise variation of the theory, proposed below, looks as follows. First we lay out the exact solution of the problem of the fundamental mode of plasma oscillation in a parabolic channel. Then this problem is solved using a linearization of the general system of equations, and the exact and approximate eigenfrequencies are shown to coincide. The reasons for this coincidence are not yet clear. However, the fact that the lowest eigenmodes calculated by different methods are the same is used as an argument toward the applicability of perturbation theory.

The specific results on the plasma oscillation spectrum of a system of electrons localized in a parabolic channel, obtained below, are very useful in interpreting the experimental data on the behavior of quasi-one-dimensional channels in a varying electric field perpendicular to the channel direction.

A. The general system of equations necessary to describe plasma oscillations in a classical electron system localized in a parabolic channel has the form

$$-m^*\dot{v}=e\varphi'(x,\ t)+V'(x), \tag{21}$$

$$\dot{n}+(nv)'=0, \tag{22}$$

$$\varphi'(x,t)=\frac{e}{\varkappa}\int\limits_{a_1(t)}^{a_2(t)} ds\frac{n(s,t)}{x-s}, \tag{23}$$

$$\int\limits_{a_1(t)}^{a_2(t)} n(s,t)\,ds=N_L. \tag{24}$$

Here v is the hydrodynamic speed in a medium with density $n(x,t)$ depending on time, $\varphi(x,t)$ is the running value of the electric potential, and $a_1(t)$, $a_2(t)$ are the boundaries of the electron density distribution, the position of which in the general case depends on time. The system (21)–(24) does not contain the coordinate y along the channel, which simplifies further calculations. Equation (23), which determines the instantaneous value of the unscreened potential $\varphi(x,t)$ through the density distribution $n(x,t)$, has a static form, that is, does not account for the effects of retardation. Such an approximation is applicable to the extent that the inequality

$$\omega\ll c/a, \tag{25}$$

is satisfied, where c is the speed of light and ω is the oscillation frequency. Usually the inequality (25) is fulfilled by a large margin.

The requirement (24) that the total number of electrons be preserved is very important and is one of the specific factors that distinguish the problem of plasma oscillations in a single channel from the analogous problem in a $2d$ system with periodically modulated density (see, for example, Ref. 18).

The equation of motion (21) describes one electron, which goes with the average-field approximation. In the static case, for $v=0$, Eq. (21) reduces to the problem of determining the equilibrium profile $n(x)$ of an electron system in a channel. In the parabolic case, answers are given by expressions (3) and (4).

The proposed exact solution of Eqs. (21)–(24) is based on the hypothesis of the existence of a self-similar solution of this system, which, for example, for a density $n(x,t)$ has the form

$$n(x,\ t)=n(x-\delta(t)). \tag{26}$$

Here $n(x)$ is the equilibrium electron density profile in the given channel, and $\delta(t)$ is the amplitude of electron density oscillations in the given mode, depending only on time. Using the definition (26), it is not hard to verify that the continuity equation (22) is exactly satisfied.

Equation (23), written to account for (26) and the obvious definitions

$$a_1(t)=-a_0+\delta(t),\quad a_2(t)=a_0+\delta(t),$$

guaranteeing preservation of the total particle number (24), transforms to

$$\varphi'(x,t)=\frac{e}{\varkappa}\int\limits_{a_1(t)}^{a_2(t)}\frac{n(s-\delta(t))}{x-s}\,ds=\frac{e}{\varkappa}\int\limits_{-a_0}^{a_0}\frac{n(\xi)}{x-\delta(t)-\xi}\,d\xi. \tag{27}$$

In other words, if $n(x,t)=n(x-\delta)$ holds, then $\varphi'(x,t)=\varphi(x-\delta(t))$ holds as well.

Up to now, the properties claimed for the self-similar solution had a general character, for an arbitrary profile $V(x)$. However, the next step, the determination of the dynamical equation for $\delta(t)$ from the general equation of motion (21), is self-consistent only in the case of a parabolic channel. To obtain this equation we note that the integral (27) is taken with the equilibrium density $n(x)$ of (4), and the definition (27) is simplified:

$$e\varphi'(x-\delta)=-k(x-\delta). \tag{27a}$$

If we substitute $\varphi'(x,t)$ of (27a) in the equation of motion and take into account the explicit form of $V(x)$ in Eq. 1, it is not difficult to obtain from (21) an equation for δ:

$$-m^*\ddot{\delta}=e\varphi'(x-\delta)+V'(x)=k\delta. \tag{28}$$

This equation does not contain the x-coordinate; that is, the hypothesis of the dependence of δ only on time is self-consistent.

In the case of an arbitrary potential $V(x)$ the integral (27) can also be reduced to a form analogous to (27a):

$$\varphi'(x,\ t)=V'(x-\delta(t)).$$

However, the difference

$$V'(x-\delta)-V'(x)$$

for an arbitrary potential $V(x)$ no longer reduces to a function of $\delta(t)$ on the whole interval $-a\leqslant x\leqslant +a$.

From (28) it follows immediately that the corresponding mode, which it is appropriate to call dipolar, has the eigenfrequency

$$\omega_1^2=k/m^*. \tag{29}$$

The result (29) looks almost trivial; a system of electrons in a parabolic channel, keeping the equilibrium form of the

profile, oscillates with a frequency ω_1 corresponding to the eigenfrequency of a single electron in the same parabolic well. The presence of such a mode in a parabolic channel is closely tied to the possibility that the boundaries of the electron distribution move; the very mobility of the boundaries permits the profile equilibrium to remain undisturbed, and means that the Coulomb interaction energy of the electrons is not disturbed. Notwithstanding the simplicity of the final result, its demonstration requires the full analysis performed above. Along the way it becomes apparent that such a self-similar solution is fully realized only for a parabolic channel.

Comparing the definition (29) of ω_1 with the evaluation (20) of ω_1 mentioned above, it is not hard to see a qualitative difference between them. The assertion of (29) says that the frequency ω_1 depends only on the curvature k of the channel and does not contain in its definition the average electron density and the channel width, as in the definition (20) of ω_1. This feature will be used below in interpreting the experimental data.

B. The problem of the plasma oscillations in a system of electrons occupying a parabolic channel can be solved in general form by linearizing the continuity equation (22). The details of this solution are in the Appendix. The final results for the spectrum have the following form:

$$\omega_l^2=lk/m^*, \quad l=1, 2, 3 \ldots . \tag{30}$$

It is apparent that the lowest mode in the spectrum (30) coincides with the definition (29) of ω_1, which indirectly indicates that it is reasonable to use the linearization procedure, at least to determine the plasma oscillation spectrum.

C. A classical description of the plasma oscillation spectrum ceases to be meaningful as the channel width a decreases. The onset of quantization of individual electron motion leads to the necessity of replacing the classical equation of motion and the classical continuity equation with their corresponding quantum equivalents. A self-consistent quantum theory produces characteristic denominators of the form

$$(\varepsilon_{l+1}-\varepsilon_l-\hbar\omega)^{-1},$$

in which the ε_l are the discrete levels of one-electron motion, and ω is the frequency of the collective oscillations. Obviously, the classical theory of plasma oscillations is reasonable if

$$\hbar\omega \gg \varepsilon_{l+1}-\varepsilon_l. \tag{31}$$

The inequality (31) can be made specific by using information on the effective potential energy $\tilde{V}(x)$ quantizing the motion of the individual electrons in a parabolic channel. As shown above [see the definition of $\tilde{V}(x)$ in (9a) and (13)], the effective potential $\tilde{V}(x)$, like the original $V(x)$ of Eq. 1, is parabolic, but is strongly renormalized by replacing $\tilde{k}$ with k. Taking account of this specific property of a parabolic channel, we can give the inequality (31) the following more explicit form:

$$k \gg \tilde{k} \tag{31a}$$

equivalent to the requirement $a \gg a_B$.

As a comment on the inequality (31a) we note that the definition (29) of ω_1 has a qualitative meaning over practically the whole interval of electron occupation of the channel. In fact, in the region $a \gg a_B$ the definition (29) of ω_1 has a clear classical meaning. In the opposite limiting case the problem should be solved quantum-mechanically, but the final result for a resonant transition of electrons between levels in the parabolic potential $V(x)$ of Eq. 1, without Coulomb renormalization, will again have the form of ω_1 in (29). Therefore, only the region of intermediate electron filling of a parabolic channel remains uninvestigated as far as determination of the frequency ω_1. But we hardly expect much deviation in this rather narrow interval of parameters of ω_1 from its value in (29).

D. The marked stability of the definition (29) of ω_1 can be favorable from the point of view of diagnostics of the properties of the parabolic channel studied. That is, the position of the plasma resonance with the lowest frequency gives, in agreement with definition (29) of ω_1, direct information on the curvature of the periodic potential for the given set of parameters of the problem. Usually this characteristic is difficult to calculate, and the possibility of its direct measurement can be extremely useful. Of course, we must take into consideration that the interaction between neighboring channels, which takes place in periodic quasi-one-dimensional systems, should change the definition of the dipole frequency. The evaluation of this effect can be carried out on the assumption that the interaction between neighboring channels also (as in the case of a single channel) takes place in the absence of dynamical screening.[3)] In this approximation the equation of motion for δ_l, a generalization of Eq. (28), has the form (nearest-neighbor approximation)

$$m^*\omega^2\delta_l=K\delta_l+\frac{Q}{\varkappa L^2}(\delta_{l+1}+\delta_{l-1}-2\delta_l), \tag{32}$$

$$K=k+2Q/\varkappa L^2, \quad Q=eN_L.$$

Here L is the distance between neighboring channels. Equation (32) has a form well known from lattice dynamics. Analysis shows that a system of channels with Coulomb interaction should have a plasma spectrum with a finite dispersion and renormalized threshold frequency ω_1^*:

$$\omega_1^*=(K/m^*)^{1/2}. \tag{33}$$

4. DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

The information above allows us to analyze in detail the observed behavior of the conductivity of a periodic system of quasi-one-dimensional channels as it depends on the frequency of an external electric field, the magnitude of V_g and the strength of the magnetic field perpendicular to the plane of the channels. A summary of the experimental data from Ref. 3 is presented in Figure 2. The curve ABC depicts schematically the resonant behavior arising in a system of one-dimensional channels acted upon by a high-frequency electric field with an electric component perpendicular to the channel axis, as a function of V_g, with magnetic field equal to zero. The curve DE shows the behavior of the characteristic frequency ω_c determining the scale of quantization of single-electron motion in the channel. This frequency is extracted from the data on Shubnikov–de Haas oscillations on the supposition that the oscillating part $\delta\sigma$ of the conductivity has

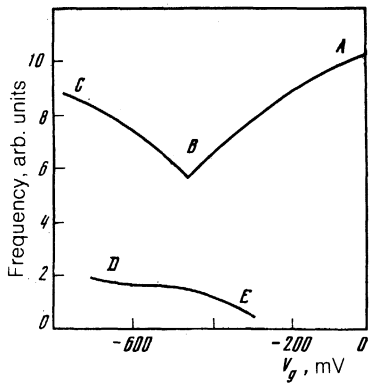

FIG. 2. Schematic data on the behavior of the high-frequency resonance (line *ABC*) and the frequency ω_c extracted from data on Shubnikov–de Haas oscillations (line *DE*), as a function of the potential V_g (for complete information see Refs. 1–3).

the structure (19a) with $\delta\mu$ independent of magnetic field and a curvature $\tilde{k}$ depending only on V_g. Both the latter assumptions, as shown above [see the comment on expression (19a)] are not generally valid. For this reason the real meaning of the curve *DE* is not very clear.

A. To interpret the high-frequency data (curve *ABC*) we assume that the presence of a resonance corresponds to the generation in the system of channels of a dipole plasma mode. This mode can arise from a uniform electric field in the plane of the $2d$ electron layer with a varying density modulation. In the region of small V_g, when the density modulation is small, the presence of a periodic perturbation leads to creation in the continuous plasma spectrum of specific mini-gaps, the position of which is determined in Refs. 19–20. The minimum threshold frequency, the generation of which can be achieved by a uniform electric field, has the structure of the frequency ω_1 in Eq. 20. As V_g grows the average density n_s of the electrons in the $2d$ system falls, and the period of the perturbation L stays fixed. As a result, the frequency ω_1 as a function of V_g should decrease; this is confirmed by the experiments of Refs. 1–3 (see the segment *AB* in Figure 2). A qualitatively correct explanation for the behavior of the resonance in the segment *AB* is found in the original work of Refs. 1–3, although a quantitative theory of the influence of arbitrary periodic density perturbations on the plasma spectrum is still lacking (the calculations of Ref. 18 contain a series of inadequately justified simplifications).

At point *B* (Fig. 2) the modulation of the electron density becomes of order unity, the continuity of the $2d$ system disappears, and a system of parallel quasi-one-dimensional channels arises, isolated from each other in the mass-transport sense. From this point, the frequency of the plasma resonances grows with increasing V_g, even though the electron density in the isolated channels continues to decrease. The estimate (20) for ω_1 becomes meaningless, as it contains two variable parameters: the electron density n_s and the channel width $a < L$. As for the definition of ω_1 in (29), which is applicable to the left of point *B*, the positive character of the derivative $\partial\omega_1/\partial V_g > 0$ here is simply explained. Notwithstanding the complex derivation of the coefficient k in the definition (1) of $V(x)$, it is clear that as V_g grows this coefficient should grow (see the table). And since the value of ω_1 in (29) is determined only by the curvature k, it is natural that in the regime where separate quasi-one-dimensional channels are created the plasma frequency becomes an increasing function of V_g in spite of the decrease in electron density in each of the channels.

B. Using the information on the curvature k as a function of V_g on the segment *BC* (Fig. 2), we can use the data on the characteristic frequency ω_c connected with the quantization of one-electron motion in a channel and depicted in Fig. 2 by the curve *DE*. Assuming in this connection that the frequency ω_c is proportional to the screened curvature $\tilde{k}$ ($\omega_c^2 = \tilde{k}/m^*$), using the definition (9a) for $\tilde{k}$ and (13) and (13a) for a and N_L, and also using curve *DE* in Fig. 2 we can, in principle, find N_L as a function of V_g on the interval *BC* in Fig. 2. However, as noted above, the extraction of the frequency ω_c from the data on the Shubnikov–deHaas oscillations, carried out in Ref. 3, is not done rigorously. For this reason the $N_L(V_g)$ dependence following from Fig. 2 has no quantitative meaning and is not cited in this work. We only note that the inequality $k \gg \tilde{k}$ following from the data of Fig. 2 in a wide interval of V_g demonstrates the quasi-classical situation and, in connection with this, the strong renormalization of the effective curvature $\tilde{k}$ in comparison with k.

C. We will sum up several results. In this work a systematic description is given of the equilibrium and simplest dynamical properties of a system of electrons localized in a parabolic channel when the channel is sufficiently filled, i.e., under the condition $l^* \gg 1$, where l^* is the maximum number of levels in the channel below the Fermi level. In our opinion, the parabolic model is the most suitable for the experimental situation in which the parameters of a single electron channel are varied over a wide interval with the aid of corresponding external fields.

The equilibrium characteristics of a channel obtained in the quasiclassical approximation [see definitions (3) and (4) for N_L, $n(x)$ and a] are in good agreement with the analogous determinations from Ref. 5, found numerically. Analysis of these characteristics shows that the Coulomb interaction of the electrons substantially influences the equilibrium properties of the channel. In particular, the effective potential energy $\tilde{V}(x)$ quantizing the motion of individual electrons in the channel is strongly renormalized compared with the bare energy $V(x)$ of Eq. 1, which forms the channel. The explicit form of $\tilde{V}(x)$ depends on the character of the approximation [see the definition (7) or (9a) of $\tilde{V}(x)$]. However, the global properties of the channel of the type that determine the total number l^* of electronic levels below the Fermi level [expression (8) or (14)] are less sensitive to the approximation of $\tilde{V}(x)$ and are consistent.

The ballistic conductivity of a single quasi-one-dimensional channel depends on the form of the channel. The discussion of the details of the conductivity carried out in the discussion of formulas (17) and (18) attests to the fact that the real channel studied in Ref. 6 is of the parabolic class.

The use of kinetic language in the calculation of the ballistic conductivity of a channel in a strong magnetic field perpendicular to the plane of the channel is an interesting possibility, markedly simplifying the derivation the final expression of the form (19a). From these definitions of $\delta\sigma$ it follows, in particular, that the usual scheme to account for the influence of the confining potential $\tilde{V}(x)$ on the Shubnikov–de Haas oscillations[1,3] is not entirely consistent. Besides

the hybridization frequency $\tilde{\omega}_c$ of (19), it is necessary to take account of the dependence of the value of N_L on H in describing the influence of a magnetic field on the Shubnikov–de Haas oscillations in a parabolic channel, in accordance with the definitions of (19a). Up to now this dependence has not been treated in working with the corresponding experimental data.

The fundamental "plasma" result is the definition (29) of ω_1, which allows us to understand qualitatively the behavior of the plasma resonance along the segment BC in Fig. 2. In addition, the interesting possibility of a complete solution of the problem of plasma oscillations is explained in the Appendix.

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APPENDIX

Suppose $\delta n(x,t) \ll n(x)$, and let the boundaries of the channel be fixed at the points $x = \pm a_0$ and the total number of electrons in the channel be constant, that is,

$$\int_{-a_0}^{a_0} \delta n(x,t)\,dx=0. \tag{A1}$$

The equation for δn can be obtained from the equation of motion (21), in which the local speed is expressed in terms of δn with the help of the linearized equation (22):

$$-n(x)v(x,t)=\int_{x}^{a_0} \delta\dot{n}(s,t)\,ds. \tag{A2}$$

The relationship (A2) behaves correctly at the ends of the interval $-a_0 \leqslant x \leqslant a_0$:

$$n(x)v(x,t)|_{x=\pm a_0}=0, \tag{A3}$$

at the end $x = -a_0$ the current goes to zero, as is guaranteed by the requirement (A1).

Taking into account (A2) and the corresponding representation of the solution of the Poisson equation for δn [see the definition (23) of $\varphi(x,t)$, written for δn in conditions of fixed limits $x = \pm a_0$], we can obtain from the equation of motion (21) the following equation for δn:

$$m^*\int_{\xi}^{1} \delta\ddot{n}\,d\xi=\frac{e^2n(0)}{\varkappa}(1-\xi^2)^{1/2}\int_{-1}^{1}\frac{\delta n(s)\,ds}{\xi-s}. \tag{A4}$$

Analysis of this equation is conveniently carried out using the properties of the Chebyshev polynomials[21] $U_l(\xi)$. We multiply both sides of equation (A4) by the polynomial $U_l(\xi)$ and integrate over the region $-1 \leqslant \xi \leqslant 1$. Taking account of the properties of the polynomials $U_l(\xi)$[21], we find from (A4)

$$m^*\omega^2\zeta_l=\frac{\pi e^2n(0)}{\varkappa}l\zeta_l, \qquad \zeta_l=\int_{-1}^{1} \delta n(\xi)T_{l+1}(\xi)\,d\xi, \tag{A5}$$

where $T_l(x)$ is the associated Chebyshev polynomial, and $n(0) = 2N_L/\pi a_0$. From this it is clear that the spectrum of oscillations of equation (A5) has the form

$$\omega_l^2=lk/m^*, \quad l=1, 2, 3\ldots, \tag{A6}$$

which is also noted in the text [see the definition (30) of ω_1].

[1)]Screening is included in the problem only to ensure uniformity in the y-direction and appears in the basic equations formulated below in the introduction of a logarithmic cutoff factor [see definition (2a)].

[2)]For the square-well model of the channel an explanation of the result (17) is complicated by the indeterminate relation between a and N_L.

[3)]In fact, the interaction between neighboring channels is severely weakened by the screening action of the gate. This influence is small, when we are talking about electrostatic calculations in the limit of a single channel (whence the large argument in the logarithm in the definition of the Coulomb energy V_c^0 in Eq. 3), and becomes substantial if we are talking of distances L at which there is interaction of neighboring channels.

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